Discerning new physics in charm meson leptonic and semileptonic decays

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Current experimental information on the charm-meson decay observables in which the $c \rightarrow s\ell\nu$ transitions occur is well compatible with the Standard Model predictions. Recent precise lattice calculations of the D_s -meson decay constant and form factors in $D \to K\ell\nu$ decays offer a possibility to search for small deviations from the Standard Model predictions in the next generation of high-intensity flavor experiments. We revisit constraints from these processes on the new physics contributions in the effective theory approach. We investigate new physics effects which might appear in the differential distributions for the longitudinally and transversely polarized K^* in $D \to K^* \ell \nu_\ell$ decays. Present constraints from these observables are rather weak, but could be used to constrain new physics effects in the future. In the case of $D \to K\ell\nu$ we identify observables sensitive to the new physics contribution coming from the scalar Wilson coefficient, namely the forward-backward asymmetry and the transversal muon polarizations. By assuming that new physics only modifies the second lepton generation, we identify the allowed region for the differential decay rate for the process $D \to K\mu\nu_\mu$ and find that it is allowed to deviate from the Standard Model prediction by only a few percent. The lepton flavor universality violation can be tested in the ratio $R_{\mu/e}(q^2) \equiv \frac{d\Gamma^{(\mu)}}{dq^2} / \frac{d\Gamma^{(\epsilon)}}{dq^2}$. If the first lepton generation behaves as in the Standard Model, we find (using the current constraint on the scalar Wilson coefficient) that the ratio $R_{\mu/e}(q^2)$ is currently allowed to be within the range (0.9,1.2), depending on the value of q^2 .

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I. INTRODUCTION

After the discovery of the Higgs boson, the main task of the LHC became the search for particles that do not belong to the Standard Model (SM). An alternative way to investigate the presence of physics beyond the SM is to explore results from high-precision experiments at low energies. Namely, a very accurate theoretical handling of processes at low energies enables the extraction of constraints on possible new physics (NP) effects in these processes. At low energies, processes driven by flavorchanging neutral currents were usually considered to be the best candidates to detect NP. However, the recent indications of the difference between the experimental result for the branching fractions for $B \to D^{(*)}\tau \nu_{\tau}$ and the theoretical predictions (see, e.g., Refs. [\[1,2\]](#page-8-0)) open a new window in searches for new physics at low energies in the processes induced by the charged currents. The $c \rightarrow s\ell\nu_{\ell}$ transition within charm mesons might offer important tests of the SM and nonperturbative QCD dynamics in particular. In the past few years, a significant effort has been made in both the theoretical and experimental study of these transitions. The precise value of the decay constant of the D_s meson is

now known from unquenched lattice QCD simulations that involve the effects of dynamical up, down, strange, and charm quarks [\[3\]](#page-8-1). The shapes of the semileptonic form factors $f_{+,0}(q^2)$ for the process $D \to K\ell\nu$ over the whole physical q^2 region were also recently calculated in lattice QCD [\[4\]](#page-8-2). On the experimental side, several new measurements of relevant branching fractions and the extraction of form-factor shapes have been performed. The Belle Collaboration recently made precision measurements of the branching fractions of leptonic modes $D_s \rightarrow \ell \nu$, where $l = \mu$, τ [\[5\]](#page-8-3). The results of measurements of the branching fractions and the form-factor shapes for the process $D \rightarrow K \ell \nu$ were reported by the FOCUS, Belle, BABAR, and CLEO collaborations [6–[10\]](#page-8-4). The analogous experimental results for the process $D \to K^*\ell\nu$ were presented in Refs. [\[11](#page-8-5)–14].

The theoretical predictions within the SM can be compared to the measured values of the total or differential branching fractions in order to extract the $|V_{cs}|$ element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. On the other hand, the constraints on the effects of NP in a given process can be derived after fixing the value of the CKM matrix element from some independent source. In 2007, the $c \rightarrow s\ell\nu$ transitions attracted a lot of interest from the point of view of searches for NP, after the disagreement between the lattice evaluations of the decay constant f_{D_s} and experimental extractions thereof at the level of around

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4σ. Several different NP scenarios were considered as explanations of that puzzle [15–[17\].](#page-8-6) Current agreements between the experimental results and the lattice evaluations offer an opportunity for a derivation of tight constraints on the NP effects in these processes. A recent analysis of this kind was performed in Refs. [\[18,19\]](#page-8-7), where the authors studied the leptonic $D_s \rightarrow \ell \nu_\ell$ and semileptonic decays $D \to K\ell\nu_{\ell}$ within the effective theory approach using two specific models. In the present article we concentrate mainly on the nonstandard (pseudo)scalar operators and include a discussion of the observables in the decays $D \to V \ell \nu$, $V = K^*$, ϕ . In Sec. [II](#page-1-0) we introduce the effective Lagrangian. Section [III](#page-1-1) is devoted to constraints on the Wilson coefficient of the pseudoscalar operator coming from the leptonic $D_s \to \ell \nu$ and semileptonic $D \to K^* \ell \nu$ decay modes. Section [IV](#page-4-0) contains the analyses of constraints on the Wilson coefficient arising from the scalar operator coming from $D \to K\ell\nu$. The branching ratios, the differential branching ratio, the forward-backward asymmetry, and the transversal muon polarizations in this process are considered. Section [V](#page-7-0) contains a brief study of the right-handed current, and the conclusions are given in Sec. [VI.](#page-7-1)

II. THE EFFECTIVE LAGRANGIAN DESCRIBING NP IN $c \rightarrow s \ell v_{\ell}$ TRANSITIONS

We assume that the relevant NP states are considerably heavier than the typical hadronic energy scale so that they can be integrated out, together with the W boson, leading to the appearance of nonstandard higher-dimensional operators in the low-energy effective description of $c \rightarrow s\ell\nu$ transitions. We choose the following normalization of the effective Lagrangian:

$$
\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{cs}^* \sum_{\ell=e,\mu,\tau} \sum_i c_i^{(\ell)} \mathcal{O}_i^{(\ell)} + \text{H.c.} \qquad (1)
$$

The usual four-fermion operator is $\mathcal{O}_{\text{SM}}^{(\ell)} = (\bar{s}\gamma_{\mu}P_Lc)$ $(\bar{\nu}_e \gamma^{\mu} P_L \ell)$ with the coefficient $c_{\text{SM}}^{(\ell)} = 1$. In this article we concentrate on the nonstandard effective operators that involve the (pseudo)scalar quark and lepton densities, while keeping only the left-handed neutrinos, namely

$$
\mathcal{O}_{R(L)}^{(\ell)} = (\bar{s}P_{R(L)}c)(\bar{\nu}_{\ell}P_R\ell). \tag{2}
$$

These operators might be induced by integrating out the beyond-the-SM charged scalar boson at the tree level. Such a boson can arise in a two-Higgs-doublet model (THDM), the extension of the SM with an additional scalar doublet; c.f. the review article [\[20\]](#page-8-8). The most studied model of this kind is the so-called type-II THDM, in which $c_{R(L)}^{(\ell)}$ can be expressed as a combination of the two real parameters: the mass of the charged scalar m_{H^+} and tan β , the ratio of the vacuum expectation values of the two doublets. Since it has a small number of free parameters, this model is actually tightly constrained by the flavor phenomenology and the new LHC results [\[21,22\]](#page-8-9). For generality, we allow the coefficients $c_{R(L)}^{(\ell)}$ to be complex valued and to depend on the flavor of the charged lepton. For example, an additional dependence (besides the factor of m_e) on the charged lepton's flavor is present in the type-III THDM [\[23\]](#page-8-10), originating from the nonholomorphic Yukawa couplings in the fermion mass basis. Another possibility is given by the aligned THDM [\[24,25\]](#page-8-11) in which the Yukawa couplings of the fermions to the neutral scalars are flavor diagonal in the fermion mass basis, while the new sources of the CP violation stem from the complex Yukawa couplings involving the charged scalar. In the following sections we constrain the values of the scalar Wilson coefficients of the operators in Eq. [\(2\)](#page-1-2) from the available measured values of the corresponding branching fractions of the (semi) leptonic decays.

It is also possible that the higher-dimensional operators modify the $W\bar{s}c$ coupling, which would be reflected in the low-energy Lagrangian by the appearance of a nonstandard admixture of the right-handed quark current,

$$
\mathcal{O}_{V,R}^{(\ell)} = (\bar{s}\gamma_{\mu}P_R c)(\bar{\nu}_{\ell}\gamma^{\mu}P_L \ell). \tag{3}
$$

We briefly study the effects of this operator in Sec. [V.](#page-7-0) The tensor operator $(\bar{s}\sigma_{\mu\nu}P_Rc)(\bar{\nu}_e\sigma^{\mu\nu}P_R\ell)$ could also appear [\[18,19\]](#page-8-7), together with the (pseudo)scalar operators, after integrating out a scalar leptoquark at the tree level. We ignore these contributions due to the present lack of reliable information on the tensor form factors.

III. THE WILSON COEFFICIENT OF THE PSEUDOSCALAR OPERATOR

A. NP in $D_s \rightarrow \ell \nu_\ell$

In this section we derive the constraints on the linear combination of the Wilson coefficients $c_{R(L)}^{(\ell)}$ from the measured branching fractions of the purely leptonic $D_s \rightarrow \ell \nu$ decay modes. The hadronic matrix element of the corresponding axial-vector current is parametrized by the decay constant f_{D_s} via $\langle 0 | \bar{s} \gamma_\mu \gamma_5 | D_s(k) \rangle = f_{D_s} k_\mu$. Using the identity $\partial_{\mu}(\bar{s}\gamma_{\mu}\gamma_5c) = i(m_s + m_c)\bar{s}\gamma_5c$, one finds that the f_{D_s} suffices to parametrize the matrix element of the pseudoscalar density,

$$
\langle 0|\bar{s}\gamma_5 c|D_s(k)\rangle = \frac{f_{D_s}m_{D_s}^2}{m_c + m_s}.
$$
 (4)

The standard formula for the branching fraction is then modified to the following form:

$$
\mathcal{B}(D_s \to \ell \nu_\ell) = \tau_{Ds} \frac{m_{Ds}}{8\pi} f_{Ds}^2 \left(1 - \frac{m_\ell^2}{m_{Ds}^2} \right)^2 G_F^2 (1 + \delta_{em}^{(\ell)})
$$

$$
\times |V_{cs}|^2 m_\ell^2 \left| 1 - c_P^{(\ell)} \frac{m_{Ds}^2}{(m_c + m_s)m_\ell} \right|^2,
$$
(5)

where the pseudoscalar combination of the couplings is $c_P^{(\ell)} \equiv c_R^{(\ell)} - c_L^{(\ell)}$. In the evaluation of the constraints we use the latest theoretical value of the decay constant $f_{D_s} = 249.0(0.3)(^{+1.1}_{-1.5})$ MeV, calculated in lattice QCD with subpercent precision by the Fermilab Lattice and MILC collaborations [\[3\].](#page-8-1) At this level of precision it is mandatory to take into account the uncertainty in the lifetime of the D_s meson (1.4%) and the electromagnetic corrections parametrized by $\delta_{em}^{(\ell)}$. A detailed study of the electromagnetic effects is out of scope of the present article; we draw attention to Ref. [\[26\]](#page-8-12) for a detailed analysis regarding the $B \to \ell \nu \gamma$ process and a comparison with the $D \to \ell \nu \gamma$ case. There are several contributions to $\delta_{em}^{(\ell)}$: the long-distance soft-photon corrections that can be studied in the approximation of point-like charged mesons and leptons, the universal short-distance electroweak corrections, and the contributions that probe the hadronic structure of the process and require the knowledge of additional hadronic form factors. Following Refs. [\[3,26\]](#page-8-1), we estimate the $\delta_{em}^{(\mu)}$ to be in the range ~ (1.3) % and $\delta_{em}^{(r)} \sim (0-1)\%$, and include these values as the new sources of the uncertainty. The leptonic branching fractions of $D_s^+ \rightarrow \tau^+ (\mu^+)\nu$ were recently measured by the Belle Collaboration [\[5\]](#page-8-3). The measured values, together with the upper limit of the yet unobserved channel $D_s^+ \rightarrow e^+ \nu$, were given as follows:

$$
\mathcal{B}(D_s \to \ell \nu_\ell) = \begin{cases} (5.7 \pm 0.21^{+0.31}_{-0.3})\%, & D_s \to \tau \nu_\tau, \\ (0.531 \pm 0.028 \pm 0.020)\%, & D_s \to \mu \nu_\mu, \\ < 1.0 \cdot 10^{-4}, 95\% \text{ C.L.}, & D_s \to e \nu_e. \end{cases} \tag{6}
$$

We use the value of $|V_{cs}|$ from the global fit of the unitary CKM matrix and given by the CKMFitter Collaboration [\[27\]](#page-8-13), $V_{cs} = 0.97317_{-0.00059}^{+0.00053}$, for we do not expect this value to be influenced by the operators in Eq. [\(2\).](#page-1-2) The resulting allowed parameter space of the corresponding NP couplings is visualized in Fig. [1](#page-2-0). The upper limit in Eq. [\(6\)](#page-2-1) leads to the constraint $|c_P^{(e)}| < 0.005$.

One could also consider the ratios of the branching fractions, i.e., $R_{\tau/\mu} = \mathcal{B}(D_s \to \tau \nu)/\mathcal{B}(D_s \to \mu \nu)$ as a test of the lepton flavor universality of the charged current. This quantity has a small theoretical error that comes from the uncertainties in the masses of the particles involved in the process; see, e.g., Ref. [\[5\]](#page-8-3). It stays unchanged with respect

FIG. 1 (color online). Allowed regions of the effective couplings $c_P^{(\tau)}$ (upper panel) and $c_P^{(\mu)}$ (lower panel), extracted from the branching fraction of the decay mode $D_s \to \tau(\mu)\nu$, respectively. The 68% (95%) C.L. regions of the parameters are shown in darker (lighter) shades.

to the SM in the natural flavor-conserving THDMs, but it could receive corrections, e.g., in the type-III THDM from the nonholomorphic Yukawa couplings in the fermion mass basis. A careful investigation of this ratio should also include the effects of the electromagnetic corrections.

B. NP in $D \to K^* \ell \nu$

The pseudoscalar Wilson coefficient $c_P^{(\ell)}$ also contributes to the semileptonic decays of the pseudoscalar to vector mesons. These processes offer a larger number of observables than the two-body leptonic decays due to the existence of the nontrivial angular distributions; see, e.g., Ref. [\[1\]](#page-8-0). Information about the helicity-suppressed contribution can be extracted experimentally by comparing the decays that involve electrons and muons in the final state. This is, however, a difficult task at present, but it could be performed more precisely in the next generation of flavor experiments [\[28,29\].](#page-8-14) The helicity-suppressed contributions are also subdominant, which implies that the sensitivity of the processes $D \to K^* \ell \nu_\ell$ and $D_s \to \phi \ell \nu_\ell$ to the coefficient $c_P^{(\ell)}$ is weaker when compared to the pure leptonic decays. Also, knowledge of the form factors in these transitions is currently less precise. Information about the decay mode $D \to K^* \ell \nu$ is reconstructed from the experimentally observed $D \to K \pi \ell \nu$ process in which the dominant vector intermediate state interferes with the scalar $K\pi$ amplitude and (to a smaller extent) with higher waves [\[30\].](#page-8-15) The extraction of the possible NP effects from the angular analysis thus requires a careful disentangling of such resonant (and also other nonresonant) contributions. Lattice simulations provide easier access to the form factors for the process $D_s \rightarrow \phi \ell \nu$, in which neither of the two mesons contains the light valence quarks and the ϕ meson can be treated as stable to a good approximation. The first results of such a calculation [including the scalar form factor $A_0(q^2)$, to be defined below] were recently presented by the HPQCD Collaboration [\[31\].](#page-8-16)

The standard parametrization of the hadronic matrix element of the vector and axial-vector currents in terms of the form factors $V(q^2)$ and $A_{0,1,2}(q^2)$ is as in Ref. [\[32\]](#page-8-17):

$$
\langle V(k',\epsilon)|\bar{s}\gamma_{\mu}(1-\gamma_{5})c|P(k)\rangle = \epsilon_{\mu\nu\alpha\beta} \frac{2iV(q^{2})}{m_{P}+m_{V}} \epsilon^{*\nu}k^{\alpha}k'^{\beta} - (m_{P}+m_{V})\left(\epsilon_{\mu} - \frac{\epsilon \cdot qq^{\mu}}{q^{2}}\right)A_{1}(q^{2}) + \epsilon \cdot q\left(\frac{(k+k')_{\mu}}{m_{P}+m_{V}} - \frac{m_{P}-m_{V}}{q^{2}}q_{\mu}\right)A_{2}(q^{2}) - 2m_{V}\frac{\epsilon \cdot qq^{\mu}}{q^{2}}A_{0}(q^{2}),
$$

$$
A_{3}(q^{2}) \equiv \frac{m_{P}+m_{V}}{2m_{V}}A_{1}(q^{2}) - \frac{m_{P}-m_{V}}{2m_{V}}A_{2}(q^{2}), \tag{7}
$$

where the spurious singularity at $q^2 = 0$ is avoided with the constraint $A_3(0) = A_0(0)$. In the above formulas the four-vector ϵ denotes the polarization vectors of a spin-1 meson, while the transferred four-momentum is $q \equiv$ $k - k' = p_{\ell} + p_{\nu}$. Contracting the above matrix element with q^{μ} , one derives the parametrization of the pseudoscalar density in terms of the form factor $A_0(q^2)$,

$$
\langle V|\bar{s}\gamma_5 c|P\rangle = \frac{2m_V \epsilon^* \cdot q}{m_c + m_s} A_0(q^2). \tag{8}
$$

The differential decay rates of the process can be conveniently expressed in terms of hadronic helicity amplitudes that are defined as projections of the matrix element of the hadronic current [\(7\)](#page-3-0) to the polarization vectors of the charged lepton-neutrino pair $\tilde{\epsilon}_m^{\mu}$, where *m* denotes the polarizations $t, 0, \pm$. These amplitudes are explicitly given in the Appendix. Note that only the helicity amplitude $H_t(q^2)$, which receives a contribution from terms with $A_0(q^2)$, is modified in the presence of the pseudoscalar Wilson coefficients,

$$
H_t \to \left(1 - c_P^{(\ell)} \frac{q^2}{m_\ell (m_c + m_s)}\right) H_t. \tag{9}
$$

The form factors are analytic functions of q^2 in the physical region and satisfy the dispersion relations by the conditions of causality and unitarity. Most of the experimental measurements of the form factors assume single-pole dominance behavior, by which the main contribution in the dispersion relations arises from the lowest pole outside the physically allowed region. In Ref. [\[33\]](#page-8-18) the form factors for $D \to K^*\ell\nu$ transitions were studied in a framework that combined the heavy-quark and chiral symmetries and included the effects of the resonances beyond the simple pole approximation. The authors of Ref. [\[34\]](#page-8-19) employed the dispersion approach within the constituent quark model. In 2005 the FOCUS Collaboration performed nonparametric measurements of the hadronic helicity amplitudes [\[11\]](#page-8-5) as functions of the lepton pair invariant mass in several bins. However, the errors in this study were too large to be used in constraining NP contributions. The latest analysis of the $D \to K \pi \ell \nu$ decays was performed by the BABAR Collaboration [\[14\]](#page-8-20). They used the simple pole parametrization of form factors and extracted the values of the ratios of the form factors for the $D \to K^*$ transition at a single kinematic point: $V(0)/A_1(0) = 1.463 \pm 0.035$, $A_2(0)/A_1(0) = 0.801 \pm 0.03$, $A_1(0) = 0.6200 \pm 0.0057$. Since only electrons and positrons were used, the analysis remained insensitive to the form factor $A_0(q^2)$.

In order to get an estimate of the allowed NP contributions in $D \to K^*\ell\nu$ we proceed by using the constraint $A_3(0) = A_0(0)$ to infer the value of $A_0(0)$ and assume that the dependence on the q^2 of the form factor $A_0(q^2)$ is well described with the simple pole parametrization. We then consider $R_{L/T}$, the ratio of the decay widths of the longitudinally and transversally polarized K^* fractions, as an observable which is sensitive to $c_P^{(\ell)}$. The differential distributions for the longitudinally and transversally polarized K^* are

$$
\frac{d\Gamma_L}{dq^2} = \mathcal{N}(q^2) \left(1 - \frac{m_{\ell}^2}{q^2} \right)^2 \left[\left(1 + \frac{m_{\ell}^2}{2q^2} \right) |H_0|^2 + \frac{3m_{\ell}^2}{2q^2} |H_t|^2 \right],
$$
\n
$$
\frac{d\Gamma_T}{dq^2} = \mathcal{N}(q^2) \left(1 - \frac{m_{\ell}^2}{q^2} \right)^2 \left[\left(1 + \frac{m_{\ell}^2}{2q^2} \right) (|H_+|^2 + |H_-|^2) \right],
$$
\n(10)

where the overall factor is given with $\mathcal{N}(q^2) =$ $G_F^2 |V_{cs}|^2 q^2 |\mathbf{q}|/(96\pi^3 m_D^2)$. We use the Particle Data

FIG. 2 (color online). Allowed regions of the effective coupling $c_p^{(\mu)}$, extracted from the ratio $R_{L/T}$. The color coding is the same as in Fig. [1.](#page-2-0)

Group (PDG) averaged value [\[35\]](#page-8-21) of the ratio $R_{L/T} =$ 1.13 ± 0.08 for the process $D^+ \rightarrow \bar{K}^{*0} \mu^+ \nu$ to extract the allowed regions of the coefficient $c_p^{(\mu)}$ coupling in Fig. [2](#page-4-1). The resulting constraint turns out to be currently much weaker than the one shown in Fig. [1](#page-2-0). It is expected that the Belle II [\[28\]](#page-8-14) and BESIII collaborations [\[29\]](#page-8-22) are going to measure the processes $D_{(s)} \to K^*(\phi) \ell \nu$ with an enhanced precision. Given the possible lattice QCD improvements, these processes could serve as a useful complementary source of information about NP in $c \rightarrow s\ell\nu$ transitions in the near future.

IV. THE WILSON COEFFICIENT OF THE SCALAR OPERATOR

The semileptonic $D \to K\ell\nu$ decays are affected by the scalar combination of the Wilson coefficients $c_S^{(\ell)}$ = $c_R^{(\ell)} + c_L^{(\ell)}$. We use the latest lattice evaluation of the corresponding form factors and measured values of the branching fractions to constrain the values of $c_S^{(\ell)}$, $\ell = e, \mu$. Then we introduce the forward-backward and transversal muon asymmetries as the observables that can be used to extract further constraints on the real and imaginary parts of the scalar Wilson coefficient, respectively.

A. NP from branching fractions $\mathcal{B}(D \to K \ell \nu_{\ell})$

The hadronic matrix element of the vector current for the $D(k) \to K(k') \ell \nu_{\ell}$ decay is parametrized by form factors $f_{+0}(q^2)$ as

$$
\langle K(k')|\bar{s}\gamma_{\mu}c|D(k)\rangle = f_{+}(q^{2})\left((k+k')_{\mu} - \frac{m_{D}^{2} - m_{K}^{2}}{q^{2}}q_{\mu}\right) + f_{0}(q^{2})\frac{m_{D}^{2} - m_{K}^{2}}{q^{2}}q_{\mu},
$$
\n(11)

with the usual kinematic constraint $f_+(0) = f_0(0)$. The partially conserved vector current identity, $\partial_{\mu}(\bar{s}\gamma_{\mu}c)$ = $i(m_s - m_c)(\bar{s}c)$, relates the matrix element of the scalar density to the form factor $f_0(q^2)$:

$$
\langle K|\bar{s}c|D\rangle = \frac{m_D^2 - m_K^2}{m_s - m_c} f_0(q^2). \tag{12}
$$

The nonvanishing hadronic helicity amplitudes for the transition $D \to K \ell \nu$ are $h_{0,t} = \tilde{\epsilon}_{0,t}^{\mu*} \langle K | J_{\mu} | D \rangle$, and are given explicitly by

$$
h_0(q^2) = \frac{\sqrt{\lambda(m_D^2, m_K^2, q^2)}}{\sqrt{q^2}} f_+(q^2),
$$

\n
$$
h_t(q^2) = \left(1 + c_S^{(\ell)} \frac{q^2}{m_\ell(m_s - m_c)}\right) \frac{m_D^2 - m_K^2}{\sqrt{q^2}} f_0(q^2),
$$
\n(13)

where λ denotes the function $\lambda(x, y, z) = x^2 + y^2 + z^2$ $2(xy + xz + yz)$. The differential decay rate of the process $D \to K \ell \nu_{\ell}$ is given by the formula

$$
\frac{d\Gamma^{(\ell)}}{dq^2} = \frac{G_F^2 |V_{cs}|^2 |\mathbf{q}| q^2}{96\pi^3 m_D^2} \left(1 - \frac{m_{\ell}^2}{q^2}\right)^2 \left[|h_0(q^2)|^2 \left(1 + \frac{m_{\ell}^2}{2q^2}\right) + \frac{3m_{\ell}^2}{2q^2} |h_t(q^2)|^2\right],\tag{14}
$$

where $|\mathbf{q}| = \sqrt{\lambda(m_D^2, m_K^2, q^2)}/2m_D$ is the magnitude of the transferred three-momentum in the rest frame of the D meson. The current average values of the branching fractions of the $D \to K\ell\nu_{\ell}$ decays can be found in the PDG review [\[35\]](#page-8-21):

$$
\mathcal{B}(D \to K\ell\nu_{\ell}) = \begin{cases} (8.83 \pm 0.22)\%, & D^{+} \to \bar{K}^{0}e^{+}\nu_{e}, \\ (9.2 \pm 0.6)\%, & D^{+} \to \bar{K}^{0}\mu^{+}\nu_{\mu}, \\ (3.55 \pm 0.04)\%, & D^{0} \to K^{-}e^{+}\nu_{e}, \\ (3.30 \pm 0.13)\%, & D^{0} \to K^{-}\mu^{+}\nu_{\mu}. \end{cases}
$$
(15)

The functional dependence on q^2 of the form factors $f_{\pm 0}$ was recently calculated in lattice OCD by the HPOCD Collaboration in Ref. [\[4\]](#page-8-2). Using their results and the measured branching fractions [\(15\),](#page-4-2) we derive the constraint on the Wilson coefficients $c_S^{(\mu)} \equiv c_R^{(\mu)} + c_L^{(\mu)}$ and represent it in Fig. [3](#page-5-0). In the case of the electron, the 95% C.L. interval reads $|c_S^{(e)}|$ < 0.2. The CLEO Collaboration measured [\[10\]](#page-8-23) the differential decay rate for the process with electrons in the final state. The corresponding constraint is not significantly more stringent than the one obtained from the full branching ratio; see Refs. [\[18,19\]](#page-8-7). In Fig. [4](#page-5-1) we present the sensitivity of the yet unmeasured differential decay rate

FIG. 3 (color online). Allowed regions of the effective coupling $c_S^{(\mu)} = c_R^{(\mu)} + c_L^{(\mu)}$ extracted from the branching fraction of the decay mode $D \to K\mu^+\nu$. The color coding is the same as in Fig. [1](#page-2-0).

 $d\Gamma^{(\mu)}/dq^2$ to the presently allowed values of the coupling $c_S^{(\mu)}$. We derive the allowed range for the ratio $R_{\mu/e}(q^2) \equiv$ $\frac{d\Gamma^{(\mu)}}{dq^2}$ / $\frac{d\Gamma^{(e)}}{dq^2}$ assuming $c_S^{(e)} = 0$ and visualize it in the right panel of Fig. [4](#page-5-1). In the future precision measurements of

FIG. 4 (color online). Upper panel: The differential decay rate for the process $D \to K \mu \nu_{\mu}$. The thin red band shows the SM prediction, while its width represents the uncertainty. The (wider) grey band corresponds to the deviations that result from the presently allowed scalar Wilson coefficient from Fig. [3.](#page-5-0) Lower panel: The SM prediction and allowed deviations in the ratio $R_{\mu/e}(q^2) \equiv \frac{d\Gamma^{(\mu)}}{dq^2} / \frac{d\Gamma^{(e)}}{dq^2}$ assuming $c_S^{(e)} = 0$.

Belle II and at the high-intensity tau-charm factories this ratio might serve as an excellent test of the lepton flavor universality.

B. NP in forward-backward asymmetry in $D \to K \ell \nu_{\ell}$

It is instructive to introduce the observables which are exclusively sensitive to the real or imaginary parts of the Wilson coefficients. We first consider the differential decay distribution over $\cos \theta_{\ell}$, where θ_{ℓ} is defined as the angle between the three-momenta of the K meson and the charged lepton in the rest frame of the lepton-neutrino pair,

$$
\frac{d^2\Gamma^{(\ell)}}{dq^2d\cos\theta_\ell} = a_\ell(q^2) + b_\ell(q^2)\cos\theta_\ell + c_\ell(q^2)\cos^2\theta_\ell.
$$
\n(16)

Note that the information carried by the function $b_{\ell}(q^2)$ is lost after integrating the above distribution over the angle θ_{ℓ} . This information can be accessed by measuring the forward-backward asymmetry in the angle θ_{ℓ} , defined as follows:

$$
A_{FB}^{(\ell)}(q^2) \equiv \frac{\int_{-1}^{0} \frac{d^2 \Gamma^{(\ell)}(q^2)}{dq^2 d \cos \theta_{\ell}} d \cos \theta_{\ell} - \int_{0}^{1} \frac{d^2 \Gamma^{(\ell)}(q^2)}{dq^2 d \cos \theta_{\ell}} d \cos \theta_{\ell}}{d \Gamma^{(\ell)}/dq^2(q^2)}
$$

$$
= -\frac{b_{\ell}(q^2)}{d \Gamma^{(\ell)}(q^2)/dq^2}.
$$
(17)

The above ratio has a small theoretical error in the full $q²$ region due to the precise evaluation of the form factors and partly due to the cancellation of the uncertainties in the numerator and the denominator. The function $b_{\ell}(q^2)$, given by

$$
b_{\ell}(q^2) = -\frac{G_F^2 |V_{cs}|^2 |\mathbf{q}| q^2}{128\pi^3 m_D^2} \left(1 - \frac{m_{\ell}^2}{q^2}\right)^2 \frac{m_{\ell}^2}{q^2} 2Re(h_0 h_t^*),\tag{18}
$$

is linearly sensitive to the real part of the coupling $c_S^{(\ell)}$. We illustrate the possible effects of the scalar operator on the forward-backward asymmetry in Fig. [5,](#page-6-0) with the values of $c_S^{(\mu)}$ taken from the 68% C.L. allowed region in Fig. [3.](#page-5-0) The thin colored (red) band represents the hadronic uncertainty in the shape of this function in the SM. The larger colored band (grey) represents the currently allowed deviations from the SM. We conclude that the large deviations from the SM in this observable are not excluded at present. The quantity $A_{FB}^{(e)}$ is highly suppressed and insensitive to the corresponding scalar Wilson coefficient due to the tiny mass of the electron. The average value of the forwardbackward asymmetry, $\langle A_{FB}^{(\ell)} \rangle$, can be calculated by performing an integration over q^2 in the numerator and

FIG. 5 (color online). Comparison of the shape of the forwardbackward asymmetry $A_{FB}^{(\mu)}(q^2)$ in the SM (red) with the deviations (grey) induced by currently allowed values of the $c_s^{(\mu)}$ couplings. Colored bands represent the form-factor uncertainties.

denominator of Eq. [\(17\).](#page-5-2) The SM value is $\langle A_{FB}^{(\mu)} \rangle = 0.055(2)$. For various values of $c_S^{(\mu)}$ from the 68% C.L. region in Fig. [3](#page-5-0), this quantity can have values in the interval $(0.0.065)$.

Some comments about the NP scenarios that could affect these observables are in order here. In the type-II THDM the Wilson coefficients that contribute to the $c \rightarrow s\ell\nu_\ell$ transitions are small:

$$
c_L^{(\ell)} = \frac{m_s m_\ell \tan^2 \beta}{m_{H^+}^2}, \qquad c_R^{(\ell)} = \frac{m_c m_\ell}{m_{H^+}^2}, \qquad (19)
$$

implying $c_S^{(\ell)} \approx -c_P^{(\ell)} = c_L^{(\ell)}$. The values of the scalar and pseudoscalar couplings are thus approximately related, so that the tight constraints from the leptonic decays imply that the forward-backward asymmetry in $D \to K \mu \nu$ does not exhibit deviations from the SM. In more general THDMs the scalar and pseudoscalar coefficients are independent. Examples of such models are the aligned THDM [\[24,25\]](#page-8-11) or the THDM with general flavor structure.

C. NP in the transversal muon polarization

The relative complex phase between the nonstandard scalar Wilson coefficient and the V_{cs} element of the CKM matrix is a possible new source of CP violation. The total decay rate does not offer independent information about such effects. One could measure the T-odd transversal polarization of the final charged lepton in the semileptonic D-meson decays [\[18,19\]](#page-8-7). It follows from CPT invariance that this observable is also CP odd. Since its value is expected to be vanishingly small in the SM, a measured nonvanishing value would be a clear sign of NP. This observable was first theoretically introduced and experimentally studied in semileptonic K-meson decays; see Refs. [36–[38\]](#page-8-24). The transversal polarization of the τ lepton in the semitauonic B decays has also been theoretically considered as a possible test of beyond-the-SM CP-violating effects; see Refs. [\[39,40\]](#page-8-25). In the case of the process with an electron in the final state, this observable remains insensitive to the corresponding scalar Wilson coefficient. We define the transversal polarization of the muon in the process $D^+ \rightarrow K^0 \mu^+ \nu$ as the ratio

$$
P_{\perp}^{(\mu)} = \frac{|A(\vec{s})|^2 - |A(-\vec{s})|^2}{|A(\vec{s})|^2 + |A(-\vec{s})|^2},
$$
\n(20)

where $\vec{s} \equiv (\vec{p}_K \times \vec{p}_e)/|\vec{p}_K \times \vec{p}_e|$ denotes the unit vector perpendicular to the $K\ell$ decay plane and $A(\pm \vec{s})$ is the amplitude for spin projections along \vec{s} . The small value of $P_{(\ell)}^{\perp}$ is generated in the SM by the final-state interactions. For example, the electromagnetic effects produce a value of the order 10^{-6} in the process $K^+ \rightarrow \pi^0 \mu^+ \nu$ [\[41\]](#page-8-26). Theoretical computations of the contributions of the final-state interactions of this observable in the semileptonic D decays is currently lacking, but we expect that it is small enough that it can be neglected. The contribution to the numerator of Eq. [\(20\)](#page-6-1) arises from the interference between the SM and the scalar amplitudes [\[37](#page-8-27)–40], namely

$$
P_{\perp}^{(\mu)}(q^2, E_{\mu}) = \left(\frac{d\Gamma}{dq^2 dE_{\mu}}\right)^{-1} \kappa(q^2, E_{\mu}) \text{Im}(h_0(q^2) h_t^*(q^2)).
$$
\n(21)

The NP contribution is encoded in the modification of the helicity amplitude $h_t(q^2)$ [see Eq. [\(13\)](#page-4-3)]. The function $\kappa(q^2, E_\mu)$ is given by

$$
\kappa(q^2, E_{\mu})
$$

= $-2\sqrt{\frac{r_{\mu}}{\lambda}} \Biggl[\Biggl(\frac{4E_{\mu}}{m_D^2} - 4r_{\mu} \Biggr) \bigl((1 - r_K - r_q)^2 - 4r_K \bigr)$
 $-4\Biggl(-\frac{2E_{\mu}}{m_D} + 2r_K + r_{\mu} + \frac{E_{\mu}(1 - r_K - r_q)}{m_D} + r_q \Biggr)^2 \Biggr]^{1/2},$
(22)

where $r_{\mu} = m_{\mu}^2 / m_D^2$, $r_K = m_K^2 / m_D^2$, $r_q = q^2 / m_D^2$ and E_{μ} is the energy of the muon in the rest frame of the decaying D meson. The average of the transversal lepton polarization over the specific kinematic region

$$
\langle P_{\perp}^{(\mu)} \rangle = \frac{\int dq^2 dE_{\mu} P_{\perp}^{(\mu)} (q^2, E_{\mu}) \frac{d^2 \Gamma}{dq^2 dE_{\mu}}}{\int dq^2 dE_{\mu} \frac{d^2 \Gamma}{dq^2 dE_{\mu}}}
$$
(23)

yields a quantity that is the measure of the difference between the number of charged leptons with their spins pointing above and below the decay plane, divided by their total number. While in the SM the value of $\langle P_{\ell}^{\perp} \rangle$ is expected to be very small (close to zero); for the presently allowed

values $c_S^{(\mu)} \approx \pm 0.1i$ we find the maximally allowed value $\langle P_{\perp}^{(\mu)} \rangle \simeq \pm 0.2$.

V. RIGHT-HANDED CURRENT

We now study the constraints on the effective operator that involves the right-handed current $\bar{s}\gamma_{\mu}P_{R}c$. The Wilson coefficient is expected to be of the form of a product of the universal coupling ϵ_R and the corresponding quark mixing matrix element in the right-handed quark sector; see, e.g., Ref. [\[42\]](#page-8-28). In the past few years the right-handed quark currents have been studied as a possibility to accommodate the tensions between the values of $|V_{ub}|$ extracted from the exclusive and inclusive (semi)leptonic decays [\[42,43\].](#page-8-28) The right-handed current would modify the extraction of $|V_{cs}|$ in the following way:

$$
|V_{cs}(1 + c_{V,R})| = |V_{cs}(D \to K\ell\nu)|_{\text{SM}/\text{exp}},
$$

$$
|V_{cs}(1 - c_{V,R})| = |V_{cs}(D_s \to \ell\nu)|_{\text{SM}/\text{exp}},
$$
 (24)

where $|V_{cs}|_{SM/exp}$ denotes the values extracted from the comparison of the experimental and predicted (in the SM) values of the branching fractions. We assume that $c_{V,R}$ is real valued, lepton universal, and a lot smaller than one, so that the above relations can be expanded to first order in this coefficient. Using the values $|V_{cs}(D \to \ell \nu)|_{\text{SM}/\text{exp}} =$ 1.010(20) (from Ref. [\[3\]](#page-8-1)) and $|V_{cs}(D \to K \ell \nu)|_{SM/\exp} =$ $0.963(15)$ (from Ref. [\[4\]](#page-8-2)), we obtain the limits

$$
|V_{cs}| = 0.987 \pm 0.013, \qquad c_{V,R} = -0.023 \pm 0.013.
$$
\n(25)

The resulting value of the $c_{V,R}$ coupling is compatible with zero at the 95% C.L., while the value of $|V_{cs}|$ is compatible with the result of the global unitarity fit [\[27\].](#page-8-13)

The $c_{V,R}$ can be further constrained in $D \to V \ell \nu$ decay modes. The HPQCD Collaboration recently calculated the ratio of the form factors $V(0)/A_1(0) = 1.72(21)$ for the process $D_s \rightarrow \phi e \nu_e$ [\[31\].](#page-8-16) This ratio is modified by the presence of the right-handed currents via

$$
V(0) \to (1 + c_{V,R})V(0), \qquad A_1(0) \to (1 - c_{V,R})A_1(0).
$$
\n(26)

A comparison of the lattice result with the value measured by the *BABAR* Collaboration, $V(0)/A_1(0) = 1.849 \pm 0.11$ [\[44\]](#page-8-29), results in the interval

$$
-0.03 \le c_{V,R} \le 0.1. \tag{27}
$$

Once the lattice results in these processes are further refined, more detailed constraints on the right-handed contributions could be performed with the use of the angular distributions, as explained in Ref. [\[45\]](#page-8-30).

VI. CONCLUSIONS

We have investigated leptonic and semileptonic $c \rightarrow$ $s\ell\bar{\nu}_\ell$ transitions of charm mesons using the effective Lagrangian approach. The most constraining processes for the pseudoscalar couplings are leptonic decays, due to the precise knowledge of the D_s -meson decay constant obtained from lattice QCD and the latest precise measurements. The branching ratios for the decay $D \to K^*\ell\nu$ and the ratio of the decay widths for the longitudinally and transversely polarized K^* have already been measured. We used the existing experimental result to look for an additional constraint on the pseudoscalar coupling. In order to obtain a better bound, one should have a precise lattice determination of the $A_0(q^2)$ form factor as well as more precise experimental results.

The scalar Wilson coefficients can be constrained from $D \rightarrow K \ell \nu$ decay modes. The most interesting observables in this respect are the forward-backward asymmetry and the CP-violating transversal muon polarization in the decay involving muons in the final state. The deviations from the SM in these observables are currently allowed. We found that the ratio $R_{\mu/e}(q^2) \equiv \frac{d\Gamma^{(\mu)}}{dq^2} / \frac{d\Gamma^{(e)}}{dq^2}$ might be used to test lepton flavor violation. By allowing the first generation of leptons to interact as in the SM and new physics to affect the second generation, we found that this ratio is currently allowed to deviate from the SM value by 10–20%, depending on q^2 . Finally, we constrained the Wilson coefficient of the right-handed current in the charm Cabibbo-allowed (semi)leptonic processes using both experimental results on $D_s \to \ell \nu$ and the lattice QCD calculation for the form-factor ratio in $D_s \rightarrow \phi e \nu_e$. Both constraints are compatible. Future experiments on charm-meson leptonic and semileptonic decays as well as lattice QCD studies will lead to very strong constraints on possible NP contributions.

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APPENDIX: HADRONIC HELICITY AMPLITUDES FOR $D \rightarrow V \ell \nu$

The nonvanishing hadronic helicity amplitudes for the $P \rightarrow V \ell \nu$ decay process are given by the following formulas:

$$
H_{\pm}(q^2) = \mp \frac{\sqrt{\lambda(m_P^2, m_V^2, q^2)}}{m_P + m_V} V(q^2) + (m_P + m_V)A_1(q^2),
$$

\n
$$
H_0(q^2) = \frac{1}{2m_V\sqrt{q^2}} \left[(m_P + m_V)(m_P^2 - m_V^2 - q^2)A_1(q^2) - \frac{\lambda(m_P^2, m_V^2, q^2)}{m_P + m_V} A_2(q^2) \right],
$$

\n
$$
H_t(q^2) = \left[1 - c_P^{(\ell)} \frac{q^2}{m_\ell(m_q + m_{\bar{q}})} \right] \frac{\sqrt{\lambda(m_P^2, m_V^2, q^2)}}{\sqrt{q^2}} A_0(q^2).
$$
\n(A1)

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