Hyperbolic inflation in the light of Planck 2015 data

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(Received 10 April 2015; published 18 May 2015)

Rubano and Barrow have discussed the emergence of a dark energy, with late-time cosmic acceleration arising from a self-interacting homogeneous scalar field with a potential of hyperbolic power type. Here, we study the evolution of this scalar-field potential back in the inflationary era. Using the hyperbolic power potential in the framework of inflation, we find that the main slow-roll parameters, like the scalar spectral index, the running of the spectral index and the tensor-to-scalar fluctuation ratio can be computed analytically. Finally, in order to test the viability of this hyperbolic scalar-field model at the early stages of the Universe, we compare the predictions of that model with the latest observational data, namely Planck 2015.

DOI: 10.1103/PhysRevD.91.103517 PACS numbers: 98.80.-k, 95.35.+d, 95.36.+x

I. INTRODUCTION

The study of cosmic microwave background (CMB) photons using the Planck [1] and BICEP2 [2] data sets has opened up a new constraint on inflationary models [3]. Specifically, the detailed analysis of Planck data [1] constrains single scalar-field models of slow-roll inflation, to have the very low tensor-to-scalar fluctuation ratio $r = n_t/n_s \ll 1$, with a scalar spectral index $n_s \simeq 0.96$ possessing no appreciable running. The upper bound set by the Planck Collaboration [1] on the tensor-to-scalar fluctuation ratio, related to the absence of the B-modes of polarization in the CMB, is $r < 0.11$, but their favored regions (higher than 95% C.L.) point towards a more stringent bound of $r \leq 10^{-3}$. From a theoretical viewpoint one can see that this is in agreement with the so-called Starobinsky-type (or R^2, with R denoting the scalar spacetime curvature) inflationary models [4].

Last year, however, the BICEP2 team [2] made an important claim: to have made the first measurement of B-mode polarization in the CMB radiation. This measurement was initially interpreted as an indication of gravitational waves at the time of the last scattering, with a tensor-to-scalar ratio $r = 0.16^{+0.06}_{-0.05}$. If such claims were confirmed, they would constitute the first experimental observation of (transverse) primordial (possibly quantum) metric fluctuations. For the scalar spectal index, it has been found that $n_s \simeq 0.96$ and $dn_s/d\ln k \simeq 0$, in agreement with the Planck data [1].

Since then a great effort has been spent in order to compare the BICEP2 tensor-to-scalar ratio with the choice of inflationary paradigm (R^2 [4], chaotic [5], inverse power law [6], hilltop [7], natural [8], supersymmetry [9], D-flation [10] and the like).

Furthermore, during this period there was an intense debate as to whether the BICEP2 signal is indubitably due to primordial gravitational waves, or is polluted by gravitationally lensed E-modes and galactic foregrounds. Recently, in Ref. [11] it was stressed that magnetized dust associated with radio loops due to supernova remnants might affect the signal received by BICEP2. In the case of polarization effects by galactic dust, Refs. [12,13] have shown that the cosmological value of the tensor-to-scalar ratio could be very small, $r \ll 0.1$. Within this framework, the recent analysis on the foreground dust in the BICEP2 region released by the Planck collaboration [14] points to a significant foreground pollution which means that the BICEP2 B-mode polarization data cannot be used as evidence for primordial CMB polarization.

Recently, the joint analysis of BICEP2/Keck Array and Planck data appeared in the literature [15]. This analysis has placed an upper bound in the tensor-to-scalar ratio, namely $r < 0.12$ at 95% significance level. In this context, the Planck team has repeated the inflationary analysis by using the Planck 2015 data and has essentially confirmed the Planck 2013 results: $n_s = 0.968 \pm 0.006$, $dn_s/d\ln k = -0.003 \pm 0.007$ and $r < 0.11$ [16].

The crux of these studies is that the potential energy of the scalar field is not really known and that one must introduce it using phenomenological arguments, starting with the simplest possibilities. There has been intense debate and speculation about the functional form of the potential energy $V(\phi)$. Various candidates have been proposed in the literature, such as a power law, inverse power law, exponential and so on (for a review see [17] and references therein) but exact solutions are not abundant and always possess special mathematical features.

Some time ago, a special solution for a spatially flat Friedmann-Lemaître-Robertson-Walker spacetime with a perfect fluid with a constant equation of state parameter $P_m = (\gamma - 1)\rho_m$ (where, for radiation $\gamma = 4/3$ and for dust $\gamma = 1$) and a scalar field with a constant equation of state

$P_\phi = (\gamma_\phi - 1)\rho_\phi$ was found in [18]. In particular, Rubano and Barrow [18] (see also [19,20]) showed that under specific conditions we can solve the Einstein equations when the potential $V(\phi)$ has the interesting hyperbolic form

$$V(\phi) = A\left[\sinh\left(\sqrt{3}\frac{(\gamma-\gamma_\phi)}{\sqrt{\gamma_\phi}}(\phi-\phi_0)\right)\right]^b, \quad (1)$$

where the constant A is

$$A = 3H_0^2(1-\Omega_{m0})\left(1-\frac{\gamma_\phi}{2}\right)\left(\frac{1-\Omega_{m0}}{\Omega_{m0}}\right)^{-b/2} \quad (2)$$

and

$$b = -\frac{2\gamma_\phi}{\gamma-\gamma_\phi} = \frac{2(1+w_\phi)}{1+w_\phi-\gamma}. \quad (3)$$

Note that H_0 and Ω_{m0} are the usual cosmological parameters (although Ω_{m0} denotes any possible matter content according to the appropriate choice of γ). If we trace the late Universe (dustlike matter $P_m = 0$) we have $\gamma = 1$. We remind the reader that w_ϕ denotes the equation of state parameter of the dark energy usually parametrized by $w_\phi = \frac{P_\phi}{\rho_\phi} = \gamma_\phi - 1$, with P_ϕ and ρ_ϕ being the pressure and density of the dark-energy fluid. Obviously the exponent b plays an important role in the cosmic dynamics, since it is related with the equation of state parameter, w_ϕ. In this context, inverting Eq. (3) one can prove that the equation of state parameter reduces to $w_\phi = 2/(b-2)$. Note that the accelerated expansion of the Universe poses the restriction $w_\phi < -1/3\Omega_{\mathrm{de}0}$ which implies $2(1-3\Omega_{\mathrm{de}0}) < b < 2$, where $\Omega_{\mathrm{de}0} = 1 - \Omega_{m0}$.

The potential (1) has some interesting geometric characteristics. Under specific conditions, it behaves either as an exponential or as a power law. If $|\lambda\phi| \gg 1$ (or $|\lambda\phi| \ll 1$), we find $V \propto e^{-b\lambda\phi}$ [or $V \propto (\lambda\phi)^b$], where $\lambda = \frac{\sqrt{3}(1-\gamma_\phi)}{\sqrt{\gamma_\phi}}$ (see also [19]). The initial motivation in [18] was to use potential (1) to describe the late-time acceleration of the Universe, but we can also apply Eq. (1) to the very early states of the cosmic evolution when the cosmic fluid is dominated by the radiation and the inflaton components. This might then provide a unified picture of inflation and dark energy in which both eras are described by the potential of Eq. (1). It is the purpose of this work to demonstrate compatibility of the Rubano and Barrow [18] scenario with the Planck 2015 data, taking into account the foreground ambiguities clouding the Planck 2015 data, as mentioned previously. The structure of the paper is as follows. The slow-roll inflation and its connection to the hyperbolic potential of Ref. [18] are reviewed in Sec. II. In Sec. III we study the performance of Eq. (1) against the Planck 2015 data. Finally, our conclusions are summarized in Sec. IV.

II. SLOW-ROLL INFLATION

Let us present here the basic ingredients in the context of single-field inflation. Assuming an inflaton field ϕ with potential energy $V(\phi)$, the slow-roll parameters are given by

$$\epsilon = \frac{M_{\mathrm{pl}}^2 V'^2}{2V^2}, \quad (4)$$

$$\eta = \frac{M_{\mathrm{pl}}^2 V''}{V}, \quad (5)$$

$$\xi = \frac{M_{\mathrm{pl}}^4 V' V'''}{V^2}, \quad (6)$$

where the prime denotes derivatives with respect to ϕ and $M_{\mathrm{pl}}^2 = 1/8\pi G$. The corresponding spectral indices are defined in terms of the slow-roll parameters, as usual [21], by

$$n_s \simeq 1 + 2\eta - 6\epsilon, \quad (7)$$

$$r \simeq 16\epsilon \simeq -8n_t, \quad (8)$$

$$n_s' = dn_s/d\ln k \simeq 16\epsilon\eta - 24\epsilon^2 - 2\xi. \quad (9)$$

In this framework, the number of e-folds is written as[1]

$$N = \int_t^{t_{\mathrm{end}}} H(t)dt \simeq \frac{1}{M_{\mathrm{pl}}^2}\int_{\phi_{\mathrm{end}}}^{\phi} \frac{V(\phi)}{V'(\phi)}d\phi, \quad (10)$$

where ϕ_{end} is the value of the inflaton field at the end of inflation, namely $\epsilon(\phi_{\mathrm{end}}) \simeq 1$. Now, let us focus on the potential of Eq. (1) which is written as

$$V(\phi) = A\sinh^b(\phi/f), \quad (11)$$

where f is the scale in units of M_{pl}. Recall that in the dark-energy era the corresponding constants are related to the cosmological parameters [see Eqs. (2) and (3)]. Therefore, in the early Universe one may expect that the corresponding constants in Eq. (11) are not necessarily equal to those derived by [18] using arguments from the late Universe where pressureless matter dominates radiation (for comparison with the observational data see [19]). When radiation dominates we have $\gamma = 4/3$ and so from the second equality of Eq. (3) we arrive at

[1]In the literature sometimes we label ϕ by $\phi_\star$, which denotes the value at the horizon crossing for which the scalar amplitude is $A_s \approx \Lambda^4/24\pi^2\epsilon M_{\mathrm{pl}}^4$, where the energy scale of inflation is $\Lambda \sim 10^{16}$ GeV.

$$b = \frac{6(1+w_\phi)}{3w_\phi - 1}. \tag{12}$$

Note that the restriction $b > 1$ implies that the potential contains a critical point.

Now inserting Eq. (11) into the slow-roll parameters we obtain after some simple algebra

$$\epsilon = \frac{b^2 M_{\rm pl}^2}{2f^2}\coth^2(\phi/f), \tag{13}$$

$$\eta = \frac{b M_{\rm pl}^2}{f^2}[(b-1)\coth^2(\phi/f)+1], \tag{14}$$

$$\xi = \frac{b^2 M_{\rm pl}^4}{f^4}\coth^2(\phi/f)[(b-1)(b-2)\coth^2(\phi/f) + (3b-2)]. \tag{15}$$

Combining Eq. (13) with Eqs. (14) and (15) we can write (η, ξ) in terms of ϵ:

$$\eta = \frac{b M_{\rm pl}^2}{f^2}\left[\frac{2f^2\epsilon}{b^2 M_{\rm pl}^2}(b-1)+1\right] \tag{16}$$

$$\xi = \frac{2M_{\rm pl}^2\epsilon}{f^2}\left[\frac{2f^2(b-1)(b-2)\epsilon}{b^2 M_{\rm pl}^2} + (3b-2)\right]. \tag{17}$$

In this case, the number of e-folds becomes

$$N \simeq \frac{f^2}{b M_{\rm pl}^2}\ln\left[\frac{\cosh(\phi/f)}{\cosh(\phi_{\rm end}/f)}\right]. \tag{18}$$

Inverting the above, we can express the inflaton as $\phi(N)$ by

$$\phi = f\cosh^{-1}[e^{NbM_{\rm pl}^2/f^2}\cosh(\phi_{\rm end}/f)]. \tag{19}$$

In order to proceed with the analysis we need to know the values of N and $\phi_{\rm end}$. First, it is natural to consider that the number of e-folds lies in the interval $[50, 60]$. Here, we set it to 55, for concreteness. Second, using the constraint $\epsilon(\phi_{\rm end}) \simeq 1$ and Eq. (13) we can estimate the value of the scalar field at the end of inflation to be

$$\phi_{\rm end} \simeq \frac{f}{2}\ln\left(\frac{\theta+1}{\theta-1}\right), \tag{20}$$

where $\theta = \sqrt{2}f/bM_{\rm pl}$. Since $\theta > 1$, the scale f obeys the restriction $f > \sqrt{2}bM_{\rm pl}/2$.

III. OBSERVATIONAL RESTRICTIONS

The point of this section is to test the viability of the potential (11) at the inflationary level, involving the latest cosmological data. In particular, the combined analysis between Planck 2015 and various data such as WMAP, high-l data and baryonic acoustic oscillations, shows that the scalar spectral index is $n_s = 0.968 \pm 0.006$. For the scalar spectral index in this work we use $n_s' = -0.003 \pm 0.007$. Furthermore, as mentioned in the Introduction, the analysis of the Planck Collaboration places an upper limit on the tensor-to-scalar ratio, $r < 0.11$, which is in agreement with the joint analysis of the BICEP2/Keck Array and Planck [15].

Let us now briefly present our results. In Figs. 1 and 2 we show the confidence contours in the (n_s, r) and (n_s, n_s') planes which are provided by the Planck team [16]. On top of that we present the area for the individual sets of (n_s, r) which are based on the potential (11), whereas in Fig. 2 we display the corresponding area in the case of (n_s, n_s'). Obviously, our results are consistent with those of Planck 2015. Specifically, as can be seen from Fig. 1, the tensor-to-scalar fluctuation ratio could reach the value of $r \simeq 0.075$, which is in a good agreement within 1σ with that of the BICEP2/Keck Array/Planck results ($r \simeq 0.05$ see Fig. 9 in [15]). Notice that in order to derive $r \simeq 0.075$ the constants in Eq. (11) need to obey the following inequalities: $1.02 \leq b \leq 1.1$ [or inverting Eq. (12) we obtain $-2.63 \leq w_\phi \leq -2.39$] and $26M_{\rm pl} \leq f \leq 39M_{\rm pl}$. Concerning the running spectral index we obtain $n_s' \simeq -0.004$, which is consistent with that of Planck 2015, namely $n_s' = -0.003 \pm 0.007$. Moreover, we find that the running of the scalar spectral index does not change significantly as a function of n_s (see Fig. 2). Regarding the inflaton field at the beginning of

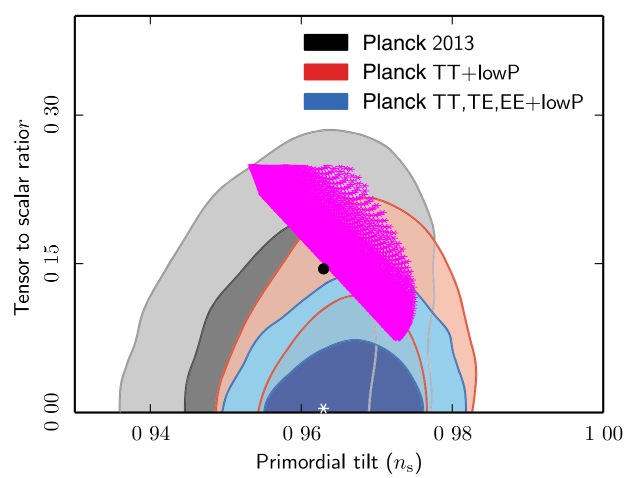

FIG. 1 (color online). The $n_s - r$ diagram for the hyperbolic potential of [19] using $N = 55$. The contours are borrowed from Planck 2015 [16]. The area which is plotted over the contours corresponds to the hyperbolic Rubano and Barrow [18] potential [see Eq. (11)]. The points correspond to chaotic (solid point) and Starobinsky (star) inflation respectively.

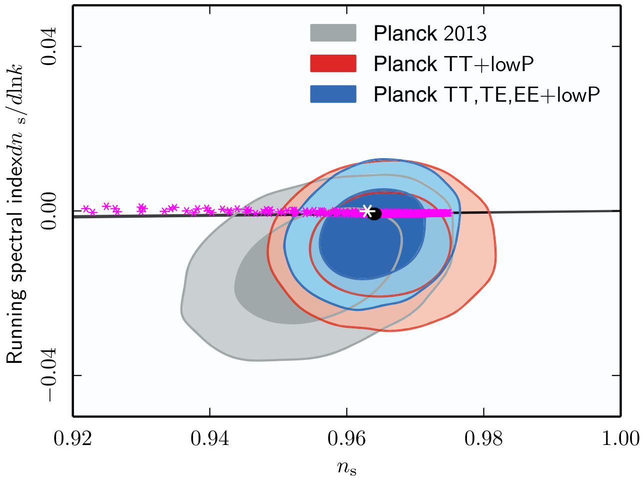

FIG. 2 (color online). The $n_s - n_s'$ diagram. The area corresponds to the hyperbolic Rubano and Barrow [18] potential (for more details see the caption of Fig. 1).

inflation, we obtain $10.6M_{\rm pl} \le \phi \le 12.9M_{\rm pl}$, while at the end of inflation we require $0.7M_{\rm pl} \le \phi_{\rm end} \le 1.1M_{\rm pl}$. Furthermore, we find that the allowed region in which our results satisfy the 2σ observational restrictions of Planck 2015 is $f \ge 11.7M_{\rm pl}$ and $1 < b \le 1.5$. Thus the data applied to slow-roll inflation place constraints on b, although the value of scale f has only a lower limit at the 2σ level.

Finally, we would like to compare our results with those found using other potentials. Specifically, in the case of the chaotic inflation $V(\phi) = \Lambda^4(\phi/M_{\rm pl})^k$ [5], the corresponding slow-roll parameters are written as $\epsilon = k/4N$, $\eta = (k-1)/2N$ and $\xi = (k-1)(k-2)/4N^2$. The latter implies $n_s = 1-(k+2)/2N$, $r = 4k/N$ and $n_s' = -(2+k)/2N^2$. Using $k = 2$ and $N = 55$ we obtain $n_s \simeq 0.964$, $r \simeq 0.145$ and $n_s' \simeq -0.0007$. This also corresponds to the slow-roll regime of intermediate inflation [22] with Hubble rate during inflation given by $H \propto t^{k/(4-k)}$, with $n_s = 1-(k+2)r/8k$, and $k = -2$ gives $n_s = 1$ exactly to first order and $n_s' = -2(n_s-1)^2/(k+2)$. On the other hand the Starobinsky inflation [4], namely $V(\phi) \propto [1 - 2e^{-B\phi/M_{\rm pl}} + \mathcal{O}(e^{-2B\phi/M_{\rm pl}})]$, leads to the following slow-roll predictions [23,24]: $n_s \approx 1 - 2/N$ and $r \approx 8/B^2N^2$, where $B^2 = 2/3$. Furthermore, for Starobinsky inflation following the notations of [24] we find that the running spectral index is given by $n_s' \approx -2/N^2$. To this end using $N = 55$ we obtain $(n_s, r, n_s') \approx (0.963, 0.004, -6.6\times10^{-4})$. The above slow-roll parameters are indicated by the solid points (chaotic inflation) in Figs. 1 and 2 while the stars represent values for the Starobinsky inflation.

IV. CONCLUSIONS

In the light of the Planck 2015 results, a debate is taking place in the literature about the best implementation of the inflationary paradigm. In the current article we would like to contribute to this discussion. Employing the hyperbolic dark-energy scalar-field potential of Rubano and Barrow [18] (see also [19]) we study the performance of this model as a description of inflation. We find that the hyperbolic inflation turns out to be quite promising in the context of the new data from Planck 2015. Specifically, using the scalar-field potential (11), we calculate the slow-roll parameters analytically and then we compare the corresponding predictions against the observational data. We find that currently hyperbolic inflation is consistent with the results provided by Planck 2015 within 1σ uncertainties. The combination of Ref. [18] with these calculations provides an overall cosmological investigation of the potential given by Eq. (11). We find that the hyperbolic structure of this potential leads to a viable model which can be used separately to understand the main properties of both inflation and dark energy in the presence of a single perfect fluid.[2] Finally, following the notations of [25] we can provide a unified picture of dark energy and inflation using the sum of the potentials (1) and (11):

$$V_{\rm tot}(\phi) = A_{\rm de}\sinh^{b_{\rm de}}[\lambda_{\rm de}(\phi-\phi_0)] + A\sinh^b(\phi/f),$$

where $\lambda_{\rm de} = \frac{\sqrt{3}(1-\gamma_\phi)}{\sqrt{\gamma_\phi}}$ and the constants $A_{\rm de}$, $b_{\rm de}$, b are given by Eqs. (2), (3) and (12). Note that similar considerations hold for the early dark-energy model [26].

ACKNOWLEDGMENTS

S. B. acknowledges support by the Research Center for Astronomy of the Academy of Athens in the context of the program "Tracing the Cosmic Acceleration." J. D. B. acknowledges STFC support. Also, we would like to thank N. E. Mavromatos for the useful comments and suggestions.

[2] If two fluids are present (for example, matter and radiation) simultaneously, then an exact solution requires the scalar field to have a more complicated potential than the power of an exponential. Our model gives the limiting form of that more complicated solution in the limiting cases where one perfect fluid dominates over the other.

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