Scalar leptoquarks and the rare B meson decays

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We study some rare decays of B mesons involving the quark-level transition $b \to q l^+ l^- (q = d, s)$ in the scalar leptoquark model. We constrain the leptoquark parameter space using the recently measured branching ratios of $B_{s,d} \to \mu^+\mu^-$ processes. Using such parameters, we obtain the branching ratios, direct CP-violation parameters, and isospin asymmetries in the $B \to K\mu^+\mu^-$ and $B \to \pi\mu^+\mu^-$ processes. We also obtain the branching ratios for some lepton-flavor-violating decays $B \to l_i^+ l_j^-$. We find that the various anomalies associated with the isospin asymmetries of the $B \to K\mu^+\mu^-$ process can be explained in the scalar leptoquark model.

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I. INTRODUCTION

The rare decays of B mesons involving flavor-changing neutral-current (FCNC) transitions $b \to s/d$ provide an excellent testing ground to look for new physics (NP). In the standard model (SM), these transitions occur at the one-loop level and, hence, are very sensitive to any new physics contributions. Though so far we have not seen any clear indication of new physics in the b sector, there appears to be some kind of tension with the SM predictions in some $b \to s$ penguin-induced transitions. It should be noted that the recent measurement by the LHCb Collaboration [1] shows several significant deviations on angular observables in the rare decay $B \to K^{*0}\mu^+\mu^-$ from their corresponding SM expectations. In particular, the most significant discrepancy of 3.7σ, which arises in the variable P_5' [2] (the analogue of S_5 in [3]), provides a high sensitivity to NP effects in $b \to s\gamma, sl^+l^-$ transitions. Further results from LHCb experiments in combination with critical assessment of the theoretical uncertainties will be necessary to clarify whether the observed deviations are a real sign of NP or simply statistical fluctuations [4,5].

Another indication of new physics is related to the recent measurement, by LHCb experiments, of isospin asymmetry in the $B \to K\mu^+\mu^-$ process [6], which gives a negative deviation from zero at the level of 4σ while taking into account the entire q^2 spectrum. The isospin asymmetry in $B \to Kll$ is expected to be vanishingly small in the SM; hence, the measured asymmetry provides another smoking gun signal for new physics.

More recently, another discrepancy occurred in the measurement, by the LHCb Collaboration, of the ratio of the branching fractions of $B \to Kl^+l^-$ decays into dimuons over dielectrons [7],

$$R_K = \frac{\mathrm{BR}(B \to K\mu^+\mu^-)}{\mathrm{BR}(B \to Ke^+e^-)}, \tag{1}$$

where the obtained value in the dilepton invariant mass-squared bin $(1 \lesssim q^2 < 6)$ GeV^2 is

$$R_K^{\mathrm{LHCb}} = 0.745^{+0.090}_{-0.074} \pm 0.036. \tag{2}$$

Combining the statistical and systematic uncertainties in quadrature, this observation corresponds to a 2.6σ deviation from its SM prediction $R_K = 1.0003 \pm 0.0001$ [8], where corrections of order α_s and $(1/m_b)$ are included. In contrast to the anomaly in the rare decay $B \to K^*\mu^+\mu^-$, which is affected by unknown power corrections, the ratio R_K is theoretically clean; this might be a sign of lepton-flavor nonuniversal physics.

Though it is conceivable that these anomalies, mostly associated with $b \to sl^+l^-$ transitions, are due to statistical fluctuations or underestimated theory uncertainties, the possible interplay of new physics could not be ruled out. These LHCb results have recently attracted much theoretical attention [2,4,5,9], in the context of new physics models as well as in model-independent ways. In this paper we would like to investigate some of the rare decay modes of B mesons involving the FCNC transitions $b \to (s,d)l^+l^-$, e.g., $B \to Kl^+l^-$, $B \to \pi l^+l^-$, and $B \to l_i^+ l_j^-$, using the scalar leptoquark (LQ) model. In particular, we would like to see whether the leptoquark model can accommodate some of the anomalies discussed above, specifically the ones associated with the $B \to Kll$ processes. It is well known that leptoquarks are color-triplet bosonic particles that can couple to a quark and a lepton at the same time and that can occur in various extensions of the standard model [10]. They can have spin-1 (vector leptoquarks), which exist in grand unified theories based on $SU(5)$, $SO(10)$, etc., or spin-0 (scalar leptoquarks). Scalar leptoquarks can exist at the TeV scale in extended technicolor models [11] as well as in quark- and lepton-composite models [12]. The phenomenology of scalar leptoquarks has been studied extensively in the literature [13–15]. It is generally assumed that the vector leptoquarks tend to couple directly to neutrinos; hence, it is expected

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that their couplings are tightly constrained from the neutrino mass and mixing data. Therefore, in this paper we consider the model where leptoquarks can couple only to a pair of quarks and leptons and, thus, may be inert with respect to proton decay. Hence, the bounds from proton decay may not be applicable for such cases, and leptoquarks may produce signatures in other low-energy phenomena [15].

The paper is organized as follows. In Sec. II we briefly discuss the effective Hamiltonian describing the process $b \to sl^+l^-$ and the new contributions that arise due to the exchange of scalar leptoquarks. The constraints on the leptoquark parameter space are obtained in Sec. III, using the recently measured branching ratios of the decay modes $B_{s,d} \to \mu^+\mu^-$ and the $B \to X_s e^+e^-$ process. The branching ratios and various asymmetries of the rare decay modes $B \to Kl^+l^-$ and $B \to \pi l^+l^-$ are discussed in Secs. IV and V, respectively. In Sec. VI we present the lepton-flavor-violating decays $B \to l_i^+ l_j^-$. Section VII contains the conclusion.

II. EFFECTIVE HAMILTONIAN FOR THE $b \to sl^+l^-$ PROCESS

In the standard model effective Hamiltonian describing the quark-level transition, $b \to sll$ is given as [16]

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \left[\sum_{i=1}^{6} C_i(\mu) O_i + C_7 \frac{e}{16\pi^2} (\bar{s}\sigma_{\mu\nu}(m_s P_L + m_b P_R) b) F^{\mu\nu} + C_9^{\rm eff} \frac{\alpha}{4\pi} (\bar{s}\gamma^\mu P_L b) \bar{l}\gamma_\mu l + C_{10} \frac{\alpha}{4\pi} (\bar{s}\gamma^\mu P_L b) \bar{l}\gamma_\mu \gamma_5 l \right], \tag{3}$$

where G_F is the Fermi constant, $V_{qq'}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, α is the fine-structure constant, $P_{L,R} = (1 \mp \gamma_5)/2$, and the C_i's are the Wilson coefficients. The values of the Wilson coefficients are calculated at the next-to-next-leading order by matching the full theory to the effective theory at the electroweak scale and, subsequently, solving the renormalization group equation to run them down to the b-quark mass scale [3]; the values used in this analysis are listed in Table I.

A. New physics contributions due to scalar leptoquark exchange

The effective Hamiltonian (3) will be modified in the leptoquark model due to the additional contributions arising from the exchange of scalar leptoquarks. Here, we will consider the minimal renormalizable scalar leptoquark models [15], which contain one single additional representation of $SU(3) \times SU(2) \times U(1)$ and which do not allow proton decay at the tree level. It has also been shown that this requirement can only be satisfied by two models; in these models, the leptoquarks can be represented as $X = (3, 2, 7/6)$ and $X = (3, 2, 1/6)$ under the gauge group $SU(3) \times SU(2) \times U(1)$. Our objective here is to consider these scalar leptoquarks, which potentially contribute to the $b \to (s, d)\mu^+\mu^-$ transitions and constrain the underlying couplings from experimental data on $B_{s,d} \to \mu^+\mu^-$. The details of these new contributions are explicitly discussed in Ref. [18]; here, we simply outline the main points.

The interaction Lagrangian for the coupling of scalar leptoquark $X = (3, 2, 7/6)$ to the fermion bilinears is given as

$$\mathcal{L} = -\lambda_u^{ij} \bar{u}_{\alpha R}^i (V_\alpha e_L^j - Y_\alpha \nu_L^j) - \lambda_e^{ij} \bar{e}_R^i (V_\alpha^\dagger u_{\alpha L}^j + Y_\alpha^\dagger d_{\alpha L}^j) + {\rm H.c.} \tag{4}$$

Using the Fierz transformation, one can obtain from Eq. (4) the contribution to the interaction Hamiltonian for the $b \to s\mu^+\mu^-$ process as

$$\mathcal{H}_{\rm LQ} = \frac{\lambda_\mu^{32}\lambda_\mu^{22*}}{8M_Y^2} [\bar{s}\gamma^\mu(1-\gamma_5)b][\bar{\mu}\gamma_\mu(1+\gamma_5)\mu] \equiv \frac{\lambda_\mu^{32}\lambda_\mu^{22*}}{4M_Y^2}(O_9 + O_{10}). \tag{5}$$

One can thus write the leptoquark effective Hamiltonian (5) analogous to its SM counterpart (3) as

$$\mathcal{H}_{\rm LQ} = -\frac{G_F \alpha}{\sqrt{2}\pi} V_{tb}V_{ts}^* (C_9^{\rm NP} O_9 + C_{10}^{\rm NP} O_{10}), \tag{6}$$

with the new Wilson coefficients

TABLE I. The SM Wilson coefficients evaluated at the scale $\mu = 4.6$ GeV [17].

C_1	C_2	C_3	C_4	C_5	C_6	$C_7^{\rm eff}$	$C_8^{\rm eff}$	C_9	C_{10}
−0.3001	1.008	−0.0047	−0.0827	0.0003	0.0009	−0.2969	−0.1642	4.2607	−4.2453

$$C_9^{\rm NP} = C_{10}^{\rm NP} = -\frac{\pi}{2\sqrt{2}G_F\alpha V_{tb}V_{ts}^*}\frac{\lambda_\mu^{32}\lambda_\mu^{22*}}{M_Y^2}. \quad (7)$$

Similarly, the interaction Lagrangian for the coupling of the $X = (3, 2, 1/6)$ leptoquark to the fermion bilinear can be expressed as

$$\mathcal{L} = -\lambda_d^{ij}\bar{d}_{\alpha R}^i(V_\alpha e_L^j - Y_\alpha \nu_L^j) + \text{H.c.} \quad (8)$$

After performing the Fierz transformation, the interaction Hamiltonian becomes

$$\begin{aligned}\mathcal{H}_{\rm LQ} &= \frac{\lambda_s^{22}\lambda_b^{32*}}{8M_V^2}[\bar{s}\gamma^\mu(1+\gamma_5)b][\bar{\mu}\gamma_\mu(1-\gamma_5)\mu] \\ &= \frac{\lambda_s^{22}\lambda_b^{32*}}{4M_V^2}(O_9^{\prime\rm NP} - O_{10}^{\prime\rm NP}),\end{aligned} \quad (9)$$

where O_9' and O_{10}' are the four-fermion current-current operators obtained from $O_{9,10}$ by making the replacement $P_L \leftrightarrow P_R$. Thus, due to the exchange of the leptoquark $X = (3, 2, 1/6)$, one can obtain the new Wilson coefficients $C_9^{\prime\rm NP}$ and $C_{10}^{\prime\rm NP}$ associated with the operators O_9' and O_{10}' as

$$C_9^{\prime\rm NP} = -C_{10}^{\prime\rm NP} = \frac{\pi}{2\sqrt{2}G_F\alpha V_{tb}V_{ts}^*}\frac{\lambda_s^{22}\lambda_b^{32*}}{M_V^2}. \quad (10)$$

The analogous new physics contributions for the $b \to d\mu^+\mu^-$ transitions can be obtained from the $b \to s\mu^+\mu^-$ process by replacing the leptoquark couplings $\lambda^{32}\lambda^{22*}$ by $\lambda^{32}\lambda^{12*}$ and the CKM elements $V_{tb}V_{ts}^*$ by $V_{tb}V_{td}^*$ in Eqs. (6)–(10). After having the idea of new physics contributions to the process $b \to (s, d)\mu^+\mu^-$ in mind, we now proceed to constrain the new physics parameter space using the recent measurement of $B_{s,d} \to \mu^+\mu^-$.

III. THE $B_{s,d} \to \mu^+\mu^-$ DECAY PROCESS

The rare leptonic decay processes $B_{s,d} \to \mu^+\mu^-$, mediated by the FCNC transition $b \to s, d$, are strongly suppressed in the standard model, as they occur at the one-loop level and also suffer from helicity suppression. These decay processes are very clean; the only nonperturbative quantity involved is the B meson decay constant, which can be reliably calculated using nonperturbative methods such as QCD sum rules, lattice gauge theory, and so forth. Therefore, these processes are considered to be one of the most powerful tools in providing important constraints on models of new physics; they have been very well studied in the literature and have recently attracted a lot of attention [19–25]. Therefore, we will here point out the main points. Note that the constraint on the leptoquark couplings from $B_s \to \mu^+\mu^-$ was recently extracted in Ref. [18].

The most general effective Hamiltonian describing these processes is given as

$$\mathcal{H}_{\rm eff} = \frac{G_F\alpha}{\sqrt{2}\pi}V_{tb}V_{tq}^*[C_{10}^{\rm eff}O_{10} + C_{10}'O_{10}'], \quad (11)$$

where $q = d$ or s, $C_{10}^{\rm eff} = C_{10}^{\rm SM} + C_{10}^{\rm NP}$, and $C_{10}' = C_{10}^{\prime\rm NP}$. The corresponding branching ratio is given as

$$\begin{aligned}\text{BR}(B_q \to \mu^+\mu^-) &= \frac{G_F^2}{16\pi^3}\tau_{B_q}\alpha^2 f_{B_q}^2 M_{B_q} m_\mu^2 |V_{tb}V_{tq}^*|^2 \\ &\quad \times |C_{10}^{\rm eff} - C_{10}'|^2\sqrt{1 - \frac{4m_\mu^2}{M_{B_q}^2}}.\end{aligned} \quad (12)$$

However, as discussed in Ref. [19], the average time-integrated branching ratios $\overline{\text{BR}}(B_q \to \mu^+\mu^-)$ depend on the details of $B_q - \bar{B}_q$ mixing, which in the SM are related to the decay widths $\Gamma(B_q \to \mu^+\mu^-)$ by a very simple relation: $\overline{\text{BR}}(B_q \to \mu^+\mu^-) = \Gamma(B_q \to \mu^+\mu^-)/\Gamma_H^q$, where Γ_H^q is the total width of the heavier mass eigenstate.

Including the corrections of $\mathcal{O}(\alpha_{em})$ and $\mathcal{O}(\alpha_s^2)$, the updated branching ratios in the standard model are calculated in [25] as

$$\begin{aligned}&\overline{\text{BR}}(B_s \to \mu^+\mu^-)|_{\rm SM} = (3.65 \pm 0.23) \times 10^{-9}, \\ &\overline{\text{BR}}(B_d \to \mu^+\mu^-)|_{\rm SM} = (1.06 \pm 0.09) \times 10^{-10}.\end{aligned} \quad (13)$$

These processes have been recently measured by CMS [26] and LHCb [27] experiments, and the current experimental world averages [28] are

$$\begin{aligned}&\overline{\text{BR}}(B_s \to \mu^+\mu^-) = (2.9 \pm 0.7) \times 10^{-9}, \\ &\overline{\text{BR}}(B_d \to \mu^+\mu^-) = (3.6^{+1.6}_{-1.4}) \times 10^{-10},\end{aligned} \quad (14)$$

which are more or less consistent with the latest SM prediction [Eq. (13)]; however, they certainly do not rule out the possibility of new physics, as the experimental errors are still quite large.

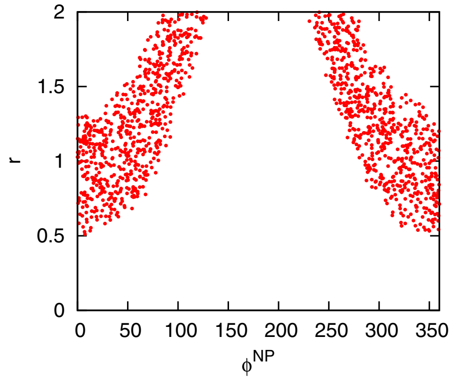
We will now consider the additional contributions that arise due to the effect of scalar leptoquarks in these modes. Including the contributions arising from scalar leptoquark exchange, one can write the transition amplitude for this process from Eq. (11) as

$$\begin{aligned}\mathcal{M}(B_q^0 \to \mu^+\mu^-) &= \langle\mu^+\mu^-|\mathcal{H}_{\rm eff}|B_q^0\rangle \\ &= -\frac{G_F}{\sqrt{2}\pi}V_{tb}V_{tq}^*\alpha f_{B_q}M_{B_q}m_\mu C_{10}^{\rm SM}P,\end{aligned} \quad (15)$$

where

$$P \equiv \frac{C_{10}^{\rm eff} - C_{10}'}{C_{10}^{\rm SM}} = 1 + \frac{C_{10}^{\rm NP} - C_{10}^{\prime\rm NP}}{C_{10}^{\rm SM}} = 1 + re^{i\phi^{\rm NP}}, \quad (16)$$

with



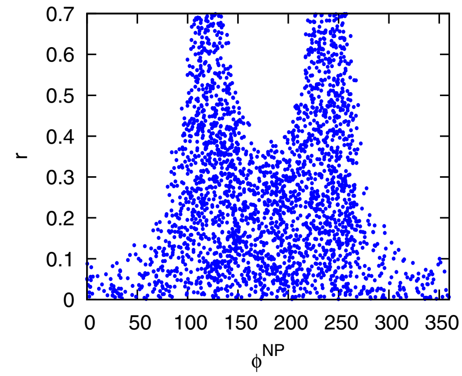


FIG. 1 (color online). The allowed regions in the $r-\phi^{\rm NP}$ parameter space obtained from $\mathrm{BR}(B_d \to \mu^+\mu^-)$ (left panel) and $\mathrm{BR}(B_s \to \mu^+\mu^-)$ (right panel).

$$re^{i\phi^{\rm NP}} = (C_{10}^{\rm NP} - C_{10}^{\prime \rm NP})/C_{10}^{\rm SM}. \tag{17}$$

The expression r denotes the magnitude of the ratio of NP to SM contributions and $\phi^{\rm NP}$ is the relative phase between them. As discussed in Sec. II, the exchange of the leptoquarks $X(3,2,7/6)$ and $X(3,2,1/6)$ gives new additional contributions to the Wilson coefficients C_{10} and C'_{10}, respectively. Thus, the branching ratio in both cases will be

$$\overline{\mathrm{BR}}(B_q \to \mu^+\mu^-) = \overline{\mathrm{BR}}(B_q \to \mu^+\mu^-)|_{\rm SM} \times (1 + r^2 - 2r\cos\phi^{\rm NP}). \tag{18}$$

Using the theoretical and experimental branching ratios from Eqs. (13) and (14), the constraints on the combination of LQ couplings can be obtained by requiring that each individual leptoquark contribution to the branching ratio does not exceed the experimental result. The allowed regions in the $r-\phi^{\rm NP}$ plane compatible with the $1-\sigma$ range of the experimental data are shown in Fig. 1 for $B_d \to \mu^+\mu^-$ (left panel) and $B_s \to \mu^+\mu^-$ (right panel). From the figure one can see that for $B_d \to \mu^+\mu^-$ the allowed range of r and $\phi^{\rm NP}$ is

$$0.5 \le r \le 1.3, \quad \text{for } (0 \le \phi^{\rm NP} \le \pi/2) \quad \text{or} \quad (3\pi/2 \le \phi^{\rm NP} \le 2\pi), \tag{19}$$

which can be translated to obtain the bounds for the leptoquark couplings using Eqs. (7), (10), and (17) as

$$1.5 \times 10^{-9}\ \mathrm{GeV}^{-2} \le \frac{|\lambda^{32}\lambda^{12*}|}{M_S^2} \le 3.9 \times 10^{-9}\ \mathrm{GeV}^{-2}, \tag{20}$$

with M_S as the leptoquark mass. For the $B_s \to \mu^+\mu^-$ process for $0 \le r \le 0.1$, the entire range for $\phi^{\rm NP}$ is allowed; i.e.,

$$0 \le r \le 0.1, \quad \text{for } 0 \le \phi^{\rm NP} \le 2\pi. \tag{21}$$

However, in our analysis we will use relatively mild constraint of

$$0 \le r \le 0.35, \quad \text{with} \quad \pi/2 \le \phi^{\rm NP} \le 3\pi/2. \tag{22}$$

This gives the constraint on leptoquark couplings as

$$0 \le \frac{|\lambda^{32}\lambda^{22*}|}{M_S^2} \le 5 \times 10^{-9}\ \mathrm{GeV}^{-2} \quad \text{for } \pi/2 \le \phi^{\rm NP} \le 3\pi/2. \tag{23}$$

One can also obtain the constraint on the leptoquark couplings $|\lambda^{32}\lambda^{22*}|/M_S^2$ from the inclusive measurements $\mathrm{BR}(\bar{B}_d^0 \to X_s\mu^+\mu^-)$. However, as shown in Ref. [18], these constraints are more relaxed than those obtained from $B_s \to \mu^+\mu^-$. Thus, in our analysis we will use the constraints obtained from $B_s \to \mu^+\mu^-$.

For other leptonic decay channels, i.e., $B_{s,d} \to e^+e^-, \tau^+\tau^-$, only the experimental upper limits exist [29]. Using now the theoretical predictions for these branching ratios from [25], we obtain the constraint on the upper limits of the various combinations of leptoquark couplings as presented in Table II. However, the constraints obtained from such processes are rather weak.

For the analysis of the $B \to Ke^+e^-$ process, we need to know the values of the leptoquark couplings $\lambda^{31}\lambda^{21*}/M_S^2$, which can be extracted from the inclusive decay rates $B \to X_s e^+e^-$. To obtain such constraints, we closely follow the procedure adopted in Ref. [18], using the

TABLE II. Constraints obtained from the leptoquark couplings from various leptonic $B_{s,d} \to l^+l^-$ decays.

Decay process	Couplings involved	Upper bound of the couplings (in GeV^{-2})
$B_d \to e^\pm e^\mp$	$\frac{\lvert\lambda^{31}\lambda^{11*}\rvert}{M_S^2}$	$<1.73 \times 10^{-5}$
$B_d \to \tau^\pm\tau^\mp$	$\frac{\lvert\lambda^{33}\lambda^{13*}\rvert}{M_S^2}$	$<1.28 \times 10^{-6}$
$B_s \to e^\pm e^\mp$	$\frac{\lvert\lambda^{31}\lambda^{21*}\rvert}{M_S^2}$	$<2.54 \times 10^{-5}$
$B_s \to \tau^\pm\tau^\mp$	$\frac{\lvert\lambda^{33}\lambda^{23*}\rvert}{M_S^2}$	$<1.2 \times 10^{-8}$

SM predictions and the corresponding experimental measurements from [30] for both low q^2 [(1–6) GeV^2] and high q^2 ($\gtrsim 14.2$ GeV^2) as follows:

$$\begin{aligned}
\mathrm{BR}(B\to X_s ee)|_{q^2\in[1,6]\ \mathrm{GeV}^2} &= (1.73\pm 0.12)\times 10^{-6}\ \text{(SM prediction)}\\
&= (1.93\pm 0.55)\times 10^{-6}\ \text{(expt)}\\
\mathrm{BR}(B\to X_s ee)|_{q^2>14.2\ \mathrm{GeV}^2} &= (0.2\pm 0.06)\times 10^{-6}\ \text{(SM prediction)}\\
&= (0.56\pm 0.19)\times 10^{-6}\ \text{(expt)}. \qquad (24)
\end{aligned}$$

In the left panel of Fig. 2, we show the allowed region in the $C_{10}^{\rm NP}-\phi^{\rm NP}$ parameter space due to the exchange of the leptoquark $X(3,2,7/6)$. The right panel depicts the allowed region in the $C_9^{\prime\rm NP}-C_{10}^{\prime\rm NP}$ space due to the exchange of the leptoquark $X(3,2,1/6)$, where the green (red) regions corresponds to high-q^2 (low-q^2) limits in both panels. Thus, it can be noticed that the bounds coming from the high-q^2 measurements are rather weak. Considering the exchange of the $X(3,2,7/6)$ leptoquark as an example, we obtain the bound on $C_{10}^{\rm NP}$ as $-2.0\le C_{10}^{\rm NP}\le 3.0$ for the entire range of ϕ, which gives the bound on r as

$$0\le r\le 0.7. \qquad (25)$$

After obtaining the bounds on various leptoquark couplings, we now proceed to study the rare decays $B\to K/\pi l^+l^-$ and $B\to l_i^+l_j^-$.

IV. THE $B\to Kl^+l^-$ PROCESS

We now consider the semileptonic decay process $B\to Kl^+l^-$, which is mediated by the quark-level transition $b\to sl^+l^-$; hence, it constitutes quite a suitable tool to look for new physics. The isospin asymmetries of $B\to K\mu^+\mu^-$ and the partial branching ratios of the decays $B^0\to K^0\mu^+\mu^-$ and $B^+\to K^+\mu^+\mu^-$ have recently been measured as functions of the dimuon mass squared (q^2) by the LHCb Collaboration [31]. In this paper we will study, using the QCD factorization approach, the process in the large recoil region, i.e., $1\le q^2\le 6$ GeV^2, in order to be well below the radiative tail of the charmonium resonances [32–34]. LHCb has measured the branching ratio in this region; the updated result is [31]

$$\mathrm{BR}(B^+\to K^+\mu^+\mu^-)|_{q^2\in[1,6]\ \mathrm{GeV}^2} = (1.19\pm 0.03\pm 0.06)\times 10^{-7}. \qquad (26)$$

This mode has also been analyzed by others [35–37]; the SM prediction is given as

$$\mathrm{BR}(B^+\to K^+\mu^+\mu^-)|^{\rm SM}_{q^2\in[1,6]\ \mathrm{GeV}^2} = (1.75^{+0.60}_{-0.29})\times 10^{-7}. \qquad (27)$$

Though there is no significant discrepancy between these two results, the SM predictions are slightly higher than the experimental measurement.

To calculate the branching ratio, one uses the effective Hamiltonian presented in Eq. (3) and obtains the transition amplitude for this process. The matrix elements of the various hadronic currents between the initial B meson and the final K meson can be parameterized in terms of the form factors f_+, f_0, and f_T as [8,38]

$$\langle K(p_K)|\bar{s}\gamma_\mu b|\bar{B}(p_B)\rangle = (2p_B-q)_\mu f_+(q^2) + \frac{M_B^2-M_K^2}{q^2}q_\mu[f_0(q^2)-f_+(q^2)], \qquad (28)$$

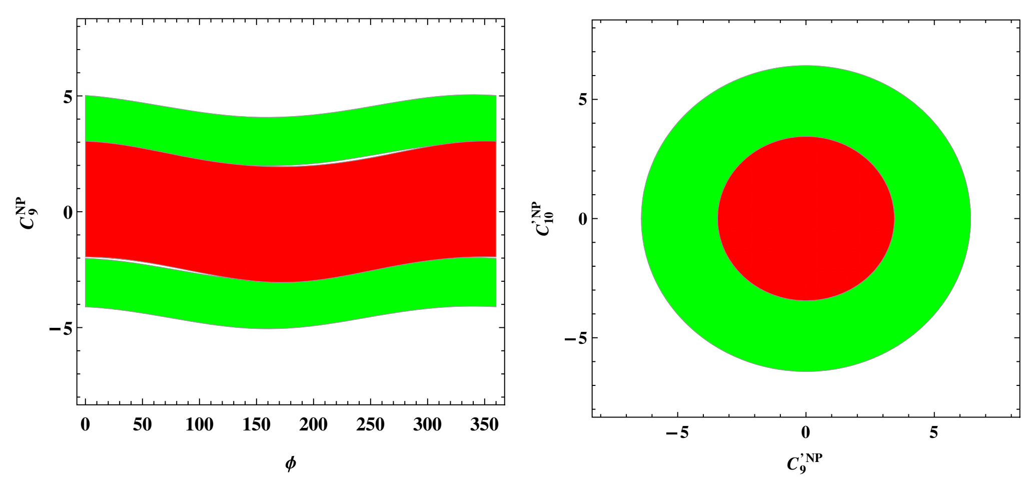


FIG. 2 (color online). The allowed regions in $C_{10}^{\rm NP}-\phi^{\rm NP}$ parameter space (left panel) and $C_9^{\prime\rm NP}-C_{10}^{\prime\rm NP}$ (right panel) obtained from $\mathrm{BR}(B\to X_s e^+e^-)$, where the green (red) region corresponds to high-q^2 (low-q^2) limits.

$$\langle K(p_K)|\bar{s}i\sigma_{\mu\nu}q^\nu b|\bar{B}(p_B)\rangle = -[(2p_B-q)_\mu q^2 - (M_B^2 - M_K^2)q_\mu]\frac{f_T(q^2)}{M_B+M_K}, \quad (29)$$

where the 4-momenta of the initial B meson and the final kaon are denoted by p_B and p_K, respectively, M_B, M_K are the corresponding masses, and q^2 is the momentum transfer. In the large recoil region, the energy of the kaon E_K is large compared to the typical size of hadronic binding energies ($\Lambda_{\rm QCD}$) and the dilepton invariant mass squared q^2 is low. As a consequence, in this region the virtual photon exchange between the hadronic part and the dilepton pair, as well as the hard gluon scattering can be treated in an expansion in $1/E_K$ [8], using either QCD factorization [33,34] or soft collinear effective theory [39]. At leading order in the $1/E_K$ expansion, all the form factors $f_{+,0,T}(q^2)$ can be related to a single form factor $\xi_P(q^2)$. Within the QCD factorization approach, the form factor $f_+(q^2)$ is chosen to be $f_+(q^2) = \xi_P(q^2)$; including subleading corrections, the other form factors can written as [8]

$$\frac{f_0}{f_+} = \frac{2E_K}{M_B}\left[1+\mathcal{O}(\alpha_s)+\mathcal{O}\left(\frac{q^2}{M_B^2}\sqrt{\frac{\Lambda_{\rm QCD}}{E_K}}\right)\right],$$

$$\frac{f_T}{f_+} = \frac{M_B+M_K}{M_B}\left[1+\mathcal{O}(\alpha_s)+\mathcal{O}\left(\sqrt{\frac{\Lambda_{\rm QCD}}{E_K}}\right)\right]. \quad (30)$$

Thus, only one soft form factor $\xi_P(q^2)$ appears in the $B\to K$ transition amplitude due to the symmetry relations in the large energy limit of QCD [32,40], and the transition amplitude for the process $B\to Kl^+l^-$ can be written as [8]

$$\mathcal{M}(\bar{B}\to K\bar{l}l) = i\frac{G_F\alpha}{\sqrt{2}\pi}V_{tb}V_{ts}^*\xi_P(q^2)[F_V p_B^\mu(\bar{l}\gamma_\mu l) + F_A p_B^\mu(\bar{l}\gamma_\mu\gamma_5 l) + F_P(\bar{l}\gamma_5 l)]. \quad (31)$$

The functions $F_{V,A,P}(q^2)$ are given as

$$F_V = C_9 + \frac{2m_b}{M_B}\frac{\mathcal{T}_P(q^2)}{\xi_P(q^2)},$$

$$F_A = C_{10},$$

$$F_P = m_l C_{10}\left[\frac{M_B^2-M_K^2}{q^2}\left(\frac{f_0(q^2)}{f_+(q^2)}-1\right)-1\right]. \quad (32)$$

The parameter $\mathcal{T}_P(q^2)$ takes into account the virtual one-photon exchange between the hadron and the lepton pair and hard scattering contribution. At lowest order, it can be expressed as

$$\mathcal{T}_P^{(0)}(q^2) = \xi_P(q^2)\left(C_7^{\rm eff(0)} + \frac{M_B}{2m_b}Y^{(0)}(q^2)\right). \quad (33)$$

The function $Y(q^2)$ denotes the perturbative part coming from the one-loop matrix elements of the four quark operators and is given in Ref. [33]. The detailed expression for $\mathcal{T}_P(q^2)$, including the subleading corrections, is presented in the Appendix.

With Eq. (31), the double differential decay rate with respect to q^2 and $\cos\theta$ for the lepton flavor l is given as [8]

$$\frac{d^2\Gamma_l}{dq^2 d\cos\theta} = a_l(q^2) + c_l(q^2)\cos^2\theta, \quad (34)$$

where

$$a_l(q^2) = \Gamma_0\sqrt{\lambda}\beta_l\xi_P^2\left[q^2|F_P|^2 + \frac{\lambda}{4}(|F_A|^2+|F_V|^2) + 2m_l(M_B^2-M_K^2+q^2){\rm Re}(F_PF_A^*) + 4m_l^2M_B^2|F_A|^2\right],$$

$$c_l(q^2) = -\Gamma_0\sqrt{\lambda}\beta_l^3\xi_P^2\frac{\lambda}{4}(|F_A|^2+|F_V|^2), \quad (35)$$

$$\lambda = M_B^4 + M_K^4 + q^4 - 2(M_B^2M_K^2 + M_B^2q^2 + M_K^2q^2),$$

$$\beta_l = \sqrt{1-4m_l^2/q^2}, \quad (36)$$

and

$$\Gamma_0 = \frac{G_F^2\alpha^2|V_{tb}V_{ts}^*|^2}{2^9\pi^5M_B^3}. \quad (37)$$

The decay rate can be expressed as

$$\Gamma_l = 2\left(A_l + \frac{1}{3}C_l\right), \quad (38)$$

where the q^2-integrated coefficients are given as

$$A_l = \int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 a_l(q^2), \qquad C_l = \int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 c_l(q^2). \quad (39)$$

The observable R_K, which is the ratio of $B\to K\mu^+\mu^-$ to $B\to Ke^+e^-$ decay rates with the same q^2 cuts, is

$$R_K \equiv \frac{\Gamma_\mu}{\Gamma_e} = \int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2\frac{d\Gamma_\mu}{dq^2} \Big/ \int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2\frac{d\Gamma_e}{dq^2}, \quad (40)$$

which probes the lepton-flavor nonuniversality effects.

Other observables can be constructed from ratios or asymmetries where the leading form factor uncertainties cancel. The CP-averaged isospin asymmetry is such an observable, defined as

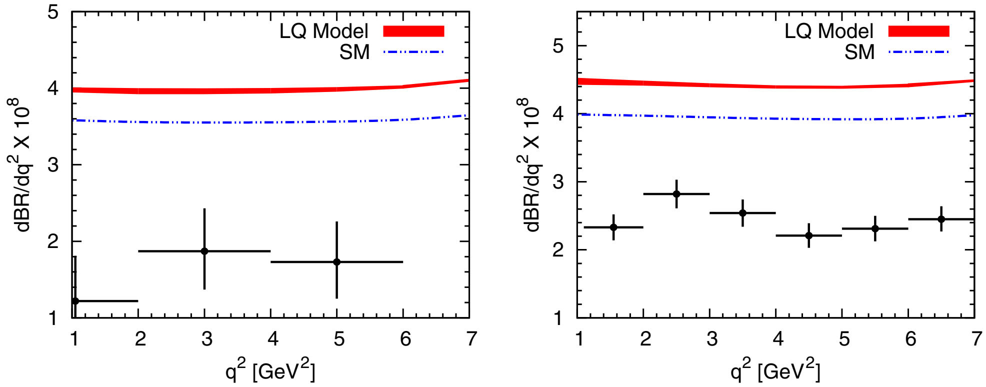


FIG. 3 (color online). The variation of branching ratios with q^2 for the decay processes $B^0 \to K^0\mu^+\mu^-$ (left panel) and $B^+ \to K^+\mu^+\mu^-$ (right panel) in the standard model and in the leptoquark model. The red band for the leptoquark model is obtained using the allowed leptoquark parameter space. The q^2-averaged (binwise) 1σ experimental results are shown by black plots, where the horizontal (vertical) line denotes the bin width (1σ error).

$$A_I(q^2) = \frac{d\Gamma(B^0 \to K^0\mu^+\mu^-)/dq^2 - d\Gamma(B^+ \to K^+\mu^+\mu^-)/dq^2}{d\Gamma(B^0 \to K^0\mu^+\mu^-)/dq^2 + d\Gamma(B^+ \to K^+\mu^+\mu^-)/dq^2}$$
$$= \frac{d\mathrm{BR}(B^0 \to K^0\mu^+\mu^-)/dq^2 - (\tau_0/\tau_+)d\mathrm{BR}(B^+ \to K^+\mu^+\mu^-)/dq^2}{d\mathrm{BR}(B^0 \to K^0\mu^+\mu^-)/dq^2 + (\tau_0/\tau_+)d\mathrm{BR}(B^+ \to K^+\mu^+\mu^-)/dq^2}. \tag{41}$$

With these formulas at hand, we now proceed with numerical estimation. To make predictions for SM observables, or to extract information about potentially new short-distance physics, one requires the knowledge of associated hadronic form factors. For this purpose we use value of the form factors $f_+(q^2) = \xi_P(q^2)$ calculated in the light-cone sum rule approach [38], where the q^2 dependence is given by simple fits as

$$f_+(q^2) = \frac{r_1}{1 - q^2/m_{\rm fit}^2} + \frac{r_2}{(1 - q^2/m_{\rm fit}^2)^2}, \tag{42}$$

and the values of the parameters used are taken from [38]. The particle masses, the lifetime of the B_s meson, and the decay constants are taken from [29], and the SM Wilson coefficients (the C_i's) are listed in Table I. For the CKM matrix elements we use the Wolfenstein parametrization with the values $A = 0.814^{+0.023}_{-0.024}$, $\lambda = 0.22537 \pm 0.00061$, $\bar\rho = 0.117 \pm 0.021$, and $\bar\eta = 0.353 \pm 0.013$ [29]. The values of the quark masses used in our analysis are as follows. The b-quark mass used in $\mathcal{T}_P$ and F_V is the potential subtracted (PS) mass $m_{\rm PS}(\mu_f)$ at the factorization scale $\mu_f \sim \sqrt{\Lambda_{\rm QCD} m_b}$ and is denoted by m_b. The function $Y(q^2)$ is evaluated by using the pole mass $m_b^{\rm pole}$, and its relation to the PS mass is given as $m_b^{\rm pole} = m_b^{\rm PS}(\mu_f) + 4\alpha_s\mu_f/3\pi$. The quark masses used are (in GeV) $m_b = 4.6$ and $m_c = 1.4$, and the fine structure coupling constant $\alpha = 1/130$. Using these values we show in Fig. 3 the variation of differential branching ratios for $B^0 \to K^0\mu^+\mu^-$ (left panel) and $B^+ \to K^+\mu^+\mu^-$ (right panel) in the standard model with respect to the dimuon invariant mass. The variations of isospin asymmetry and R_K are shown in Fig. 4. The total branching ratios integrated over the range $q^2 \in [1, 6]$ GeV^2 are summarized in Table III. Our predictions for branching fractions are in agreement with the previous predictions [35], and the slight difference can be attributed to the difference in the values of input parameters used in the calculation. These predictions, however, are slightly larger than the experimental values. The q^2-averaged isospin asymmetry, $\langle A_I(q^2)\rangle$, can be obtained from Eq. (41) by replacing the differential branching fractions with the corresponding integrated values.

In the leptoquark model, these processes will receive additional contributions arising from the leptoquark exchange; hence, the Wilson coefficients $C_{9,10}$ will receive additional contributions $C_{9,10}^{\rm NP}$. In addition, new Wilson coefficients $C'_{9,10}$ associated with the chirally flipped operators $O'_{9,10}$ will also be present, as already discussed in Sec. II. The bounds on these new Wilson coefficients can be obtained from the constraint on r [Eq. (22)] that has been extracted from the experimental results on $\mathrm{BR}(B_s \to \mu^+\mu^-)$. Thus, for the leptoquarks $X = (3, 2, 7/6)$ and $X = (3, 2, 1/6)$, we obtain the value of $r \lesssim 0.35$ for $\pi/2 \le \phi^{\rm NP} \le 3\pi/2$, which can be translated with Eqs. (7), (10), and (17) to give the value of the new Wilson coefficients as

$$|C_9^{\rm LQ}| = |C_{10}^{\rm LQ}| \le |rC_{10}^{\rm SM}| \quad [\text{for } X = (3, 2, 7/6)],$$
$$|C_9'^{\rm LQ}| = |C_{10}'^{\rm LQ}| \le |rC_{10}^{\rm SM}| \quad [\text{for } X = (3, 2, 1/6)]. \tag{43}$$

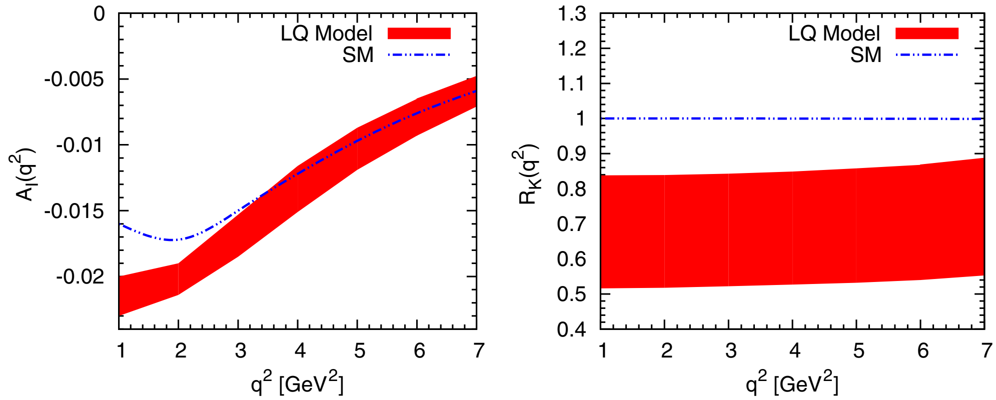


FIG. 4 (color online). The variation of isospin asymmetry (left panel) for $B \to K\mu\mu$ and of R_K (right panel) with q^2.

Using these values, we show the variation of differential branching ratios, as well as isospin asymmetry and R_K, for $X = (3, 2, 7/6)$ in Figs. 3 and 4. For the calculation of the $B^+ \to K^+ e^+ e^-$ in the determination of R_K, we have used the constraint on the leptoquark couplings obtained from the $B \to X_s ee$ inclusive decay rate. From these figures it can be seen that there is a slight deviation in the $B \to K\mu\mu$ branching ratios from their SM values. The isospin asymmetry also has a slight deviation from its SM prediction; this deviation is substantial in the low-q^2 region. The R_K value, however, deviates significantly, and it is possible to accommodate the observed experimental value in the leptoquark model. The integrated branching ratios and the isospin asymmetries are presented in Table III.

V. THE $B \to \pi ll$ PROCESS

In this section we study the decay mode $B \to \pi\mu^+\mu^-$, which is mediated by the quark-level transition $b \to dl^+l^-$. This decay mode has recently been observed by the LHCb Collaboration [41], and the measured branching ratio is

$$\mathrm{BR}(B^+ \to \pi^+\mu^+\mu^-) = (2.3 \pm 0.6(\mathrm{stat}) \pm 0.1(\mathrm{syst})) \times 10^{-8}, \tag{44}$$

at 5.2σ significance.

TABLE III. The predicted values for the integrated branching ratios and isospin asymmetry in the range $q^2 \in [1, 6]$ GeV^2 for the decay mode $B \to K\mu\mu$ and the value of R_K, in the SM and in the leptoquark model.

Observables	SM predictions	Values in LQ model
$B_d \to K^0\mu^+\mu^-$	1.82×10^{-7}	$(2.04–2.16) \times 10^{-7}$
$B^+ \to K^+\mu^+\mu^-$	1.99×10^{-7}	$(2.2–2.3) \times 10^{-7}$
$B^+ \to K^+e^+e^-$	1.82×10^{-7}	$(2.3–3.7) \times 10^{-7}$
$\langle A_I \rangle$	-0.03	$-0.036 \to -0.024$
R_K	1.09	0.62–0.96

For the calculation of the branching ratio, we closely follow [17]; here, we preview only the main results. In the standard model the effective Hamiltonian for the $b \to dl^+l^-$ transition is given by

$$\mathcal{H}_{\mathrm{eff}} = -\frac{G_F}{\sqrt{2}}[\lambda_t^{(d)}\mathcal{H}_{\mathrm{eff}}^{(t)} + \lambda_u^{(d)}\mathcal{H}_{\mathrm{eff}}^{(u)}] + \mathrm{H.c.}, \tag{45}$$

where $\lambda_q^{(d)} = V_{qb}V_{qd}^*$ and

$$\mathcal{H}_{\mathrm{eff}}^{(u)} = C_1(O_1^c - O_1^u) + C_u(O_2^c - O_2^u),$$
$$\mathcal{H}_{\mathrm{eff}}^{(t)} = C_1 O_1^c + C_2 Q_2^c + \sum_{i=3}^{10} C_i O_i. \tag{46}$$

It should be noted that for $b \to dl^+l^-$ transitions, $\lambda_u^{(d)}$ and $\lambda_t^{(d)}$ are comparable in magnitude with the phase difference $\phi_2 = \arg(-V_{td}V_{tb}^*/V_{ud}V_{ub}^*)$. Thus, one can write the transition amplitude for the process $B \to \pi l^+l^-$ as

$$\mathcal{M}(B(p) \to \pi(p')l^+l^-) = \frac{G_F\alpha}{2\sqrt{2}\pi}c_\pi^{-1}\xi_\pi[(\lambda_t C_{9,\pi}^{(t)} + \lambda_u C_{9,\pi}^{(u)})(p+p')^\mu(\bar{l}\gamma_\mu l) + \lambda_t C_{10}(p+p')^\mu(\bar{l}\gamma_\mu\gamma_5 l)], \tag{47}$$

where $c_\pi = \sqrt{2}$ for π^0 and 1 for $\pi^\pm$, and

$$C_{9,\pi}^{(t)}(q^2) = C_9 + \frac{2m_b}{M_B}\frac{\mathcal{T}_\pi^{(t)}(q^2)}{\xi_\pi(q^2)},$$
$$C_{9,\pi}^{(u)}(q^2) = \frac{2m_b}{M_B}\frac{\mathcal{T}_\pi^{(u)}(q^2)}{\xi_\pi(q^2)}. \tag{48}$$

The differential branching ratio is given as

$$\frac{d\mathrm{BR}}{dq^2}(B\to\pi l^+l^-) = S_\pi\tau_B\frac{G_F^2M_B^3}{96\pi^3}\left(\frac{\alpha}{4\pi}\right)^2\lambda_\pi^3\xi_\pi(q^2)^2|\lambda_t|^2 \times(|C_{9,\pi}^{(t)}(q^2)-R_{ut}e^{i\phi_2}C_{9,\pi}^{(u)}(q^2)|^2+|C_{10}|^2), \tag{49}$$

where

$$\lambda_\pi(q^2,m_\pi^2)=\left[\left(1-\frac{q^2}{M_B^2}\right)^2-\frac{2m_\pi^2}{M_B^2}\left(1+\frac{q^2}{M_B^2}\right)+\frac{m_\pi^4}{M_B^4}\right]^{1/2} \tag{50}$$

with $S_\pi=1/c_\pi^2$ and $\lambda_u^{(d)}/\lambda_t^{(d)}=-R_{ut}e^{i\phi_2}$. The branching ratio for the CP conjugate mode can be obtained by changing the sign of the weak phase ϕ_2. One can then define the q^2 dependence of the direct CP asymmetries as

$$A_{CP}^+(q^2)=\frac{d\mathrm{BR}(B^-\to\pi^-ll)/dq^2-d\mathrm{BR}(B^+\to\pi^+ll)/dq^2}{d\mathrm{BR}(B^-\to\pi^-ll)/dq^2+d\mathrm{BR}(B^+\to\pi^+ll)/dq^2},$$
$$A_{CP}^0(q^2)=\frac{d\mathrm{BR}(\bar{B}^0\to\pi^0ll)/dq^2-d\mathrm{BR}(B^0\to\pi^0ll)/dq^2}{d\mathrm{BR}(\bar{B}^0\to\pi^0ll)/dq^2+d\mathrm{BR}(B^0\to\pi^0ll)/dq^2}. \tag{51}$$

The q^2-dependent isospin asymmetry is defined as

$$A_I(q^2)=\frac{\tau_{B^0}}{2\tau_{B^\pm}}\frac{d\overline{\mathrm{BR}}(B^+\to\pi^+l^+l^-)/dq^2}{d\overline{\mathrm{BR}}(B^0\to\pi^0l^+l^-)/dq^2}-1, \tag{52}$$

where $\overline{\mathrm{BR}}$ is the CP-averaged branching ratio.

The $B\to\pi$ form factor can be obtained using the light-cone QCD sum rule approach

$$\xi_\pi(q^2)=\frac{\xi_\pi(0)}{(1-q^2/m_{B^*}^2)(1-\alpha_{BK}q^2/m_B^2)}, \tag{53}$$

where the numerical value for the normalization constant is $\xi_\pi(0)=0.26_{-0.03}^{+0.04}$ and the slope parameter $\alpha_{BK}=0.53\pm0.06$. The light-cone distribution amplitude is given by

$$\phi_\pi(u)=6u(1-u)[1+a_2^\pi C_2^{(3/2)}(2u-1) + a_4^\pi C_4^{(3/2)}(2u-1)+\cdots], \tag{54}$$

where $C_n^{3/2}(x)$ are Gegenbauer polynomials and the coefficients a_i^π are related to the moments of distribution amplitudes. The numerical values of these coefficients are $a_2^\pi=0.25\pm0.15$, $a_2^\pi+a_4^\pi=0.1\pm0.1$, as given in Refs. [17,38].

The B meson light-cone distribution amplitudes can be given as

$$\Phi_{B,+}(\omega)=\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0},\qquad \Phi_{B,-}(\omega)=\frac{1}{\omega_0}e^{-\omega/\omega_0}, \tag{55}$$

with $\omega_0=2\bar{\Lambda}_{\mathrm{HQET}}/3$ and $\bar{\Lambda}_{\mathrm{HQET}}=m_B-m_b$. These enter only through the moments

$$\lambda_{B,+}^{-1}=\int_0^\infty d\omega\frac{\Phi_{B,+}(\omega)}{\omega}=\omega_0^{-1}, \tag{56}$$

$$\lambda_{B,-}^{-1}(q^2)=\int_0^\infty d\omega\frac{\Phi_{B,-}(\omega)}{\omega-q^2/M_B-i\epsilon} =\frac{e^{-q^2/M_B\omega_0}}{\omega_0}[-Ei(q^2/M_B\omega_0)+i\pi], \tag{57}$$

where $Ei(z)$ is the exponential integral function. Using these formulas, we show in Fig. 5 the differential branching ratios for $\bar{B}^0\to\pi^0\mu^+\mu^-$ (left panel) and $B^-\to\pi^-\mu^+\mu^-$ (right panel) in both the SM and in the leptoquark model, for which we have used the constraints on leptoquark couplings as extracted from $B_d\to\mu^+\mu^-$ [Eq. (19)]. In this case, the branching ratios in the leptoquark model have significant deviations from their corresponding SM values.

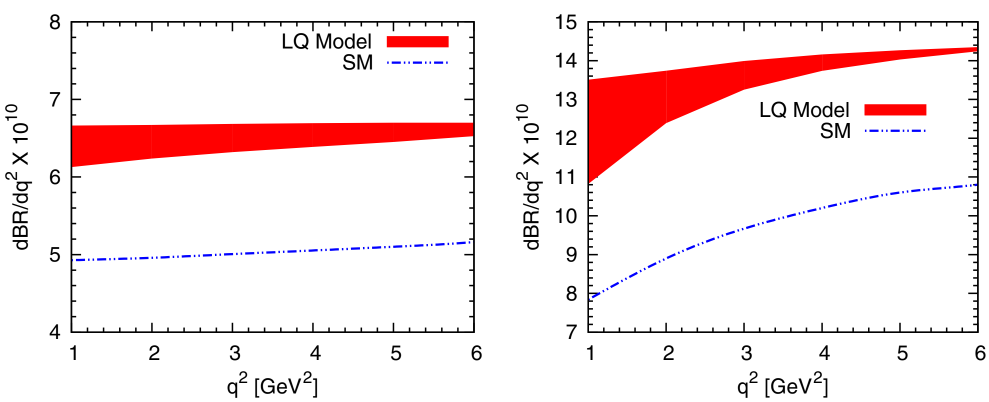


FIG. 5 (color online). The differential branching ratios for $\bar{B}^0\to\pi^0\mu^+\mu^-$ (left panel) and $B^-\to\pi^-\mu^+\mu^-$ (right panel) in the SM and the leptoquark model.

TABLE IV. The predicted branching ratios, q^2-averaged CP asymmetries, and the isospin asymmetry for the $B \to \pi\mu^+\mu^-$ process.

Observables	SM predictions	Values in LQ model
$B_d \to \pi^0\mu^+\mu^-$	2.6×10^{-9}	$(3.2\text{–}3.4) \times 10^{-9}$
$B^+ \to \pi^+\mu^+\mu^-$	5.6×10^{-9}	$(7.2\text{–}7.3) \times 10^{-9}$
$\langle A_{CP}^0\rangle$	-0.103	$-0.04 \to -0.065$
$\langle A_{CP}^+\rangle$	-0.268	$-0.11 \to -0.06$
$\langle A_I\rangle$	0.078	0.04–0.07

The integrated branching ratios in the range $q^2 \in [1, 6]$ GeV^2 are presented in Table IV. Similarly, the variation of CP asymmetries are shown in Fig. 6, and the CP asymmetries averaged over the q^2 range are given in Table IV. The variation of isospin asymmetry is shown in the left panel of Fig. 7.

The ratio of the branching ratios of $B^+ \to \pi^+\mu^+\mu^-$ and $B^+ \to K^+\mu^+\mu^-$ has recently been measured by LHCb experiments [41] as

$$\frac{\mathrm{BR}(B^+ \to \pi^+\mu^+\mu^-)}{\mathrm{BR}(B^+ \to K^+\mu^+\mu^-)} = 0.053 \pm 0.014(\mathrm{stat}) \pm 0.001(\mathrm{syst}). \tag{58}$$

We define

$$R_+(q^2) = \frac{d\mathrm{BR}(B^+ \to \pi^+ ll)/dq^2}{d\mathrm{BR}(B^+ \to K^+ ll)/dq^2} \tag{59}$$

and show the variation of $R_+(q^2)$ with dimuon invariant mass in the right panel of Fig. 7.

VI. LEPTON-FLAVOR-VIOLATING DECAYS $B_{s,d} \to l_i^+ l_j^-$

It is very well known that in the standard model the family lepton numbers (L_e, L_μ, L_τ) are exactly conserved. However, the experimental observation of neutrino oscillation implies that the family lepton numbers are no longer conserved quantum numbers and must be violated. Because of the violation of these lepton numbers, FCNC processes in the lepton sector could in principle occur, analogous to the quark sector. Some examples of FCNC transitions in the lepton sector are $l_i \to l_j\gamma$, $l_i \to l_j l_k \bar{l}_k$, and $B \to l_i \bar{l}_j$, etc., where l_i is any charged lepton. Though no direct conclusive experimental evidence for such processes has so far been observed, severe constraints exist on some of these lepton-flavor violation (LFV) decay modes [29]. The LFV decays are well studied in the literature in various scenarios beyond the standard model. Here we would like to investigate the effect of scalar leptoquarks in predicting the branching ratios for the LFV decays $B_{s,d} \to l_i^+ l_j^-$. These decay modes have been previously investigated in [42].

The effective Hamiltonian for the $B_{s,d} \to l_i^+ l_j^-$ process will have a structure analogous to that of $B_{s,d} \to l^+ l^-$, which is given in the leptoquark model as

$$\mathcal{H}_{\mathrm{LQ}} = [G_V(\bar{s}\gamma^\mu P_L b)\bar{l}_i\gamma_\mu l_j + G_A(\bar{s}\gamma^\mu P_L b)\bar{l}_i\gamma_\mu\gamma_5 l_j], \tag{60}$$

where the constants G_V and G_A are given as

$$G_V = G_A = \frac{\lambda^{j3}\lambda^{i2*} + \lambda^{j2}\lambda^{i3*}}{8M_Y^2}. \tag{61}$$

Here we consider the exchange of the leptoquark as $X(3, 2, 7/6)$; for $X(3, 2, 1/6)$, one will have operators that are chirality flipped.

This gives the branching ratio as

$$\begin{aligned} \mathrm{BR}(B_{s,d} \to l_i^+ l_j) &= \frac{|\mathbf{p}|}{4\pi m_B^2}|F_V f_{B_q}|^2[(m_j - m_i)^2(m_B^2 - (m_i + m_j)^2) \\ &\quad + (m_j + m_i)^2(m_B^2 - (m_i - m_j)^2)], \end{aligned} \tag{62}$$

where

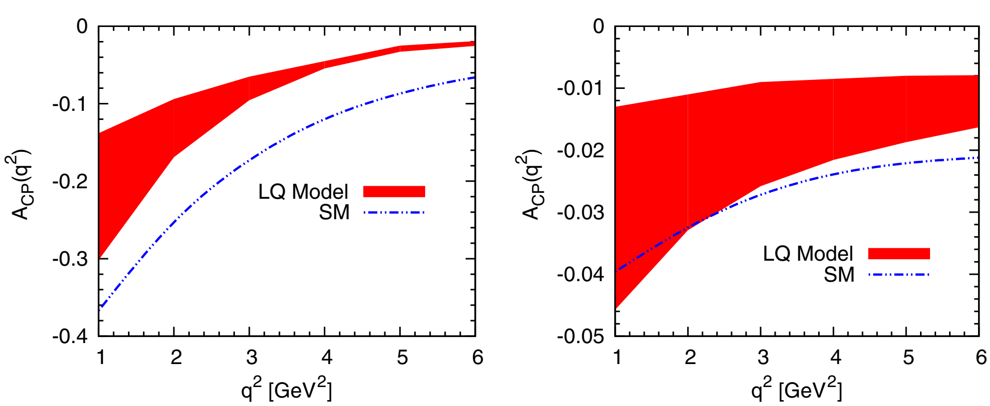


FIG. 6 (color online). Variation of $A_{CP}(q^2)$ for $B^0(\bar{B}^0) \to \pi^0\mu^+\mu^-$ (left panel) and $B^\pm \to \pi^\pm\mu^+\mu^-$ (right panel).

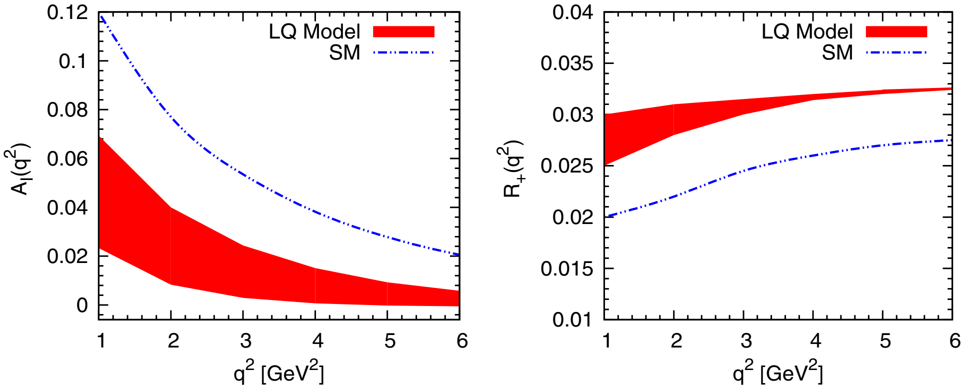


FIG. 7 (color online). The variation of isospin asymmetry for the $B \to \pi\mu^+\mu^-$ process and $R_+(q^2)$ in both the SM and the LQ model.

$$|\mathbf{p}| = \frac{\sqrt{(m_B^2 - m_i^2 - m_j^2)^2 - 4m_i^2 m_j^2}}{2m_B} \tag{63}$$

is the center-of-mass momentum of the outgoing leptons in the initial $B_{s,d}$ rest frame.

For numerical estimation we need to know the values of the different couplings involved in the expression for the branching ratio. We assume that the leptoquarks have full-strength coupling to a lepton and a quark of the same generation, and that the coupling with the quarks and leptons of different generations are assumed to be Cabibbo suppressed. We use the values of these couplings extracted from the leptonic decays $B_{s,d} \to \mu^+\mu^-$ as the benchmark values, and determine the other required couplings assuming that the couplings between different generation of quarks and leptons follow the simple scaling law, i.e., $\lambda^{ij} = (m_i/m_j)^{1/4}\lambda^{ii}$ with $j > i$. This assumption follows from the fact that in the quark sector the expansion parameter of the CKM matrix in the Wolfenstein parameterization can be related to the down-type quark masses as $\lambda \sim \sqrt{m_d/m_s}$ whereas in the lepton sector one can have the same order for λ with the relation $\lambda \sim (m_e/m_\mu)^{1/4}$. With this simple ansatz, the predicted values of the branching ratios for various LFV decays are listed in Table V, which are consistent with present experimental upper limits [29].

TABLE V. The predicted branching ratios for various lepton-flavor violating $B_{s,d}$ decays.

Decay process	Couplings involved	Predicted BR	Expt. upper limit [29]
$B_d \to \mu^\pm e^\mp$	$\frac{\lambda^{31}\lambda^{12*}}{M_S^2}$	$(9.5 \times 10^{-13} - 6.4 \times 10^{-12})$	$<2.8 \times 10^{-9}$
	$\frac{\lambda^{32}\lambda^{11*}}{M_S^2}$	$(2.0 \times 10^{-10} - 1.3 \times 10^{-9})$	
$B_d \to \mu^\pm \tau^\mp$	$\frac{\lambda^{32}\lambda^{13*}}{M_S^2}$	$(7.5 \times 10^{-10} - 5.1 \times 10^{-9})$	$<2.2 \times 10^{-5}$
	$\frac{\lambda^{33}\lambda^{12*}}{M_S^2}$	$(1.3 \times 10^{-8} - 8.5 \times 10^{-8})$	
$B_d \to e^\pm \tau^\mp$	$\frac{\lambda^{31}\lambda^{13*}}{M_S^2}$	$(5.2 \times 10^{-11} - 3.5 \times 10^{-10})$	$<2.8 \times 10^{-5}$
	$\frac{\lambda^{33}\lambda^{11*}}{M_S^2}$	$(1.8 \times 10^{-7} - 1.2 \times 10^{-6})$	
$B_s \to \mu^\pm e^\mp$	$\frac{\lambda^{32}\lambda^{21*}}{M_S^2}$	$<1.5 \times 10^{-11}$	$<1.1 \times 10^{-8}$
	$\frac{\lambda^{31}\lambda^{22*}}{M_S^2}$	$<3.2 \times 10^{-9}$	
$B_s \to \mu^\pm \tau^\mp$	$\frac{\lambda^{32}\lambda^{23*}}{M_S^2}$	$<1.2 \times 10^{-8}$	...
	$\frac{\lambda^{33}\lambda^{22*}}{M_S^2}$	$<2.0 \times 10^{-7}$	
$B_s \to e^\pm \tau^\mp$	$\frac{\lambda^{31}\lambda^{23*}}{M_S^2}$	$<8.5 \times 10^{-10}$	...
	$\frac{\lambda^{33}\lambda^{21*}}{M_S^2}$	$<2.9 \times 10^{-6}$	

VII. CONCLUSION

In this paper we have studied the effect of the scalar leptoquarks in the rare decays of $B \to K l^+ l^-$ and $B \to \pi\mu^+\mu^-$, and the lepton-flavor-violating decays $B \to l_i^+ l_j^-$. We considered the simple renormalizable leptoquark models in which proton decay is prohibited at tree level. The leptoquark parameter space has been constrained using the recent measurements on $\mathrm{BR}(B_{s,d} \to \mu^+\mu^-)$ and the value of $\mathrm{BR}(\bar{B}_d^0 \to X_s e^+ e^-)$. Using such parameters, we obtained the bounds on the product of leptoquark couplings and then estimated the branching ratios and isospin asymmetries for the $B \to K\mu^+\mu^-$ process. We found that the observed anomaly of R_K can be explained in the leptoquark model. This is because the couplings of leptoquarks are family dependent, and one can have lepton-flavor interaction in this model. For $B \to \pi\mu^+\mu^-$, we studied the effect of leptoquarks on branching ratios, CP asymmetry parameters, the isospin asymmetry parameter A_I, and the R_+ parameter that corresponds to the ratio of the branching ratios of $B^+ \to \pi^+\mu^+\mu^-$ and $B^+ \to K^+\mu^+\mu^-$. For $B \to \pi\mu^+\mu^-$ decays, these observables deviate significantly from their corresponding SM values. We have also obtained the branching ratios for various lepton-flavor-violating decays $B \to l_i^+ l_j^-$. Some of these decay modes, e.g., $B_d \to \mu^\pm\tau^\mp$, are expected to have branching ratios that are within the reach of LHCb, the observation of which would provide the hints for possible existence of leptoquarks.

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APPENDIX: AMPLITUDE FOR THE $B \to P\gamma^*$ PROCESS

Here we will present the expressions for $B \to P\gamma^*$ amplitudes from Refs. [33,34]. Including corrections $\mathcal{O}(\alpha_s)$, the $B \to P\gamma^*$ amplitude in the heavy quark limit is given by

$$\mathcal{T}_P^{(i)} = \xi_P C_P^{(i)} + \zeta_P \sum_{\pm} \int_0^\infty \frac{d\omega}{\omega} \Phi_{B,\pm}(\omega) \times \int_0^1 du \phi_P(u) T_{p,\pm}^{(i)}(u,\omega), \tag{A1}$$

where

$$\zeta_P = \frac{\pi^2}{N_C} \frac{f_B f_P}{M_B}. \tag{A2}$$

The expressions for the coefficient functions $C_P^{(i)}$ and $\mathcal{T}_{P,\pm}^{(i)}$ are given as

$$C_P^{(i)} = C_P^{(0,i)} + \frac{\alpha_s C_F}{4\pi} C_P^{(1,i)}, \tag{A3}$$

$$T_{P,\pm}^{(i)}(u,\omega) = T_{P,\pm}^{(0,i)}(u,\omega) + \frac{\alpha_s C_F}{4\pi} T_{P,\pm}^{(1,i)}(u,\omega), \tag{A4}$$

and $i = t, u$. The $B \to P\gamma^*$ amplitude can be related to the $B \to V_\parallel \gamma^*$ amplitude as

$$C_P^{(i)} = -C_\parallel^{(i)}, \qquad T_{P,\pm}^{(i)}(u,\omega) = -T_{\parallel,\pm}^{(i)}(u,\omega). \tag{A5}$$

The expressions for the $\bar{B} \to P\gamma^*$ amplitudes are

$$\mathcal{T}_P^{(t)} = \xi_P \left(C_P^{(0,t)} + \frac{\alpha_s C_F}{4\pi} [C_P^{(f,t)} + C_P^{(nf,t)}] \right) + \zeta_P \lambda_{B,-}^{-1} \int du \phi_P(u) \hat{T}_{P,-}^{(0,t)} + \frac{\alpha_s C_F}{4\pi} \zeta_P \left(\lambda_{B,+}^{-1} \int du \phi_P(u) [T_{P,+}^{(f,t)}(u) + T_{P,+}^{(nf,t)}(u)] + \lambda_{B,-}^{-1} \int du \phi_P(u) \hat{T}_{P,-}^{(nf,t)}(u) \right), \tag{A6}$$

$$\mathcal{T}_P^{(u)} = \xi_P \left(C_P^{(0,u)} + \frac{\alpha_s C_F}{4\pi} [C_P^{(nf,u)}] \right) + \zeta_P \lambda_{B,-}^{-1} \int du \phi_P(u) \hat{T}_{P,-}^{(0,u)} + \frac{\alpha_s C_F}{4\pi} \zeta_P \left(\lambda_{B,+}^{-1} \int du \phi_P(u) T_{P,+}^{(nf,u)}(u) + \lambda_{B,-}^{-1} \int du \phi_P(u) \hat{T}_{P,-}^{(nf,u)}(u) \right), \tag{A7}$$

where use has been made of

$$T_{P,-}^{(0,i)}(u,\omega) = \frac{M_B \omega}{M_B \omega - q^2 - i\epsilon} \hat{T}_{P,-}^{(0,i)}, \qquad T_{P,-}^{(nf,i)}(u,\omega) = \frac{M_B \omega}{M_B \omega - q^2 - i\epsilon} \hat{T}_{P,-}^{(nf,i)}(u). \tag{A8}$$

The form factor terms including the $\mathcal{O}(\alpha_s^0)$ contributions are

$$C_P^{(0,t)} = C_7^{\mathrm{eff}} + \frac{M_B}{2m_b} Y(q^2), \qquad C_P^{(0,u)} = \frac{M_B}{2m_b} Y^{(u)}(q^2), \tag{A9}$$

where

$$Y^{(u)}(q^2) = \left(\frac{4}{3}C_1 + C_2\right)[h(s, m_c) - h(s, 0)]. \quad \text{(A10)}$$

The first-order corrections $C_P^{(1,i)}$ are divided into a factorizable and a nonfactorizable term and can be written as

$$C_P^{(1,i)} = C_P^{(f,i)} + C_P^{(nf,i)}. \quad \text{(A11)}$$

The factorizable and nonfactorizable terms including $\mathcal{O}(\alpha_s)$ corrections are

$$C_P^{(f,t)} = \left(\ln\frac{m_b^2}{\mu^2} + 2L + \Delta M\right)C_7^{\rm eff}, \quad \text{(A12)}$$

where L and ΔM are defined in [33,34], and

$$C_F C_P^{(nf,t)} = -\bar{C}_2 F_2^{(7)} - C_8^{\rm eff} F_8^{(7)} - \frac{M_B}{2m_b}\left[\bar{C}_2 F_2^{(9)} + 2\bar{C}_1\left(F_1^{(9)} + \frac{1}{6}F_2^{(9)}\right) + C_8^{\rm eff}F_8^{(9)}\right], \quad \text{(A13)}$$

$$C_F C_P^{(nf,u)} = -\bar{C}_2(F_2^{(7)} + F_{2,u}^{(7)}) - \frac{M_B}{2m_b}\left[\bar{C}_2(F_2^{(9)} + F_{2,u}^{(9)}) + 2\bar{C}_1\left((F_1^{(9)} + F_{1,u}^{(9)}) + \frac{1}{6}(F_2^{(9)} + F_{(2,u)}^{(9)})\right)\right]. \quad \text{(A14)}$$

The longitudinal amplitude receives a contribution from weak annihilation topology, where the photon couples to the spectator quark in the B meson, and the $\mathcal{O}(\alpha_s^0)$ contributions to hard-spectator scattering from the weak annihilation diagrams are

$$\hat{T}_{P,-}^{(0,t)} = e_q\frac{4M_B}{m_b}C_q^{34}, \qquad \hat{T}_{P,-}^{(0,u)} = -e_q\frac{4M_B}{m_b}C_q^{12}, \quad \text{(A15)}$$

where e_q is the charge of the spectator quark $q = u, d$ in the B meson, and

$$C_q^{34} = C_3 + \frac{4}{3}(C_4 + 12C_5 + 16C_6),$$
$$C_q^{12} = 3\delta_{qu}C_2 - \delta_{qd}\left(\frac{4}{3}C_1 + C_2\right). \quad \text{(A16)}$$

The first-order corrections can also be divided into a factorizable and a nonfactorizable term, as

$$\mathcal{T}_{P,\pm}^{(1,i)} = \mathcal{T}_{P,\pm}^{(f,i)} \pm \mathcal{T}_{P,\pm}^{(nf,i)}. \quad \text{(A17)}$$

The factorizable and nonfactorizable terms including $\mathcal{O}(\alpha_s)$ corrections to the hard-spectator scattering term are given by

$$T_{P,+}^{(f,t)}(u) = -C_7^{\rm eff}\frac{4M_B}{\bar{u}E}, \quad \text{(A18)}$$

$$T_{P,+}^{(nf,t)}(u) = -\frac{M_B}{m_b}[e_u t_\parallel(u.m_c)(\bar{C}_2 + \bar{C}_4 - \bar{C}_6) + e_d t_\parallel(u, m_b)(\bar{C}_3 + \bar{C}_4 - \bar{C}_6) + e_d t_\parallel(u, 0)\bar{C}_3], \quad \text{(A19)}$$

$$T_{P,+}^{(nf,u)}(u) = -e_u\frac{M_B}{m_b}\left(C_2 - \frac{1}{6}C_1\right)[t_\parallel(u, m_c) - t_\parallel(u, 0)], \quad \text{(A20)}$$

$$\hat{T}_{P,-}^{(nf,t)}(u) = -e_q\left[\frac{8C_8^{\rm eff}}{\bar{u} + uq^2/M_B^2} + \frac{6M_B}{m_b}\left\{h(\bar{u}M_B^2 + uq^2, m_c)(\bar{C}_2 + \bar{C}_4 + \bar{C}_6) + h(\bar{u}M_B^2 + uq^2, m_b)(\bar{C}_3 + \bar{C}_4 + \bar{C}_6) + h(\bar{u}M_B^2 + uq^2, 0)(\bar{C}_3 + 3\bar{C}_4 + 3\bar{C}_6) - \frac{8}{27}(\bar{C}_3 - \bar{C}_5 - 15\bar{C}_6)\right\}\right], \quad \text{(A21)}$$

$$\hat{T}_{P,-}^{(nf,t)}(u) = -e_q\frac{6M_B}{m_b}\left(C_2 - \frac{1}{6}C_1\right)[h(\bar{u}M_B^2 + uq^2, m_c) - h(\bar{u}M_B^2 + uq^2, 0)], \quad \text{(A22)}$$

where $\bar{C}_i$ (for $i = 1, \ldots, 6$) are defined by

$$\bar{C}_1 = \frac{1}{2}C_1, \quad \bar{C}_2 = C_2 - \frac{1}{6}C_1, \quad \bar{C}_3 = C_3 - \frac{1}{6}C_4 + 16C_5 - \frac{8}{3}C_6,$$
$$\bar{C}_4 = \frac{1}{2}C_4 + 8C_6, \quad \bar{C}_5 = C_3 - \frac{1}{6}C_4 + 4C_5 - \frac{2}{3}C_6, \quad \bar{C}_6 = \frac{1}{2}C_4 + 2C_6. \quad \text{(A23)}$$

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