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Infrared Abelian dominance without Abelian projection

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Maximal Abelian gauge has been a particular choice to study dynamical generation of off-diagonal gluon masses in QCD. This gauge is a special case of an Abelian projection. Abelian dominance is characterized by off-diagonal gluons acquiring masses in the relevant phase. Here we propose a gauge condition which is quadratic in fields and which does not fall in the class of an Abelian projection. We explore the possible vacua of the gauge-fixed effective action of the theory and find evidence that ghost bilinears may be subject to condensation, which would signal acquisition of masses by off-diagonal gluons. Such a vacuum satisfies the requirement of Abelian dominance, providing an example of the hypothesis through a mechanism other than Abelian projection.

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I. INTRODUCTION

One of the burning questions to be answered definitively in QCD is, "what is the physical mechanism by which quarks and gluons are confined?" A classical model for confinement is a linear potential between static quarks. In the mid 1970s, a dual version of the type-II superconductor was proposed by Nambu [1], 't Hooft [2], and Mandelstam [3]. In the type-II superconductor, the magnetic field is trapped in the form of one-dimensional Abrikosov vortex tubes inside the superconductor, i.e., inside the medium of condensed electric charges [4]. In the same way but dually the idea of these proposals for the quark confinement mechanism is that the electric field due to quarks is trapped in the form of vortex tubes in a phase in which magnetic monopoles are condensed. This gives rise to forces characterized by a constant string tension and a linear potential between static quarks. Here the key point is that the dual picture is based on the Abelian gauge theory whereas QCD is non-Abelian, therefore one needs to demonstrate that QCD reduces to an effective Abelian theory at an infra-red scale. Secondly, it requires a new concept of condensed magnetic monopoles. Need for such an effective theory led to the concept of "Abelian dominance" [5].

According to the proposal of Abelian dominance, at a low energy scale, QCD can be effectively expressed in terms of Abelian gauge degrees of freedom [5]. It is usually discussed in terms of off-diagonal gluons i.e., gluons not associated with the Cartan subalgebra of $SU(N)$. In the $SU(N)$ gauge theory, there are $N(N-1)$ off-diagonal gluons. These gluons attaining large dynamical masses is presumed to provide the required evidence of existence of Abelian dominance. In the infra-red limit, the off-diagonal gluons decouple, leaving behind the massless diagonal gluons as the only dominant degrees of freedom. Thus, one gets $N-1$ copies of an Abelian gauge theory, one for each diagonal gluon. As far as we know, the occurrence of off-diagonal gluon masses and infra-red Abelian dominance have been studied mostly in the Maximal Abelian gauge, a few of the references being [6–9], which is a particular case of an Abelian projection [2]. A few studies have relied on unconventional gauges such as the Laplacian Abelian gauge (Abelian projection) and the Landau gauge [10,11] also in restrictive settings.

An Abelian projection [2] is a partial gauge fixing which leaves the maximal torus group of a group G unbroken. For $SU(N)$, the gauge condition takes the form of a variable $X(x)$ satisfying following conditions:

(i) It takes values in the Lie algebra of $SU(N)$.

(ii) It transforms by adjoint action

$$X(x) \to U(x)X(x)U(x)^{-1} \tag{1}$$

With a suitable gauge ransformation $X(x)$ can be diagonalized,

$$\widehat{X(x)} = V(x)X(x)V(x)^{-1}; \qquad V(x) \in SU(N) \tag{2}$$

such that $\widehat{X(x)}$ is invariant under $U(1)^{N-1}$, the maximal torus group of $SU(N)$. Hence each such variable $X(x)$ defines an Abelian projection. A prime example being the Lie algebra valued field strength. Here we propose "a quadratic gauge" which does not fall in the class of Abelian projections but possesses strong hints for a dynamical mass generation for off-diagonal gluons, possibly providing another route to Abelian dominance.

II. QUADRATIC GAUGE AND THE EFFECTIVE LAGRANGIAN

Consider Quadratic Gauge condition, specified as follows,

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$$F^a[A^\mu(x)] = A^a_\mu(x)A^{\mu a}(x) = f^a(x);\ \text{for each}\ a \tag{3}$$

where $f^a(x)$ is an arbitrary function of x. We note that this is not an Abelian projection as the gauge condition does not take values in the Lie algebra. Furthermore, the condition of an Abelian projection is stipulated to be covariant in order to ensure survival of an Abelian component. The above gauge condition does not meet this requirement either. The resulting effective Lagrangian density contains gauge fixing and ghost terms as follows,

$$\mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{ghost}} = -\frac{1}{2\zeta}\sum_a (A^a_\mu A^{\mu a})^2 - \sum_a \overline{c^a} A^{\mu a}(D_\mu c)^a \tag{4}$$

Where ζ is an arbitrary gauge fixing parameter and $(D_\mu c)^a = \partial_\mu c^a - g f^{abc} A^b_\mu c^c$. Now onwards, we shall drop the summation symbol, but the summation over an index a will be understood when it appears repeatedly, including when repeated *thrice* as in the ghost terms above. In particular,

$$-\overline{c^a} A^{\mu a}(D_\mu c)^a = -\overline{c^a} A^{\mu a}\partial_\mu c^a + g f^{abc}\overline{c^a} c^c A^{\mu a} A^b_\mu \tag{5}$$

where the summation over indices a, b and c each runs independently over 1 to $N^2 - 1$. With this understanding, we write the full effective Lagrangian density as

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} - \frac{1}{2\zeta}(A^a_\mu A^{\mu a})^2 - \overline{c^a} A^{\mu a}(D_\mu c)^a \tag{6}$$

where the first term is the usual Yang-Mills Lagrangian with $F^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) - g f^{abc} A^b_\mu(x) A^c_\nu(x)$.

III. OFF-DIAGONAL GLUON MASSES AND ABELIAN DOMINANCE

Although we do not intend to derive perturbative rules for the S-matrix here, the intuitive understanding in terms of the properties of quanta that can in principle occur in the asymptotic states is the most convenient in taking the discussion forward. With this in mind, we examine the degrees of freedom in the gauge-fixed action, which suggest that the off-diagonal gluons acquire masses if certain ghost bilinears are replaced by c-numbers. As such we now proceed to identify propagators for the various degrees of freedom, which will be instrumental in setting up a case for ghost condensation.

A. Mass generation due to ghost condensation

The Gluon propagator is formally given by

$$(\mathcal{O}^{-1}_{\text{ab}})_{\mu\nu}(p) = \delta_{ab}(\eta^{\mu\nu} p^2 - p^\mu p^\nu)^{-1} \tag{7}$$

We impose the Feynman gauge only on the diagonal gluons as we anticipate the off diagonal gluons to acquire masses and are not relevant to immediate discussion. This amounts to adding the following gauge fixing and ghost Lagrangians into (6), where the index j will designate the diagonal indices.

$$\mathcal{L}'_{\text{eff}} = \mathcal{L}_{\text{eff}} - \frac{1}{2\xi}(\partial_\mu A^{\mu j})^2 + \partial_\mu \overline{c^j}(D_\mu c)^j \tag{8}$$

Therefore, the diagonal gluon propagators are

$$(\mathcal{O}^{-1})^{jb}_{\mu\nu}(p) = -\frac{i\,\delta^{jb}}{p^2}\eta_{\mu\nu} \tag{9}$$

and the diagonal ghost propagators are

$$G^{jb}(p) = \frac{i\,\delta^{jb}}{p^2} \tag{10}$$

Consider any one of the ghost terms in the partially gauge-fixed Lagrangian (6):

$$-\overline{c^a} A^{\mu a}(D_\mu c)^a = -\overline{c^a} A^{\mu a}\partial_\mu c^a + g f^{abc}\overline{c^a} c^c A^{\mu a} A^b_\mu. \tag{11}$$

We see that in the second term on the right, if $\overline{c^a} c^c$ are replaced by c-numbers, the term provides a gluon mass matrix. This condition would be realized if the ghost bilinears underwent condensation. Such a possibility was elaborated in Ref. [6]. We shall here show that a similar computation lends strength to this hypothesis, i.e., an effective potential can be derived whose minima occur at

$$\langle \overline{c^m} c^n \rangle \neq 0 \quad \text{for all}\ m \quad \text{and} \quad n. \tag{12}$$

Consider a pair of anti-ghost and ghost, $\overline{c^m}$ and c^n, and introduce for them an auxiliary scalar field σ^{mn}, which is their putative condensate. We insert the following identity into the gauge-fixed path integral,

$$1 = \int \prod_{m=1,n=m}^{N^2-1,N^2-1} (\mathcal{D}\sigma^{mn}) \prod_{m=1,n=m}^{N^2-1,N^2-1} e^{-i\int d^4x \frac{1}{2\kappa}(\sigma^{mn} - \beta\overline{c^3}c^3 - \alpha\overline{c^m}c^n)^2}, \tag{13}$$

where α and β are couplings and κ is an arbitrary parameter. Ghost field c^3 is chosen as an example, it could be any one of the diagonal ghosts. This amounts to the following effective Lagrangian

$$\begin{aligned}\mathcal{L}(A,\sigma) = \mathcal{L}'_{\text{eff}} + \sum_{m=1,n=m}^{N^2-1,N^2-1}\Big[&\frac{1}{2\kappa}(\sigma^{mn})^2 - \frac{\beta}{\kappa}\sigma^{mn}\overline{c^3}c^3 \\ &- \frac{\alpha}{\kappa}\sigma^{mn}\overline{c^m}c^n + \frac{\alpha\beta}{\kappa}\overline{c^m}c^n\overline{c^3}c^3 \\ &+ \frac{\beta^2}{2\kappa}(\overline{c^3}c^3)^2 + \frac{\alpha^2}{2\kappa}(\overline{c^m}c^n)^2\Big]\end{aligned} \tag{14}$$

The dimensionless parameters α, β and κ make their appearance in the effective mass matrix of the gluons in Eq. (25), to be determined in terms of a mass parameter μ, and can be adjusted to reproduce the confinement scale within the Abelian dominance hypothesis.

The effective potential for σ^{mn} may now be computed within the standard strategy of Coleman-Weinberg mechanism in which one-loop diagrams give the leading quantum correction. In the present case, this consists of all one-loop c^3 diagrams.

$$V^{1-\mathrm{loop}}(\sigma^{mn}) = \int \frac{d^4p}{2i(2\pi)^4} \sum_{k=1}^{\infty} -\frac{1}{k} (\sigma^{mn})^k \frac{(i)^k}{(p^2)^k} \left(\frac{-i\eta}{\kappa}\right)^k = \int \frac{d^4p}{2i(2\pi)^4} \ln\left(\frac{p^2 - \frac{\eta}{\kappa}\sigma^{mn}}{p^2}\right) \tag{15}$$

Here $\eta = \alpha + \beta$ if $m = n = 3$, else $\eta = \beta$. Therefore, the effective potential is

$$V(\sigma^{mn}) = \frac{1}{2\kappa}(\sigma^{mn})^2 + \int \frac{d^4p}{2i(2\pi)^4} \ln\left(\frac{p^2 - \frac{\eta}{\kappa}\sigma^{mn}}{p^2}\right) \tag{16}$$

The extremum of the potential is given by the zero of the gap equation

$$V'(\sigma^{mn}) = \frac{\sigma^{mn}}{\kappa} - \frac{\eta}{\kappa} \int \frac{d^4p}{2i(2\pi)^4} \frac{1}{p^2 - \frac{\eta}{\kappa}\sigma^{mn}} = 0 \tag{17}$$

In the minimal subtraction scheme of the dimensional regularization, the gap equation can be written as

$$\frac{\sigma^{mn}}{\kappa} + \frac{1}{32\pi^2} \frac{\eta^2}{\kappa^2} \sigma^{mn} \left(\ln\left(\frac{\frac{\eta}{\kappa}\sigma^{mn}}{4\pi\mu^2}\right) + \gamma - 1 \right) = 0, \tag{18}$$

where γ is the Euler constant, 0.57721…, and μ is an arbitrary scale mass.

Apart from a trivial solution, it has a nontrivial solution,

$$\sigma_0^{mn} = \frac{\kappa}{\eta} 4\pi\mu^2 e^{(1-\gamma)} \exp\left(\frac{-32\pi^2\kappa}{\eta^2}\right), \tag{19}$$

which corresponds to the minimum of the potential because

$$V''(\sigma_0^{mn}) = \frac{1}{32\pi^2} \frac{\eta^2}{\kappa^2} > 0. \tag{20}$$

From the equation of motion for σ^{mn}, we can deduce that

$$\langle \overline{c^3} c^3 \rangle \sim \frac{\sigma_0^{33}}{\alpha + \beta} \quad \text{and} \tag{21}$$

$$\langle \overline{c^m} c^n \rangle \sim \frac{\sigma_0^{mn} - \beta \langle \overline{c^3} c^3 \rangle}{\alpha}. \tag{22}$$

Thus, we have laid out a mechanism and set up a case in which all the condensates are nonzero real and, except $\langle \overline{c^3} c^3 \rangle$, all of them are equal, reflecting an approximate $SU(N)$ symmetric vacuum. We needed $\frac{N^2(N^2-1)}{2}$ number of auxiliary fields only because condensates are real. Additional terms due to unity in the path integral and additional Feynman gauge fixing do not affect conclusions of the theory; therefore, we won't consider them in further discussions and take into account the original effective Lagrangian only.

In the proposed ghost-condensed phase, the second term of Eq. (11) gives us off-diagonal components of the gluon mass matrix,

$$(M^2)^{ab}_{\mathrm{dyn}} = 2g \sum_{c=1}^{N^2-1} f^{abc} \langle \overline{c^a} c^c \rangle, \tag{23}$$

whereas diagonal components of M^2_{dyn} are zero since $f^{aac} = 0$. To obtain a spectrum of the theory, i.e., to obtain masses of gluons, we must diagonalize the matrix and find eigenvalues. Required demonstration is simple in the approximate $SU(N)$ symmetric state which we argued, where all, except $\langle \overline{c^3} c^3 \rangle$, ghost-anti ghost condensates are identical, i.e.,

$$\langle \overline{c^3} c^3 \rangle \neq \langle \overline{c^1} c^1 \rangle = \cdots = \langle \overline{c^1} c^{N^2-1} \rangle = \cdots = \langle \overline{c^{N^2-1}} c^1 \rangle = \cdots = \langle \overline{c^{N^2-1}} c^{N^2-1} \rangle = K. \tag{24}$$

Thus,

$$(M^2)^{ab}_{\mathrm{dyn}} = 2g \sum_{c=1}^{N^2-1} f^{abc} K, \tag{25}$$

which is an antisymmetric matrix due to the antisymmetry of the structure constants. We proceed to show that this matrix has only $N(N-1)$ nonzero eigenvalues and, thus, has nullity $N-1$. We define the matrix

$$J^{ab} = i\left(\sum_{c=1}^{N^2-1} [T^{ab}]^c\right) = \left[\sum_{c=1}^{N^2-1} f^{abc}\right], \tag{26}$$

which belongs to a $(N^2-1) \times (N^2-1)$ adjoint representation of $\mathfrak{g}$, the Lie algebra vector space. For the case of $SU(N)$, the coadjoint representation $[T_{ab}]^*_c$ which is dual of the adjoint is the same as the adjoint. Elements g of $SU(N)$ act on $\mathfrak{g}^*$, the dual vector space, by conjugation,

$$\{Ad^*F = g^{-1}Fg, F \in \mathfrak{g}^*\}.$$

The orbit $\mathcal{O}_F = \{Ad^*F, \forall g \in SU(N)\}$, passing through F, is known as the coadjoint orbit. This action has a stabilizer, i.e., a set of elements in $SU(N)$ that leave the elements F

invariant. The stabilizer happens to be the maximal torus group for the case of a compact connected Lie group. Thus for $SU(N)$, it is the maximal torus group $U(1)^{N-1}$. Thus, the coadjoint orbits of $SU(N)$ group are isomorphic to a manifold $SU(N)/U(1)^{N-1}$, i.e., $\mathcal{O}_F \sim SU(N)/U(1)^{N-1} \sim \mathbb{C}P^{N-1} \otimes \mathbb{C}P^{N-2} \otimes \cdots \otimes \mathbb{C}P^1$. It is an $N(N-1)$ dimensional symplectic manifold with a symplectic form $\mathrm{Tr}(F[X,Y])$, where $F \in \mathfrak{g}^*$ and $X, Y \in \mathfrak{g}$ [12]. In the present problem, that is the form of the expression under consideration, $J^{ab} = \frac{-i}{N}\mathrm{Tr}(F[T^a, T^b])$, with $F = \sum_{c=1}^{N^2-1} T^c$ and T^a, T^b, T^c being basis generators. The rank of a symplectic form is always equal to a dimension of the coadjoint orbit. Since J^{ab} is a normal matrix, its rank and number of nonzero eigenvalues are equal. So, the rank and, therefore, the number of nonzero eigenvalues are $N(N-1)$ and, thus, nullity is $N-1$. Thus, we have proved at tree level that in an approximate $SU(N)$ invariant vacuum, $N(N-1)$ off-diagonal gluons acquire masses and $N-1$ diagonal gluons remain massless. Therefore, the off-diagonal gluon acquires the massive propagator in this phase, which is of the form

$$(\mathcal{O}^{-1}_{\mathrm{ofd}})^{ab}_{\mu\nu}(p) = -\frac{i\,\delta^{ab}}{p^2 - M^2_{\mathrm{gluon}}}\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{M^2_{\mathrm{gluon}}}\right). \qquad (27)$$

The nonzero eigenvalues thus identified, being eigenvalues of an antisymmetric matrix, are purely imaginary and occur in conjugate pairs, viz., $M^2_{\mathrm{gluon}} = \pm i m^2$ (m^2 positive real). Now since M^2_{gluon} is purely imaginary, the off-diagonal gluon propagator does not have any singularity on a real p^2 axis, which is a sufficient condition for the confinement [13]. This implies that masses of these gluons $M_{\mathrm{gluon}} = \frac{1}{\sqrt{2}}(1 \pm i)m$ or $\frac{1}{\sqrt{2}}(-1 \mp i)m$. We ignore the latter choice since it gives $\mathrm{Re}(M_{\mathrm{gluon}})$ negative, which is not physical. Although we are not interested in an S-matrix interpretation for these degrees of freedom, *prima facie* there is no danger from these eigenvalues being purely imaginary. However, to retain the intuitive appeal of the arguments, it is necessary to check that we have not departed too far from their interpretation as quanta and, in principle, an S-matrix interpretation. This is what we shall do in the next section.

In this description, the off-diagonal gluons acquire masses with the positive real parts, which makes them short-ranged. Only the diagonal gluons mediate long-range interactions, strongly suggesting Abelian dominance.

IV. HERMITICITY OF THE EFFECTIVE LAGRANGIAN IN A QUADRATIC GAUGE

In order to retain the appeal to a picture of this ground state in terms of quanta, it is useful to check that it will not conflict with nominally expected restrictions on the manner in which such degrees of freedom enter into the S matrix. Here we will show that while in the normal phase the effective Lagrangian is manifestly Hermitian, a condition indispensable for the S-matrix unitarity as per [14], the effective Lagrangian obeys an extended Hermiticity condition in the ghost-condensed phase, but the S-matrix continues to remain unitary.

The effective Lagrangian in the normal phase is given in Eq. (6),

$$\mathcal{L}_{\mathrm{eff}} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} - \frac{1}{2\zeta}(A^a_\mu A^{\mu a})^2 - \overline{c^a}A^{\mu a}(D_\mu c)^a. \qquad (28)$$

The Hermiticity property of fields is given by [14]

$$\begin{aligned} A^{a\dagger}_\mu &= A^a_\mu \\ c^{a\dagger} &= c^a \\ \overline{c^a}^\dagger &= -\overline{c^a}. \end{aligned} \qquad (29)$$

It is easy to check that this Lagrangian is Hermitian under the Hermitian conjugation of fields since

$$(\overline{c^a}c^c)^\dagger = -c^c\overline{c^a} = \overline{c^a}c^c$$
$$(\overline{c^a}\partial_\mu c^c)^\dagger = -\partial_\mu c^c\overline{c^a} = \overline{c^a}\partial_\mu c^c$$

(we have used an anticommutativity of ghost fields).

In the ghost-condensed phase, $\langle\overline{c^a}c^c\rangle$, the derivative terms like $\langle\overline{c^a}\partial_\mu c^c\rangle$ are zero since we have assumed the condensate to be the same at every space-time point x. Hence, the effective Lagrangian now becomes

$$\mathcal{L}_{\mathrm{eff}} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} - \frac{1}{2\zeta}(A^a_\mu A^{\mu a})^2 + M^2_a A^a_\mu A^{\mu a}. \qquad (30)$$

Here $M^2_a = 0$ when a indexes the diagonal gluons, e.g., for $SU(3)$, $M^2_3 = M^2_8 = 0$. While for the off-diagonal gluons, $M^2_1 = +im^2_1$, $M^2_2 = -im^2_1$, $M^2_4 = +im^2_2$, $M^2_5 = -im^2_2$, $M^2_6 = +im^2_3$, $M^2_7 = -im^2_3$ (m^2_1, m^2_2, m^2_3 are positive real). Hence, for $SU(3)$, the last term of the effective Lagrangian in Eq. (30) would be

$$\begin{aligned} M^2_a A^a_\mu A^{\mu a} = &+ im^2_1 A^1_\mu A^{\mu 1} - im^2_1 A^2_\mu A^{\mu 2} + im^2_2 A^4_\mu A^{\mu 4} \\ &- im^2_2 A^5_\mu A^{\mu 5} + im^2_3 A^6_\mu A^{\mu 6} - im^2_3 A^7_\mu A^{\mu 7}. \end{aligned} \qquad (31)$$

Now, taking the Hermitian conjugate of a Lagrangian in Eq. (30) will alter no term except the mass term and will interchange the sign of mass terms between "conjugate" gluons, e.g, in Eq. (31),

$$\begin{aligned} (M^2_a A^a_\mu A^{\mu a})^\dagger = &- im^2_1 A^1_\mu A^{\mu 1} + im^2_1 A^2_\mu A^{\mu 2} - im^2_2 A^4_\mu A^{\mu 4} \\ &+ im^2_2 A^5_\mu A^{\mu 5} - im^2_3 A^6_\mu A^{\mu 6} + im^2_3 A^7_\mu A^{\mu 7}. \end{aligned} \qquad (32)$$

We now invoke an inner automorphism of the gauge group to help resolve the issue. Consider the individual Lagrangian term L_i, which can be $-\frac{1}{4}F^i_{\mu\nu}F^{\mu\nu i}$, $-\frac{1}{2\zeta}(A^i_\mu A^{\mu i})^2$, $im^2 A^i_\mu A^{\mu i}$, $-\overline{c^i}A^{\mu i}(\partial_\mu c)^i$ for each i, where the last term, although zero in the ghost-condensed phase, has been retained for the sake of generality. We propose an inner automorphism $\mathfrak{T}$ such that

$$\begin{aligned} &\mathfrak{T}L_1\mathfrak{T}^\dagger = L_2 \quad \mathfrak{T}L_4\mathfrak{T}^\dagger = L_5 \quad \mathfrak{T}L_6\mathfrak{T}^\dagger = L_7 \quad \mathfrak{T}L_3\mathfrak{T}^\dagger = L_8 \\ &\mathfrak{T}L_2\mathfrak{T}^\dagger = L_1 \quad \mathfrak{T}L_5\mathfrak{T}^\dagger = L_4 \quad \mathfrak{T}L_7\mathfrak{T}^\dagger = L_6 \quad \mathfrak{T}L_8\mathfrak{T}^\dagger = L_3, \end{aligned} \tag{33}$$

with the property

$$\mathfrak{T}^2 = \mathfrak{T}^{\dagger 2} = 1. \tag{34}$$

The inner automorphism is essentially exchanging group indices, i.e., $1 \leftrightarrow 2$, $4 \leftrightarrow 5$, $6 \leftrightarrow 7$, $3 \leftrightarrow 8$. In the adjoint representation, it is given by

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

It can be seen that the inner automorphism operation does not cause any change in any of the terms except the mass term in Eq. (30) and further, when operated on Eq. (32), it can be easily verified that

$$\mathfrak{T}(M_a^2 A^a_\mu A^{\mu a})^\dagger \mathfrak{T}^\dagger = M_a^2 A^a_\mu A^{\mu a}. \tag{35}$$

The rest of the Lagrangian, which includes interacting parts, is Hermitian and the role of an inner automorphism remains trivial for it. Therefore, the interaction Hamiltonian H_I is also Hermitian. Thus, the vacuum may be considered to furnish a nontrivial representation of an inner automorphism. So, as a result, the $\mathcal{L}_{\text{eff}}$ in Eq. (30) is invariant under the Hermitian conjugation followed by an inner automorphism,

$$\mathfrak{T}\mathcal{L}^\dagger_{\text{eff}}\mathfrak{T}^\dagger = \mathcal{L}_{\text{eff}}. \tag{36}$$

Now, the S matrix, in general, is given as follows:

$$S = T\exp\left(-i\int H_I(t)dt\right). \tag{37}$$

This interaction Hamiltonian H_I is Hermitian and the free quadratic part plays no role when the S matrix is expanded perturbatively. Therefore, even though the effective Lagrangian is not pure Hermitian, the usual unitarity condition for the S matrix, $S^\dagger S = SS^\dagger = 1$, still holds in the ghost-condensed phase. We may understand the inclusion of an inner automorphism symmetry in ensuring the Hermiticity of the effective Lagrangian $\mathcal{L}_{\text{eff}}$ to be a refinement over the usual discrete internal symmetry C, the charge conjugation symmetry.

V. CONCLUSION

The gauge-fixed Lagrangian with the choice of quadratic gauge $A^a_\mu A^{\mu a} = f^a(x)$, for each a, contains ghost-gauge field interaction terms $gf^{abc}\overline{c^a}c^c A^{\mu a}A^b_\mu$. It is possible to interpret the latter as providing masses to the off-diagonal gluons in a vacuum respecting an approximate $SU(N)$ invariance, which we argue is signaled by ghost condensation. This interpretation provides a clue to the existence of Abelian dominance at infrared energies in the quadratic gauge in QCD. We also showed that the effective Lagrangian in the absence of condensation is Hermitian, giving us the usual unitarity condition for the S matrix, and proposed an extended Hermiticity in the suggested ghost-condensed phase which would ensure unitarity in the latter phase.

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