Solutions to the problem of Elko spinor localization in brane models

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Two different solutions to the problem of the zero-mode localization of the Elko spinor are presented. The first solution is given by the introduction of a mass term and by coupling the spinor with the brane through a delta function. The second solution is reached by a Yukawa geometrical coupling with the Ricci scalar. These two models consistently change the boundary condition at infinity and at the origin. For the case of the geometrical coupling, we are able to show that the zero mode is localized for any smooth version of the Randall-Sundrum model.

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I. INTRODUCTION

The idea that space-time has more than four dimensions garnered much attention following the theory of Kaluza and Klein regarding compact extra dimensions. The main idea is that the extra dimensions are so tiny that they cannot be observed, and the "escape" of the fields to the extra dimensions becomes a small correction [1]. Depending on the compactification and the number of dimensions, different kinds of fields in the lower-dimensional theory can be obtained [2]. In fact, these models provide us with a plethora of massive states, the zero-mode being just one particular case.

Models considering the Universe as a brane in a higher-dimensional space-time regained attention in the late 1990s [3]. A scenario considering our world as a shell was proposed in Ref. [4] and further developed in Refs. [5–7]. Probably inspired by Ref. [4], Randall and Sundrum (RS) proposed another scenario with four-dimensional branes in a five-dimensional (5D) anti–de Sitter bulk. In this scenario two different models were considered. In the first model (RSI), two branes in a compact space with Z_2 symmetry were used to solve the hierarchy problem [8]. Because the model has a compact dimension, the dimensional reduction works in a very similar way to the Kaluza and Klein model. In the second model (RSII), just one brane embedded in a large extra-dimensional space is considered. The extra dimension is curved by a warp factor such that the model has been considered as an alternative to compactification [9]. As it is a model with large extra dimensions, the issue of the zero-mode localization of fields is non-trivial. In fact, in the last ten years the zero-mode localization of gauge fields has become a drawback in these models. Localization is necessary since in four-dimensional space-time no fields propagating into the bulk are observed. Moreover, it has been found that the zero modes of gravity and scalar fields are localized [9,10] in a positive-tension brane. However, due to its conformal invariance the vector field is not localized, which is a serious problem for a realistic model.

Many solutions to the above-mentioned problem have been found. For example, some authors introduced a dilaton coupling to solve the problem [11], while others suggested that a strongly coupled gauge theory in five dimensions can generate a massless photon in the brane, like in Ref. [12]. Generalizations considering antisymmetric fields can also be found in the literature [13–20]. Most of these models introduced other fields or nonlinearities in the gauge field [21]. However, there are two models that do not introduce new fields or nonlinearities. One introduces a mass term and a delta interaction with the brane [22]. This kind of model has been generalized to consider smooth branes and to p-forms by the present authors in Ref. [23]. The other one has a gauge field coupled to the Ricci scalar, which is called a geometrical coupling [24,25]. Similar ideas have been used before, but using a coupling with the field strength [26]. Another interesting approach to solve the problem is related to models where membranes are smoothed out by topological defects [27–30]. The advantage of these models is that the δ-function singularities—which are generated by the brane in the RS scenario—are eliminated. This kind of generalization also provides methods for finding analytical solutions [31,32].

Studies of localization and the resonances of matter fields with half spin have also been considered for cases with and without an interaction term in Refs. [33–43]. Another kind of field that can be considered is the Elko spinor. This field has spin $1/2$ and mass dimension one, and it can be considered as a first-principle candidate for dark matter [44–48]. (For a more detailed study, see, for example, Ref. [49].) In the framework of a brane-world scenario, the localization of Elko spinors on branes was considered in Ref. [50]. However, the present authors have shown in Ref. [51] that their conclusion stating that the zero

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mode of this field is localized is not correct. This conclusion reopened the problem of the localization of the Elko spinor.

Here we show that the previous proposals of the present authors to solve the problem of p-form field localization in Refs. [23,24] can also be applied to this problem. We also show that for specific values of the coupling constant there can be localized zero modes for the Elko spinor. This raises the same interesting question about the origin of this kind of coupling.

This paper is organized as follows. Section II gives a review of how to obtain the mass equation for the Elko spinors. Section III is devoted to finding the solution to the problem by using a mass term and a quadratic interaction with a delta function. In Sec. IV we use the geometrical coupling as another solution to the localization of the zero mode. Finally, in Sec. V we discuss the conclusions and perspectives.

II. REVIEW OF THE LOCALIZATION OF 5D ELKO SPINORS WITH A COUPLING TERM ON A MINKOWSKI BRANE

In this section we present a review of the localization of the 5D Elko spinor, based on Ref. [50]. The action used in the cited article for the Elko spinor is

$$S = \int d^5x\sqrt{-g}\left[-\frac{1}{4}(D_M\lambda D^M\bar{\lambda} + D_M\bar{\lambda}D^M\lambda) - \eta F(z)\bar{\lambda}\lambda\right], \tag{1}$$

where η in a coupling constant, $F(z)$ is a scalar function of the conformal extra dimension z, and D_M is the covariant derivative defined as

$$D_M\lambda = \partial_M\lambda + \Omega_M\lambda. \tag{2}$$

As shown in Ref. [50], the nonvanishing components of the spin connection are

$$\Omega_\mu = \frac{1}{2}A'(z)\gamma_\mu\gamma_5, \tag{3}$$

where the prime denotes a derivative with respect to the argument, $A(z)$ is the conformal warp factor $g_{MN} = \mathrm{e}^{2A(z)}\eta_{MN}$, and $\eta_{MN} = \mathrm{Diag}(-1,1,1,1,1)$. Taking the variation of the action with respect to $\bar{\lambda}$, we obtain the equation of motion,

$$D_M[\sqrt{-g}D^M\lambda] - 2\eta\sqrt{-g}F(z)\lambda = 0. \tag{4}$$

Using the metric and the nonvanishing components of the spin connection, we can write the above equation in the form

$$\eta^{\mu\nu}\partial_\mu\partial_\nu\lambda - A'(z)\gamma_5\eta^{\mu\nu}\gamma_\mu\partial_\nu\lambda - A'^2\lambda + \mathrm{e}^{-3A}(\mathrm{e}^{3A}\lambda')' - 2\eta\mathrm{e}^{2A}F(z)\lambda = 0. \tag{5}$$

Due to the term $A'(z)\gamma_5\eta^{\mu\nu}\gamma_\mu\partial_\nu\lambda$ the authors of Ref. [50] proposed a decomposition of the Elko field as $\lambda = \lambda_+ + \lambda_-$, with

$$\lambda_\pm = \mathrm{e}^{-3A/2}\sum_n\alpha_n(z)\tilde{\lambda}^n_\pm(x) = \mathrm{e}^{-3A/2}\sum_n\alpha_n(z)[\varsigma^n_\pm(x) + \tau^n_\pm(x)], \tag{6}$$

where $\varsigma^n_\pm(x)$ and $\tau^n_\pm(x)$ are two independent four-dimensional (4D) Elko fields satisfying the Klein-Gordon equations $\Box\tau^n_\pm(x) = m_n^2\tau^n_\pm(x)$, $\Box\varsigma^n_\pm(x) = m_n^2\varsigma^n_\pm(x)$ and the relations $\gamma^5\tau_\pm = \mp\varsigma_\pm$, $\gamma^5\varsigma_\pm = \pm\tau_\pm$. The decomposition in Eq. (5) leads to

$$\alpha''_n(z) - \left(\frac{13A'^2}{4} + \frac{3A''}{2} - m_n^2 + im_nA'(z) + 2\eta\mathrm{e}^{2A}F(z)\right)\alpha_n(z) = 0, \tag{7}$$

with the normalization condition

$$\int\alpha^*_n\alpha_m dz = \delta_{mn}. \tag{8}$$

Equation (7) is the general equation of the localization coefficients α_n. In the following sections the coefficients for some $F(z)$ will be derived.

III. LOCALIZATION OF 5D ELKO SPINORS WITH 4D AND 5D MASS TERMS

In this section we explicitly calculate the solution of Eq. (7), where $F(z)$ is a 5D mass term plus a 4D one, i.e.,

$$\eta F(z) = \frac{1}{2}(M^2 + c\delta(z)). \tag{9}$$

These kinds of couplings have used for p-form fields and provided an efficient mechanism to trap the zero modes [23]. In this case, the equation of α_n can be written as

$$\alpha''_n(z) - \left[\left(\frac{19k^2}{4} + M^2\right)[k|z|+1]^{-2} + (c-3k)\delta(z) - m_n^2 - im_nk\,\mathrm{sgn}(z)[k|z|+1]^{-1}\right]\alpha_n(z) = 0, \tag{10}$$

where we used the conformal Randall-Sundrum warp factor

$$A(z) = -\ln[k|z|+1]. \tag{11}$$

For the zero mode $m_0 = 0$, Eq. (10) provides the solution

$$a_0 = C_+(k|z|+1)^{1/2+\nu} + C_-(k|z|+1)^{1/2-\nu}, \tag{12}$$

where C_+ and C_- are constants and ν is given by

$$\nu = \sqrt{5 + M^2/k^2}. \tag{13}$$

The boundary condition at the origin imposes a relation between the constants,

$$(2k(2+\nu) - c)C_+ + (2k(2-\nu) - c)C_- = 0. \tag{14}$$

To obtain a convergent solution we must fix

$$c = 2k(2-\nu). \tag{15}$$

This procedure leads to the normalized convergent solution

$$a_0 = \frac{(k|z|+1)^{1/2-\nu}}{\sqrt{k(\nu-1)}}. \tag{16}$$

As we can see from Eqs. (13) and (15), it is not possible to get rid of M and c at the same time and keep the solution convergent. In this regard the result of Ref. [50] is incomplete, because its does not satisfy the boundary condition at $z = 0$.

For massive modes, the field (10) can be written in the form

$$a_n''(z) - \left[\left(\frac{19k^2}{4} + M^2\right)[k|z|+1]^{-2} + (c-3k)\delta(z) - m_n^2 - im_n k\,\mathrm{sgn}(z)[k|z|+1]^{-1}\right]a_n(z) = 0. \tag{17}$$

The above equation has the solution

$$\begin{aligned} a_n(z) &= C_1\theta(z)M_{1/2,\nu}(i2m_n/k(k|z|+1)) + C_2\theta(-z)M_{-1/2,\nu}(i2m_n/k(k|z|+1)) \\ &\quad + C_3\theta(z)W_{1/2,\nu}(i2m_n/k(k|z|+1)) + C_4\theta(-z)W_{-1/2,\nu}(i2m_n/k(k|z|+1)), \end{aligned} \tag{18}$$

where C_1, C_2, C_3, and C_2 are constants satisfying the following boundary conditions:

$$C_1M_{1/2,\nu}(2im_n/k) - C_2M_{-1/2,\nu}(2im_n/k) + C_3W_{1/2,\nu}(2im_n/k) - C_4W_{-1/2,\nu}(2im_n/k) = 0, \tag{19}$$

$$\begin{aligned} &4im_n[C_1M'_{1/2,\nu}(u) + C_2M'_{-1/2,\nu}(u)] - (c-3k)(C_1M_{1/2,\nu}(im_n/k) + C_2M_{-1/2,\nu}(im_n/k)) \\ &\quad + 4im_n[C_3W'_{1/2,\nu}(u) + C_4W'_{-1/2,\nu}(u)] - (c-3k)(C_3W_{1/2,\nu}(im_n/k) + C_4W_{-1/2,\nu}(im_n/k)) = 0. \end{aligned} \tag{20}$$

At this point, it is interesting to try to localize a specific massive mode, in order to model the dark matter as the Elko spinor without the Higgs mechanism. Because the argument of the Whittaker function is complex, it is not possible to find a coupling constant c that localizes any massive mode. Thus, the Higgs mechanism is still necessary in order to keep the Elko spinor as a candidate for dark matter.

IV. ELKO SPINOR WITH GEOMETRICAL COUPLING

In this section a geometrical coupling based on a Yukawa interaction with the Ricci scalar is introduced. This kind of coupling has been used for p-forms and provides a second efficient method to localize the zero modes [24,25]. In the RS brane it provides a natural source for the coupling term used in the previous section. The action is given by

$$S = \int d^5x\sqrt{-g}\left[-\frac{1}{4}(D_M\lambda D^M\bar{\lambda} + D_M\bar{\lambda}D^M\lambda) - \eta R\lambda\bar{\lambda}\right], \tag{21}$$

i.e., it is equivalent to making $F(z) = R$ in Eq. (1). In conformal coordinates the Ricci scalar is given by

$$R = -4(2A'' + 3A'^2)e^{-2A}, \tag{22}$$

and the field equation (7) can be written as

$$\begin{aligned} a_n''(z) - \Bigg[\left(\frac{13}{4} - 24\eta\right)A'^2 + \left(\frac{3}{2} - 16\eta\right)A'' \\ - m_n^2 + im_nA'(z)\Bigg]a_n(z) = 0. \end{aligned} \tag{23}$$

As in Refs. [24,25], we propose a solution for the zero mode in the form

$$a_0 \propto e^{\gamma A}, \tag{24}$$

so that the field equation provides the condition for this kind of solution,

$$\left(\gamma^2 - \frac{13}{4} + 24\eta\right)A'^2 + \left(\gamma - \frac{3}{2} + 16\eta\right)A'' = 0. \quad (25)$$

As we can see from Eq. (24), one needs $\gamma > 0$ in order to obtain a convergent solution. Using this restriction, Eq. (25) provides the solution $\gamma = 2$ and $\eta = -1/32$. Therefore, in this model the localization of the zero mode is guaranteed for any smooth version of the RS model. This solves the problem of the zero-mode localization of Elko spinors. To obtain explicit solutions, as well as the massive modes, we must use specific scenarios, i.e., explicit warp factors. This is what we will compute in the following subsections.

A. Randall-Sundrum brane

First, we will compute the explicit solution for the Randall-Sundrum brane scenario. In this case, the warp factor in conformal coordinates is given by

$$A(z) = -\ln[k|z| + 1]. \quad (26)$$

Inserting this warp factor into Eq. (24), we obtain the normalized convergent solution for the zero mode,

$$a_0 = \sqrt{\frac{2}{3k}}(k|z| + 1)^{-2}. \quad (27)$$

Comparing this with the solution obtained in Sec. III, we conclude that the geometrical coupling is equivalent to making $M^2/k^2 = 5/4$, and from Eq. (15), $c = -k$. This correspondence occurs due to the fact that the geometrical coupling in a Randall-Sundrum case reduces to

$$\eta R\lambda\bar{\lambda} = \frac{1}{2}\left(\frac{5}{4}k^2 - k\delta(z)\right)\lambda\bar{\lambda}, \quad (28)$$

i.e., it is equivalent to the term used in Sec. III. Due to this equivalence, the massive modes are given by

$$\begin{aligned} a_n(z) = {} & \theta(z)[A_1 M_{1/2,5/2}(i2m_n/k(k|z|+1)) + B_1 W_{1/2,5/2}(i2m_n/k(k|z|+1))] \\ & + \theta(-z)[A_2 M_{-1/2,5/2}(i2m_n/k(k|z|+1)) + B_2 W_{-1/2,5/2}(i2m_n/k(k|z|+1))], \end{aligned} \quad (29)$$

where A_1, A_2, B_1, and B_2 are constants satisfying the following conditions:

$$A_1 M_{1/2,5/2}(u_0) - A_2 M_{-1/2,5/2}(u_0) + B_1 W_{1/2,5/2}(u_0) - B_2 W_{-1/2,5/2}(u_0) = 0, \quad (30)$$

$$\begin{aligned} & u_0[A_1 M'_{1/2,5/2}(u) + A_2 M'_{-1/2,5/2}(u)]|_{u=u_0} + 2[A_1 M_{1/2,5/2}(u_0) + A_2 M_{-1/2,5/2}(u_0)] \\ & + u_0[B_1 W'_{1/2,5/2}(u) + B_2 W'_{-1/2,5/2}(u)]|_{u=u_0} + 2[B_1 W_{1/2,5/2}(u_0) + B_2 W_{-1/2,5/2}(u_0)] = 0, \end{aligned} \quad (31)$$

where $u_0 = 2im_n/k$. Due the asymptotic behavior of the Whittaker functions with complex arguments, is not possible to find a convergent solution, i.e., the massive modes are nonlocalized.

B. Smooth brane

Here we use the smooth warp factor [52,53]

$$A(z) = -\frac{1}{2n}\ln[(kz)^{2n} + 1], \quad (32)$$

which recovers the Randall-Sundrum metric at large z for integer n. Inserting this warp factor into Eq. (24), we find the normalized convergent solution for the zero mode,

$$a_0 = \sqrt{\frac{k}{2}\frac{\Gamma(2/n)}{\Gamma(3/2n)\Gamma(1+1/2n)}}[(kz)^{2n} + 1]^{-1/n}. \quad (33)$$

For massive modes the field equation can be written as

$$\begin{aligned} a_n''(z) - \Bigg[& 6\frac{(kz)^{2n-2}k^2}{[(kz)^{2n}+1]} - 4(n+1)\frac{(kz)^{2n-2}k^2}{[(kz)^{2n}+1]^2} \\ & - m_n^2 - im_n\frac{(kz)^{2n-1}k}{[(kz)^{2n}+1]}\Bigg]a_n(z) = 0. \end{aligned} \quad (34)$$

The solution of the massive modes cannot be determined analytically. Since the asymptotic behavior is the same as in the Randall- Sundrum case, it is not possible to find a convergent solution. A conclusive result needs a numerical analysis, and this will be the subject of a future work. Due to the imaginary massive term it is not possible to use the transference matrix method to study the unstable massive modes.

V. CONCLUSION

In this paper we have used two different methods to localize the zero mode of the Elko spinor. This problem was reopened in Ref. [50], which showed that the conclusions about localization in Ref. [51] were incorrect. The first method discussed here was the inclusion of 4D and 5D mass terms. We found the relation between these two mass terms that localizes the zero mode. This result shows that it is not possible to get rid of booth contributions for a localized solution, in disagreement with the results obtained in Ref. [50]. This disagreement occurs because the authors of the cited article found a convergent solution, but they do not take into account the boundary condition at the origin. We also showed that the massive modes are nonlocalized for this new model. The reason for this is that the solution is given by the Whittaker function with a complex argument, so it is not possible to find a coupling constant c that localizes a specific massive mode in order to model the dark matter as Elko spinors without the Higgs mechanism.

The other method used to localize the zero mode of the Elko spinor uses a geometrical coupling. In this model we computed the coupling constant that localizes the zero mode for all warp factors with Randall-Sundrum asymptotic behavior. We computed the explicit solution for the Randall-Sundrum case and for a specific smooth brane scenario. For the first one we conclude that the massive modes are nonlocalized, and for the smooth scenario the asymptotic behavior indicates that these modes are nonlocalized. A numerical analyses is needed to provide a final answer for the nonlocalization of massive modes in the smooth scenario, and this is the subject of future work. Due to the complex term in the potential, the present authors have not been able to compute resonances, and this is left to a future work.

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