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Masses and axial currents of the doubly charmed baryons

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The chiral dynamics of the doubly heavy baryons is solely governed by the light quark. In this paper, we have derived the chiral corrections to the mass of the doubly heavy baryons up to N^{3}LO. The mass splitting of Ξ_{cc} and Ω_{cc} at the N^{2}LO depends on one unknown low energy constant c_7. By fitting the lattice masses of $\Xi_{cc}(3520)$, we estimate the mass of Ω_{cc} to be around 3.726 GeV. Moreover, we have also performed a systematical analysis of the chiral corrections to the axial currents and axial charges of the doubly heavy baryons. The chiral structure and analytical expressions will be very useful to the chiral extrapolations of the future lattice QCD simulations of the doubly heavy baryons.

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I. INTRODUCTION

As one of the most important groups in the baryon family, the doubly charmed baryons are composed of two charmed quarks and one light quark (the doubly heavy baryons Ξ_{cc}^{++}, Ξ_{cc}^{+} and Ω_{cc}^{+} with quark components ccu, ccd and ccs, respectively), which were predicted in the quark model (see Ref. [1] for a detailed review). In the past decades, there have been some experimental efforts in the search of the doubly charmed baryons [2–5]. The SELEX collaboration announced the first observation of the doubly charmed baryon $\Xi_{cc}^{+}(3520)$ with the mass $M = 3519 \pm 1$ MeV and width $\Gamma = 3$ MeV [2], where the observed decay mode is $\Xi_{cc}^{+} \to \Lambda_c^{+} K^{-} \pi^{+}$. Later, $\Xi_{cc}^{+}(3520)$ was confirmed by SELEX in the $pD^{+}K^{-}$ decay channel with the mass 3518.7 ± 1.7 MeV [4]. Although SELEX also reported $\Xi_{cc}^{+}(3520)$, these results were not confirmed by FOCUS [6], *BABAR* [7], Belle [8] and LHCb collaborations [9].

The doubly charmed baryons have been extensively studied with different theoretical approaches. The Ξ_{cc} mass was predicted to be 3.48~3.74 GeV in the quark model, while the Ω_{cc} mass is estimated to be 3.59~3.86 GeV [10–22]. The Lattice QCD groups also studied these systems [23–27], where the predicted mass of Ξ_{cc} is 3.51~3.67 GeV and the mass of Ω_{cc} is 3.68~3.76 GeV.

The mass splittings of baryons within the same multiplet encode important information on their inner structure. For example, the mass splittings of the light baryons were reviewed in Refs. [28,29]. In Refs. [30,31], the mass splitting of the singly heavy baryons was studied within the framework of the chiral perturbation theory. In Ref. [32], the authors investigated the mass splitting of the doubly heavy baryons by considering the heavy diquark symmetry. Besides the baryon mass, the axial current and axial charge of the baryons are also very important observables, which attract lots of attention [33–55].

The experimental search of the doubly charmed baryons is full of challenges and opportunities. In this paper, we adopt the chiral perturbation theory to calculate the chiral corrections to the doubly charmed baryon masses and their mass splittings, which will be helpful to further experimental exploration of the doubly charmed baryons. Under the same framework, we also study the chiral corrections to the axial charge and axial current of the doubly charmed baryons, which may be measured through the semileptonic decays of the doubly charmed baryons in the future.

Chiral perturbation theory (χPT) is an elegant framework to deal with the low energy process in hadron physics. With the help of the chiral power counting scheme proposed by Weinberg *et al.* [56,57], one can consider the chiral corrections to the physical observables order by order.

In the baryon sector, the baryon mass does not vanish in the chiral limit. This inherent mass scale breaks the naive chiral power counting. To solve this issue, various schemes were proposed such as the heavy baryon χPT, infrared baryon χPT, and extended on-mass-shell method etc.

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In the heavy baryon χPT, the baryon is treated to be extremely heavy and acts as a static source [54], which allows us to take the nonrelativistic limit of the fully relativistic theory and make expansion in powers of the inverse baryon mass. For the case of infrared regularization, the loop integral can be separated into infrared regular part and infrared singular one [58,59], where the later one conserves the Weinberg's power counting rule. In the extended on-mass-shell method, the power counting breaking terms are subtracted and the low energy constants are redefined [60–63]. In our paper, we use the heavy baryon χPT approach to investigate the chiral corrections to the masses and axial currents of the doubly charmed baryons.

This paper is organized as follows. After the Introduction, we introduce the chiral Lagrangians of the doubly charmed baryons and its nonrelativistic reduction in Sec. II. Then we present the calculation details of the chiral corrections to the masses and axial currents of the doubly charmed baryons and the corresponding numerical results in Secs. III and IV respectively. This paper ends with a summary in Sec. V. We collect the N^3LO chiral corrections to the masses and some lengthy expressions in the appendix.

II. THE CHIRAL LAGRANGIANS OF THE DOUBLY CHARMED BARYONS

In order to calculate the chiral corrections to the masses and axial currents, we need to construct the chiral effective Lagrangians of the doubly charmed baryons with the help of chiral, parity and charge conjugation symmetries. We first introduce the notations U and u to describe the pseudoscalar meson field, which have the relation

$$U = u^2 = \exp\left(i\frac{\phi(x)}{F_0}\right), \tag{1}$$

where $\phi(x)$ has the definition

$$\phi(x) = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}. \tag{2}$$

The doubly heavy baryon field ψ with spin $\frac{1}{2}$ is a column vector in the flavor space, i.e.,

$$\psi = \begin{pmatrix} \Xi_{cc}^{++} \\ \Xi_{cc}^{+} \\ \Omega_{cc}^{+} \end{pmatrix}, \tag{3}$$

where the quark contents of Ξ_{cc}^{++}, Ξ_{cc}^{+}, and Ω_{cc}^{+} are ccu, ccd, and ccs, respectively.

TABLE I. The properties of the building blocks under the $SU(3)_L \times SU(3)_R$ (CH), parity (P) and charge conjugation (C) transformations.

	U	u	χ	$f_{\mu\nu}^R$	$f_{\mu\nu}^L$	$D_\mu\psi$
CH	$V_RUV_L^\dagger$	$V_RuK^\dagger$	$V_R\chi V_L^\dagger$	$V_Rf_{\mu\nu}^RV_R^\dagger$	$V_Lf_{\mu\nu}^LV_L^\dagger$	$KD_\mu\psi$
P	$U^\dagger$	$u^\dagger$	$\chi^\dagger$	$f^{L\mu\nu}$	$f^{R\mu\nu}$	$\gamma^0D^\mu\psi$
C	U^T	u^T	χ^T	$-(f_{\mu\nu}^L)^T$	$-(f_{\mu\nu}^R)^T$	$CD_\mu'^T\bar{\psi}^T$
	ψ	$\bar{\psi}$	$\chi_\pm$	$f_{\mu\nu}^\pm$	u_μ	Γ_μ
CH	$K\psi$	$\bar{\psi}K^\dagger$	$K\chi_\pm K^\dagger$	$Kf_{\mu\nu}^\pm K^\dagger$	$Ku_\mu K^\dagger$	$K\Gamma^\mu K^\dagger$ $-\partial^\mu KK^\dagger$
P	$\gamma^0\psi$	$\bar{\psi}\gamma^0$	$\pm\chi_\pm$	$\pm f^{\pm\mu\nu}$	$-u^\mu$	Γ^μ
C	$C\bar{\psi}^T$	ψ^TC	$\chi_\pm^T$	$\mp(f_{\mu\nu}^\pm)$	$(u_\mu)^T$	$-(\Gamma_\mu)^T$

In Table I, we show the transformation properties of the building blocks, which include $\chi, \chi_\pm, f_{\mu\nu}^R, f_{\mu\nu}^L, f_{\mu\nu}^\pm, u_\mu, \Gamma_\mu$, D_μ and D_μ' with the definitions

$$\chi = 2B_0(s + ip), \tag{4}$$

$$\chi_\pm = u^\dagger\chi u^\dagger \pm u\chi^\dagger u, \tag{5}$$

$$f_{\mu\nu}^R = \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu], \tag{6}$$

$$f_{\mu\nu}^L = \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu], \tag{7}$$

$$f_{\mu\nu}^\pm = u^\dagger f_{\mu\nu}^R u \pm u f_{\mu\nu}^L u^\dagger, \tag{8}$$

$$u_\mu = i[u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_u)u^\dagger], \tag{9}$$

$$\Gamma_\mu = \frac{1}{2}[u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_u)u^\dagger], \tag{10}$$

$$D_\mu = \partial_\mu + \Gamma_\mu - iv_\mu^{(s)}, \tag{11}$$

$$D_\mu' = \partial_\mu - \Gamma_\mu + iv_\mu^{(s)}, \tag{12}$$

where $r_\mu = v_\mu + a_\mu$, $l_\mu = v_\mu - a_\mu$, and $v_\mu, v_\mu^{(s)}, a_\mu, s, p$ are external c-number fields. Considering the transformation properties listed in Table I, the chiral Lagrangian of the doubly heavy baryon can be constructed order by order, i.e.,

$$\mathcal{L}^{(1)} = \bar{\psi}\left(iD\!\!\!/ - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu\right)\psi, \tag{13}$$

$$\mathcal{L}^{(2)} = c_1\bar{\psi}\langle\chi_\pm\rangle\psi - \left\{\frac{c_2}{8m^2}\bar{\psi}\langle u_\mu u_\nu\rangle\{D^\mu, D^\nu\}\psi + \text{H.c.}\right\} - \left\{\frac{c_3}{8m^2}\bar{\psi}\{u_\mu, u_\nu\}\{D^\mu, D^\nu\}\psi + \text{H.c.}\right\} + \frac{c_4}{2}\bar{\psi}\langle u^2\rangle\psi + \frac{c_5}{2}\bar{\psi}u^2\psi + \left\{\frac{ic_6}{4}\bar{\psi}\sigma^{\mu\nu}[u_\mu, u_\nu]\psi + \text{H.c.}\right\} + c_7\bar{\psi}\hat{\chi}_+\psi + \frac{c_8}{8m}\bar{\psi}\sigma^{\mu\nu}f^+_{\mu\nu}\psi + \frac{c_9}{8m}\bar{\psi}\sigma^{\mu\nu}\langle f^+_{\mu\nu}\rangle\psi, \tag{14}$$

$$\mathcal{L}^{(3)} = \bar{\psi}\left\{\frac{h_1}{2}\gamma^\mu\gamma_5\langle\chi_+\rangle u_\mu + \frac{h_2}{2}\gamma^\mu\gamma_5\{\hat{\chi}_+, u_\mu\} + \frac{h_3}{2}\gamma^\mu\gamma_5\langle\hat{\chi}_+ u_\mu\rangle + \cdots\right\}\psi, \tag{15}$$

$$\mathcal{L}^{(4)} = e_1\bar{\psi}\langle\chi_+\rangle\langle\chi_+\rangle\psi + e_2\bar{\psi}\hat{\chi}_+\langle\chi_+\rangle\psi + e_3\bar{\psi}\langle\hat{\chi}_+\hat{\chi}_+\rangle\psi + e_4\bar{\psi}\hat{\chi}_+\hat{\chi}_+\psi + e_5\bar{\psi}\langle\chi_-\rangle\langle\chi_-\rangle\psi + e_6\bar{\psi}\hat{\chi}_-\langle\chi_-\rangle\psi + e_7\bar{\psi}\langle\hat{\chi}_-\hat{\chi}_-\rangle\psi + e_8\bar{\psi}\hat{\chi}_-\hat{\chi}_-\psi + \cdots, \tag{16}$$

where $\hat{A} = A - \frac{1}{3}\langle A\rangle$ and $\langle A\rangle$ denotes the trace of A in the flavor space. In the above Lagrangians, c_i $(i = 1, \ldots, 9)$, h_j $(j = 1, \ldots, 3)$, and e_k $(k = 1, \ldots, 8)$ are the effective coupling constants. They are sometimes denoted as the low energy constants (LECs).

Since the doubly heavy baryons are very heavy, we can take the nonrelativistic limit of the fully relativistic theory and expands the Lagrangian in the power of the inverse of the doubly heavy baryon mass. The four-momentum of the doubly heavy baryon can be written as

$$p_\mu = mv_\mu + l_\mu, \tag{17}$$

where v_μ is the four-velocity and l_μ the small off-shell momentum, which satisfies $v\cdot l \ll m$. The baryon field is decomposed into the light and heavy components $\psi = e^{-imv\cdot x}(H + h)$, where $\not{v}H = H$, $\not{v}h = -h$.

The generating functional for the relativistic theory reads

$$\exp iZ[\eta, \bar{\eta}, v, a, s, p] = \int [d\psi][d\bar{\psi}][du] \times \exp\left\{i\left[S + \int d^4x(\bar{\eta}\psi + \bar{\psi}\eta)\right]\right\}, \tag{18}$$

where

$$S = \int d^4x\mathcal{L}. \tag{19}$$

In the terms of the fields H and h, we can rewrite the original Lagrangian

$$\mathcal{L} = \bar{H}\mathcal{A}H + \bar{h}\mathcal{B}H + \bar{H}\gamma^0\mathcal{B}^\dagger\gamma^0 h - \bar{h}\mathcal{C}h. \tag{20}$$

$\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ in Eq. (20) can be expanded in series of terms of different orders of q^i, where q is the low energy momentum,

$$\mathcal{A} = \mathcal{A}_{(1)} + \mathcal{A}_{(2)} + \cdots, \tag{21}$$

$$\mathcal{B} = \mathcal{B}_{(1)} + \mathcal{B}_{(2)} + \cdots, \tag{22}$$

$$\mathcal{C} = \mathcal{C}_{(1)} + \mathcal{C}_{(2)} + \cdots. \tag{23}$$

The expressions of $\mathcal{A}, \mathcal{B}, \mathcal{C}$ are collected in the Appendix B. With the replacement

$$R = \frac{1}{2}(1 + \not{v})e^{imv\cdot x}\eta,$$
$$\rho = \frac{1}{2}(1 - \not{v})e^{imv\cdot x}\eta,$$

we have

$$\bar{\eta}\psi + \bar{\psi}\eta = \bar{R}H + \bar{H}R + \bar{\rho}h + \bar{h}\rho. \tag{24}$$

With $h' = h - \mathcal{C}^{-1}(\mathcal{B}H + \rho)$ and after integrating out the heavy degrees of freedom, the generating functional becomes

$$\exp iZ[R, \bar{R}, \rho, \bar{\rho}, v, a, s, p] = \int [dH][d\bar{H}][du]\Delta_h \exp i\left[S' + \int d^4x(\bar{R}H + \bar{H}R)\right], \tag{25}$$

where

$$S' = \int d^4x\bar{H}[\mathcal{A} + (\gamma_0\mathcal{B}^\dagger\gamma_0)\mathcal{C}^{-1}\mathcal{B}]H \tag{26}$$

and Δ_h is a constant. Then, one expands the matrix $\mathcal{C}^{-1}$ in terms of $1/m$

$$\mathcal{C}^{-1} = \frac{1}{2m} - \frac{i(v\cdot D) + g_A S_v\cdot u}{(2m)^2} - \frac{\mathcal{C}_{(2)}}{(2m)^2} + \frac{(iv\cdot D + g_A S_v\cdot u)^2}{(2m)^3} + \cdots. \tag{27}$$

Finally, the nonrelativistic Lagrangians corresponding to the action S' can be expressed as

$$\mathcal{L}' = \mathcal{L}'_{(1)} + \mathcal{L}'_{(2)} + \mathcal{L}'_{(3)} + \mathcal{L}'_{(4)} + \cdots \tag{28}$$

with $\mathcal{L}'_{(i)} = \bar{H}T_{(i)}H$ $(i = 1, 2, 3, 4, \ldots)$, where

$$T_{(1)} = i(v\cdot D) + g_A S_v\cdot u, \tag{29}$$

$$T_{(2)} = c_1\langle\chi_+\rangle + \frac{c_2}{2}\langle(v\cdot u)^2\rangle + c_3(v\cdot u)^2 + \frac{c_4}{2}\langle u^2\rangle + \frac{c_5}{2}u^2 + \frac{c_6}{2}[S_v^\mu, S_v^\nu][u_\mu, u_\nu] + c_7\hat{\chi}_+ - \frac{ic_8}{4m}[S_v^\mu, S_v^\nu]f_{\mu\nu}^+ - \frac{ic_9}{4m}[S_v^\mu, S_v^\nu]\langle f_{\mu\nu}^+\rangle + \frac{2}{m}(S_v\cdot D)^2 - \frac{ig_A}{2m}\{S_v\cdot D, v\cdot u\} - \frac{g_A^2}{8m}(v\cdot u)^2 + \cdots, \tag{30}$$

$$T_{(3)} = h_1 S_v^\mu\langle\chi_+\rangle u_\mu + h_2 S_v^\mu\{\hat{\chi}_+, u_\mu\} + h_3 S_v^\mu\langle\hat{\chi}_+ u_\mu\rangle - \frac{c_8}{4m^2}S_v\cdot D(v^\mu S_v^\nu - v^\nu S_v^\mu)f_{\mu\nu}^+ - \frac{c_9}{4m^2}S_v\cdot D(v^\mu S_v^\nu - v^\nu S_v^\mu)\langle f_{\mu\nu}^+\rangle + \frac{ig_A c_8}{16m^2}(v^\mu S_v^\nu - v^\nu S_v^\mu)[v\cdot u, f_{\mu\nu}^+]\frac{c_8}{4m^2}(v^\mu S_v^\nu - v^\nu S_v^\mu)f_{\mu\nu}^+ S_v\cdot D + \frac{c_9}{4m^2}(v^\mu S_v^\nu - v^\nu S_v^\mu)\langle f_{\mu\nu}^+\rangle S_v\cdot D - \frac{i}{m^2}S_v\cdot Dv\cdot DS_v\cdot D + \cdots, \tag{31}$$

$$\begin{aligned} T_{(4)} = {} & e_1\langle\chi_+\rangle\langle\chi_+\rangle + e_2\hat{\chi}_+\langle\chi_+\rangle + e_3\langle\hat{\chi}_+\hat{\chi}_+\rangle + e_4\hat{\chi}_+\hat{\chi}_+ + e_5\langle\chi_-\rangle\langle\chi_-\rangle + e_6\hat{\chi}_-\langle\chi_-\rangle + e_7\langle\hat{\chi}_-\hat{\chi}_-\rangle + e_8\hat{\chi}_-\hat{\chi}_- \\ & - \frac{c_8^2}{32m^3}(v^\mu S_v^\nu - v^\nu S_v^\mu)f_{\mu\nu}^+(v^\alpha S_v^\beta - v^\beta S_v^\alpha)f_{\alpha\beta}^+ - \frac{c_8 c_9}{32m^3}(v^\mu S_v^\nu - v^\nu S_v^\mu)f_{\mu\nu}^+(v^\alpha S_v^\beta - v^\beta S_v^\alpha)\langle f_{\alpha\beta}^+\rangle \\ & - \frac{c_8 c_9}{32m^3}(v^\mu S_v^\nu - v^\nu S_v^\mu)\langle f_{\mu\nu}^+\rangle(v^\alpha S_v^\beta - v^\beta S_v^\alpha)f_{\alpha\beta}^+ - \frac{c_9^2}{32m^3}(v^\mu S_v^\nu - v^\nu S_v^\mu)\langle f_{\mu\nu}^+\rangle(v^\alpha S_v^\beta - v^\beta S_v^\alpha)\langle f_{\alpha\beta}^+\rangle \\ & + \frac{ic_8}{(2m)^3}S_v\cdot Dv\cdot D(v^\mu S_v^\nu - v^\nu S_v^\mu)f_{\mu\nu}^+ + \frac{ic_9}{(2m)^3}S_v\cdot Dv\cdot D(v^\mu S_v^\nu - v^\nu S_v^\mu)\langle f_{\mu\nu}^+\rangle \\ & - \frac{ic_8}{(2m)^3}(v^\mu S_v^\nu - v^\nu S_v^\mu)f_{\mu\nu}^+ v\cdot DS_v\cdot D - \frac{ic_9}{(2m)^3}(v^\mu S_v^\nu - v^\nu S_v^\mu)\langle f_{\mu\nu}^+\rangle v\cdot DS_v\cdot D \\ & + \frac{c_7}{m^2}S_v\cdot D\hat{\chi}_+ S_v\cdot D + \frac{c_1}{m^2}S_v\cdot D\langle\chi_+\rangle S_v\cdot D - \frac{ic_8}{4m^3}S_v\cdot D[S_v^\mu, S_v^\nu]f_{\mu\nu}^+ S_v\cdot D \\ & - \frac{ic_9}{4m^3}S_v\cdot D[S_v^\mu, S_v^\nu]\langle f_{\mu\nu}^+\rangle S_v\cdot D - \frac{1}{2m^3}S_v\cdot D(v\cdot D)^2 S_v\cdot D + \cdots, \end{aligned} \tag{32}$$

where

$$S_v^\mu = \frac{i}{2}\gamma_5\sigma^{\mu\nu}v_\nu. \tag{33}$$

The above Lagrangians will be employed to calculate the chiral correction to the mass and axial current of the doubly charmed baryons.

III. THE CHIRAL CORRECTION TO THE MASS OF THE DOUBLY HEAVY BARYON

With the notations $\eta = v\cdot p$ and $\xi = (p - mv)^2$, the full propagator of the doubly heavy baryon is written as

$$G = \frac{i}{v\cdot p - m_0 - \Sigma_B(\eta,\xi)} = \frac{iZ_N}{v\cdot p - m - Z_N\tilde{\Sigma}_B(\eta,\xi)}, \tag{34}$$

where $\Sigma_B(\eta,\xi)$ denotes the high order contributions to the self-energy, which are from Figs. 1(a)–1(h).

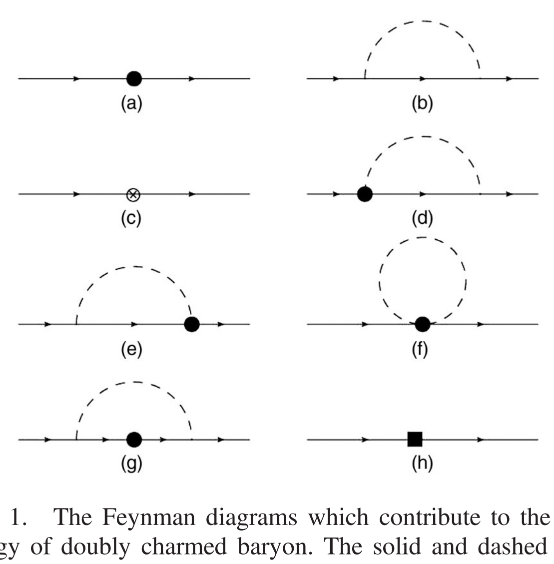

FIG. 1. The Feynman diagrams which contribute to the self-energy of doubly charmed baryon. The solid and dashed lines denote the doubly charmed baryons and goldstone bosons. The solid dot, circle-cross and black box denote the vertices from the $\mathcal{O}(p^2, p^3, p^4)$ Lagrangians respectively.

The mass of the doubly charmed baryon is

$$m = m_0 + \Sigma_B(0,0). \tag{35}$$

The renormalization constant reads

$$Z_N = \frac{1}{1 - \Sigma'_B(0,0)} \tag{36}$$

with

$$\Sigma'_B(0,0) = \frac{\partial \Sigma_B(\eta,\xi)}{\partial \eta}\bigg|_{(\eta,\xi)=(0,0)}. \tag{37}$$

The chiral contribution to the self-energy up to next-to-next-to-leading order (N^2LO) includes three pieces

$$\Sigma_B^{(a)} = -\left\{2c_1\langle\chi\rangle + 2c_7\hat{\chi}_{ii} - \frac{2}{m} S_v^\mu S_v^\nu k_\mu k_\nu\right\}, \tag{38}$$

$$\Sigma_{B,P}^{(b)} = iC_{BP}^{(b)} \int \frac{d^4q}{(2\pi)^4}\left[\frac{g_A}{F_{P0}} S_v\cdot q\right]\frac{i}{v\cdot(k-q)+i\epsilon} \times \frac{i}{q^2 - M_P^2 + i\epsilon}\left[-\frac{g_A}{F_{P0}} S_v\cdot q\right]$$
$$= -C_{BP}^{(b)} \frac{g_A^2}{(4\pi F_{P0})^2}\left\{\frac{v\cdot k_u}{4}\left([3M_P^2 - 2(v\cdot k_u)^2] \times \left[R + \ln\left(\frac{M_P^2}{\mu^2}\right)\right] - 2[M_P^2 - (v\cdot k_u)^2]\right) + [M_P^2 - (v\cdot k_u)^2]^{3/2} \arccos\left(-\frac{v\cdot k_u}{M_P}\right)\right\}, \tag{39}$$

$$\Sigma_B^{(c)} = -\frac{1}{m^2}(S_v\cdot k)(v\cdot k)(S_v\cdot k), \tag{40}$$

which correspond to Figs. 1(a)–1(c), respectively.

$$\Sigma_B = \Sigma_B^{(a)} + \sum_P \Sigma_{B,P}^{(b)} + \Sigma_B^{(c)} \tag{41}$$

with the subscripts $B = \Xi_{cc}^{++}, \Xi_{cc}^{+}, \Omega_{cc}^{+}$ and $P = \pi^{\pm,0}, K^{\pm,0}, \bar{K}^0, \eta$. $F_{\pi0}$, F_{K0} and $F_{\eta0}$ are the decay constants of π, K and η, which are 0.092, 0.112 and

TABLE II. The values of the coefficients $(C_{BP}^{(b/d/g)})^{1/2}$, $(C_{1BP}^{(f)})^{1/2}$, $C_{2BP}^{(f)}$, $C_{3BP}^{(f)}$, $C_{4BP}^{(f)}$, and $C_{5BP}^{(f)}$ in Eqs. (38)–(40) and (A1)–(A6).

$(C_{BP}^{(b/d/g)})^{1/2}$	π^+	π^0	π^-	K^+	K^0	$\bar{K}^0$	K^-	η
Ξ_{cc}^{++}	$\sqrt{2}$	1	0	$\sqrt{2}$	0	0	0	$\frac{1}{\sqrt{3}}$
Ξ_{cc}^{+}	0	−1	$\sqrt{2}$	0	$\sqrt{2}$	0	0	$\frac{1}{\sqrt{3}}$
Ω_{cc}^{+}	0	0	0	0	0	$\sqrt{2}$	$\sqrt{2}$	$-\frac{2}{\sqrt{3}}$
$(C_{1BP}^{(f)})^{1/2}$	π^+	π^0	π^-	K^+	K^0	$\bar{K}^0$	K^-	η
Ξ_{cc}^{++}	$\sqrt{2}$	1	0	$\sqrt{2}$	0	0	0	$\frac{1}{\sqrt{3}}$
Ξ_{cc}^{+}	0	−1	$\sqrt{2}$	0	$\sqrt{2}$	0	0	$\frac{1}{\sqrt{3}}$
Ω_{cc}^{+}	0	0	0	0	0	$\sqrt{2}$	$\sqrt{2}$	$-\frac{2}{\sqrt{3}}$
$C_{2BP}^{(f)}$	π^+	π^0	π^-	K^+	K^0	$\bar{K}^0$	K^-	η
$\Xi_{cc}^{++}/\Xi_{cc}^{+}/\Omega_{cc}^{+}$	2	2	2	2	2	2	2	2
$C_{3BP}^{(f)}$	π^+	π^0	π^-	K^+	K^0	$\bar{K}^0$	K^-	η
Ξ_{cc}^{++}	$4B_0m_d$	$2B_0(m_u)$	0	$4B_0m_s$	0	0	0	$\frac{2}{3}B_0(m_u)$
Ξ_{cc}^{+}	0	$2B_0(m_d)$	$4B_0m_u$	0	0	0	$\frac{2}{3}B_0(m_d)$	
Ω_{cc}^{+}	0	0	0	0	0	$4B_0(m_d)$	$4B_0(m_u)$	$\frac{8}{3}B_0(m_s)$
$C_{4BP}^{(f)}$	π^+	π^0	π^-	K^+	K^0	$\bar{K}^0$	K^-	η
Ξ_{cc}^{++}	$4B_0m_u$	$2B_0(m_u)$	0	$4B_0m_u$	0	0	0	$\frac{2}{3}B_0(m_u)$
Ξ_{cc}^{+}	0	$2B_0(m_d)$	$4B_0m_d$	0	$4B_0(m_d)$	0	0	$\frac{2}{3}B_0(m_d)$
Ω_{cc}^{+}	0	0	0	0	0	$4B_0(m_s)$	$4B_0(m_s)$	$\frac{8}{3}B_0(m_s)$
$C_{5BP}^{(f)}$	π^+	π^0	π^-	K^+	K^0	$\bar{K}^0$	K^-	η
$\Xi_{cc}^{++}/\Xi_{cc}^{+}/\Omega_{cc}^{+}$	$4B_0m_u$	$2B_0(m_u+m_d)$	$4B_0m_d$	$4B_0m_u$	$4B_0m_d$	$4B_0m_s$	$4B_0m_s$	$\frac{2}{3}B_0(m_u+m_d+4m_s)$

0.110 GeV, respectively. In addition, the coefficients $C_{BP}^{(b)}$ are given in Table II.

We notice that there are three low energy constants c_1, c_7, and g_A in Eqs. (38)–(40). Among these LECs, c_1 appearing in the next-to-leading order Lagrangian can be absorbed into the bare mass term. Thus, c_7 and g_A are the two unknown constants. Due to the absence of the corresponding experimental information, we have to fix these unknown constants based on the other theoretical calculations. In Ref. [64], Hu and Mehen constructed Lagrangian with the following form:

$$\mathcal{L} = \mathrm{Tr}[T_a^\dagger (iD_0)_{ba} T_b] - g\mathrm{Tr}[T_a^\dagger T_b \vec{\sigma} \cdot \vec{A}_{ba}] + \cdots \quad (42)$$

by considering the heavy diquark symmetry, where $T_{a,i\beta} = \sqrt{2}(\Xi^*_{a,i\beta} + \frac{1}{\sqrt{3}} \Xi_{a,\gamma} \sigma^i_{\gamma\beta})$. In Eq. (42), the coupling $g = 0.6$ is determined by fitting the D^{*+} width. Comparing our effective Lagrangian with that in Eq. (42), we get $g_A = g$. In the following, we take $g_A = 0.6$.

The LEC c_7 and bare mass m_0 are still unknown. We try to fix these two unknown constants by fitting the lattice data with pion mass up to 0.4 GeV in Ref. [27].

The masses of Ξ_{cc} are given for different m_π and m_c in Ref. [27]. We assume only the bare mass (m_0) depends on the mass (m_c) of the valence charm quark, and the dependence respects the heavy quark expansion

$$m_0 = \tilde{m}_0 + 2m_c + \alpha/m_c + O(1/m_c^2). \quad (43)$$

The physical mass $m_c|_{\mathrm{phy}}$ is tuned to reproduce the mass of the D meson at the physical point in Ref. [27]:

$$m_c|_{\mathrm{phy}} = 0.591 \pm 0.028 \ \mathrm{GeV}. \quad (44)$$

We give the fitted results with $\chi^2_{\mathrm{dof}} \lesssim 1$ in Table III. Generally speaking, it indicates the lattice data are overfitted if a result with $\chi^2_{\mathrm{dof}} \lesssim 1$ could be obtained. One can fit the lattice data well with any c_7 lying in the range (−6.0, 0.6) from the table. Therefore the current lattice data of $m_{\Xi_{cc}}$ are not enough to constrain c_7 yet. However, c_7 being around −0.2 might be a real solution considering that the mass of Ω_{cc} is 3.68–3.76 GeV by lattice QCD groups [23–27].

We plot the best fitted results with supposing $c_7 = -0.2$ in Fig. 2. The best fitting needs $\tilde{m}_0 = 3.460$ and $\alpha = -0.488$ and predicts

$$m_{\Xi_{cc}} = 3.665^{+0.093}_{-0.097} \ \mathrm{GeV}, \quad m_{\Omega_{cc}} = 3.726^{+0.093}_{-0.097} \ \mathrm{GeV}. \quad (45)$$

We have also obtained the mass correction of the doubly charmed baryon up to the next-next-next-leading order (N^{3}LO), which is collected in Appendix A. Unfortunately there appear too many unknown LECs which cannot be fixed by experimental or theoretical approaches. We are unable to use the N^{3}LO mass formula to compare with the current experimental data. However, the chiral structure and expression of the mass at the N^{3}LO will be helpful to the chiral extrapolation of the lattice data in the lattice QCD simulation.

TABLE III. Parameters for fitting the lattice data from Ref. [27] and the physical masses of doubly charmed baryons with the corresponding fitted parameters. The errors of the masses are from the error of $m_c|_{\mathrm{phy}}$.

c_7	$\tilde{m}_0$	α	χ^2_{dof}	$m_{\Xi_{cc}}$	$m_{\Omega_{cc}}$
0.6	3.314	−0.518	1.0	$3.710^{+0.096}_{-0.100}$	$3.045^{+0.096}_{-0.100}$
0.3	3.363	−0.505	0.8	$3.690^{+0.095}_{-0.099}$	$3.297^{+0.095}_{-0.099}$
0.0	3.450	−0.510	0.7	$3.677^{+0.095}_{-0.099}$	$3.557^{+0.095}_{-0.099}$
−0.1	3.472	−0.509	0.6	$3.672^{+0.095}_{-0.099}$	$3.642^{+0.095}_{-0.099}$
−0.2	3.460	−0.488	0.6	$3.665^{+0.093}_{-0.097}$	$3.726^{+0.093}_{-0.097}$
−0.3	3.517	−0.506	0.5	$3.661^{+0.095}_{-0.099}$	$3.813^{+0.095}_{-0.099}$
−0.4	3.541	−0.506	0.5	$3.655^{+0.095}_{-0.099}$	$3.898^{+0.095}_{-0.099}$
−0.5	3.562	−0.503	0.4	$3.650^{+0.095}_{-0.098}$	$3.983^{+0.095}_{-0.098}$
−1.0	3.552	−0.427	0.4	$3.618^{+0.089}_{-0.092}$	$4.405^{+0.089}_{-0.092}$
−2.0	3.900	−0.484	0.1	$3.567^{+0.093}_{-0.097}$	$5.261^{+0.093}_{-0.097}$
−4.0	4.351	−0.458	0.4	$3.457^{+0.091}_{-0.095}$	$6.966^{+0.091}_{-0.095}$
−6.0	4.801	−0.431	1.6	$3.347^{+0.089}_{-0.092}$	$8.671^{+0.089}_{-0.092}$

IV. THE CHIRAL CORRECTION TO THE AXIAL CURRENT

In the following, we discuss the chiral correction to the axial current of the doubly charmed baryon. Using Lagrangian $\mathcal{L}'_{(1)}$ and $\mathcal{L}'_{(3)}$ in Eq. (A1), we obtain the axial current at the tree level:

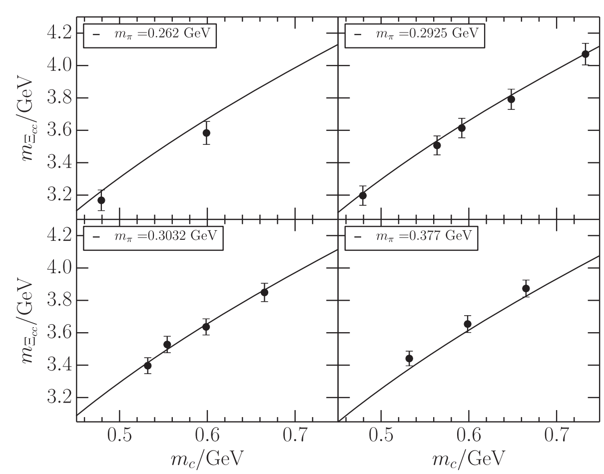

FIG. 2. The masses of Ξ_{cc} as a function of m_c for different masses of pion. The lattice data are from Ref. [27], and the solid curves are our fitted results up to next to the leading order with $c_7 = -0.2$, $\tilde{m}_0 = 3.460$, $\alpha = -0.488$, and $\chi^2_{\mathrm{dof}} = 0.6$.

$$
\begin{aligned}
A^{k,\mu} &= \frac{\partial\mathcal{L}}{\partial r_k^\mu} - \frac{\partial\mathcal{L}}{\partial l_k^\mu} \\
&= \frac{1}{2} v^\mu \bar{H}(u^\dagger T^k u - u T^k u^\dagger)H + g_A \bar{H} S_v^\mu (u^\dagger T^k u + u T^k u^\dagger)H + h_1 \bar{H} S_v^\mu \langle\chi_+\rangle (u^\dagger T^k u + u T^k u^\dagger)H \\
&\quad + h_2 \bar{H} S_v^\mu \{\hat{\chi}_+, (u^\dagger T^k u + u T^k u^\dagger)\} H + h_3 \bar{H} S_v^\mu \langle \hat{\chi}_+ (u^\dagger T^k u + u T^k u^\dagger)\rangle H. \qquad (46)
\end{aligned}
$$

We collect the diagrams contributing to the renormalization of the axial currents in Fig. 3.

The renormalized matrix element of $A^{k,\mu}$ between the doubly heavy baryon states can be written as

$$
\begin{aligned}
\langle B_d | A^{k,\mu} | B_a \rangle = \Bigg\{ & g_A 2T_{ad}^k \left[1 - \sum_P \frac{g_A^2}{(4\pi F_{P0})^2} \left(\frac{3M_P^2}{4} \left(R + \ln\frac{M_P^2}{\mu^2}\right) + \frac{1}{2} M_P^2 \right) (C_{B_aP}^{(b)} + C_{B_dP}^{(b)}) \frac{1}{2} \right] \\
& + h_1 \langle\chi_+\rangle 2T_{ad}^k + h_2 \sum_i [2\chi_{ai} T_{id}^k + 2T_{ai}^k \chi_{id}] + h_3 2\langle\chi_+ T^k\rangle + \sum_{P,b,c} \Sigma_{bc}^{\text{current}} \Bigg\} \bar{u}_a S_v^\mu u_d. \qquad (47)
\end{aligned}
$$

In the above equations, we have

$$
\Sigma^{\text{current}} = \Sigma_{(1)}^{\text{current}} + \Sigma_{(2)}^{\text{current}} + \Sigma_{(3)}^{\text{current}} + \Sigma_{(4)}^{\text{current}},
$$

where

$$
\Sigma_{(1)}^{\text{current}} = 0, \qquad (48)
$$

$$
\Sigma_{(2)}^{\text{current}} = 0, \qquad (49)
$$

$$
\begin{aligned}
\Sigma_{(3)}^{\text{current}} = -\frac{g_A^3}{6F_{P0}^2} C_{\tilde{P}ab} \delta_{cd} \delta_{ab} T_{bc}^k \Bigg\{ & -2(v\cdot k)\left[-4\frac{1}{32\pi^2}\left(R - \frac{2}{3}\right) \times \frac{v\cdot k}{8\pi^2}\left(1 - 2\ln\frac{M_{\tilde{P}}}{\mu}\right) - \frac{1}{4\pi^2}\sqrt{M_{\tilde{P}}^2 - (v\cdot k)^2} \right. \\
& \left. \times \arccos\frac{-v\cdot k}{M_{\tilde{P}}} \right] + [M_{\tilde{P}}^2 - (v\cdot k)^2]\left[-4\frac{1}{32\pi^2}\left(R - \frac{2}{3}\right) + \frac{1}{8\pi^2}\left(1 - 2\ln\frac{M_{\tilde{P}}}{\mu}\right) + \frac{v\cdot k}{2\pi^2}[M_{\tilde{P}}^2 - (v\cdot k)^2]^{-1/2} \right. \\
& \left. \times \arccos\frac{-v\cdot k}{M_{\tilde{P}}} - \frac{1}{4\pi^2} \right] - 2M_{\tilde{P}}^2 \left(\frac{1}{32\pi^2}\left(R - \frac{2}{3}\right) + \frac{1}{16\pi^2}\ln\frac{M_{\tilde{P}}}{\mu} \right) \Bigg\}, \qquad (50)
\end{aligned}
$$

$$
\Sigma_{(4)}^{\text{current}} = -\frac{i g_A}{4F_{P0}^2} [-C_{\tilde{P}bc} T_{ab}^k \delta_{bd} + 2C_{\tilde{P}ab} T_{bc}^k \delta_{ad}\delta_{bc} - T_{cd}^k C_{\tilde{P}ab}\delta_{ac}] \frac{M_{\tilde{P}}^2}{(4\pi)^2} \left[-R + \ln\frac{\mu^2}{M_{\tilde{P}}^2} \right], \qquad (51)
$$

which correspond to Fig. 3 [(a)–(d)], respectively. Since the doubly heavy baryons are very heavy, we do not take into account the small recoil corrections in the present paper. The contribution from Fig. 3 [(a) and (b)] vanishes as shown in Eqs. (48) and (49). In the above equations, the matrices $C_{\tilde{P}}$ ($\tilde{P} = \pi, K, \eta$) are defined as

$$
C_\pi = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad C_K = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 2 \\ 2 & 2 & 0 \end{pmatrix},
$$

$$
C_\eta = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{4}{3} \end{pmatrix}. \qquad (52)
$$

The wave function renormalization constant is expressed as

$$
Z_{N,BP} = 1 - C_{BP}^{(b)} \frac{g_A^2}{(4\pi F_{P0})^2} \left\{ \frac{3M_P^2}{4}\left(R + \ln\frac{M_P^2}{\mu^2}\right) + \frac{1}{2}M_P^2 \right\}. \qquad (53)
$$

We define the axial charge of the heavy baryon through the matrix element

$$
\langle B_d | A^{k,\mu} | B_a \rangle = g_{ad}^A \bar{u}_d S_v^\mu u_a, \qquad (54)
$$

where g_{ad}^A is the axial charge.

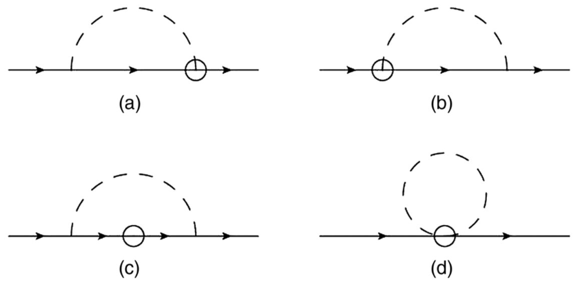

FIG. 3. The diagrams contributing to the renormalization of the axial current. The circle represents an insertion of the axial current.

In Eq. (47), there exist three low energy constants h_1, h_2 and h_3. The LEC h_1 can be absorbed into the g_A term. There remain two unknown constants h_2 and h_3. At present, there is not enough information to fix h_2 and h_3. As a crude approximation, we simply parametrize h_2 and h_3 as $h_2 = h_3 = \frac{1}{\lambda^2}$, where λ is the typical energy scale around the mass of the doubly heavy baryons. Taking the typical value $\lambda = \pm 3.6$ GeV, we obtain $g^{1+i2}_{\Xi^{++}_{cc}\Xi^{+}_{cc}} = 1.15$ and $g^{4+i5}_{\Xi^{++}_{cc}\Omega^{+}_{cc}} = 1.18$. If only considering the tree level contribution, we get $g^{1+i2}_{\Xi^{++}_{cc}\Xi^{+}_{cc}} = g^{4+i5}_{\Xi^{++}_{cc}\Omega^{+}_{cc}} = 1.2$. If λ varies from 2 to 5 GeV, the range of $g^{1+i2}_{\Xi^{++}_{cc}\Xi^{+}_{cc}}$ and $g^{4+i5}_{\Xi^{++}_{cc}\Omega^{+}_{cc}}$ will be $1.16 - 1.14$ and 1.35–1.14 respectively. We also consider the case $h_{2,3} \sim -\frac{1}{\lambda^2}$. And $g^{1+i2}_{\Xi^{++}_{cc}\Xi^{+}_{cc}}$ and $g^{4+i5}_{\Xi^{++}_{cc}\Omega^{+}_{cc}}$ will be 1.12–1.14 and 0.86–1.07 respectively, when λ is in the range 2–5 GeV.

V. SUMMARY

Although the doubly heavy baryons have not been established experimentally, these systems are particularly interesting. To a large extent, they are even simpler than the light baryons such as nucleons where the interaction among the three light quarks is very complicated. In contrast, the presence of the two heavy quarks acts as a static color source in the heavy quark limit. For example, the chiral dynamics of the doubly heavy baryons is solely governed by the light quark. We can gain valuable insights into the light quark chiral behavior through the chiral perturbation theory study of the doubly heavy baryons.

In this paper, we have constructed the chiral effective Lagrangians describing the interactions of light mesons and doubly charmed baryons. We further make the nonrelativistic reduction and obtain the chiral Lagrangians up to $O(p^4)$ in the heavy baryon limit. We have derived the chiral corrections to the mass of the doubly heavy baryons up to N^{3}LO. Unfortunately there exist too many unknown low energy constants. We are forced to perform the numerical analysis at the N^{2}LO. The mass splitting of Ξ_{cc} and Ω_{cc} at the next-to-next-to-leading order depends on one unknown low energy constant c_7. By fitting the lattice data for Ξ_{cc} from Ref. [27] and supposing $c_7 = -0.2$, we estimate the mass of Ω_{cc} to be around 3.726 GeV.

Moreover, we have also performed a systematical analysis of the chiral corrections to the axial currents and axial charges of the doubly heavy baryons, which may be measured through the semileptonic decays of the heavy baryons in the future.

The chiral corrections to the mass of the doubly heavy baryons have been derived up to N^{3}LO and the axial charge to N^{2}LO. The chiral structure and analytical expressions will be very useful to the chiral extrapolations of the future lattice QCD simulations of the doubly heavy baryons.

The exploration of the doubly charmed baryons is still an important and intriguing research topic, which can deepen our understanding of hadron spectrum and nonperturbative QCD. We are looking forward to more developments from both experimental and theoretical studies. There is a very good chance that these doubly charmed baryons will be observed at facilities such as LHC and BelleII.

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APPENDIX A: THE N^{3}LO CONTRIBUTION TO THE MASS OF THE DOUBLY CHARMED BARYON

We list the N^{3}LO chiral corrections to the mass of the doubly charmed baryon, i.e.,

$$\begin{aligned}\Sigma^{(d)}_{B,P} &= C^{(d)}_{BP} i \int \frac{d^4 q}{(2\pi)^4} (-) \frac{g_A}{2mF_0} S^{\mu}_{v} v^{\nu} (-q_{\mu} q_{\nu} + 2 q_{\nu} k_{\mu}) \\ &\quad \times \frac{i}{q^2 - M^2 + i\epsilon} \frac{i}{v \cdot (k-q) + i\epsilon} (-) \frac{g_A}{F_0} S^{\alpha}_{v} q_{\alpha} \\ &= C^{(d)}_{BP} \frac{g_A^2}{2mF_0^2} (S_v \cdot S_v) \left\{ \frac{1}{4} \left(\frac{M^2}{4\pi} \right)^2 \left[-R + \ln\left(\frac{\mu^2}{M^2} \right) + \frac{1}{2} \right] \right. \\ &\quad \left. + v \cdot k C_{21}(v \cdot k, M^2) \right\} \end{aligned} \tag{A1}$$

from Fig. 1(d), where

$$C_{21}(v\cdot k, M^2) = \frac{1}{n-1}\{(v\cdot k)I(0) + [M^2 - (v\cdot k)^2]J(0, v\cdot k)\},$$
$$I(0) = \frac{M^2}{16\pi^2}\left[R + \ln\left(\frac{M^2}{\mu^2}\right)\right] + O(n-4),$$
$$J(0,\omega) = \frac{\omega}{8\pi^2}\left[R + \ln\left(\frac{M^2}{\mu^2}\right) - 1\right] + \frac{1}{4\pi^2}\sqrt{M^2-\omega^2}\arccos\left(-\frac{\omega}{M}\right) + O(n-4),$$
$$R = \frac{2}{n-4} - [\ln(4\pi) + \Gamma'(1) + 1].$$

We also have the relation

$$\Sigma^{(e)}_{B,P} = \Sigma^{(d)}_{B,P}, \tag{A2}$$

where $\Sigma^{(e)}_{B,P}$ comes from Fig. s(e). The corrections from Figs. 1(f) and 1(g) are

$$\begin{aligned}\Sigma^{(f)}_{B,P} = {}& i\int\frac{d^4q}{(2\pi)^4} i\left\{-\frac{c_1}{F_0^2}C^{(f)}_{5BP} + \frac{1}{2}C^{(f)}_{2BP}\frac{c_2}{F_0^2}v^\mu v^\nu iq_\mu(-)iq_\nu + C^{(f)}_{1BP}\left(c_3 - \frac{g_A^2}{8m}\right)\frac{1}{F_0^2}v^\mu v^\nu iq_\mu(-iq_\nu)\right.\\ &\left.+\frac{1}{2}C^{(f)}_{2BP}\frac{c_4}{F_0^2}iq^\mu(-iq_\mu) + C^{(f)}_{1BP}\frac{c_5}{2F_0^2}iq^\mu(-iq_\mu) - \frac{c_7}{2F_0^2}C^{(f)}_{3BP} - \frac{c_7}{4F_0^2}C^{(f)}_{4BP} - \frac{c_7}{4F_0^2}C^{(f)}_{4BP} + \frac{c_7}{3F_0^2}C^{(f)}_{5BP}\right\}\frac{i}{q^2-M^2+i\epsilon}\\ ={}& \left[\left(-\frac{c_1}{F_0^2} + \frac{c_7}{3F_0^2}\right)C^{(f)}_{5BP} - \frac{1}{2}C^{(f)}_{2BP}\frac{c_7}{2F_0^2}C^{(f)}_{4BP} + -\frac{c_7}{2F_0^2}C^{(f)}_{3BP}\right]\left(\frac{M}{4\pi}\right)^2\left(\frac{2}{\epsilon} - r_E + 1 + \ln\left(\frac{4\pi\mu^2}{M^2}\right) + O(\epsilon)\right)\\ &+\frac{1}{4}\left[\frac{c_2}{2F_0^2}C^{(f)}_{2BP} + \left(c_3 - \frac{g_A^2}{8m}\right)\frac{1}{F_0^2}C^{(f)}_{1BP}\right]\times\left(\frac{M^2}{4\pi}\right)^2\left(\frac{2}{\epsilon} - r_E + \frac{3}{2} + \ln\left(\frac{4\pi\mu^2}{M^2}\right) + O(\epsilon)\right)\\ &+\left[\frac{c_4}{2F_0^2}C^{(f)}_{2BP} + \frac{c_5}{2F_0^2}C^{(f)}_{1BP}\right]\left(\frac{M^2}{4\pi}\right)^2\times\left(\frac{2}{\epsilon} - r_E + 1 + \ln\left(\frac{4\pi\mu^2}{M^2}\right) + O(\epsilon)\right),\end{aligned} \tag{A3}$$

and

$$\begin{aligned}\Sigma^{(g)}_{B,P} = {}& i\int\frac{d^4q}{(2\pi)^4} i(-)\frac{g_A}{F_0}S^\mu_v iq_\mu\frac{i}{q^2-M^2+i\epsilon}\frac{i}{v\cdot(k-q)+i\epsilon}\times\left[i2c_1\langle\chi\rangle + i2\chi_{jj} + i\frac{2}{m}S^\alpha_v S^\beta_v i(k-q)_\alpha i(k-q)_\beta\right]\\ &\times\frac{i}{v\cdot(k-q)}i(-)\frac{g_A}{F_0}S^\nu_v(-i)q_\nu C^{(g)}_{BP}\\ ={}& -\frac{1}{2}(c_1\langle\chi\rangle + c_7\chi_{jj})C^{(g)}_{BP}\frac{g_A^2}{F_0^2}\{G_2(v\cdot k) - nG_2(v\cdot k)\} + \frac{1}{8m}C^{(g)}_{BP}\frac{g_A^2}{F_0^2}\{v^\mu v^\nu\Delta_{\mu\nu} - 2\Delta^\mu_\mu + k^2\Delta\\ &+(v\cdot k)^2r^2G_0(v\cdot k) + (v\cdot k)^2(nG_2(v\cdot k) + G_3(v\cdot k)) - 2(v\cdot k)^3G_1(v\cdot k) + 2(v\cdot k)^2k^2J_0(v\cdot k)\\ &+2(v\cdot k)(nJ_2(v\cdot k) + J_3(v\cdot k)) - 4(v\cdot k)^2J_1(v\cdot k) - M^2k^2G_0(v\cdot k) - M^2(nG_2(v\cdot k) + G_3(v\cdot k))\\ &+2M^2(v\cdot k)G_1(v\cdot k),\end{aligned} \tag{A4}$$

respectively, where

$$\Delta_{\mu\alpha} = -g_{\mu\alpha}\frac{M^2}{4}\left(\Delta - \frac{M^2}{32\pi^2}\right), \tag{A5}$$

and G_i and J_i ($i = 0, 1, 2, 3$) are defined in the Appendix of Ref. [29],

$$\Sigma_B^{(h)} = -4e_1\langle\chi\rangle\langle\chi\rangle - e_2\left[4\chi_{i_Bi_B}\langle\chi\rangle - \frac{4}{3}\langle\chi\rangle\langle\chi\rangle\right] - e_3\left[4\langle\chi\chi\rangle - \frac{8}{3}\langle\chi\rangle\langle\chi\rangle + \frac{4}{9}\langle\chi\rangle\langle\chi\rangle\right] - e_4\left[4\sum_j\chi_{i_Bj}\chi_{ji_B} - \frac{8}{3}\chi_{i_Bi_B}\langle\chi\rangle + \frac{4}{9}\langle\chi\rangle\langle\chi\rangle\right] - \frac{c_7}{m^2}S_v\cdot(ik)\left[\chi_{i_Bi_B} - \frac{1}{3}\langle\chi\rangle\right]S_v\cdot(ik) - \frac{c_1}{m^2}S_v\cdot(ik)\langle\chi\rangle S_v\cdot(ik) + \frac{1}{2m^3}S_v\cdot(ik)[v\cdot(ik)]^2S_v\cdot(ik) \tag{A6}$$

corresponding to Fig. 1(h). Here $i_{\Xi_{cc}^{++}} = 1$, $i_{\Xi_{cc}^{+}} = 2$, $i_{\Omega_{cc}^{+}} = 3$. Some coefficients in the above expressions are listed in Table II.

APPENDIX B: SOME EXPRESSIONS

The expressions of $\mathcal{A}_{(i)}$, $\mathcal{B}_{(j)}$ and $\mathcal{C}_{(k)}$ that appear in Eqs. (21)–(23) are

$$\mathcal{A}_{(1)} = iv\cdot D + g_AS_v\cdot u, \tag{B1}$$

$$\mathcal{A}_{(2)} = c_1\langle\chi_+\rangle + \frac{c_2}{2}\langle(v\cdot u)^2\rangle + c_3(v\cdot u)^2 + \frac{c_4}{2}\langle u^2\rangle + \frac{c_5}{2}u^2 + \frac{c_6}{2}[S_v^\mu, S_v^\nu][u_\mu, u_\nu] + c_7\hat{\chi}_+ - \frac{ic_8}{4m}[S_v^\mu, S_v^\nu]f_{\mu\nu}^+ - \frac{ic_9}{4m}[S_v^\mu, S_v^\nu]\langle f_{\mu\nu}^+\rangle, \tag{B2}$$

$$\mathcal{A}_{(3)} = h_1S_v^\mu\langle\chi_+\rangle u_\mu + h_2S_v^\mu\{\hat{\chi}_+, u_\mu\} + h_3S_v^\mu\langle\hat{\chi}_+u_\mu\rangle + \cdots, \tag{B3}$$

$$\mathcal{A}_{(4)} = e_1\langle\chi_+\rangle\langle\chi_+\rangle + e_2\hat{\chi}_+\langle\chi_+\rangle + e_3\langle\hat{\chi}_+\hat{\chi}_+\rangle + e_4\hat{\chi}_+\hat{\chi}_+ + e_5\langle\chi_-\rangle\langle\chi_-\rangle + e_6\hat{\chi}_-\langle\chi_-\rangle + e_7\langle\hat{\chi}_-\hat{\chi}_-\rangle + e_8\hat{\chi}_-\hat{\chi}_-, \tag{B4}$$

$$\mathcal{B}_{(1)} = -2i\gamma_5S_v\cdot D - \frac{g_A}{2}\gamma_5v\cdot u, \tag{B5}$$

$$\mathcal{B}_{(2)} = -\frac{c_6}{2}\gamma_5(v^\mu S_v^\nu - v^\nu S_v^\mu)[u_\mu, u_\nu] + \frac{ic_8}{4m}\gamma_5(v^\mu S_v^\nu - v^\nu S_v^\mu)f_{\mu\nu}^+ + \frac{ic_9}{4m}\gamma_5(v^\mu S_v^\nu - v^\nu S_v^\mu)\langle f_{\mu\nu}^+\rangle, \tag{B6}$$

$$\mathcal{B}_{(3)} = -\frac{h_1}{2}\gamma_5\langle\chi_+\rangle v\cdot u - \frac{h_2}{2}\gamma_5\{\hat{\chi}_+, v\cdot u\} - \frac{h_3}{2}\gamma_5\langle\hat{\chi}_+v\cdot u\rangle + \cdots, \tag{B7}$$

$$\mathcal{C}_{(1)} = iv\cdot D + 2m + g_AS_v\cdot u, \tag{B8}$$

$$\mathcal{C}_{(2)} = -c_1\langle\chi_+\rangle - \frac{c_2}{2}\langle(v\cdot u)^2\rangle - c_3(v\cdot u)^2 - \frac{c_4}{2}\langle u^2\rangle - \frac{c_5}{2}u^2 - \frac{c_6}{2}[S_v^\mu, S_v^\nu][u_\mu, u_\nu] - c_7\hat{\chi}_+ + \frac{ic_8}{4m}[S_v^\mu, S_v^\nu]f_{\mu\nu}^+ + \frac{ic_9}{4m}[S_v^\mu, S_v^\nu]\langle f_{\mu\nu}^+\rangle, \tag{B9}$$

$$\mathcal{C}_{(3)} = h_1S_v^\mu\langle\chi_+\rangle u_\mu + h_2S_v^\mu\{\hat{\chi}_+, u_\mu\} + h_3S_v^\mu\langle\hat{\chi}_+u_\mu\rangle + \cdots, \tag{B10}$$

$$\mathcal{C}_{(4)} = -e_1\langle\chi_+\rangle\langle\chi_+\rangle - e_2\hat{\chi}_+\langle\chi_+\rangle - e_3\langle\hat{\chi}_+\hat{\chi}_+\rangle - e_4\hat{\chi}_+\hat{\chi}_+ - e_5\langle\chi_-\rangle\langle\chi_-\rangle - e_6\hat{\chi}_-\langle\chi_-\rangle - e_7\langle\hat{\chi}_-\hat{\chi}_-\rangle - e_8\hat{\chi}_-\hat{\chi}_-. \tag{B11}$$

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