Dark matter origins of neutrino masses PHYSICAL REVIEW D 91, 093011 (2015)
Dark matter origins of neutrino masses

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We propose a simple scenario that directly connects the dark matter (DM) and neutrino mass scales. Based on an interaction between the DM particle χ and the neutrino ν of the form $\chi\chi\nu\chi/\Lambda^2$, the DM annihilation cross section into the neutrino is determined and a neutrino mass is radiatively induced. Using the observed neutrino mass scale and the DM relic density, the DM mass and the effective scale Λ are found to be of the order MeV and GeV, respectively. We construct an ultraviolet-complete toy model based on the inverse seesaw mechanism which realizes this potential connection between DM and neutrino physics.

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<u>I. I. I. II. I. I. I. II</u>. The Standard Model (SM) is not able to explain the existence of dark matter (DM) in the Universe as well as the finite masses of neutrinos. Experimentally, both phenomena are firmly established. The two neutrino mass-squared differences are very well measured in neutrino oscillation experiments [\[1\].](#page-3-0) Together with the upper limit on the sum of the neutrino masses, $\sum m_{\nu} \lesssim 0.66$ eV, derived from cosmological observations [\[2\],](#page-3-1) they imply that the heaviest active neutrino has a mass of 0.05 to 0.22 eV. While the DM mass is largely unconstrained, the crucially important DM relic abundance is very well measured at $\Omega h^2 = 0.12$ [\[2\]](#page-3-1).

Connections between DM physics and the origin and size of the neutrino masses have been proposed in the literature in the context of radiative neutrino mass models, for example in Refs. [\[3](#page-3-2)–5], where the neutrino mass is induced radiatively with DM particles and heavy neutrinos in the loop. In these models, the neutrino mass scale depends on the DM and heavy neutrino masses as well as various coupling constants. This implies that the DM mass cannot be uniquely determined given the observations, unless other model parameters are fixed. An alternative scenario was proposed in Refs. [\[6,7\]](#page-3-3). Similar to our case, it connects neutrino physics with a MeV scale DM particle, although the underlying model is quite different.

In this work, we propose a simple scenario that connects the DM particle and neutrino mass scales. We start with an effective six-dimensional operator

$$
\frac{\chi\chi\nu\nu}{\Lambda^2},\tag{1}
$$

where γ refers to a gauge singlet Majorana DM particle, while ν is the SM neutrino.¹ Here and in the following, we use the two-component Weyl spinor notation for all fermionic fields. We implicitly assume that χ is odd under a Z_2 symmetry to ensure its stability. Assuming that this operator is the only one coupling DM to SM particles, the DM annihilation cross section times the DM relative velocity v_{rel} is approximated by $\sigma v_{\text{rel}} \approx m_{\chi}^2 / (\pi \Lambda^4)$. This implies a DM relic abundance of $\Omega h^2 \approx 8.2 \times$ $10^{-10} \text{ GeV}^{-2} / (\sigma v_{\text{rel}})$. On the other hand, the neutrino receives a radiative mass by contracting two χ fields in the interaction operator, $m_{\nu} \approx 55 m_{\chi}^3 / (\pi^2 \Lambda^2)$. Using the experimental data on the DM relic abundance and the light neutrino mass scale, the DM mass m_x and the scale Λ of the interaction operator can be determined easily,

$$
m_{\chi} \approx 0.4 \text{ MeV} \left(\frac{m_{\nu}}{0.1 \text{ eV}} \right)^{1/2} \left(\frac{\Omega h^2}{0.12} \right)^{1/4},
$$
 (2)

$$
\Lambda \approx 1.5 \text{ GeV} \left(\frac{m_{\nu}}{0.1 \text{ eV}}\right)^{1/4} \left(\frac{\Omega h^2}{0.12}\right)^{3/8}.
$$
 (3)

Naturally, m_{γ} and Λ are of the order MeV and GeV, respectively. The effective operator scale Λ is far below the electroweak (EW) scale, which is why the operator is not invariant under the SM gauge group. It also naturally implies the existence of at least one more particle lighter than the EW scale in order to obtain the interaction operator $\chi\chi\nu\upsilon/\Lambda^2$. We proceed by constructing a possible model that realizes the previous operator in two steps: firstly by discussing an effective Lagrangian, and then a possible fully UV-complete toy model.

II. EFFECTIVE LAGRANGIAN

The natural scale of the operator [\(1\)](#page-0-2) is GeV and in order to discuss a possible SM effective model we have to introduce another light particle that connects the DM sector with the SM. In addition, we assume that the only source of lepton number violation (LNV), that generates the DM Majorana mass, is situated in the hidden sector. We do not specify this source of LNV but it could for example result

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¹We neglect the flavor structure of the three neutrinos and work with one Majorana neutrino field with mass scale $m_{\nu} \approx 0.1 \text{ eV}$.

WEI-CHIH HUANG AND FRANK F. DEPPISCH PHYSICAL REVIEW D 91, 093011 (2015)

FIG. 1. Loop diagrams generating a Majorana neutrino mass. The arrow represents chirality; if the arrow direction is the same as that of the momentum, it represents the left-handed chirality. One of the contributions (right panel) involves a chirality flip.

from a seesawlike mechanism in the hidden sector. Note that one has to make sure that in the UV-complete theory, the hidden sector does not couple to the SM directly; i.e. it has to go through the DM particle χ . Therefore, any other effective operators have to conserve lepton number which for example forbids the Weinberg operator LHLH.

We introduce a complex scalar Φ with two units of lepton number, $L(\Phi) = 2$, which connects the DM and SM sectors,

$$
\mathcal{L} \supset c_2 \Phi \chi \chi + \frac{\Phi^* L H L H}{\Lambda_*^2} + \text{H.c.} \tag{4}
$$

Here, L and H are the SM lepton and Higgs boson doublets, respectively. Choosing $L(\chi) = -1$, the Lagrangian [\(4\)](#page-1-0) conserves lepton number. After integrating out Φ and EW breaking, $H = (0, v)^T$, one obtains

$$
\mathcal{L} \supset \frac{\chi \chi \nu \nu}{\Lambda^2} + \text{H.c.},\tag{5}
$$

where $\Lambda = \Lambda_* m_{\Phi}/(\sqrt{c_2} v)$.

Due to the Majorana nature of the DM particle χ , the light neutrino ν will obtain a loop-induced Majorana mass as shown in Fig. [1.](#page-1-1) Treating Λ as a dimensionful coupling constant instead of a cutoff scale, the neutrino mass becomes

$$
m_{\nu} = \frac{m_{\chi}^{3}}{2\pi^{2}\Lambda^{2}} \left(6 \ln \frac{m_{\chi}}{\mu} - 1 \right),
$$
 (6)

using the dimensional regularization scheme with modified minimal subtraction,² renormalized at the scale μ . We take μ to be the neutrino mass m_{ν} , with the incoming momentum p set to zero.

On the other hand, the relic abundance of χ is determined by the same effective operator.³ The DM annihilation cross section reads, up to v_{rel}^2 ,

FIG. 2 (color online). Neutrino mass m_{ν} and DM relic abundance Ωh^2 as a function of the DM mass m_x and the effective DM-neutrino interaction scale Λ. The red curve corresponds to the correct DM density, while the blue (purple) line refers to the upper (lower) limit on the heaviest active neutrino mass, as denoted. At the intersection, $m_v \approx 0.5$ MeV and $\Lambda \approx 2$ GeV.

$$
\sigma v_{\rm rel} = \frac{m_{\chi}^2}{\pi \Lambda^4} \left(1 + \frac{1}{2} v_{\rm rel}^2 \right). \tag{7}
$$

We base the computation of the DM relic density on the thermally averaged annihilation cross section $\langle \sigma v_{\text{rel}} \rangle$, where we include the fact that the number of relativistic degrees of freedom is much smaller for MeV DM as opposed to GeV DM, as described in Ref. [\[8\].](#page-3-4)

Given the observed neutrino mass scale and the DM relic density, the DM mass m_{χ} is around the sub-MeV scale while $\Lambda \approx 2$ GeV, as estimated in the previous section. The general relation between the model parameters m_{γ} , Λ and the observables m_{ν} , Ωh^2 Ωh^2 is shown in Fig. 2. The red curve denotes the observed relic abundance $\Omega h^2 = 0.12$ while the blue (purple) line corresponds to the upper (lower) limit on the heaviest active neutrino mass. The fact that Λ is much smaller than the EW scale justifies the explicit EW symmetry breaking of Eq. [\(5\)](#page-1-3) and it implies the existence of the light particle Φ in this scenario.

II. UV-COMPLETE TOY MODEL

As a final step, we construct a UV-complete toy model that in turn generates the effective Lagrangian and the low energy DM-neutrino interaction, as shown in Fig. [3](#page-2-0). The corresponding Lagrangian reads

$$
\mathcal{L} \supset \frac{c_1}{2} (\Phi_\chi + \langle \Phi_\chi \rangle) \chi \chi + c_2 \Phi \chi \chi + c_3 \Phi^* \xi \xi + yLHN - m_{\Phi_\chi} \Phi_\chi \Phi_\chi^* - m_{\Phi} \Phi \Phi^* - m_N N \xi + \text{H.c.}, \quad (8)
$$

where Φ_{γ} and Φ are scalar fields with lepton number $L = 2$, N and ξ are heavy Dirac neutrinos with opposite L. The vacuum expectation value (VEV) $\langle \Phi_{\chi} \rangle$ of Φ_{χ} generates the

²As we shall see later, this is well justified in the UV-complete model which has exactly the same loop structure.

³One has to include the contribution from $\chi^{\dagger} \chi^{\dagger} \nu^{\dagger} \gamma^{\dagger} \Lambda^2$, which involves a different chirality. The interference between the two different chirality contributions is tiny, being proportional to the very small neutrino mass m_{ν} .

FIG. 3. Diagram generating the effective DM-neutrino interaction in the UV-complete toy model. For illustration, we only show the diagram with a right-handed χ .

DM mass $m_{\chi} = c_1 \langle \Phi_{\chi} \rangle$. In principle, Φ_{χ} could be very heavy compared to $\langle \Phi_{\chi} \rangle$. For instance, Φ_{χ} may couple to another scalar ϕ such that $\langle \Phi_{\chi} \rangle = \langle \phi \rangle^2 / m_{\Phi_{\chi}} \ll \langle \phi \rangle$, $m_{\Phi_{\chi}}$, similar to the type-II seesaw mechanism. In other words, the small VEV $\langle \Phi_{\chi} \rangle$ can in this case be triggered by the VEV of ϕ , suppressed by the heavy Φ_{γ} mass, $m_{\Phi_{\gamma}}$. Moreover, the massless Majoron from ϕ could be removed by gauging $B - L$. The quantum numbers of the various fields are listed in Table [I](#page-2-1). The Z_2 symmetry is imposed to guarantee the stability of DM and forbid the mixing between DM and the SM neutrino. Lepton number is spontaneously broken after Φ_{χ} obtains a VEV, giving a Majorana mass to χ . Moreover, χ induces the mixing between Φ_{γ} and Φ such that LNV is transferred to the heavy neutrino N and finally to ν via the heavy-light neutrino mixing. Integrating out the heavy particles, we obtain the effective DM-neutrino interaction $\chi \chi \nu \nu/\Lambda^2$, with

$$
\Lambda = \frac{1}{\sqrt{c_2 c_3}} \frac{m_N m_\Phi}{y v}.
$$
\n(9)

Alternatively, the *χ*-induced mixing between Φ_{χ} and Φ gives rise to a linear term in Φ once Φ_{γ} acquires a VEV. This linear term will induce a small VEV of Φ as

$$
\langle \Phi \rangle = c_2 \chi \overline{\chi} / m_{\Phi}^2, \tag{10}
$$

where $\overline{\chi}\overline{\chi}$ represents the χ -loop of mass dimension 3. It in turn gives a small Majorana mass term to ξ , $c_3\langle \Phi \rangle \xi \xi$. The full neutrino mass matrix in the basis (ν, N, ξ) reads

TABLE I. Particle content and corresponding quantum numbers in the toy model.

Field				γ		
$[SU(2)L]$ _Y	$2_{-1/2}$	$2_{1/2}$	1 ₀			1 ₀

$$
\begin{pmatrix} 0 & yv & 0 \\ yv & 0 & m_N \\ 0 & m_N & 2\frac{c_2c_3\chi\chi}{m_{\Phi}^2} \end{pmatrix},
$$
 (11)

which is exactly the inverse seesaw [\[9\].](#page-3-5) The resulting light neutrino mass will be $m_{\nu} = 2c_2c_3y^2v^2\chi^2\chi/(m_{\phi}^2m_N^2)$, which implies Eq. [\(9\)](#page-2-2) from Eq. [\(5\)](#page-1-3). In addition, for $\Lambda \approx 1$ GeV one has $m_N m_\Phi \approx 100 \text{ GeV}^2$ if all couplings are of $\mathcal{O}(1)$. This also means that m_N is bounded from above, $m_N \lesssim 100$ TeV, since m_{Φ} is required to be larger than m_{χ} ; otherwise Φ cannot be regarded as heavy enough to generate the effective operator $\chi \chi \nu \nu/\Lambda^2$. On the other hand, m_N is also bounded from below $m_N \gtrsim 100 \text{ GeV}$ for $y = \mathcal{O}(1)$ due to constraints from EW precision and flavor changing neutral currents data [\[10](#page-3-6)–12].

The UV-complete toy model satisfies two very important requirements necessary for this mechanism to work: Firstly, LNV arises in the hidden sector, and it is mediated to the SM sector (including right-handed neutrinos) only by the DM particle χ . Secondly, the heavy particles that are being integrated do *not* enter the χ -loop, which radiatively induces the light neutrino mass. It is a distinctive feature of the model, setting it apart from the existing literature, for instance Refs. [13–[15\],](#page-4-0) where heavy particles exist inside loops that give rise to radiative neutrino masses. It is this feature that renders our model more predictive.

IV. CONCLUSIONS IV. CONCLUSIONS

In this paper, we propose a simple scenario that directly connects the physics of DM and neutrino masses. The introduced operator $\chi \chi \nu \nu/\Lambda^2$ induces a radiative Majorana neutrino mass and leads to the DM annihilation. Given the observed DM density and the neutrino mass scale, the DM mass m_{χ} and the operator scale Λ are uniquely fixed to be of order MeV and GeV, respectively. In a UV-complete toy model, we postulate the breaking of lepton number in a hidden sector that is mediated via a DM loop to the visible sector and thus to the light neutrinos, generating the effective Weinberg operator. To our knowledge this has not been explored in the literature but we find this possibility rather suggestive and intriguing; it would for example motivate why lepton number is only slightly broken in the visible sector and the DM loop mediation is quite natural with the presence of a Z_2 symmetry to ensure the DM stability, only allowing the DM particle to interact in pairs.

In our paper we focus purely on the relation between the neutrino mass generation and the DM annihilation. As an outlook, we would like to comment on other potential signatures of the model. The DM annihilation cross section is S-wave dominated without velocity suppression. This implies a neutrino flux due to ongoing DM annihilation, for example, from the Galactic center. The DM mass and hence the energy scale of the neutrino flux are of order MeV, in the vicinity of the energy threshold of neutrino experiments such as Super-Kamiokande [\[16\],](#page-4-1) KamLAND [\[17,18\]](#page-4-2), SNO [\[19\]](#page-4-3) and Borexino [\[20\]](#page-4-4). We estimate the expected monochromatic neutrino flux as $\Phi_{\nu} \approx 300 \text{(MeV}/m_{\nu})^2 \text{ cm}^{-2} \text{ s}^{-1}$ using the calculation of Ref. [\[21\]](#page-4-5). Such a flux would give rise to a few events for an exposure of a Mton \cdot yr.

The fact that the effective scale Λ is naturally of order GeV implies the existence of light exotic states. With regard to direct searches at colliders, it is difficult to make a general statement without fully specifying a UV-complete model. For a TeV scale neutrino N, $m_{\Phi} \approx$ GeV. The scalar Φ only couples indirectly through N and is hardly constrained by collider searches. If Φ couples to the SM Higgs via $\Phi^* \Phi H(H)$, invisible Higgs decays without phase space suppression would be generated. The mass range of the heavy quasi-Dirac neutrino N (and ξ) is confined to be 100 GeV $\leq m_N \leq 100$ TeV with a relatively large heavylight neutrino mixing. Therefore, the heavy neutrino production cross section could be sizable at the LHC.

Finally, we would like to point out that the MeV scale DM particle can contribute to the entropy of the Universe during the time of big bang nucleosynthesis (BBN). In our scenario, DM is still in thermal equilibrium with neutrinos after they decouple from the thermal bath around the temperature $T = 2.3$ MeV [\[22\].](#page-4-6) The DM particles subsequently become nonrelativistic and transfer entropy to the neutrinos, thereby reheating the neutrino temperature with respect to that of photons. This leads to a larger number of relativistic degrees of freedom N_{ν} during the time of last scattering producing the cosmic microwave background (CMB). MeV scale Majorana DM coupling to the neutrinos will result in $N_{\nu}^{\text{BBN}} = 4$ and $N_{\nu}^{\text{CMB}} = 4.4$ [\[23,24\].](#page-4-7) This is consistent with the BBN observation, $1.8 < N_{\nu}^{\rm BBN} < 4.5$ [\[1,25\]](#page-3-0), but it is in tension with $N_{\nu}^{\text{CMB}} = 3.36 \pm 0.34$ from the CMB data alone [\[2\].](#page-3-1) Potentially more severe is the constraint from the determination of the primordial deuterium abundance. The observationally determined value, expressed relative to the hydrogen abundance, is $D/H =$ $(2.53 \pm 0.04) \times 10^{-5}$ [\[1\].](#page-3-0) On the other hand, a Majorana DM particle with mass $m_{\gamma} \approx 0.5$ MeV and annihilating to neutrinos as in our scenario would result in a value $D/H \approx$ 3×10^{-5} [\[24\]](#page-4-8). Compatibility with observation in this case requires a DM mass $m_{\gamma} \gtrsim 7$ MeV.

The apparent tension of our toy model with astrophysical data can be avoided or at least alleviated in many ways. We would like to highlight two possible options that can be implemented in a more realistic model. Firstly, to realize the observed neutrino mixing pattern and mass-squared differences, at least two generations of (N, ξ) are needed. In this case, one can have a larger DM mass \gtrsim 7 MeV while having cancellations in the flavor structure between two contributions to the neutrino masses to keep m_{ν} small, such that the CMB and BBN constraints do not apply. The corresponding neutrino mass with $m_{\chi} \gtrsim 7$ MeV would be around 20 eV, necessitating a 1% tuning by accident or symmetry to achieve the required $m_{\nu} \approx 0.2$ eV. Secondly, one can add a Dirac component to the DM χ mass, effectively turning χ into a quasi-Dirac particle. This would increase the DM mass while the Majorana neutrino mass, proportional to the Majorana DM mass component, could be kept constant. The two solutions, however, will modify the DM and neutrino mass link but they could be implemented in a controllable way via the help of a symmetry such that the connection between DM and neutrino physics still exists.

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