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Top quark induced effective potential in a composite Higgs model

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We consider nonperturbative aspects of a composite Higgs model that serves as a prototype for physics beyond the Standard Model, in which a new strongly interacting sector undergoes chiral symmetry breaking, and generates the Higgs particle as a pseudo-Nambu-Goldstone boson. In addition, the top quark couples linearly to baryons of the new strong sector, thereby becoming partially composite. We study the dynamics leading to the top quark Yukawa coupling as well as the top quark contribution to the effective potential for the Higgs, obtaining expressions for these couplings in terms of baryonic correlation functions in the underlying strongly interacting theory. We then show that a large-N limit exists in which the top quark contribution to the Higgs effective potential overcomes that of the weak gauge bosons, inducing electroweak symmetry breaking. The same large-N limit also suggests that the baryons that couple to the top quark may be relatively light. This composite Higgs model, and similar ones, can be studied on the lattice with the methods developed for lattice QCD.

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I. INTRODUCTION

Since the discovery of a light Higgs boson at the LHC, interest in beyond-the-Standard-Model scenarios has focused on models in which the Higgs is naturally light compared to the typical scale of new physics. One approach postulates the existence of a new strongly interacting sector, which we will refer to as hypercolor in this paper. The Higgs doublet of the Standard Model (SM) emerges among the Nambu-Goldstone bosons (NGBs) originating from dynamical symmetry breaking of the flavor symmetry group G of the hypercolor theory. The electroweak gauge bosons as well as the SM fermions then couple to these NGBs, breaking the symmetry group G explicitly to a smaller group, thereby generating an effective potential for the NGBs. Under suitable conditions, this radiatively induced effective potential leads to electroweak symmetry breaking, with the Higgs field acquiring an expectation value as in the SM. This framework still allows for many different possibilities. For reviews that span the evolution of this field, as well as for generic features of these models, we refer to Refs. [1–5].

We will be interested in composite Higgs models in which the sector external to the hypercolor gauge theory, which includes the SM gauge bosons and fermions, is as simple as possible. For instance, we do not wish to introduce any weakly coupled gauge bosons besides the electroweak gauge bosons, as in little Higgs models [1]. The electroweak gauge bosons have to stay massless at the dynamical symmetry breaking scale of the hypercolor theory, and therefore they have to couple to generators in the unbroken flavor subgroup $H \subset G$. As a result, the effective potential generated for the hypercolor NGBs by the electroweak gauge bosons will not lead to electroweak symmetry breaking, a phenomenon often referred to as vacuum alignment [6].

Electroweak symmetry breaking must therefore originate in the effective potential generated by the top quark, being the SM fermion with the strongest coupling to the Higgs, and, hence, to the hypercolor theory. We will postulate that the top quark couples linearly to hyperbaryons (the baryons of the hypercolor theory), as first proposed in Ref. [7]. This idea is attractive from the point of view of CP violation and flavor-changing neutral currents (FCNCs) [4]. Here, we will limit ourselves to a discussion of the top quark sector, where the main concerns are to generate the experimentally measured value of the top quark's mass naturally,[1] together with a Higgs potential that triggers electroweak symmetry breaking. It is generally acknowledged that the mass of the top quark sets it apart from the other SM fermions as it is the only SM fermion with a mass of the order of the electroweak symmetry breaking scale, $v \sim 250$ GeV. This suggests that the top quark may play an essential role in generating electroweak symmetry breaking, whereas the origin of the other SM fermion masses, and the strength of other symmetry breakings such as CP violation and FCNCs, might be very different.

The concrete hypercolor theory we will study in this article was proposed in Ref. [8]. It was preceded by a general study that highlighted what makes that theory particularly attractive [10].[2] The hypercolor theory is a

[1]Without causing problems for $Z \to b\bar{b}$ decays [8,9].
[2]See also Ref. [11].

vectorlike $SU(4)$ gauge theory with fermions in two different irreps (irreducible representations). One of these irreps, the six-dimensional two-index antisymmetric irrep, is real. With five Majorana (or Weyl) fermions in this irrep, dynamical symmetry breaking in that sector of the theory gives rise to an $SU(5)/SO(5)$ nonlinear sigma model as its low-energy effective theory. As we will see in detail below, the Higgs field lives in this nonlinear sigma model.

Generally speaking, composite Higgs models often rely on an $SU(N_w)/SO(N_w)$ nonlinear sigma model, which can arise from chiral symmetry breaking in a theory containing N_w Weyl (or Majorana) fermions in a real irrep. An alternative coset structure is $SU(N_w)/Sp(N_w)$, for which the N_w Weyl fermions should be in a pseudoreal irrep [6]. For recent lattice work involving the pseudoreal fundamental irrep of $SU(2)$ gauge theory we refer to Refs. [12,13]. For a review on beyond-the-SM lattice work, see Ref. [14].

The most familiar example of a real irrep is the adjoint representation, which occurs for example in supersymmetric theories [15]. However, five Majorana fermions in the adjoint irrep would most likely push the theory from being confining to being conformal, even before the introduction of any fermions in another irrep.[3] Avoiding the adjoint irrep, the smallest instance of a real irrep is the sextet of $SU(4)$.

Our goals are as follows. First, a gauge theory such as this $SU(4)$ hypercolor theory is amenable to investigations using the methods of lattice gauge theory. The effective theory below the hypercolor scale, relevant for SM phenomenology, can be parametrized in terms of low-energy couplings (LECs). These LECs can be expressed in terms of correlation functions in the hypercolor theory, which, in turn, allows for their computation on the lattice. While this is well understood for the electroweak gauge sector, a similar careful derivation of the LECs controlling the top sector has to our knowledge not been given to date. We derive the necessary correspondence using spurion techniques.

Second, once the connection between the effective theory and the hypercolor theory has been established, we find that it is possible to obtain semiquantitative estimates of the size of these LECs, using large-N methods and factorization. In particular, we show that the contribution of the top quark to the Higgs effective potential indeed drives electroweak symmetry breaking in a particular large-N limit.

We expect that the techniques developed in this article can be easily extended to similar hypercolor models. In this sense, our choice of the model of Ref. [8] should be considered as a useful example.

This article is organized as follows. Section II introduces and reviews the hypercolor theory [8], including its field content, symmetries, and the effective nonlinear fields that will be needed for the low-energy effective theory. In Sec. III we briefly discuss the contribution of the electroweak gauge bosons to the Higgs effective potential. The main part of this article is Sec. IV, where we discuss the top quark sector in detail. We introduce the top quark spurions and the hyperbaryons in Sec. IV A. We discuss the top Yukawa coupling in Sec. IV B, and the top quark contribution to the Higgs effective potential in Sec. IV C. In Sec. IV D we define a large-N limit of the model, and show that for large enough N the top-induced Higgs potential will lead to electroweak symmetry breaking. For simplicity, we assume a minimal explicit breaking of the flavor group of the hypercolor theory by the couplings to the SM. In Sec. IV E we briefly comment on the more general situation that arises if we relax this assumption. In Sec. V we discuss the similarities between the hypercolor theory and QCD, thus arguing that techniques developed to study QCD on the lattice should be sufficient for hypercolor theories as well. Section VI contains our conclusions. A short appendix collects some of our conventions.

II. FERRETTI'S MODEL

In Ref. [10], several requirements were put forward for a class of composite Higgs models based on a hypercolor gauge theory as an UV completion. We begin by listing these requirements. The gauge group is assumed to be simple, and the dynamical symmetry breaking pattern, $G \to H$, to be such that

$$\begin{aligned} H &\supset SU(3)_{\text{color}} \times SU(2)_L \times SU(2)_R \times U(1)_X \\ &\supset SU(3)_{\text{color}} \times SU(2)_L \times U(1)_Y, \end{aligned} \tag{2.1}$$

with the SM gauge group in the last line. The group $SU(2)_R$ is the familiar custodial symmetry of the SM, and the hypercharge is $Y = T_R^3 + X$. The SM Higgs doublet, with quantum numbers $(\mathbf{1},\mathbf{2},\mathbf{2})_0$ under $SU(3)_{\text{color}} \times SU(2)_L \times SU(2)_R \times U(1)_X$, should be contained in the NGB multiplet associated with the symmetry breaking $G \to H$. In order to accommodate a partially composite top quark [7], i.e., for the top quark to acquire its mass through linear couplings to hyperbaryons, there must exist hyperbaryons with quantum numbers that match those of the SM quarks. This includes a set of right-handed, spin-1/2 hyperbaryons with quantum numbers $(\mathbf{3},\mathbf{2})_{1/6}$ of the SM gauge group $SU(3)_{\text{color}} \times SU(2)_L \times U(1)_Y$, which serve as partners of the SM quark doublet q_L, and left-handed, spin-1/2 hyperbaryons with the quantum numbers $(\mathbf{3},\mathbf{1})_{2/3}$, to serve as partners of the SM quark singlet t_R. Finally, the hypercolor theory should be asymptotically free, and both the hypercolor gauge group and the SM gauge group should be free of anomalies.

The hypercolor model with the smallest gauge group that satisfies all these requirements is an $SU(4)$ gauge theory [10]. The hyperfermion content consists of five Majorana fermions χ_i, $i = 1,\ldots,5$, transforming in the six-dimensional two-index antisymmetric irrep of hypercolor,

[3] See Ref. [14] and references therein.

which is a real representation, and three Dirac fermions ψ_a, $a = 1, 2, 3$, in the fundamental representation. The Majorana field χ can be written in terms of a Weyl fermion Υ as

$$\chi_{ABi} = \begin{pmatrix} \Upsilon_{ABi} \\ \frac{1}{2}\epsilon_{ABCD}\epsilon(\bar{\Upsilon}_i^{CD})^T \end{pmatrix}, \tag{2.2a}$$

$$\bar{\chi}_i^{AB} = \frac{1}{2}\epsilon^{ABCD}\chi_{CDi}^T C = \begin{pmatrix} -\frac{1}{2}\epsilon^{ABCD}(\Upsilon_{CDi})^T\epsilon & \bar{\Upsilon}_i^{AB} \end{pmatrix}. \tag{2.2b}$$

We use capital letters for the $SU(4)$ hypercolor indices, with lower indices for the fundamental irrep, and upper indices for the antifundamental irrep. Several lower or upper indices will always be fully antisymmetrized. A Dirac fermion ψ in the fundamental irrep can be written in terms of two right-handed Weyl fermions, Ψ in the fundamental irrep and $\tilde{\Psi}$ in the antifundamental, as

$$\psi_{Aa} = \begin{pmatrix} \Psi_{Aa} \\ \epsilon\bar{\tilde{\Psi}}_{Aa}^T \end{pmatrix}, \qquad \bar{\psi}_a^A = \begin{pmatrix} -(\tilde{\Psi}_a^A)^T\epsilon & \bar{\Psi}_a^A \end{pmatrix}. \tag{2.3}$$

We suppress spinor indices. C is the charge-conjugation matrix, $\epsilon = i\sigma_2$ is the two-dimensional ϵ tensor acting on the Weyl spinor index, and the superscript T denotes the transpose in spinor space. With the lattice in mind, we work in Euclidean space, choosing our Dirac matrices to be Hermitian and using the chiral representation; see the Appendix.

The hypercolor theory possesses a flavor symmetry group

$$G = SU(5)\times SU(3)\times SU(3)'\times U(1)_X\times U(1)', \tag{2.4}$$

with quantum numbers $(\mathbf{5},\mathbf{1},\mathbf{1})_{(0,-1)}$ for Υ, $(\mathbf{1},\bar{\mathbf{3}},\mathbf{1})_{(1/3,5/3)}$ for Ψ, and $(\mathbf{1},\mathbf{1},\mathbf{3})_{(-1/3,5/3)}$ for $\tilde{\Psi}$.[4]

We assume that dynamical symmetry breaking takes place, generating a condensate $\langle\bar{\chi}_i\chi_j\rangle \propto \delta_{ij}$ that breaks $SU(5) \to SO(5)$. Consistent with the general considerations of Ref. [6], the Majorana bilinear $\bar{\chi}_i\chi_j$ is antisymmetric on its spinor indices and symmetric on its hypercolor indices, and so it is symmetric on its flavor indices. In addition, there is a condensate $\langle\bar{\psi}_a\psi_b\rangle \propto \delta_{ab}$ that breaks $SU(3)\times SU(3)'$ to its diagonal subgroup, which we identify with $SU(3)_{\text{color}}$. Both condensates also break $U(1)'$. The unbroken group is

$$H = SO(5)\times SU(3)_{\text{color}}\times U(1)_X. \tag{2.5}$$

For heuristic arguments supporting this pattern of symmetry breaking, see Refs. [6,8]. Of course, whether this is the actual symmetry breaking pattern is something that can be investigated on the lattice. Indeed the symmetry breaking pattern of the Dirac fermions, with $SU(3)\times SU(3)'$ breaking to the diagonal $SU(3)$ subgroup, is consistent with all known lattice results. A first study of the real- irrep symmetry breaking pattern, in a similar theory except with four, instead of five, Majorana fermions, has recently appeared in Ref. [16].

The effective theory at energy scales much below the hypercolor scale Λ_{HC} thus contains NGBs parametrizing the $U(1)'$ group manifold, and the cosets $SU(3)\times SU(3)'/SU(3)_{\text{color}}$ and $SU(5)/SO(5)$, amounting to 1, 8, and 14 NGBs for each of these factors, respectively. These NGBs are massless when all couplings of the hypercolor theory to the SM are turned off. A nontrivial effective potential is induced both by the SM gauge bosons, as we briefly review in Sec. III, and by the coupling to the third-generation quarks. The latter, which is the main subject of this paper, will be studied in Sec. IV.

The Higgs doublet is a subset of the NGB multiplet parametrizing the coset $SU(5)/SO(5)$. In more detail, the 14 NGBs corresponding to the generators in this coset are described by a nonlinear field $\Sigma \in SU(5)/SO(5)$ obtained by considering fluctuations around the vacuum $\langle\Sigma\rangle = \Sigma_0 = \mathbf{1}$,

$$\Sigma = u\Sigma_0 u^T = \exp(i\Pi/f)\Sigma_0\exp(i\Pi/f)^T = \exp(2i\Pi/f), \tag{2.6}$$

with[5]

$$\Sigma = \Sigma^T \Rightarrow \Pi = \Pi^T. \tag{2.7}$$

Under $g \in SU(5)$, Σ transforms as $\Sigma \to g\Sigma g^T$.

At the level of the algebra, $SU(2)_L\times SU(2)_R$ in Eq. (2.1) is equivalent to the $SO(4) \subset SO(5)$ associated with the first four rows and columns. The explicit form of the generators is given in the Appendix. With this choice, the field Π can be written as

$$\Pi = \Theta + \Theta^\dagger + \Phi_0 + \Phi_+ + \Phi_+^\dagger + \eta, \tag{2.8}$$

with Θ containing the Higgs doublet $H = (H_+, H_0)^T$,

$$\Theta = \begin{pmatrix} 0 & 0 & 0 & 0 & -iH_+/\sqrt{2} \\ 0 & 0 & 0 & 0 & H_+/\sqrt{2} \\ 0 & 0 & 0 & 0 & iH_0/\sqrt{2} \\ 0 & 0 & 0 & 0 & H_0/\sqrt{2} \\ -iH_+/\sqrt{2} & H_+/\sqrt{2} & iH_0/\sqrt{2} & H_0/\sqrt{2} & 0 \end{pmatrix}. \tag{2.9}$$

[4]Compare Table I of Ref. [8].

[5]Note that in Ref. [8], the notation Σ is used for the field u of Eq. (2.6).

For the explicit parametrization of the rest of Π, we refer to Ref. [8], as we will not need it here. The Higgs doublet comprises four of the NGBs, and the $SU(2)_L$ triplets ϕ_0, ϕ_+, and $\phi_- = (\phi_+)^\dagger$ comprise nine more NGBs.[6] Finally, Π has a component η proportional to the generator $\mathrm{diag}(1,1,1,1,-4)$, which is neutral with respect to the entire SM model gauge group, and which completes the multiplet of 14 NGBs.

While they play only a small role, we will need also the nonlinear fields associated with the other broken symmetries. We account for the $SU(3)\times SU(3)'/SU(3)_{\text{color}}$ coset by a nonlinear field $\Omega\in SU(3)$ transforming as $\Omega \to g\Omega h^\dagger$ for $g\in SU(3)$ and $h\in SU(3)'$. A similar nonlinear field arises in the familiar chiral Lagrangian of three-flavor QCD, but the reader should keep in mind the different physical roles of the various $SU(3)$ groups in the case at hand. Throughout most of this paper, we will assume that the only source of *explicit* breaking of $SU(3)\times SU(3)'$ to $SU(3)_{\text{color}}$ arises from the coupling of the $SU(3)_{\text{color}}$ currents to the SM gluons. A wider range of possibilities than what is the main focus of this article is allowed if we relax this assumption. This is briefly discussed in Sec. IV E. The eight NGBs associated with the nonlinear Ω field transform in the adjoint irrep of $SU(3)_{\text{color}}$. They are singlets under both $SU(2)_L$ and $U(1)_Y$.

Finally, to account for the spontaneous breaking of $U(1)'$ we introduce a nonlinear field $\Phi\in U(1)$ with unit charge under $U(1)'$. The associated NGB η' is neutral under the SM gauge interactions. Using an $\sim$ sign to indicate identical transformation properties under the entire flavor group G, we thus have

$$\Phi^{-2}\Sigma_{ij} \sim \bar\chi_i P_R \chi_j \sim \epsilon^{ABCD}(\Upsilon_{CDi})^T \epsilon \Upsilon_{ABj}, \tag{2.10a}$$

$$\Phi^{-10/3}\Omega_{ab} \sim \bar\psi_a P_L \psi_b \sim \bar\Psi^A_a \epsilon (\bar{\bar\Psi}_{Ab})^T. \tag{2.10b}$$

III. HIGGS EFFECTIVE POTENTIAL FROM ELECTROWEAK GAUGE BOSONS

In this section, we briefly review the contribution from the SM gauge bosons to the effective potential for the NGBs, starting with the effective potential for the $SU(5)/SO(5)$ nonlinear field Σ generated by the electroweak gauge bosons. This part of the effective potential takes the form

$$V^{\text{EW}}_{\text{eff}}(\Sigma) = C_{LR}\sum_Q \mathrm{tr}(Q\Sigma Q^*\Sigma^*), \tag{3.1}$$

if we work to leading (i.e., quadratic) order in the SM gauge couplings. The sum over Q runs over the $SU(2)_L$ generators gT^a_L with T^a_L given in Eq. (A5), and the hypercharge generator $g'Y = g'(T^3_R + X)$, with $X = 0$ for the Π field. Here

$$C_{LR} = \frac{3}{(4\pi)^2}\int_0^\infty dq^2 q^2 \Pi_{LR}(q^2), \tag{3.2}$$

and

$$(q^2\delta_{\mu\nu} - q_\mu q_\nu)\Pi_{LR}(q^2) = \int d^4x e^{iqx}\mathrm{tr}\langle \gamma_\mu P_R[\chi(x)\bar\chi(0)]\gamma_\nu P_L[\chi(0)\bar\chi(x)]\rangle, \tag{3.3}$$

where $[\chi(x)\bar\chi(y)]$ is the Majorana-fermion propagator for the field χ of Eq. (2.2), and $\langle\ldots\rangle$ indicates the expectation value with respect to the hypercolor gauge fields. This type of effective potential goes back to the well-known formula for the mass difference between the charged and neutral pions in QCD. For further explanations and a derivation of this result in the present context we refer to the review article Ref. [4] and to the Appendix of Ref. [17].[7]

The proof of Ref. [19] that $C_{LR} > 0$ applies also in this case. Using the explicit form (A5), the minimum of $V^{\text{EW}}_{\text{eff}}(\Sigma)$ is equal to $-C_{LR}(3g^2 + g'^2)$, which is attained at $\Sigma = \mathbf{1}$. This part of the effective potential does not rotate the vacuum of the hypercolor theory, exhibiting the phenomenon of vacuum alignment [6].

Expanding Σ to quadratic order in Π inside $V^{\text{EW}}_{\text{eff}}$, we find the mass terms

$$V^{\text{EW}(2)}_{\text{eff}} = \frac{C_{LR}}{f^2}\left\{(3g^2+g'^2)\left(2H^\dagger H + \frac{16}{3}\phi^\dagger_+\phi_+\right) + 8g^2\phi^\dagger_0\phi_0\right\}. \tag{3.4}$$

The explicit symmetry breaking by the electroweak gauge bosons produces a positive mass squared for all components of the NGB multiplet Π except η.

To summarize, when we couple the hypercolor theory to the SM gauge bosons, an effective potential for the nonlinear field Σ is generated. At lowest order, it is proportional to the squares of the electroweak couplings $g_{\text{EW}} = g$ for $SU(2)_L$, or $g_{\text{EW}} = g'$ for $U(1)_Y$, where it is understood that all SM couplings are evaluated at the hypercolor scale Λ_{HC}. Because of vacuum alignment, the expectation value Σ_0 will remain equal to 1, but the Higgs doublet and the three $SU(2)_L$ triplets will acquire a mass proportional to $g_{\text{EW}}f$, while the singlet η will remain massless. To avoid

[6] In the notation of Ref. [8], $\phi_0 = (\phi_0^-, \phi_0^0, \phi_0^+)$ with ϕ_0^0 real and $\phi_0^+ = (\phi_0^-)^*$, while $\phi_+ = (\phi_+^-, \phi_+^0, \phi_+^+)$, with all components complex.

[7] The factor of 3 in Eq. (3.2) comes from tracing over the transversal projector. This factor is erroneously missing in the published versions of Eqs. (A11a) and (A11b) of Ref. [17], and Eq. (A5) of [18]. Also, in the published versions, the term $gW_\mu\, Q_a\, J_{\mu A}$ in Eq. (A2) of Ref. [17] should be multiplied by i, and similarly for the term $gW_\mu(\, Q^L_a\, J^L_{\mu a} + Q^R_a\, J^R_{\mu a})$ in Eq. (A2) of Ref. [18].

confusion, the contribution to the effective potential from the top quark has not yet been included, and will be discussed in the next section.

Similarly, when we turn on the QCD interactions, an effective potential for the Ω nonlinear field is generated,

$$V_{\rm eff}^{\rm QCD}(\Omega) = -C_{LR}^{\rm QCD}\sum_{Q}\mathrm{tr}(Q\Omega Q\Omega^\dagger), \qquad (3.5)$$

where now Q runs over the eight generators $g_s\lambda_a$ of $SU(3)_{\rm color}$, and g_s is the QCD coupling (at the hypercolor scale). In the underlying hypercolor theory, $C_{LR}^{\rm QCD}$ has a representation analogous to Eq. (3.2), except that the Majorana-fermion propagator in Eq. (3.3) is replaced by the Dirac-fermion propagator $[\psi(x)\bar{\psi}(y)]$. Once again there is vacuum alignment nailing down the vacuum at $\langle\Omega_{ab}\rangle = \delta_{ab}$, and giving the octet of NGBs a mass of order $g_s f$. Thus, as long as f is much larger than the electroweak scale, both the $SU(2)_L$ triplet NGBs and the color-octet NGBs are much heavier than the electroweak gauge bosons or the top quark.

As already noted, the NGB η' of the spontaneously broken $U(1)'$ is inert under all the SM gauge interactions. Moreover, the coupling of the hypercolor sector to the SM considered in the next section does not break $U(1)'$ explicitly. Therefore, no effective potential will be generated for the associated nonlinear field Φ, and η' will remain exactly massless.

IV. THE TOP QUARK SECTOR

We now proceed to the main part of this article, which is the study of the dynamics arising from the coupling of the top quark to the hypercolor theory. There are two aspects of interest: the contribution of the top quark to the effective potential for the nonlinear field Σ containing the Higgs field, analogous to the contribution from the weak gauge bosons in Eq. (3.1), and the mass of the top quark itself.

We will begin with the coupling of the top quark to the hypercolor theory at the "microscopic" level, which involves only the elementary fields: the hypercolor gauge fields, and the hyperfermions of Eqs. (2.2) and (2.3). We introduce fermionic spurions which transform in complete representations of the flavor group G of Eq. (2.4), and which contain the $SU(2)_L$ doublet q_L of the left-handed top and bottom quarks t_L and b_L, as well as the right-handed top quark t_R. Following the mechanism proposed in Ref. [7], the spurions will be coupled linearly to suitable hyperbaryon fields, which are three-fermion operators in the hypercolor theory. We demand that the spurion-hyperbaryon interactions are invariant under G, because the microscopic theory does not know about the dynamical breaking $G \to H$.

The spurion-hyperbaryon interaction terms are four-fermion operators, and are assumed to arise from some extended hypercolor (EHC) sector with a dynamical scale $\Lambda_{\rm EHC} \gg \Lambda_{\rm HC}$, the origin of which we will not specify. We will return to this point in the conclusion section. The hyperbaryon operators and the four-fermion couplings are constructed in Sec. IV A.

We then turn to the effective low-energy theory. We demand that also the effective theory is invariant under G, but it can now depend on the effective fields: the $SU(5)/SO(5)$ coset field Σ, which plays a central role since it contains the Higgs field, as well as the $SU(3)\times SU(3)'/SU(3)_{\rm color}$ field Ω and the $U(1)$-valued field Φ. Note that we do not allow the effective theory to contain any effective fields for hyperbaryons. This strategy generalizes the standard construction of the chiral Lagrangian for QCD.[8]

We proceed in two steps. First, in Sec. IV B, we consider the coupling of the SM quarks q_L and t_R to the effective nonlinear fields, integrating out all other states in the hypercolor theory. This will lead to an expression for the top Yukawa coupling in terms of a hyperbaryon two-point function in the hypercolor theory.

Next, in Sec. IV C, we consider the contribution to the effective potential obtained by integrating also over the third-generation quarks to leading order in the top Yukawa coupling. This involves restricting the spurions to their SM values, in which all components except those corresponding to $q_L = (t_L, b_L)$ and to t_R are set equal to zero, and integrating over q_L and t_R. Like the coupling to the SM gauge bosons (Sec. III), this breaks explicitly the flavor group G. However, in the approximation in which we work, only $SU(5)$ is broken explicitly, whereas all other factors in Eq. (2.4) are not. As a result, no effective potential is generated for Ω or Φ.

The explicit breaking of $SU(5)$ generates an effective potential for Σ. This new contribution is parametrized by one new LEC $C_{\rm top}$ analogous to C_{LR} in Eq. (3.1). We will show that $C_{\rm top}$ can be expressed as an integral over a hyperbaryon four-point function convoluted with two free, massless fermion propagators.

Up to this point, our analysis is from first principles. In Sec. IV D we turn to physical but nonrigorous considerations. We show that a large-N limit exists [where $N = 4$ for the hypercolor group $SU(4)$ we consider here] in which the hyperbaryon four-point function factorizes, leading to a simple result for $C_{\rm top}$, and ultimately to a nontrivial expectation value for the Higgs field.

Finally, in Sec. IV E we comment on the phenomenological consequences of our analysis. This includes a brief discussion of the more general situation where the explicit breaking of $SU(3)\times SU(3)'$ to its diagonal subgroup $SU(3)_{\rm color}$ is allowed to come from sources other than the QCD gluons.

A. Top quark spurions and hyperbaryons

We begin with introducing the top quark spurions, a left-handed spurion T_L, which we choose in the **5** irrep of

[8]See for instance Ref. [20].

TABLE I. Local hyperbaryon operators. The leftmost column gives the Weyl-fermion content, and the rightmost column is the notation used for the operator. The remaining columns list the quantum numbers.

	$SU(5)$	$SU(3)\times SU(3)'$	$SU(3)_c$	$U(1)_X$	$U(1)'$	
$\Upsilon(\Psi\Psi)$	$\mathbf{5}$	$(\bar{\mathbf{3}},\mathbf{1})\times(\bar{\mathbf{3}},\mathbf{1})\to(\mathbf{3},\mathbf{1})$	$\mathbf{3}$	2/3	7/3	B_R
$\Upsilon(\tilde{\Psi}\tilde{\Psi})$	$\mathbf{5}$	$(\mathbf{1},\bar{\mathbf{3}})\times(\mathbf{1},\bar{\mathbf{3}})\to(\mathbf{1},\mathbf{3})$	$\mathbf{3}$	2/3	−13/3	B'_R
$\bar{\Upsilon}(\bar{\Psi}\bar{\Psi})$	$\bar{\mathbf{5}}$	$(\mathbf{3},\mathbf{1})\times(\mathbf{3},\mathbf{1})\to(\bar{\mathbf{3}},\mathbf{1})$	$\bar{\mathbf{3}}$	−2/3	−7/3	$\bar{B}_R$
$\bar{\Upsilon}(\bar{\tilde{\Psi}}\bar{\tilde{\Psi}})$	$\bar{\mathbf{5}}$	$(\mathbf{1},\mathbf{3})\times(\mathbf{1},\mathbf{3})\to(\mathbf{1},\bar{\mathbf{3}})$	$\bar{\mathbf{3}}$	−2/3	13/3	$\bar{B}'_R$
$\bar{\Upsilon}(\Psi\Psi)$	$\bar{\mathbf{5}}$	$(\bar{\mathbf{3}},\mathbf{1})\times(\bar{\mathbf{3}},\mathbf{1})\to(\mathbf{3},\mathbf{1})$	$\mathbf{3}$	2/3	13/3	B_L
$\bar{\Upsilon}(\bar{\tilde{\Psi}}\bar{\tilde{\Psi}})$	$\bar{\mathbf{5}}$	$(\mathbf{1},\bar{\mathbf{3}})\times(\mathbf{1},\bar{\mathbf{3}})\to(\mathbf{1},\mathbf{3})$	$\mathbf{3}$	2/3	−7/3	B'_L
$\Upsilon(\bar{\Psi}\bar{\Psi})$	$\mathbf{5}$	$(\mathbf{3},\mathbf{1})\times(\mathbf{3},\mathbf{1})\to(\bar{\mathbf{3}},\mathbf{1})$	$\bar{\mathbf{3}}$	−2/3	−13/3	$\bar{B}_L$
$\Upsilon(\bar{\tilde{\Psi}}\bar{\tilde{\Psi}})$	$\mathbf{5}$	$(\mathbf{1},\mathbf{3})\times(\mathbf{1},\mathbf{3})\to(\mathbf{1},\bar{\mathbf{3}})$	$\bar{\mathbf{3}}$	−2/3	7/3	$\bar{B}'_L$

$SU(5)$, and a right-handed spurion T_R, which we choose in the $\bar{\mathbf{5}}$ irrep. Both irreps reduce to the $\mathbf{5}$ of $SO(5)$. This choice ensures that terms like $\bar{T}_L T_R$ in the effective potential are disallowed by $SU(5)$, but allowed by $SO(5)$.[9] Both T_L and T_R will have $U(1)_X$ charge 2/3, as this will yield the correct hypercharges for q_L and t_R. The SM values for these spurions are

$$T_L = \hat{T}_L \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} ib_L \\ b_L \\ it_L \\ -t_L \\ 0 \end{pmatrix}, \qquad T_R = \hat{T}_R \equiv \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ it_R \end{pmatrix}. \quad (4.1)$$

Using Eq. (A5), it is straightforward to verify that the pair (t_L, b_L) transforms as an $SU(2)_L$ doublet, and has hypercharge $Y = 1/6$. The $SU(2)_L$ singlet t_R has hypercharge $Y = 2/3$. The quantum numbers of the spurions under the remaining flavor symmetries will be discussed shortly.

The spurions couple to the hypercolor theory through the G-invariant Lagrangian

$$\mathcal{L}_{\rm EHC} = \lambda_1 \bar{T}_L B_R + \lambda_1^* \bar{B}_R T_L + \lambda_2 \bar{T}_R B_L + \lambda_2^* \bar{B}_L T_R, \quad (4.2)$$

where $B_{L,R}$ are hyperbaryon fields with appropriate quantum numbers. Setting the spurions $T_{L,R}$ equal to their SM values (4.1) then tells us how the SM quarks t_L, b_L, and t_R couple to the hypercolor theory. Since we will use three-hyperfermion local interpolating fields for the hyperbaryons, the four-fermion interactions in $\mathcal{L}_{\rm EHC}$ have engineering dimension six. $\mathcal{L}_{\rm EHC}$ originates from some other sector with scale $\Lambda_{\rm EHC} \gg \Lambda_{\rm HC}$, with effective couplings $\lambda_{1,2} \sim O(\Lambda_{\rm EHC}^{-2})$ just below that scale.

[9]We may switch $\mathbf{5}$ with $\bar{\mathbf{5}}$, but the key point is that the two spurions are chosen to be in different $SU(5)$ irreps.

Limiting ourselves to local hyperbaryon fields, all operators that can be used in the construction of a G-invariant $\mathcal{L}_{\rm EHC}$ are listed in Table I. The schematic structure in terms of Weyl fields is indicated in the first column of the table, followed by the quantum numbers under the flavor group G. The column labeled as $SU(3)_c$ gives the $SU(3)_{\rm color}$ irrep. The spinor index of the hyperbaryon field is always carried by the Majorana fermion χ. Using Eqs. (2.2) and (2.3), explicit expressions for the unprimed operators in Table I are

$$B_{Ria} = -\frac{1}{2}\epsilon^{ABCD}\epsilon_{abc} P_R \chi_{ABi} (\psi_{Cb}^T C P_R \psi_{Dc}) \quad (4.3a)$$

$$= \frac{1}{2}\epsilon^{ABCD}\epsilon_{abc} \Upsilon_{ABi} (\Psi_{Cb}^T \epsilon \Psi_{Dc}),$$

$$\bar{B}_{Ria} = \frac{1}{2}\epsilon_{ABCD}\epsilon_{abc} \bar{\chi}_i^{AB} P_L (\bar{\psi}_b^C C P_L (\bar{\psi}_c^D)^T) \quad (4.3b)$$

$$= \frac{1}{2}\epsilon_{ABCD}\epsilon_{abc} \bar{\Upsilon}_i^{AB} (\bar{\Psi}_b^C \epsilon (\bar{\Psi}_c^D)^T),$$

$$B_{Lia} = -\frac{1}{2}\epsilon^{ABCD}\epsilon_{abc} P_L \chi_{ABi} (\psi_{Cb}^T C P_R \psi_{Dc}) \quad (4.3c)$$

$$= \epsilon_{abc}\epsilon (\bar{\Upsilon}_i^{AB})^T (\Psi_{Ab}^T \epsilon \Psi_{Bc}),$$

$$\bar{B}_{Lia} = \frac{1}{2}\epsilon_{ABCD}\epsilon_{abc} \bar{\chi}_i^{AB} P_R (\bar{\psi}_b^C C P_L (\bar{\psi}_c^D)^T) \quad (4.3d)$$

$$= \epsilon_{abc} \Upsilon_{ABi}^T \epsilon (\bar{\Psi}_b^A \epsilon (\bar{\Psi}_c^B)^T). \quad (4.3e)$$

The primed operators in Table I are obtained from Eq. (4.3) by interchanging $P_R \leftrightarrow P_L$ inside the $\psi\psi$ and $\overline{\psi}\overline{\psi}$ bilinears.

When the spurions $T_{L,R}$ are restricted to their SM values, the phases of $\lambda_{1,2}$ in Eq. (4.2) can be removed by (nonanomalous) $SU(2)_L$ and $SU(2)_R$ transformations on the spurion fields, implying that, from now on, we may take $\lambda_{1,2}$ to be real and positive. This allows us to require that the Lagrangian (4.2) be CP invariant. The CP transformation acts as

$$\psi \to \gamma_2 \bar{\psi}^T, \qquad \bar{\psi} \to \psi^T \gamma_2, \quad (4.4)$$

for both Dirac and Majorana fermions (see the Appendix for our Dirac matrices conventions). The sign choices we have made in Eq. (4.3) imply that a CP transformation applied to the elementary fields χ, ψ, and $\bar{\psi}$ induces a CP transformation of the same form on the hyperbaryon fields as well, thereby ensuring the CP invariance of $\mathcal{L}_{\rm EHC}$.

The unprimed fields in Table I transform nontrivially under $SU(3)$ and are singlets under $SU(3)'$, whereas for the primed fields the opposite is true. Choosing either the primed or the unprimed version for each hyperbaryon field gives rise to a total of four different possibilities for $\mathcal{L}_{\rm EHC}$. The quantum numbers of the spurions T_L and T_R are chosen accordingly, so as to ensure the G invariance of

$\mathcal{L}_{\rm EHC}$. The SM quark fields q_L and t_R are endowed with the same $SU(3)\times SU(3)'\times U(1)_X\times U(1)'$ quantum numbers as their parent spurion. This construction is consistent with the SM, since the resulting quantum numbers under $SU(3)_{\rm color}$ will always be the same. In addition, it follows that the entire theory, including the hypercolor sector, the SM Lagrangian, and their coupling via $\mathcal{L}_{\rm EHC}$, is invariant under both $SU(3)$ and $SU(3)'$, provided that the QCD interactions can be neglected. This is indeed the case in this section, because we calculate the Higgs effective potential to second order in all SM couplings, and since the result will be quadratic in the top Yukawa coupling, any corrections that involve an additional dependence on the QCD coupling g_s are neglected.

Note that we cannot generalize Eq. (4.2) to include, simultaneously, terms that couple a given spurion to both of the unprimed and primed hyperbaryons, as this will not allow for any consistent assignment of $SU(3)\times SU(3)'$ quantum numbers. For example, $\bar{T}_L$ can couple to either B_R or B'_R, but not to both. It is because of this fact that $\mathcal{L}_{\rm EHC}$ depends on only two coupling constants $\lambda_{1,2}$. This will lead to considerable simplification in our analysis. In Sec. IV E we briefly comment on the more general case, where $\mathcal{L}_{\rm EHC}$ is restricted only by $SU(3)_{\rm color}$.

B. The top quark Yukawa coupling

Our next task is to construct the electroweak effective field theory. As a first step, we integrate only over the gauge fields and fermions of the hypercolor theory, and obtain an effective theory that depends on the spurions T_L and T_R, and on the nonlinear fields, including in particular the $SU(5)/SO(5)$ field Σ. We assume that the electroweak scale $m_W \sim m_t$ is much smaller than the hypercolor scale $f \sim M \sim \Lambda_{\rm HC}$, where M is of order the mass of the hyperbaryons which are assumed to couple to the top quark in Eq. (4.2). This provides us with a power counting, and, in particular, the effective theory can be organized according to a derivative expansion. We will be concerned with the lowest nontrivial order in this expansion.

Demanding full G invariance, the leading-order spurion potential is

$$V_{\rm top} = \mu_L \Phi^2 \bar{T}_R \Sigma^* T_L + \mu_R \Phi^{-2} \bar{T}_L \Sigma T_R. \tag{4.5}$$

Terms like $\bar{T}_L T_L$ vanish because of chiral projectors, while terms like $\bar{T}_L \bar{T}_R^T$ are not allowed by $U(1)_X$ symmetry. Bilinear terms independent of Σ are possible, but thanks to $SU(5)$ invariance, they have an LL or RR structure, and need an insertion of γ_μ. Therefore, they contain at least one derivative, and their role is to renormalize the kinetic terms for the top and bottom quarks, which are present when these SM fields are made dynamical. It can be checked that the correction is of order y, where y is the top quark Yukawa coupling introduced in Eq. (4.12) below.

$V_{\rm top}$ depends on two effective fields, Σ and Φ. The role of Φ is to reinstate $U(1)'$ invariance [cf. Eq. (2.10)]. When we choose both hyperbaryons in Eq. (4.2) to be unprimed ones, the hyperbaryons and the spurions transform nontrivially only under $SU(3)$, and are singlets of $SU(3)'$. Therefore, $V_{\rm top}$ is invariant under $SU(3)\times SU(3)'$ as it stands, without having to introduce any dependence on the effective field Ω.

If we set $\Sigma = \Phi = 1$, and substitute the SM values $\hat{T}_L$ and $\hat{T}_R$ defined in Eq. (4.1) for T_L and T_R, we find that $V_{\rm top}$ vanishes. Σ will need to develop a nontrivial expectation value for the SM top quark to acquire a nonzero mass. This will be discussed in Sec. IV C below.

In order to find the LECs $\mu_{L,R}$ in Eq. (4.5), we consider the second derivatives

$$\frac{\partial^2}{\partial T_L(y)\partial \bar{T}_R(x)} \log Z, \qquad \frac{\partial^2}{\partial T_R(y)\partial \bar{T}_L(x)} \log Z, \tag{4.6}$$

where Z is the partition function of either the effective or the microscopic theory. Requiring the effective theory to match the microscopic theory (and noting that the fermionic spurions are Grassmann) yields the relations

$$\begin{aligned} -\mu_L P_L \langle \Phi^2 \Sigma^* \rangle \delta(x-y) + \cdots &= \lambda_1\lambda_2 P_L \langle B_L(x)\bar{B}_R(y)\rangle P_L, \\ -\mu_R P_R \langle \Phi^{-2} \Sigma \rangle \delta(x-y) + \cdots &= \lambda_1\lambda_2 P_R \langle B_R(x)\bar{B}_L(y)\rangle P_R. \end{aligned} \tag{4.7}$$

The ellipses on the left-hand side indicate that the leading-order low-energy theory given by $V_{\rm top}$ reproduces the correlation functions on the right-hand side only to leading order in a derivative expansion.

By assumption, symmetry breaking in the hypercolor theory yields $\langle \bar{\chi}_i \chi_j \rangle \propto \langle \Sigma_{ij} \rangle = \delta_{ij}$, up to a symmetry transformation. (When the SM fields become dynamical we may in general have $\langle \Sigma_{ij} \rangle \neq \delta_{ij}$, but these corrections are of higher order in the SM gauge and Yukawa couplings.) Setting $\langle \Sigma_{ij} \rangle = \delta_{ij}$ and $\langle \Phi \rangle = 1$ in Eq. (4.7) provides us with expressions for the parameters $\mu_{L,R}$. Assembling the chiral baryon fields together as

$$B = B_R + B_L, \qquad \bar{B} = \bar{B}_R + \bar{B}_L, \tag{4.8}$$

where B is a Dirac field with quantum numbers $(\mathbf{5},\mathbf{3})$ under the unbroken $SO(5)\times SU(3)_{\rm color}$, and writing

$$\begin{aligned} \delta(x-y) &= \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)}, \\ \langle B(x)\bar{B}(y)\rangle &= \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} S_B(p), \end{aligned} \tag{4.9}$$

we find, to leading order in the momentum expansion of the effective theory,

$$\mu_L P_L = -\lambda_1\lambda_2 P_L S_B(0) P_L,$$
$$\mu_R P_R = -\lambda_1\lambda_2 P_R S_B(0) P_R. \quad (4.10)$$

Apart from the chiral projectors, these two expressions must be equal, because any hyperbaryon fields occurring in Eq. (4.2) can only have a Dirac mass; Majorana masses (such as $B_R^T \epsilon B_R$ or $B_L^T \epsilon B_L$) are forbidden by $U(1)_X$ symmetry. Therefore,

$$\mu = \mu_L = \mu_R = -\lambda_1\lambda_2 S_B(0), \quad (4.11)$$

where we have used that at zero momentum $S_B(0)$ is proportional to the unit matrix in spinor space.

We may now introduce the top quark Yukawa coupling y by writing

$$\mu = yf/2, \quad (4.12)$$

with f the decay constant of the hypercolor theory. If the Higgs field will now develop a nonzero expectation value,

$$\langle H_0 \rangle = \langle H_0^\dagger \rangle = h/\sqrt{2}, \quad (4.13)$$

this will induce a top quark mass

$$m_t = \frac{1}{2\sqrt{2}} yf \sin(2h/f) \approx \frac{1}{\sqrt{2}} yh, \quad (4.14)$$

where we have used Eqs. (2.6), (2.9), (4.1), and (4.5), and the approximate equality holds for $h/f \ll 1$.

We may introduce an effective hyperbaryon field $\tilde{B}$ which is canonically normalized by writing

$$B = f^3 \sqrt{Z_B}\tilde{B}. \quad (4.15)$$

We define M as the zero-momentum mass of the canonically normalized $\bar{B}$ field corresponding to a term $M\bar{\tilde{B}}\tilde{B}$. In other words, $S_B(0) = f^6 Z_B/M$. This gives

$$y = -2\lambda_1\lambda_2 Z_B f^5/M. \quad (4.16)$$

We comment that the field B does not necessarily correspond to any baryon mass eigenstate of the hypercolor theory. Still, generically we might expect it to couple to the lightest hyperbaryon with quantum numbers that match those of the SM quarks, in which case M will be a quantity of the order of this smallest hyperbaryon mass.

We conclude this subsection with a technical comment. If in Eq. (4.2) we choose one unprimed and one primed hyperbaryon field, this implies that one of the spurions transforms nontrivially under $SU(3)$ while the other under $SU(3)'$. In this case, $V_{\rm top}$ will depend on Ω. For definiteness, replacing B_L in Eq. (4.2) by B'_L implies that now T_R transforms nontrivially under $SU(3)'$, and Eq. (4.5) gets replaced by

$$V_{\rm top} = \mu_L \Phi^{-14/3} \bar{T}_R \Sigma^* \Omega^\dagger T_L + \mu_R \Phi^{14/3} \bar{T}_L \Sigma \Omega T_R. \quad (4.17)$$

This hardly changes our analysis, because, in order to obtain expressions for the parameters $\mu_{L,R}$ in Eq. (4.5) we are setting all nonlinear fields equal to the identity anyway.

In the next subsection, we will work out the effective potential after integrating over the third-generation quarks. In this calculation, any dependence on both Ω and Φ will drop out regardless of our choice of hyperbaryon fields in Eq. (4.2), as it must be because both $SU(3)\times SU(3)'$ and $U(1)'$ are not explicitly broken in the top sector to the order we work, and therefore no effective potential can be generated for those nonlinear fields. In particular, when $V_{\rm top}$ depends on Ω as in Eq. (4.17), then expression (4.19c) below, which is the only term that will contribute to the effective potential, gets multiplied by $\mathrm{tr}(\Omega\Omega^\dagger) = \mathrm{tr}\mathbf{1}$, showing that indeed the Ω dependence cancels out.

C. Higgs effective potential induced by the top quark

We now integrate over the SM top quark in order to obtain the associated contribution $V_{\rm eff}^{\rm top}(\Sigma)$ to the effective potential. Adding this to Eq. (3.1) gives the complete effective potential for Σ to second order in the SM gauge and Yukawa couplings. We will disregard all the other SM fermions, including the bottom quark, on the grounds that their Yukawa couplings are much smaller, and so their contribution to the effective potential will be much smaller as well.

We begin by splitting the spurions $T_{L,R}$ as follows[10]:

$$T_L(x) = t_L(x) v_L, \qquad T_R(x) = t_R(x) v_R. \quad (4.18)$$

The new global spurions $v_{L,R}$ carry the $SU(5)$ quantum numbers, which contain the SM symmetry $SU(2)_L$. We also assign $U(1)_X$ to these spurions, because the hypercharge Y is the sum of the charge X and the third component of $SU(2)_R$, with the latter being a subgroup of $SU(5)$ as well. The (Grassmann) fields $t_{L,R}$ carry the spin index. They also inherit the $SU(3)$ and $SU(3)'$ quantum numbers from $T_{L,R}$. We promote $t_{L,R}$ to dynamical fields by adding tree-level kinetic terms $\bar{t}_L \partial\!\!\!/ t_L + \bar{t}_R \partial\!\!\!/ t_R$.

The effective potential $V_{\rm eff}^{\rm top}$ at order y is obtained by substituting Eq. (4.18) into $V_{\rm top}$ of Eq. (4.5), and integrating over the top quark, leading to a contribution with the form of $\Phi^{-2}\bar{v}_L \Sigma v_R + \mathrm{H.c.}$ This contribution vanishes, however, because the only nonzero tree-level top propagators are $\langle t_L \bar{t}_L \rangle$ and $\langle t_R \bar{t}_R \rangle$.

The leading contribution to $V_{\rm eff}^{\rm top}$ is of order y^2. It involves four global spurions. Momentarily suppressing any dependence on the Ω and Φ fields, the possible terms that depend on Σ are

[10]Whether or not t_L in Eq. (4.18) coincides with the component with the same name of $\hat{T}_L$ in Eq. (4.1) depends on the value we choose for v_L, as we will see below.

$$(\bar{v}_L \Sigma v_R)^2 + \text{H.c.}, \tag{4.19a}$$

$$(\bar{v}_R \Sigma^* v_L)^2 + \text{H.c.}, \tag{4.19b}$$

$$(\bar{v}_L \Sigma v_R)(\bar{v}_R \Sigma^* v_L). \tag{4.19c}$$

The tree-level top propagators allow for the generation of the last term only. If the effective potential (4.19c) arises as the product of the two interactions in Eq. (4.5), the Φ dependence evidently cancels out. Moreover, as we have explained in the previous section, regardless of the choice of hyperbaryon fields we make in Eq. (4.2), $V_{\text{eff}}^{\text{top}}$ will be independent of Ω and Φ, because, to the order we are working, $SU(3)$, $SU(3)'$, and $U(1)'$ are not broken explicitly.

Promoting the fields t_L and t_R in Eq. (4.1) to be dynamical amounts to setting

$$v_L = \hat{v}_L \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ i \\ -1 \\ 0 \end{pmatrix}, \qquad v_R = \hat{v}_R \equiv \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ i \end{pmatrix}, \tag{4.20}$$

with $\hat{\bar{v}}_{L,R} = \hat{v}_{L,R}^\dagger$. The resulting contribution to the effective potential is

$$y^2 C_{\text{top}} (\hat{\bar{v}}_L \Sigma \hat{v}_R)(\hat{\bar{v}}_R \Sigma^* \bar{v}_L) = \frac{y^2}{2} C_{\text{top}} (\Sigma_{35} - i\Sigma_{45})(\Sigma_{35}^* + i\Sigma_{45}^*). \tag{4.21}$$

C_{top} is a new LEC. We have factored out the square of the Yukawa coupling y to make explicit the order at which we work.

However, we are not done yet. In order to arrive at this result we have used Eq. (4.20) for the global spurions, which projects onto a particular component of the $SU(2)_L$ doublet q_L, the one denoted t_L in Eq. (4.1). In order to add the contribution of the other component denoted b_L, we replace $\hat{v}_L$ of Eq. (4.20) by $(i, 1, 0, 0, 0)^T/\sqrt{2}$ (the right-handed singlet spurion $\hat{v}_R$ is unchanged). Adding the two contributions together we arrive at the $SU(2)_L$ invariant effective potential

$$V_{\text{eff}}^{\text{top}} = \frac{y^2}{2} C_{\text{top}} (|\Sigma_{35} - i\Sigma_{45}|^2 + |\Sigma_{15} + i\Sigma_{25}|^2). \tag{4.22}$$

This is the leading contribution of dynamical third-generation quarks to the effective potential. Expanding the nonlinear Σ field to quadratic order in the NGB fields gives

$$V_{\text{eff}}^{\text{top}(2)} = 4y^2 \frac{C_{\text{top}}}{f^2} H^\dagger H. \tag{4.23}$$

When the Higgs field $H = (H_+, H_0)^T$ acquires an expectation value, conventionally it is assigned to the lower component H_0, as in Eq. (4.13). This selects t_L (rather than any other linear combination of the doublet fields t_L and b_L) as the left-handed field that, together with the right-handed field t_R, forms the physical top quark.

Let us pause to consider these results. The full effective potential $V_{\text{eff}}(\Sigma)$ is the sum of Eqs. (3.1) and (4.22). If C_{top} is positive, the global minimum is attained for $\Sigma = \Sigma_0 = \mathbf{1}$ (with $\Sigma_{i5} = 0$ for $i = 1, \ldots, 4$). For electroweak symmetry breaking to take place, C_{top} must therefore be negative. To second order in the NBG fields, the effective potential is the sum of Eqs. (3.4) and (4.23). As already observed in Ref. [8], the curvature at the origin can become negative only in the direction of the H field. This happens when

$$\frac{C_{\text{top}}}{C_{LR}} < -\frac{3g^2 + g'^2}{2y^2} = -\frac{2m_W^2 + m_Z^2}{m_t^2} \approx -0.7, \tag{4.24}$$

triggering a nonzero expectation value for the Higgs field.

If we use Eq. (4.13) and, moreover, assume that all other NGB fields in Eq. (2.8) remain zero, the total effective potential is

$$V_{\text{eff}}(h) = -C_{LR}(3g^2 + g'^2)\cos^2(h/f) + \frac{y^2}{2} C_{\text{top}} \sin^2(2h/f). \tag{4.25}$$

While the global minimum of $V_{\text{eff}}(\Sigma)$ must occur at nonzero h if Eq. (4.24) is satisfied, due to the complexity of $V_{\text{eff}}(\Sigma)$ we have not been able to prove that, given arbitrary values of the SM couplings or the LECs, the global minimum will never involve nonzero expectation values for any other NGBs.

We next turn to the calculation of the low-energy constant C_{top}. As in the previous subsection, this is done by matching the effective potential (4.22) to the underlying theory with top-hyperbaryon couplings as given in Eq. (4.2). The difference is that now we also integrate over the SM top quark field. After splitting the $T_{L,R}$ spurions as in Eq. (4.18), the matching will involve taking four derivatives, one with respect to each of the global spurions $v_{L,R}$ and $\bar{v}_{L,R}$.[11] Once again we will set $\Sigma = \mathbf{1}$. This implies that we must take into account terms independent of Σ that have a similar dependence on the global spurions as Eq. (4.19c). There are two such terms,

$$y^2 C_1 (\bar{v}_L v_L)(\bar{v}_R v_R) + y^2 C_2 (\bar{v}_L \bar{v}_R^T)(v_R^T v_L), \tag{4.26}$$

where we introduced new LECs C_1 and C_2, and, for convenience, separated out a factor of y^2, as we did in

[11] We will discuss later on what values to choose for the global spurions in Eq. (4.18), or equivalently, with respect to which component of each global spurion one would choose to differentiate.

Eq. (4.21). Taking the four derivatives in the effective theory we find, after setting $\Sigma = \mathbf{1}$,

$$\frac{\partial^4}{\partial\bar{v}_{Li}\partial v_{Rj}\partial\bar{v}_{Rk}\partial v_{L\ell}}\log Z_{\rm eff} = -y^2V(C_{\rm top}\delta_{ij}\delta_{k\ell} + C_1\delta_{i\ell}\delta_{jk} + C_2\delta_{ik}\delta_{j\ell}), \tag{4.27}$$

where V is the volume. In the microscopic theory we find

$$\begin{aligned}&\frac{\partial^4}{\partial\bar{v}_{Li}\partial v_{Rj}\partial\bar{v}_{Rk}\partial v_{L\ell}}\log Z\\ &= (\lambda_1\lambda_2)^2\int d^4x_1d^4x_2d^4x_3d^4x_4\\ &\quad\times\langle(\bar{B}_{R\ell}t_L)(x_4)(\bar{t}_LB_{Ri})(x_1)(\bar{B}_{Lj}t_R)(x_2)(\bar{t}_RB_{Lk})(x_3)\rangle\\ &= -(\lambda_1\lambda_2)^2\int d^4x_1d^4x_2d^4x_3d^4x_4\\ &\quad\times\int\frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{p_\mu}{p^2}\frac{q_\nu}{q^2}e^{ip(x_4-x_1)+iq(x_2-x_3)}\\ &\quad\times\langle(\bar{B}_{R\ell}(x_4)\gamma_\mu P_RB_{Ri}(x_1))(\bar{B}_{Lj}(x_2)\gamma_\nu P_LB_{Lk}(x_3))\rangle.\end{aligned} \tag{4.28}$$

In the last equality we have integrated over the top quark, substituting free massless fermion propagators for its two-point functions. The remaining expectation value on the last line is to be computed in the pure hypercolor theory. We may now project onto the $C_{\rm top}$ term in Eq. (4.27) by choosing $i = j \neq k = \ell$, obtaining

$$\begin{aligned}C_{\rm top} &= \frac{(\lambda_1\lambda_2)^2}{y^2}\frac{1}{V}\int d^4x_1d^4x_2d^4x_3d^4x_4\\ &\quad\times\int\frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{p_\mu}{p^2}\frac{q_\nu}{q^2}e^{ip(x_4-x_1)+iq(x_2-x_3)}\\ &\quad\times\langle(\bar{B}_{Rk}(x_4)\gamma_\mu P_RB_{Ri}(x_1))(\bar{B}_{Li}(x_2)\gamma_\nu P_LB_{Lk}(x_3))\rangle_{i\neq k}.\end{aligned} \tag{4.29}$$

In terms of the Fourier transform

$$\begin{aligned}&\langle(\bar{B}_{R\ell}(k_4)\gamma_\mu P_RB_{Ri}(k_1))(\bar{B}_{Lj}(k_2)\gamma_\nu P_LB_{Lk}(k_3))\rangle\\ &= \int d^4x_1d^4x_2d^4x_3d^4x_4e^{-ik_1x_1+ik_2x_2-ik_3x_3+ik_4x_4}\\ &\quad\times\langle(\bar{B}_{R\ell}(x_4)\gamma_\mu P_RB_{Ri}(x_1))(\bar{B}_{Lj}(x_2)\gamma_\nu P_LB_{Lk}(x_3))\rangle,\end{aligned} \tag{4.30}$$

we may write this in momentum space as[12]

$$C_{\rm top} = \frac{(\lambda_1\lambda_2)^2}{y^2}\frac{1}{V}\int\frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{p_\mu}{p^2}\frac{q_\nu}{q^2}\times\langle(\bar{B}_{Rk}(p)\gamma_\mu P_RB_{Ri}(p))(\bar{B}_{Li}(q)\gamma_\nu P_LB_{Lk}(q))\rangle_{i\neq k}. \tag{4.31}$$

This is our main result.

The top sector effective potential Eq. (4.22) depends on the experimentally known top Yukawa coupling, and on $C_{\rm top}$. Using Eq. (4.16), we may reexpress the ratio $\lambda_1\lambda_2/y$ in Eq. (4.31) in terms of quantities that are calculable in the pure hypercolor theory. This determines $V_{\rm eff}^{\rm top}$ completely. The dependence on the extended hypercolor sector coming from the couplings $\lambda_{1,2}$ has dropped out.

Diagrammatically, each term on the right-hand side of Eq. (4.27) originates from diagrams of the microscopic theory with a distinct topology. This is shown in Fig. 1, where we have kept only the propagators of the top quark (solid lines) and of the Majorana fermions χ (dashed-dot lines). All other fields including the Dirac fermions in the fundamental irrep of hypercolor have been suppressed. With these conventions, $C_{\rm top}$ arises from the class of diagrams represented by Fig. 1(a), while C_1 and C_2 arise from Figs. 1(b) and 1(c) respectively.

D. Large-N estimate of $y^2C_{\rm top}$

Determining $C_{\rm top}$ using Eq. (4.31) requires knowledge of the ratio $\lambda_1\lambda_2/y$, and a strong-coupling calculation that can be done using lattice gauge theory. Such a lattice calculation, however, would be a major undertaking (see Sec. V). In this subsection, we resort to analytic techniques hoping to shed some light on the most interesting question, which is whether $C_{\rm top}$ could indeed be negative, and large enough in size to cause electroweak symmetry breaking.

We will first consider what can be said if we assume that the hyperbaryon four-point function in Eq. (4.31) factorizes into the product of two hyperbaryon two-point functions. We will show that $C_{\rm top}$ is negative in this case. We will then argue that a large-N limit exists in which the factorized contribution dominates, and thus the Higgs field acquires a nonzero expectation value.

Assuming factorization, and using[13]

$$\begin{aligned}P_R\langle B_{Ri}(p)\bar{B}_{Lj}(q)\rangle P_R &= (2\pi)^4\delta(p-q)\delta_{ij}P_RS_B(p)P_R,\\ P_L\langle B_{Li}(q)\bar{B}_{Rj}(p)\rangle P_L &= (2\pi)^4\delta(p-q)\delta_{ij}P_LS_B(p)P_L,\end{aligned} \tag{4.32}$$

where $S_B(p)$ was defined in Eqs. (4.8) and (4.9), Eq. (4.31) leads to

[12]In finite volume, the momentum integral $\int d^4p/(2\pi)^4$ is to be understood as a momentum average, $V^{-1}\sum_{p_\mu}$. Alternatively, in infinite volume, V is to be interpreted as $(2\pi)^4\delta(0)$ in momentum space.

[13]On the right-hand side, δ_{ij} follows from $SO(5)$ invariance.

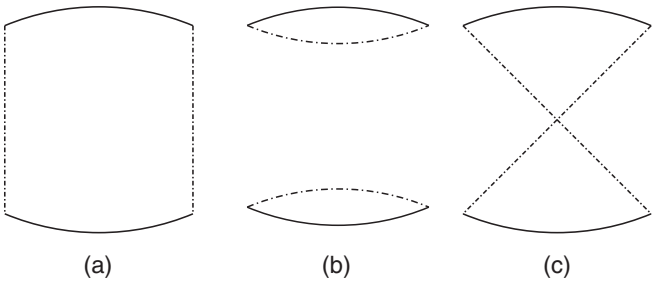

FIG. 1. The three possible Majorana-fermion contractions contributing to Eq. (4.28), corresponding to (a) $C_{\rm top}$, (b) C_1, and (c) C_2. Only the top quark (solid lines) and Majorana fermions (dashed lines) are shown. The vertices $x_1,\ldots,x_4$ of Eq. (4.28) correspond to a clockwise motion starting at the lower left corner.

$$C_{\rm top}^{\rm fact}=-\frac{(\lambda_1\lambda_2)^2}{y^2}\int\frac{d^4p}{(2\pi)^4}\frac{p_\mu p_\nu}{(p^2)^2}{\rm tr}(\gamma_\mu P_R S_B(p)\gamma_\nu P_L S_B(p)). \tag{4.33}$$

Using a dispersive representation for the hyperbaryon propagator,

$$S_B(p)=\int_0^\infty\frac{ds}{2\pi}\rho(s)\frac{-i\not{p}+\sqrt{s}}{p^2+s}, \tag{4.34}$$

with $\rho(s)\geq 0$ for all $s\geq 0$, this becomes

$$C_{\rm top}^{\rm fact}=-\frac{1}{8\pi^2}\frac{(\lambda_1\lambda_2)^2}{y^2}\int_0^\infty\frac{ds}{2\pi}\int_0^\infty\frac{dt}{2\pi}\rho(s)\rho(t)\frac{\sqrt{ts}}{t-s}\log\frac{t}{s}. \tag{4.35}$$

This result is negative, because the integrand is manifestly positive.[14] As a simple example, if we take $\rho(s)=2\pi f^6 Z_B\delta(s-M^2)$, Eq. (4.35) reduces to

$$C_{\rm top}^{\rm fact}=-\frac{1}{8\pi^2}\frac{(\lambda_1\lambda_2)^2}{y^2}f^{12}Z_B^2=-\frac{1}{32\pi^2}f^2M^2, \tag{4.36}$$

where we used Eq. (4.16).

We next consider a large-N limit in which factorization can be shown to hold. Of course, in the model of Ref. [8], the number of (hyper)colors is $N=4$. What makes this generalization nontrivial is that, unlike the $SU(4)$ case where the two-index antisymmetric irrep is real, for any $N>4$ this irrep is complex. This means that the Majorana condition (2.2b) cannot be imposed without violating gauge invariance. In order to cope with this, in addition to Weyl fields Υ_{ABi} in the antisymmetric representation of $SU(N)$, we introduce Weyl fields $\tilde{\Upsilon}_i^{AB}$ belonging to the irrep made out of the antisymmetrized product of two antifundamentals.[15] Instead of Majorana fermions, we now construct Dirac fermions out of these Weyl fields according to

$$\tilde{\omega}_{ABi}=\begin{pmatrix}\Upsilon_{ABi}\\ \epsilon\bar{\tilde{\Upsilon}}^T_{ABi}\end{pmatrix},$$
$$\bar{\tilde{\omega}}_i^{AB}=\begin{pmatrix}-(\tilde{\Upsilon}_i^{AB})^T\epsilon & \bar{\Upsilon}_i^{AB}\end{pmatrix}, \tag{4.37}$$

as well as their charge conjugates

$$\omega_i^{AB}=C(\bar{\tilde{\omega}}_i^{AB})_i^T=\begin{pmatrix}\tilde{\Upsilon}_i^{AB}\\ \epsilon(\bar{\Upsilon}_i^{AB})^T\end{pmatrix},$$
$$\bar{\omega}_{ABi}=\tilde{\omega}^T_{ABi}C=\begin{pmatrix}-(\Upsilon_{ABi})^T\epsilon & \bar{\tilde{\Upsilon}}_{ABi}\end{pmatrix}. \tag{4.38}$$

The index $i=1,\ldots,N_f$ now counts the number of Dirac fermions. Going back to the $SU(4)$ theory, we have been forced to consider an even number, $2N_f$, of Majorana fermions. There is no large-N generalization that would involve the desired odd number of five Majorana fermions for the $SU(4)$ theory. Even more, for any $N>4$ the symmetry breaking pattern becomes that of complex- irrep Dirac fermions, namely, $SU(N_f)\times SU(N_f)\to SU(N_f)$ [6].

We have shown in Fig. 1 the contractions of the antisymmetric- irrep fermions that contribute to $C_{\rm top}$. According to Eq. (4.31), we should choose fixed values $i\neq k$. The minimal number of Dirac fermions we need in order to distinguish Fig. 1(a) from Figs. 1(b) and 1(c) is two, and so we will take $N_f=2$ Dirac fermions in the antisymmetric irrep. For $N=4$, this is equivalent to a theory with four Majorana fermions (instead of five). This is the best we can do in terms of a large-N generalization. According to Eq. (4.31), for the left side of Fig. 1(a) we need the contraction $P_R\langle\omega_i\bar{\omega}_i\rangle P_R$, whereas for the right side we need $P_L\langle\omega_k\bar{\omega}_k\rangle P_L$. Once these contractions have been fixed we can drop the indices i and k, and forget about the flavor index. This suggests that the number of flavors is not crucial if our goal is to obtain a large-N estimate of the class of diagrams depicted in Fig. 1(a), and thus that the necessary transition from Majorana fermions to Dirac fermions for $N>4$ is inconsequential.

We are now ready to give the generalization of the hyperbaryon operators. In the $N>4$ theory with $N_f=2$ Dirac flavors they are defined by

$$B_{Ria}=\epsilon_{abc}P_R\omega_i^{AB}(\Psi^T_{Ab}\epsilon\Psi_{Bc}),$$
$$\bar{B}_{Ria}=\epsilon_{abc}\bar{\omega}_{iAB}P_L(\bar{\Psi}^{Ab}\epsilon(\bar{\Psi}^{Bc})^T),$$
$$B_{Lia}=\epsilon_{abc}P_L\omega_i^{AB}(\Psi^T_{Ab}\epsilon\Psi_{Bc}),$$
$$\bar{B}_{Lia}=\epsilon_{abc}\bar{\omega}_{iAB}P_R(\bar{\Psi}^{Ab}\epsilon(\bar{\Psi}^{Bc})^T). \tag{4.39}$$

[14]Only the factorizable contribution appears to have been considered in Refs. [4,8].

[15]This is the same as the antisymmetric product of $N-2$ fundamentals, since $\tilde{\Upsilon}^{AB}\sim\epsilon^{A_1A_2\ldots A_{N-2}AB}\Upsilon'_{A_1A_2\ldots A_{N-2}}$.

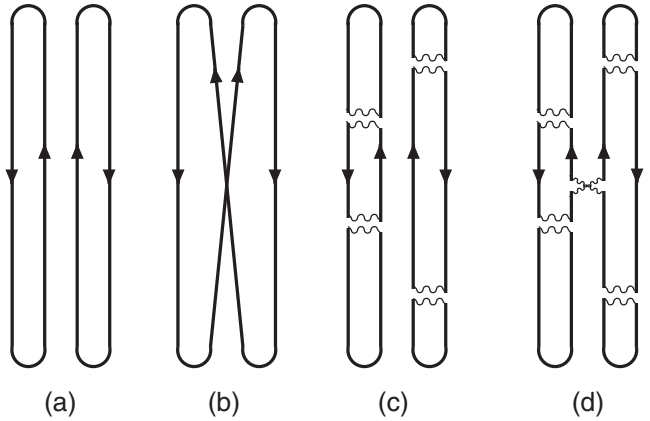

FIG. 2. A hyperbaryon in large N: diagrams without hypergluons (a, b) and with them (c, d). Diagrams (a) and (c) are leading, while (b) and (d) are subleading.

We have used index conventions similar to the previous sections. For $N = 4$ we may impose the Majorana condition $\omega = \tilde{\omega}$, and then these definitions reproduce Eq. (4.3).

Before we work out the more complicated case of Fig. 1(a), let us consider the behavior of a single hyperbaryon in large N. The hyperbaryons are bound by interchanging hypergluons between their elementary constituents. The situation here is different from the conventional large-N limit of baryons made only of fundamental-irrep fermions, where the number of constituents grows linearly with N [21]. For our hyperbaryons, the number of elementary constituents (as well as the number of hypercolor indices of each field) is fixed. This resembles the behavior of mesons within the usual large-N treatment.

In Fig. 2 we show a few examples. The bottom end of each diagram represents a hyperbaryon, say B_R [first line of Eq. (4.39)], and the top end the corresponding antibaryon, say $\bar{B}_L$ [last line of Eq. (4.39)]. Starting with Fig. 2(a) the two vertical lines in the middle represent the propagation of the double-indexed fermion of the antisymmetric irrep, and the lines on the sides the propagation of the two fundamental- irrep fermions, one on each side. The lines are oriented: the arrows point from a superscript index to a subscript index. Figure 2(b) shows an alternative index contraction, still without hypergluon fields. Note that Fig. 2(a) dominates over Fig. 2(b) in large N, since the former is of order N^2 and the latter of order N. In Fig. 2(c) we have added hypergluon interactions. Introducing the 't Hooft coupling $\lambda = g^2 N$ it follows that the planar diagram is again of order N^2. The factors of $g = \sqrt{\lambda/N}$ from each hypergluon vertex are compensated by a matching increase in the number of index loops. While the diagram appears disconnected to the eye, this is really not the case, because the two central vertical lines correspond to the two-point function of a single two-index fermion, $\langle \omega^{AB} \bar{\omega}_{CD} \rangle$. Figure 2(d) shows a different arrangement of hypergluon interactions. The hypergluon that is exchanged at the center of the diagram represents a self-energy correction for $\langle \omega^{AB} \bar{\omega}_{CD} \rangle$, which, to be consistent with the directionality of the index lines, gives rise to a nonplanar diagram. This diagram is subleading in the large-N counting.

The upshot is that, in large N, the dominant diagrams that bind the hyperbaryon are planar diagrams of order N^2, such as for example those in Figs. 2(a) and 2(c). The hyperbaryon two-point function in Eq. (4.9) will exhibit this large-N behavior, much like the two-point function of NGBs made out of antisymmetric- irrep fermions [16], which, in turn, leads to $f \sim N$ for large N.[16] Using Eqs. (4.15) and (4.16), it follows that M is independent of N, while $Z_B \sim 1/N^4$, and the top Yukawa coupling behaves like $y \sim N$. In contrast to QCD, where m_{nucleon}/f_π grows like $\sqrt{N}$, we find that M/f decreases like $1/N$. If M is indeed related to the mass of the lightest hyperbaryon in the theory, this suggests that the lightest hyperbaryon could be relatively light compared to Λ_{HC}.

With the contractions of the double-index hyperfermions fixed to be those in Fig. 1(a), let us now study the large-N behavior of the various possible contractions of the single-index, fundamental- irrep fields. Since $U(1)_X$ is not broken spontaneously, the Wick contractions have to comply with this symmetry. Let us start, for example, with the two Ψ fields of the B_R hyperbaryon at the bottom left of Fig. 1(a). They can be contracted with the corresponding fields in the top left or the bottom right, but not with those in the top right. There are three possibilities: (1) both Ψ fields are contracted with those at the top left, (2) both are contracted with those at the bottom right, or (3) one is contracted with a field at the top left and the other with a field at the bottom right.

Let us consider these three cases in turn. First we contract both Ψ fields at the lower left corner with the $\bar{\Psi}$ fields of the $\bar{B}_L$ hyperbaryon at the upper left corner, i.e., case (1) above. Remembering that also the two-index field at the lower left corner of Fig. 1(a) is contracted with the two-index field at the upper left corner, this gives a contribution proportional to

$$(\delta^A_C \delta^B_D - \delta^A_D \delta^B_C)(\delta^C_A \delta^D_B - \delta^D_A \delta^C_B) = 2N(N-1), \qquad (4.40)$$

where we label the $SU(N)$ indices as $\omega^{AB} \Psi_A \Psi_B$ at the lower left corner and as $\bar{\omega}_{CD} \bar{\Psi}^C \bar{\Psi}^D$ at the upper left corner. We get a similar factor from the right side of the diagram, so that the total diagram is of order $(N(N-1))^2 \sim N^4$. In Fig. 3(a) we show as an example the order-N^4 contribution coming from using twice the diagram of Fig. 2(a).

For case (2), the Ψ fields at the lower left corner are both contracted with the corresponding fields at the lower right corner. Using $SU(N)$ indices AB at the lower left, CD at the

[16]Note the difference with QCD, where the decay constant f_π of the fundamental- irrep NGBs scales like $\sqrt{N}$.

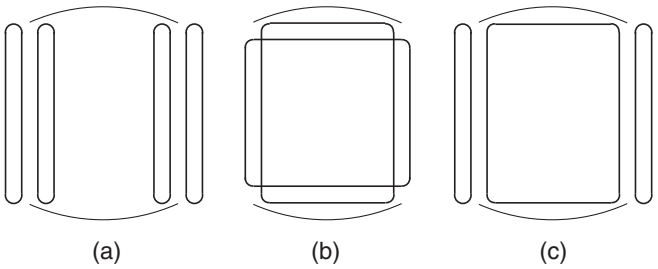

FIG. 3. Different large-N contributions to Fig. 1(a). The curved lines at the top and the bottom represent the two top quark propagators. The diagrams scale like (a) N^4, (b) N^2, and (c) N^3.

upper left, EF at the lower right, and GH at the upper right corners, this contraction leads to a contribution of order

$$\delta_C^{[A}\delta_D^{B]}\delta_E^{[G}\delta_F^{H]}\delta_A^{[E}\delta_B^{F]}\delta_G^{[C}\delta_H^{D]} \sim N^2. \tag{4.41}$$

The notation $[\ldots]$ denotes antisymmetrization in the pair of indices inside the brackets. An example is shown in Fig. 3(b).

Finally we consider the mixed case (3), where one of the Ψ fields is contracted with the lower right corner, and the other with the upper left one. It is straightforward to see that this gives a contribution

$$\delta_C^{[A}\delta_D^{B]}\delta_E^{[G}\delta_F^{H]}\delta_A^{[C}\delta_G^{D]}\delta_B^{[E}\delta_H^{F]} \sim N^3. \tag{4.42}$$

An example is shown in Fig. 3(c).

We should also consider diagrams "dressed" with hypergluons. The interesting case is that of Fig. 3(a), where hypergluons exchanged between the two hyperbaryons on the left and on the right make these hyperbaryons interact. Such interactions would spoil the factorization of the four-point function in Eq. (4.29). However, these interactions are suppressed in large N for the same reason that meson-meson interactions are suppressed in large-N QCD. Any hypergluon connecting the left and right sides of Fig. 3(a) will reduce the number of hypercolor loops by one, in addition to adding a factor of g^2/N. The key point here is that the number of hyperfermion constituents of the hyperbaryons is fixed to three, in contrast to the case of baryons in QCD, where the number of constituent quarks grows linearly with N.

We conclude that the factorizable part of Fig 1(a), which grows like N^4, is the dominant large-N contribution of the top quark sector. $C_{\rm top}$ itself grows like N^2, since $V_{\rm eff}^{\rm top}$ is proportional to $y^2 C_{\rm top}$ and y scales like N [this is consistent with the large-N behavior of the right-hand side of Eq. (4.36)]. According to the discussion in the beginning of this subsection, this produces a negative value for $C_{\rm top}$, which, in turn, generates a negative curvature for the Higgs field in Eq. (4.25).

We recall that the other contribution to the effective potential (4.25) for the Higgs field is coming from a single electroweak gauge-boson exchange. This contribution, which is parametrized by C_{LR}, always produces a positive-curvature term in the effective potential. It is easily seen that C_{LR} is subleading in the large-N counting. The electroweak gauge group $SU(2)_L$ is a subgroup of $SU(5)$, whereas hypercharge is a subgroup of $SU(5)\times U(1)_X$. Therefore all the electroweak gauge bosons interact with the antisymmetric- irrep hyperfermions, from which the $SU(5)/SO(5)$ coset fields are formed. C_{LR} involves a single closed fermion loop, and so for the antisymmetric-irrep fields it is of order N^2. (For the fundamental irrep, C_{LR} would be of order N. Observe that the electroweak gauge bosons do not carry hypercolor, and therefore their exchange has no effect on the large-N counting.) This is subleading to the factorizable contribution of the top sector, which is of order N^4. It follows that, in large N, Eq. (4.25) is dominated by the contribution of the top quark sector, developing a negative curvature at the origin that triggers electroweak symmetry breaking.

E. Phenomenological consequences

Our analysis in this section was based on the Lagrangian (4.2). While $\mathcal{L}_{\rm EHC}$ depends on two coupling constants λ_1 and λ_2, only their product enters the determination of the top Yukawa coupling in Sec. IV B, and of the induced effective potential for the Higgs studied in Sec. IV C. An additional free parameter of the model is the scale of the hypercolor theory itself, $\Lambda_{\rm HC}$. Using the results of Secs. IV B and IV C, we may in principle perform a lattice calculation that will fix the values of $\lambda_1\lambda_2$ and of $\Lambda_{\rm HC}$ in terms of the experimentally known values of the top Yukawa coupling y, and of the Higgs field's expectation value, provided that $C_{\rm top}$ turns out to be large enough compared to C_{LR} to trigger condensation [see Eq. (4.24)].[17]

The only remaining uncertainty then arises from the four different choices for the hyperbaryon fields (that each can be a primed or an unprimed one, cf. Table I) in Eq. (4.2). This gives rise to a discrete fourfold ambiguity in the predicted values of $\lambda_1\lambda_2$ and of $\Lambda_{\rm HC}$. For each of these four possibilities, one can proceed to compare other predictions of the hypercolor theory with experimental constraints, which can in principle rule out, or rule in, that particular version of the hypercolor theory.

A less-constrained hypercolor model can be obtained by relaxing the assumption that $SU(3)\times SU(3)'$ is broken explicitly to $SU(3)_{\rm color}$ only by the QCD interactions. There are at least two alternative ways to introduce such an explicit breaking.

First, one can introduce a Dirac mass term $m\bar{\psi}\psi$ for the fundamental- irrep hyperfermions, with $m \gtrsim \Lambda_{\rm HC}$. We may think of m as arising from the expectation value of a three by three global spurion $M_{ab} = m\delta_{ab}$, where M_{ab} has

[17]We are assuming that none of the $SU(2)_L$ triplet NGBs acquire an expectation value.

the same transformation properties as the $SU(3)\times SU(3)'/SU(3)_{\rm color}$ coset field Ω_{ab}. Using the global spurion M_{ab} allows for the coupling of each SM spurion field $T_{L,R}$ to both of the unprimed and primed hyperbaryon fields from Table I. In fact, if the hyperbaryon field is assumed to have well-defined transformation properties under $SU(3)_{\rm color}$ only, some additional operators besides those shown in Table I may occur. For their structure, see Table II of Ref. [8]. The bottom line is that $\mathcal{L}_{\rm EHC}$ can now depend on four (or more) coupling constants, instead of just two.

An alternative mechanism involves the introduction of two more spurions $T'_{L,R}$, and assuming that the unprimed hyperbaryon fields and spurions transform under $SU(3)$, while the primed ones transform under $SU(3)'$. Twice as many terms are then allowed by $SU(3)\times SU(3)'$ invariance of $\mathcal{L}_{\rm EHC}$, with each term involving either unprimed or primed fields. The explicit breaking to $SU(3)_{\rm color}$ then occurs by assigning to all spurions, both primed and unprimed, the same SM values as in Eq. (4.1).

In any one of these more general schemes, the matching of the top Yukawa coupling and of $C_{\rm top}$ to the underlying theory can be done using the same techniques as before. However, the predictive power will be reduced, since this analysis would still provide just two constraints on the larger set of parameters, which includes $\Lambda_{\rm HC}$ and all the coupling constants that may now occur in $\mathcal{L}_{\rm EHC}$.

Finally, note that if we expand both Σ and Φ in Eq. (4.5) to first order in the NGB fields, and use the SM values $\hat{T}_{L,R}$, we obtain the dimension-five operator $(y/f')\eta'\bar{t}_R H_i \epsilon_{ij} q_{Lj}$ and its Hermitian conjugate, where f' is the decay constant of η'. This couples η' to the SM fields. If we set the Higgs field to its vacuum expectation value, the above operator reduces to $(m_t/f')\eta'\bar{t}_R t_L$. The phenomenological implications of these interactions have to be looked into. If they turn out to be incompatible with experiment, this would necessitate the introduction of an explicit breaking of $U(1)'$ that makes the η' sufficiently heavy. One possible source for this explicit breaking is the Dirac mass term $m\bar{\psi}\psi$ discussed above.

V. LATTICE ASPECTS

In this short section, we explain why a lattice computation of $C_{\rm top}$ would be a "QCD-like" computation. The question to address is why the lattice formulation of hypercolor theories such as considered here resembles the lattice formulation of QCD, in view of the well-known complications with fermion doubling and chirality on the lattice. The fermion doubling problem has its roots in the observation that a single Weyl fermion cannot live on the lattice [22,23]. Since the hypercolor theory contains an odd number of Weyl fermions in the two-index antisymmetric irrep, this might seem to imply that the model we consider here cannot be easily discretized.

The first observation is that the integration over the SM quark fields q_L and t_R is done analytically, as we did in this article. The results for the top Yukawa coupling y [Eqs. (4.11) and (4.12)] and for $C_{\rm top}$ [Eq. (4.31)] are obtained in terms of pure hypercolor correlation functions. Hence only the hypercolor theory needs to be considered on the lattice, and the lattice action does not contain the four-fermion Lagrangian $\mathcal{L}_{\rm EHC}$ of Eq. (4.2).

Let us start from the sector that resembles QCD most closely, namely, the Dirac fermions ψ_a in the fundamental irrep of $SU(4)$ hypercolor. These can be treated in exactly the same way as the quark fields of N_f-flavor QCD. In the Wilson formulation of the theory, the gauge invariant Wilson mass term removes the fermion doublers at the price of breaking the symmetry group $SU(N_f)_L\times SU(N_f)_R$ of the continuum theory explicitly to its diagonal $SU(N_f)$ subgroup. Tuning the bare mass appropriately, one then recovers the massless theory with the full chiral symmetry group in the continuum limit [22]. In the hypercolor theory we have $N_f = 3$ Dirac fermions. The chiral flavor group $SU(3)_L\times SU(3)_R$ has been renamed $SU(3)\times SU(3)'$, and the unbroken diagonal subgroup was identified with $SU(3)_{\rm color}$.

We next turn to the novel feature of the hypercolor theory, which is the five real- irrep Weyl fermions Υ_{ABi}. The key point is that each of these Weyl fermions can be assembled together with its antifermion into a Majorana-fermion field χ_{ABi}. A Majorana mass term of the form $\bar{\chi}_i^{AB}\chi_{ABi}$ is allowed by the $SU(4)$ gauge symmetry, just like a Dirac mass term is allowed in the familiar case of fundamental- irrep fermions. Therefore, a Wilson formulation of the Majorana fermions is possible, with a Wilson mass term that once again removes fermion doublers without breaking gauge invariance. The hypercolor theory is chiral with respect to the $SU(5)$ flavor symmetry of the Majorana or Weyl fields, and consequently, the Majorana-Wilson mass term breaks $SU(5)$ explicitly down to $SO(5)$. Again, the full $SU(5)$ chiral symmetry should be recovered in the continuum limit after appropriate tuning of the bare mass.[18]

The situation with respect to (complex-irrep) Dirac fermions and to (real-irrep) Majorana fermions is thus completely parallel. In both cases, what we anticipate as the spontaneous symmetry breaking pattern of the continuum theory turns into an explicit breaking in the Wilson formulation. The explicit breaking disappears, and the full chiral symmetry group can be recovered, in the continuum limit by tuning the bare mass terms. In particular, the flavor group $SO(5)$ of the Majorana-Wilson fermion action in the hypercolor theory enlarges to $SU(5)$ in the continuum limit, much like the diagonal $SU(N_f)$ symmetry of

[18]The lattice formulation of an adjoint Majorana fermion was studied extensively in the context of supersymmetric theories; see, e.g., Ref. [15].

the Dirac-Wilson action of QCD enlarges to the full $SU(N_f)_L \times SU(N_f)_R$ symmetry in the continuum limit.

In short, it is precisely the unbroken flavor symmetry group H of Eq. (2.5) that is preserved in a lattice formulation with Wilson fermions. It is actually possible to gauge the SM group $SU(3)_{\text{color}} \times SU(2)_L \times U(1)_Y$ in this lattice formulation, because this group is contained in H. The difficulties caused on the lattice with chiral gauge symmetries[19] would only appear were one to couple also the SM fermions to this model.

The lattice formulation of the hypercolor theory is not without technical challenges. First, two fermion irreps need to be introduced simultaneously. While clearly a coding task, this is something that to our knowledge has not been done to date. Also, the computation of a four-point function as in Eq. (4.29) would be very demanding. Because there are two independent momentum variables, the cost is expected to grow like the square of the 4-volume of the lattice. The computation of $S_B(0)$ in Eq. (4.11), and of the factorizable contribution to C_{top}, would already be interesting. Here only the hyperbaryon two-point function is required, and the cost grows linearly with the 4-volume.

VI. CONCLUSION

In this article, we discussed a recently proposed composite Higgs model [8], concentrating on the top quark sector. This "hypercolor" model is an $SU(4)$ gauge theory, with a quintuplet of two-index antisymmetric Majorana fermions and a color triplet of $SU(4)$-fundamental Dirac fermions. We considered the top Yukawa coupling and the top contribution to the effective potential for the Higgs, showing how to match the relevant low-energy constants $\mu = yf/2$ and C_{top} to correlation functions of the hypercolor theory.

This matching is the starting point for any nonperturbative evaluation of the low-energy constants. The needed computations can in principle be done on the lattice, using methods that are much the same as those employed in lattice QCD. The main differences are that now the gauge group is $SU(4)$ instead of $SU(3)$, and that there are fermions in more than one irrep of the gauge group. For the Higgs effective potential, a four-point hyperbaryon correlation function needs to be considered; this appears to have been overlooked in at least some of the literature on models of this type. A lattice calculation of this four-point function would be very demanding, and how to do it efficiently is a question that goes beyond the scope of this paper. As mentioned in Sec. V, only a two-point function is needed for the low-energy constant μ, as well as for the factorizable contribution to C_{top}. This computation would be comparable in scope with a lattice computation of C_{LR}, the LEC controlling the contribution to the Higgs effective potential from the SM gauge bosons (cf. Sec. III).

We found that a large-N limit exists in which the factorizable contribution to the hyperbaryon four-point function dominates C_{top}. We also showed that the factorizable contribution generates a negative curvature at the origin. This is a necessary condition for electroweak symmetry breaking, because the effective potential induced by the SM gauge bosons does not break the SM symmetries, a manifestation of vacuum alignment.

For very large N, the top sector dominates the whole Higgs effective potential. This maximizes the symmetry breaking, with the minimum of $V_{\text{eff}}(h)$ presumably occurring for $\sin(2h/f) = 1$ in Eq. (4.25). Phenomenologically, this is not allowed [4,8]. The hope is that for $N = 4$, electroweak symmetry breaking with a phenomenologically acceptable value of h/f takes place. Only the lattice can address this question quantitatively.

In order to couple the SM fermions to the hypercolor theory, an extended hypercolor sector is necessary. While the detailed structure of the EHC theory is very important for phenomenology, the lattice setup is largely blind to these details. At energy scales much below Λ_{EHC}, the coupling of SM and hypercolor fermions is summarized by the four-fermion Lagrangian $\mathcal{L}_{\text{EHC}}$ of Eq. (4.2). Much like the familiar treatment of hadronic matrix elements of the electroweak interactions, $\mathcal{L}_{\text{EHC}}$ is not taken as part of the lattice action. Instead, one evaluates the hypercolor-theory correlation functions that arise from working to leading order in the four-fermion Lagrangian $\mathcal{L}_{\text{EHC}}$.

In the most constrained case, which is the one we have worked out in detail in this paper, $\mathcal{L}_{\text{EHC}}$ depends on only two couplings $\lambda_{1,2}$. A lattice computation can then in principle determine their product $\lambda_1\lambda_2$, together with the hypercolor scale λ_{HC}, in terms of the experimental values of the top Yukawa coupling and the Higgs expectation value, up to a fourfold ambiguity. In a less-constrained setup, a similar lattice computation would supply two constraints among the parameters of the hypercolor theory.

The parameters $\lambda_{1,2}$ have mass dimension two. According to naive dimensional analysis, their values would be of order $(\Lambda_{\text{HC}}/\Lambda_{\text{EHC}})^2$, making the top Yukawa coupling of order $y \sim (\Lambda_{\text{HC}}/\Lambda_{\text{EHC}})^4$. For comparison, we recall that in classic (walking) technicolor, the top Yukawa coupling is naively of order $(\Lambda_{\text{TC}}/\Lambda_{\text{ETC}})^2$, where Λ_{TC} and Λ_{ETC} are the scales of the technicolor and of the extended technicolor theories. Naively, the case for a partially composite top seems even worse than for technicolor. On the other hand, it might be that the experimental constraints allow Λ_{EHC} to be much closer to Λ_{HC} than Λ_{ETC} to Λ_{TC}. We also found that the hyperbaryons that couple to the top quark in $\mathcal{L}_{\text{EHC}}$ might be relatively light, as suggested by large-N counting, and this helps boost the value of the top Yukawa y as well. The key reason for the possible lightness of these hyperbaryons is that they are composed of hyperfermions in

[19]For a review, see [24].

two different irreps of the gauge group. As far as large-N counting goes, these hyperbaryons behave more like mesons than like baryons in the usual large-N limit of QCD.

Yet another feature that may be needed for a phenomenologically viable partial-compositeness model is large anomalous dimensions for the hyperbaryon fields [4]. It should be possible to study on the lattice whether or not this is the case.

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APPENDIX: CONVENTIONS

We choose our γ matrices to be Hermitian, and we use the chiral representation

$$\gamma_k = \begin{pmatrix} 0 & i\sigma_k \\ -i\sigma_k & 0 \end{pmatrix}, \qquad \gamma_4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{A1}$$

with σ_k, $k = 1, 2, 3$, the Pauli matrices. The chiral projectors are $P_R = (1+\gamma_5)/2$, $P_L = (1-\gamma_5)/2$, where

$$\gamma_5 = -\gamma_1\gamma_2\gamma_3\gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A2}$$

The charge-conjugation matrix occurring in Eq. (2.2) is $C = -\gamma_2\gamma_4$. It satisfies

$$C\gamma_\mu = -\gamma_\mu^T C, \tag{A3}$$

and $C^{-1} = C^\dagger = C^T = -C$.

For the invariant $SU(2)$ subgroups of $SO(4)$ we may choose the generators as the following tensor products of Pauli matrices and the 2×2 identity matrix I,

$$\begin{aligned} 2T_L^1 &= \sigma_2\times\sigma_1, \\ 2T_L^2 &= -\sigma_2\times\sigma_3, \\ 2T_L^3 &= I\times\sigma_2, \\ 2T_R^1 &= \sigma_1\times\sigma_2, \\ 2T_R^2 &= \sigma_2\times I, \\ 2T_R^3 &= \sigma_3\times\sigma_2. \end{aligned} \tag{A4}$$

Identifying $SO(4)$ with the upper-left 4×4 block, the $SU(2)_L$ generators and the third $SU(2)_R$ generator (which is used to construct the hypercharge Y) are given explicitly by

$$T_L^1 = \frac{i}{2}\begin{pmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad T_L^2 = \frac{i}{2}\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$T_L^3 = \frac{i}{2}\begin{pmatrix} 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad T_R^3 = \frac{i}{2}\begin{pmatrix} 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{A5}$$

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