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Comment on "Generalized black diholes"

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We show that a recent solution published by Cabrera-Munguia *et al.* is physically inconsistent since the quantity σ it involves does not have a correct limit $R \to \infty$.

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In the paper [1] we considered a 4-parameter solution from the Ernst-Manko-Ruiz (EMR) family of equatorially antisymmetric electrovac spacetimes [2] describing a pair of counterrotating Kerr-Newman (KN) sources [3] endowed with opposite electric charges—a stationary dihole. At the end of that paper we presented and briefly discussed a generalization of our 4-parameter model written in physical parametrization to the case when the two sources, in addition to electric charges, could also carry arbitrary magnetic opposite charges, the resulting 5-parameter dihole dyonic configuration being defined by the constant quantity σ of the form [cf. Eq. (40) of [1]]

$$\sigma = \sqrt{M^2 - \left(\frac{M^2a^2[(R+2M)^2 + 4(Q^2+\mathcal{B}^2)]}{[M(R+2M)+Q^2+\mathcal{B}^2]^2} + Q^2 + \mathcal{B}^2\right)\frac{R-2M}{R+2M}}, \qquad a = \frac{J}{M}, \tag{1}$$

where M, J, a, Q and $\mathcal{B}$ are, respectively, the mass, angular momentum, angular momentum per unit mass, electric charge and magnetic charge of the *upper* constituent (the characteristics of the *lower* constituent are correspondingly M, $-J$, $-a$, $-Q$, $-\mathcal{B}$), while R is the separation distance (see Fig. 1). Later, after learning about our results, Cabrera-Munguia *et al.* [4] have published a similar representation of the 5-parameter EMR metric which only slightly differs from ours in the form of σ: their σ is obtainable from (1) via the substitution

$$M^2a^2 \equiv J^2 \quad \text{to } (J - Q\mathcal{B})^2, \tag{2}$$

thus acquiring some additional terms compared to (1). Therefore, a question naturally arises: which version of the formula for σ is correct? Unfortunately, the issue of discrepancy between two σ's was not touched in the paper of Cabrera-Munguia *et al.*, although logically this should have been the main subject of that paper. Moreover, the authors of [4] made reference to our article exclusively in the context of the 4-parameter solution, with no mention of our 5-parameter dyonic model. In the present Comment we will show that the expression for σ obtained by Cabrera-Munguia *et al.* with the aid of an "enhanced" mass relation is in effect physically inconsistent.

First of all, we would like to remark that one might naively think that, since the 5-parameter solutions from [1,4] differ in the form of σ only, then the physically incorrect solution should not satisfy the field equations identically. However, this is not the case because in the solution construction procedure employed in the two papers the quantity σ is an arbitrary constant which may in principle be chosen in the infinite number of very exotic unphysical ways without violating the field equations. Hence, some other, less straightforward criteria must be

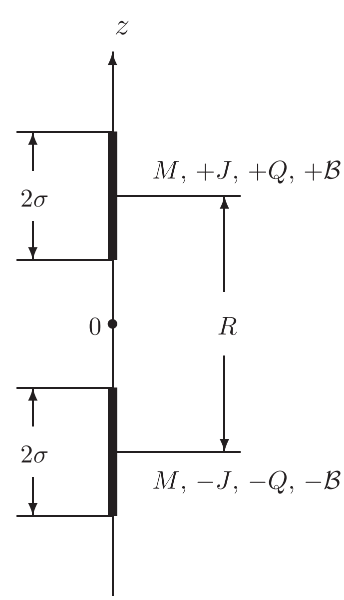

FIG. 1. Location of subextreme KN sources on the symmetry axis and the parameters associated with each source.

applied in order to check the physical relevance of the solutions. Fortunately, the physical inconsistency of formula (1) after performing the substitution (2) can be trivially established by considering the limit $R \to \infty$ (infinite separation of the KN sources when the interaction is absent), which leads to the expression

$$\sigma = \sqrt{M^2 - \frac{(J - Q\mathcal{B})^2}{M^2} - Q^2 - \mathcal{B}^2}, \tag{3}$$

and one can see that the above formula is manifestly different from the corresponding well-known σ defining the event horizon of an isolated KN black hole endowed with both electric and magnetic charges [see, e.g., Eq. (6.1) of [5]],

$$\sigma = \sqrt{M^2 - \frac{J^2}{M^2} - Q^2 - \mathcal{B}^2} = \sqrt{M^2 - a^2 - Q^2 - \mathcal{B}^2}. \tag{4}$$

To make things worse, the expression (3) is not invariant under the sign change $J \to -J$, $Q \to -Q$, $\mathcal{B} \to -\mathcal{B}$, the latter transformation converting the term $(J - Q\mathcal{B})^2$ into $(J + Q\mathcal{B})^2$, which clearly violates the symmetry of the particular two-body problem under consideration. Moreover, the nonlimiting expression for σ must be also invariant under the above sign change; however, a simple check shows that the required invariance is absent in the formula for σ given by Cabrera-Munguia *et al.*

Lastly, it might be worth mentioning that in view of the physical deficiency of the generic 5-parameter solution [4] it turns out that the specific 4-parameter metric earlier presented by Cabrera-Munguia *et al.* [6], besides its unphysical character pointed out in [1], must be also inevitably plagued by an incorrect expression for σ, because the latter was obtained by means of the same enhanced authors' formula as the more general σ from [4].

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