Magnetic brane solutions of Lovelock gravity with nonlinear electrodynamics

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In this paper, we consider logarithmic and exponential forms of nonlinear electrodynamics as a source and obtain magnetic brane solutions of the Lovelock gravity. Although these solutions have no curvature singularity and no horizon, they have a conic singularity with a deficit angle. We investigate the effects of nonlinear electrodynamics and the Lovelock gravity on the value of the deficit angle and find that various terms of Lovelock gravity do not affect the deficit angle. Next, we generalize our solutions to spinning cases with maximum rotating parameters in arbitrary dimensions and calculate the conserved quantities of the solutions. Finally, we consider nonlinear electrodynamics as a correction of the Maxwell theory and investigate the properties of the solutions.

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I. INTRODUCTION

Nonsingular solutions are playing an increasingly important role in physics. The cosmological singularity at the early Universe corresponds to an infinite energy density state, and therefore it may probably be essential to consider the quantum gravity to understand the initial state of the Universe. Hence, from the cosmological point of view, nonsingular models of the Universe have a special position for scientists [1]. From a gravitational viewpoint, various regular solutions, such as gravitational instantons, solitons, and horizonless magnetic branes (string) solutions, have become the subject of interest in recent years [2–11].

On the other hand, considering four-/higher-dimensional spacetimes, the cosmic strings/branes are topological defects that are inevitably formed during phase transitions in the early Universe [12]. Investigation of the horizonless magnetic solutions and their relations to the topological defects help us to think about the origin of cosmic magnetic fields [13,14]. Besides, from a geometric point of view, these structures are fascinating objects, which have no curvature singularity and no horizon but have a conic singularity. One of the important motivations for investigating the horizonless magnetic stings/branes comes from the fact that these kinds of solutions may be interpreted as cosmic strings/branes. The horizonless solutions of Einstein and higher-derivative gravity theories in the absence and presence of the Maxwell and dilaton fields have been studied in the literature [5,6]. An extension to include the nonlinear electrodynamics has also been done [7–11].

The purpose of the present paper is constructing a new class of static and spinning magnetic brane solutions that produces a longitudinal magnetic field in the background of anti-de Sitter spacetime. These solutions are the generalization of the solutions of Ref. [11] to higher dimensions and higher-derivative gravity.

In order to have better description of phenomena in our Universe, physicists have introduced various theories. It has been confirmed that most of phenomena in the nature are inherently chaotic and may be described with nonlinear theories. In electrodynamics domain, although the Maxwell theory is in agreement with experimental results, it fails regarding some important issues such as the self-energy of pointlike charges, which motivates us to regard nonlinear electrodynamics (NED). NED theories may be created from various viewpoint and motivations. For more explanations of some motivations, we refer the reader to the following brief examples: solving the problem of a pointlike charge self-energy, being compatible with AdS/CFT correspondence and string theory frames, understanding the nature of different complex systems, obtaining more information and insight regarding to quantum gravity, describing pair creation for Hawking radiation, and the behavior of the compact astrophysical objects such as neutron stars and pulsars [15–17]. These evidences motivate one to consider NED theories.

Through the last decades, different classes of the nonlinear theories have been introduced [18–26]. Among the NED theories, the so-called Born–Infeld (BI) type theories are quite special, the Lagrangians of which may be originated from the string theory. It has been shown that the low-energy limit of heterotic string theory on the electrodynamics side leads to a quartic correction of the Maxwell field strength [27]. Moreover, one finds that all-order loop corrections may be summed up as a BI-type Lagrangian [28–30]. Recently, two kinds of BI-type Lagrangians have been considered to examine the

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possibility of black hole solutions [20–25]. Although there are some analogs between the BI-type theories, one can find that there exist some differences between them.

In recent years, a renewed interest has grown in higher-dimensional spacetime as well as higher-dimensional gravity [31]. The main reason comes from the fact that these theories emerge in the effective low-energy action of string theory on the gravitational side [32–35]. One of the special classes of higher-derivative modifications of Einstein (EN) gravity is the Lovelock theory [36], which is a ghost-free model [37,38]. Regarding the postulates of general relativity, most physicists believe that the Lovelock Lagrangian is a natural generalization of the EN gravity to higher dimensions. Besides, Lovelock gravity may solve some of the problems of the Einstein theory such as the normalization problem, and hence it is a well-defined model [39–41]. In this paper, we consider the Lovelock gravity in the presence of two classes of BI-type NED models and obtain their horizonless solutions. We also investigate the effect of NED as a correction to the Maxwell theory.

The layout of this paper will be as follows. First, we introduce the suitable field equations regarding the Lovelock gravity coupled with different magnetic sources in which we are interested. Next, we obtain static solutions for the metric function. Then, we will consider a spinning magnetic string, and by employing the counterterm method, we calculate conserved quantities. The last section will be devoted to closing remarks.

II. STATIC SOLUTIONS

Recently, Dias and Lemos [4] have introduced an interesting spacetime with a magnetic brane interpretation that is horizonless. The mentioned metric in d dimensions may be written as

$$ds^2 = -\frac{\rho^2}{l^2}dt^2 + \frac{d\rho^2}{f(\rho)} + l^2 f(\rho)d\phi^2 + \frac{\rho^2}{l^2}dX^2, \tag{1}$$

where $dX^2 = \sum_{i=1}^{d_3} dx_i^2$ is the Euclidean metric on the d_3-dimensional submanifold [hereafter, we denote $(d-i)$ with d_i]. The angular coordinate ϕ is dimensionless and ranges in $[0, 2\pi]$, while x_i range in $(-\infty, \infty)$. This metric provides us horizonless solutions that are of our interest. Now, we are going to obtain the solutions of first, second, and third order of the Lovelock gravity in the presence of NED with the field equations

$$\partial_a(\sqrt{-g}L_F F^{ab}) = 0, \tag{2}$$

$$\begin{aligned}\Lambda g_{ab} + G_{ab}^{(1)} + \alpha_2 G_{ab}^{(2)} + \alpha_3 G_{ab}^{(3)} \\ = \frac{1}{2}g_{ab}L(F) - 2L_F F_{ac}F_b^c,\end{aligned} \tag{3}$$

where $L_F = \frac{dL(F)}{dF}$, in which $L(F)$ is the Lagrangian of NED; $\Lambda = -\frac{d_1 d_2}{2l^2}$ and $G_{ab}^{(1)} = R_{ab} - \frac{1}{2}g_{ab}R$ are, respectively, the cosmological constant and the Einstein tensor; α_i's are the Lovelock coefficients; and

$$\begin{aligned}G_{\mu\nu}^{(2)} = {} & 2(R_{\mu\sigma\kappa\tau}R_{\nu}{}^{\sigma\kappa\tau} - 2R_{\mu\rho\nu\sigma}R^{\rho\sigma} - 2R_{\mu\sigma}R^{\sigma}{}_{\nu} + RR_{\mu\nu}) \\ & - \frac{\mathcal{L}^{(2)}}{2}g_{\mu\nu},\end{aligned} \tag{4}$$

$$\begin{aligned}G_{\mu\nu}^{(3)} = {} & -3(4R^{\tau\rho\sigma\kappa}R_{\sigma\kappa\lambda\rho}R^{\lambda}{}_{\nu\tau\mu} - 8R^{\tau\rho}{}_{\lambda\sigma}R^{\sigma\kappa}{}_{\tau\mu}R^{\lambda}{}_{\nu\rho\mu} + 2R_{\nu}{}^{\tau\sigma\kappa}R_{\sigma\kappa\lambda\rho}R^{\lambda\rho}{}_{\tau\mu} - R^{\tau\rho\sigma\kappa}R_{\sigma\kappa\tau\rho}R_{\nu\mu} + 8R^{\tau}{}_{\nu\sigma\rho}R^{\sigma\kappa}{}_{\tau\mu}R^{\rho}{}_{\kappa} + 8R^{\sigma}{}_{\nu\tau\mu}R^{\tau\rho}{}_{\sigma\mu}R^{\kappa}{}_{\rho} \\ & + 4R_{\nu}{}^{\tau\sigma\kappa}R_{\sigma\kappa\mu\rho}R^{\rho}{}_{\tau} - 4R_{\nu}{}^{\tau\sigma\kappa}R_{\sigma\kappa\tau\rho}R^{\rho}{}_{\mu} + 4R^{\tau\rho\sigma\kappa}R_{\sigma\kappa\tau\mu}R_{\nu\rho} + 2RR_{\nu}{}^{\kappa\tau\rho}R_{\tau\rho\kappa\mu} + 8R^{\tau}{}_{\nu\mu\rho}R^{\rho}{}_{\sigma}R^{\sigma}{}_{\tau} - 8R^{\sigma}{}_{\nu\tau\rho}R^{\tau}{}_{\sigma}R^{\rho}_{\mu} \\ & - 8R^{\tau\rho}{}_{\sigma\mu}R^{\sigma}{}_{\tau}R_{\nu\rho} - 4RR^{\tau}{}_{\nu\mu\rho}R^{\rho}{}_{\tau} + 4R^{\tau\rho}R_{\rho\tau}R_{\nu\mu} - 8R^{\tau}{}_{\nu}R_{\tau\rho}R^{\rho}{}_{\mu} + 4RR_{\nu\rho}R^{\rho}{}_{\mu} - R^2R_{\nu\mu}) - \frac{\mathcal{L}^{(3)}}{2}g_{\mu\nu},\end{aligned} \tag{5}$$

where $\mathcal{L}^{(2)}$ and $\mathcal{L}^{(3)}$ denote the Lagrangians of the Gauss–Bonnet (GB) and third-order Lovelock (TOL) gravities, given as

$$\mathcal{L}^{(2)} = R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta} - 4R_{\mu\nu}R^{\mu\nu} + R^2, \tag{6}$$

$$\begin{aligned}\mathcal{L}^{(3)} = {} & 2R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\rho\tau}R^{\rho\tau}{}_{\mu\nu} + 8R^{\mu\nu}{}_{\sigma\rho}R^{\rho\tau}{}_{\mu\kappa} + 24R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\nu\rho}R^{\rho}{}_{\mu} \\ & + 3RR^{\mu\nu\sigma\kappa}R_{\sigma\kappa\mu\nu} + 24R^{\mu\nu\sigma\kappa}R_{\sigma\mu}R_{\kappa\nu} + 16R^{\mu\nu}R_{\nu\sigma}R^{\sigma}{}_{\mu} \\ & - 12RR^{\mu\nu}R_{\mu\nu} + R^3.\end{aligned} \tag{7}$$

In this work, we take into account the recently proposed interesting NED models [21]. One of them is the Soleng model, which is in logarithmic form of nonlinear electromagnetic field (LNEF), and another one has an exponential form of nonlinear electromagnetic field (ENEF), which was proposed by Hendi with the following explicit forms

$$L(F) = \begin{cases} \beta^2[\exp(-\frac{F}{\beta^2}) - 1] & \text{ENEF} \\ -8\beta^2\ln(1 + \frac{F}{8\beta^2}) & \text{LNEF} \end{cases}, \tag{8}$$

where β is the nonlinearity parameter and the Maxwell invariant is $F = F_{ab}F^{ab}$, in which $F_{ab} = \partial_a A_b - \partial_b A_a$ is the electromagnetic field tensor and A_a is the gauge potential. It is easy to show that the electric field comes from the time component of the vector potential (A_t), while the magnetic field is associated with the angular component (A_ϕ). Since we are looking for the magnetic solutions, we consider the following form of the gauge potential:

$$A_\mu = h(\rho)\delta_\mu^\phi. \tag{9}$$

Using Eq. (9) with the mentioned NED, one can show that the electromagnetic field equation (2) reduces to the differential equations

$$\begin{cases} (\rho l^2\beta^2 - 4\rho h'^2)h'' + d_2 l^2\beta^2 h' = 0 & \text{ENEF} \\ (4\rho l^2\beta^2 - rh'^2)h''^2 + 4d_2 h'(l^2\beta^2 + \frac{1}{4}h'^2) = 0 & \text{LNEF} \end{cases}, \tag{10}$$

where the prime and the double prime denote the first and second derivatives with respect to ρ. Solving these equations, one obtains

$$h(\rho) = \begin{cases} \frac{l\beta}{2}\int\sqrt{-L_{W1}}d\rho & \text{ENEF} \\ \frac{\beta^2\rho^{d_1}}{qd_1} - \frac{\beta^2}{q}\int\Gamma_1\rho^{d-2}d\rho & \text{LNEF} \end{cases}, \tag{11}$$

where q is an integration constant that is related to the electric charge, $L_{W1} = \mathrm{Lambert}W(-(\frac{4ql}{\beta\rho^{d_2}})^2)$, and $\Gamma_1 = \sqrt{1-(\frac{2ql}{\beta\rho^{d_2}})^2}$. Taking into account the mentioned gauge potential, one finds the nonzero components of the electromagnetic field are

$$F_{\phi\rho} = -F_{\rho\phi} = \begin{cases} \frac{2ql^2}{\rho^{d_2}}\exp(-\frac{L_{W1}}{2}), & \text{ENEF} \\ \frac{\beta^2\rho^{d_2}}{q}(1-\Gamma_1), & \text{LNEF} \end{cases}. \tag{12}$$

To obtain real solutions for the electromagnetic field, we should restrict the coordinate ρ with a lower bound ρ_0. It means

$$\rho > \rho_0 = \begin{cases} (\frac{4ql}{\beta})^{1/d_2}\exp(\frac{1}{2d_2}), & \text{ENEF} \\ (\frac{2ql}{\beta})^{1/d_2}, & \text{LNEF} \end{cases}.$$

We should note that for large values of β all relations reduce to the corresponding relations of the Maxwell theory. Besides, one can find that obtained results of electromagnetic fields reduce to those of Ref. [11] in four dimensions.

To obtain the metric function, $f(\rho)$, one can use nonzero components of the gravitational field equation (3). After cumbersome calculations, we find that there are two different differential equations with the explicit forms

$$e_t = \mathcal{K}_1 + \alpha_2\mathcal{K}_2 + \alpha_3\mathcal{K}_3 = 0, \tag{13}$$

$$e_\rho = \mathcal{K}_{11} + \alpha_2\mathcal{K}_{22} + \alpha_3\mathcal{K}_{33} = 0, \tag{14}$$

where

$$\mathcal{K}_1 = -\rho^6\left(\frac{\rho\mathcal{A}'}{d_2} + \mathcal{A}\right) - \beta^2\rho^6 \times \begin{cases} 1-\exp(\frac{-2h'^2}{l^2\beta^2}), & \text{ENEF} \\ -8\ln(\frac{4l^2\beta^2}{4l^2\beta^2+h'^2}), & \text{LNEF} \end{cases},$$

$$\mathcal{K}_2 = d_3d_4\rho^4\left[2ff'' + 2f'^2 + \frac{4d_5ff'}{\rho} + \frac{d_5d_6f^2}{\rho^2}\right],$$

$$\mathcal{K}_3 = -d_3d_4d_5d_6d_7d_8f\rho^2\left[\frac{3ff''+6f'^2}{d_7d_8} + \frac{6ff'}{d_8\rho} + \frac{f^2}{\rho^2}\right],$$

$$\mathcal{K}_{11} = \rho^6\mathcal{A} - \beta^2\rho^6 \times \begin{cases} \frac{4h'^2}{l^2\beta^2\rho}\exp(\frac{-2h'^2}{l^2\beta^2}) + \exp(\frac{-2h'^2}{l^2\beta^2}) - 1, & \text{ENEF} \\ 8\left(\frac{2}{1+(\frac{2l\beta}{h'})^2} + \ln[1+(\frac{h'}{2l\beta})^2]\right), & \text{LNEF} \end{cases},$$

$$\mathcal{K}_{22} = -d_2d_3d_4d_5f\rho^2\left(f + \frac{2\rho f'}{d_5}\right),$$

$$\mathcal{K}_{33} = d_2d_3d_4d_5d_6d_7f^2\left(f + \frac{3\rho f'}{d_7}\right).$$

Now, we desire to obtain higher-dimensional magnetic brane solutions in the EN, GB and TOL gravities, separately. One can set $\alpha_3 = 0$ to obtain the GB solutions, and for $\alpha_2 = \alpha_3 = 0$, we obtain magnetic solutions of the EN gravity. After some simplifications, we obtain

$$f_{\mathrm{EN}} = \frac{2ml^3}{\rho^{d_3}} - \frac{2\Lambda\rho^2}{d_1d_2} + \begin{cases} \frac{8\beta^2\rho^2}{d_1d_2} + \frac{-8\beta^2(\int\rho^{d_2}[\Gamma_1+\ln(\frac{\beta^2\rho^{2d_2}(1-\Gamma)}{2l^2q^2})]d\rho)}{d_2\rho^{d_3}} & \text{LNEF} \\ -\frac{\beta^2\rho^2}{d_1d_2} + \frac{4lq\beta(\int[\sqrt{-L_{W1}}+\frac{1}{\sqrt{-L_{W1}}}]d\rho)}{d_2\rho^{d_3}} & \text{ENEF} \end{cases}, \tag{15}$$

$$f_{\mathrm{GB}} = \frac{\rho^2}{2d_3d_4\alpha_2}(1-\Psi^{1/2}), \tag{16}$$

$$f_{\mathrm{TOL}} = \frac{\rho^2}{d_3d_4\alpha_2}(1-\Psi^{1/3}), \tag{17}$$

where

$$\Psi = 1 + \frac{2\chi d_3d_4\alpha_2}{d_1d_2}\left(\Lambda - \frac{d_1d_2l^3m}{\rho^{d_1}} + \mathcal{W}\right), \tag{18}$$

with

$$\mathcal{W}=\begin{cases}4\beta^2\left\{\ln\left(\frac{\beta^2\rho^{2d_2}}{2l^2q^2}[1-\Gamma_1]\right)-\frac{(2d_2+1)}{d_1}\Gamma_1\right\}+\frac{16d_2^2l^2q^2}{\rho^{2d_2}d_1d_3}\mathcal{F} & \text{LNEF}\\ \beta^2\left[\frac{1}{2}+\frac{2d_1ql}{\beta\rho^{d_1}}\int\left(\sqrt{-L_{W1}}+\frac{1}{\sqrt{-L_{W1}}}\right)d\rho\right] & \text{ENEF}\end{cases}, \tag{19}$$

in which $\chi=4$ and 3 for the GB theory and the TOL gravity, respectively; $\mathcal{F}$ is ${}_2F_1([\frac{1}{2},\frac{d_3}{2d_2}],[\frac{3d_2-1}{2d_2}],\frac{4l^2q^2}{\beta^2\rho^{2d_2}})$; m is an integration constant related to total finite mass of the solutions; and we set $\alpha_3=\frac{d_3d_4}{3d_5d_6}\alpha_2^2$ for more simplifications of TOL gravity solutions.

A. Properties of solutions

At the first step, we are going to discuss the geometric properties of the solutions. To do this, we look for possible black hole solutions by obtaining the curvature singularities and their horizons. We usually calculate the Kretschmann scalar, $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$, to achieve essential singularity. Considering the mentioned spacetime (1), it is easy to show that

$$R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}=f''^2+2d_2\left(\frac{f'}{\rho}\right)^2+2d_2d_3\left(\frac{f}{\rho^2}\right)^2. \tag{20}$$

Inserting the metric function, $f(\rho)$, in Eq. (20) and using numerical analysis, one finds that the Kretschmann scalar diverges at $\rho=\rho_0$ and it is finite for $\rho>\rho_0$, and naturally one may think that there is a curvature singularity located at $\rho=\rho_0$. In what follows, we state an important point, which confirms that the spacetime never achieves $\rho=\rho_0$. As one can confirm, easily, the metric function has a positive value for large values of $\rho\gg\rho_0$. So, two cases may occur. For the first case, $f(\rho)$ is a positive definite function with no root. Since we are not interested in naked singularity, we give up this case. We consider the second case, in which the metric function has one or more real positive root(s) larger than ρ_0.

From Figs. 1–3, we find that there is a $\rho_{\min}$ ($\rho_{\min}=\rho_0$) in which for $\rho\geq\rho_{\min}$ the metric function is real. These figures show that increasing the nonlinearity parameter leads to decreasing $\rho_{\min}$. Since we are looking for the metric function with at least one real root, we should adjust the metric parameters with a suitable range of the nonlinearity parameter to obtain $f(\rho=\rho_{\min})\leq 0$.

Moreover, Fig. 3 indicates that, although metric function of the TOL gravity is real for arbitrary ρ, in GB gravity, one encounters an imaginary interval for some values of the GB parameter. In other words, in GB gravity, we should adjust the metric parameters with a suitable interval of α to obtain a real metric function with at least one real root. Besides, Fig. 3 shows that the root of metric function does not depend on the Lovelock parameters.

Now, we denote r_+ as the largest real positive root of $f(\rho)$. The metric function is negative for $\rho<r_+$ and positive for $\rho>r_+$ and hence the metric signature may change from $(-++++\cdots+)$ to $(---++\cdots+)$ in the range $0<\rho<r_+$. Taking into account this apparent change of signature of the metric, we conclude that one cannot extend the spacetime to $\rho<r_+$. To get rid of this incorrect extension, one may use the following suitable transformation by introducing a new radial coordinate r:

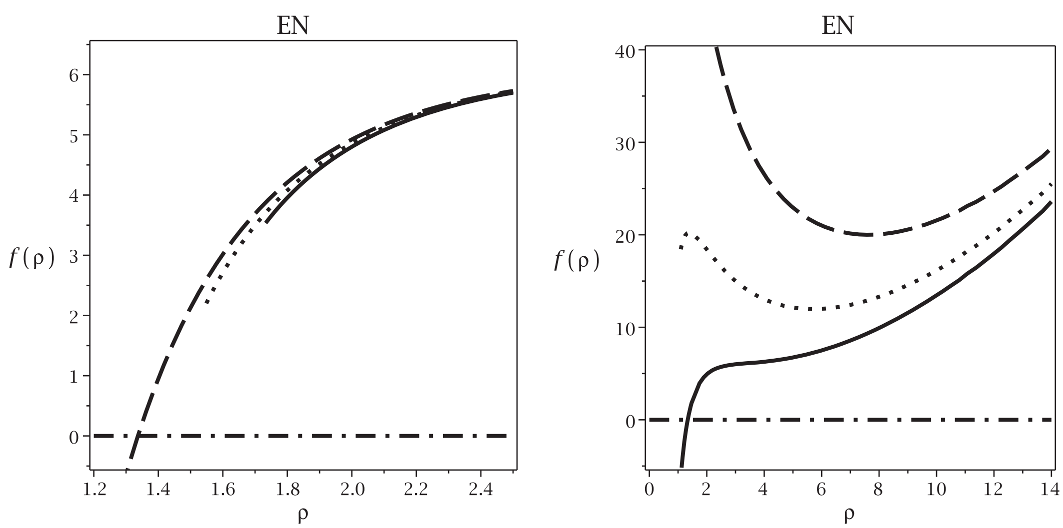

FIG. 1. LNEF branch of EN gravity: $f_{\mathrm{EN}}(\rho)$ vs ρ for $l=3$, $q=1$, and $d=4$. Left panel: $m=0.5$, $\beta=2$ (continuous line), $\beta=2.5$ (doted line), and $\beta=5$ (dashed line). Right panel: $\beta=5$, $m=0.5$ (continuous line), $m=1$ (doted line), and $m=2$ (dashed line).

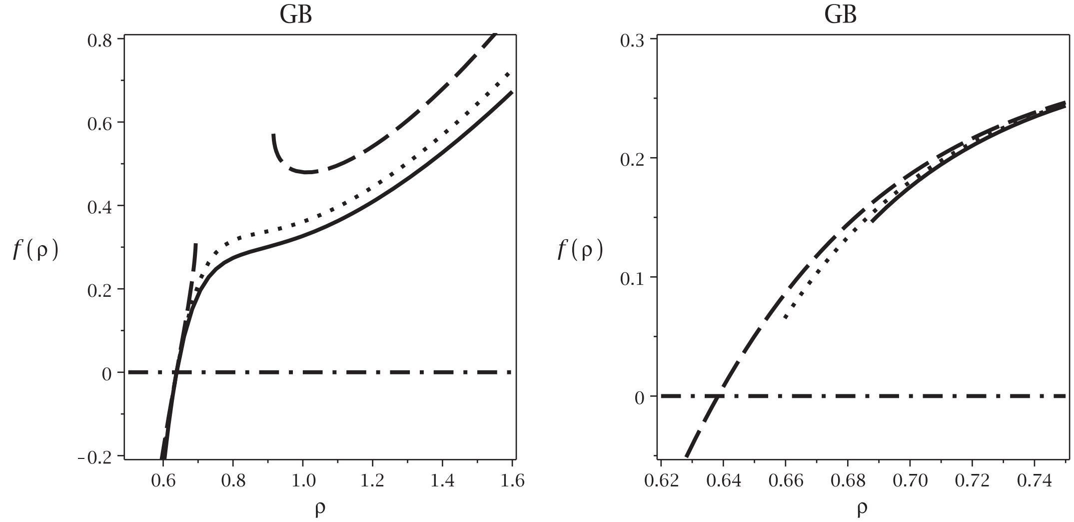

FIG. 2. LNEF branch of GB gravity: $f_{\mathrm{GB}}(\rho)$ vs ρ for $l=2$, $q=0.1$, $m=0.005$, and $d=7$. Left panel: $\beta=10$, $\alpha_2=0.01$ (continuous line), $\alpha_2=0.03$ (doted line), and $\alpha_2=0.06$ (dashed line). Right panel: $\alpha_2=0.01$, $\beta=2.6$ (continuous line), $\beta=3.2$ (doted line), and $\beta=10$ (dashed line).

$$
\begin{aligned}
&r^2=\rho^2-r_+^2,\\
&\rho\geq r_+\Leftrightarrow r\geq 0.
\end{aligned}
\tag{21}
$$

Using the mentioned transformation with $d\rho=\frac{r}{\sqrt{r^2+r_+^2}}dr$, one finds that the metric (1) should change to

$$
ds^2=-\frac{r^2+r_+^2}{l^2}dt^2+\frac{r^2}{(r^2+r_+^2)f(r)}dr^2+l^2f(r)d\phi^2+\frac{r^2+r_+^2}{l^2}dX^2.
\tag{22}
$$

It is worthwhile to mention that with this new coordinate, the electromagnetic field, and the metric functions lead to the form

$$
F_{r\phi}=\begin{cases}\frac{2ql^2}{(r^2+r_+^2)^{\frac{d_2}{2}}}\exp\left(-\frac{L_W}{2}\right), & \text{ENEF}\\ \frac{\beta^2(r^2+r_+^2)^{\frac{d_2}{2}}}{q}(1-\Gamma), & \text{LNEF}\end{cases},
\tag{23}
$$

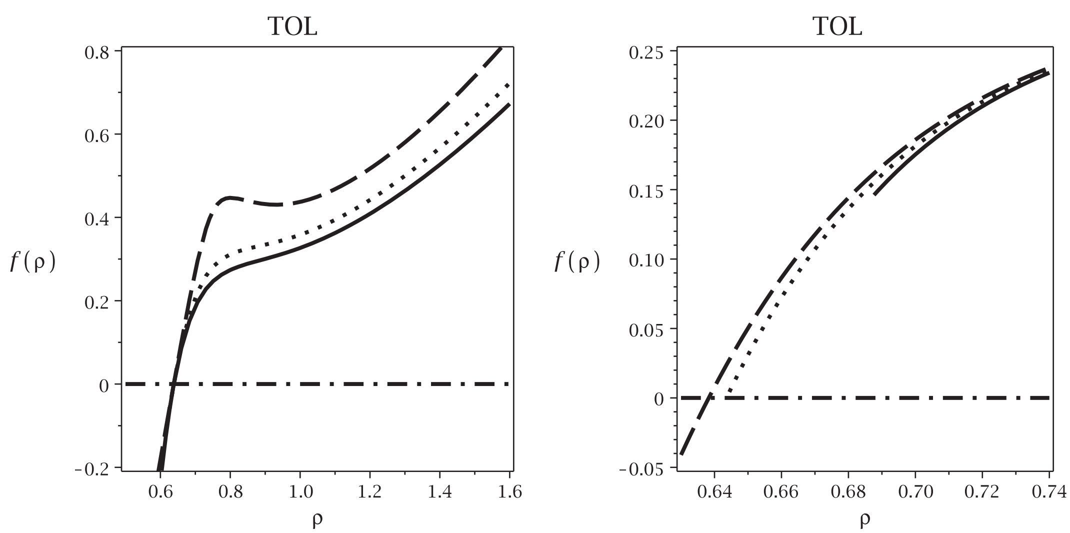

FIG. 3. LNEF branch of TOL gravity: $f_{\mathrm{TOL}}(\rho)$ vs ρ for $l=2$, $q=0.1$, $m=0.005$, and $d=7$. Left panel: $\beta=10$, $\alpha_2=0.01$ (continuous line), $\alpha_2=0.03$ (doted line), and $\alpha_2=0.06$ (dashed line). Right panel: $\alpha_2=0.01$, $\beta=2.6$ (continuous line), $\beta=3.2$ (doted line), and $\beta=10$ (dashed line).

$$f_{\mathrm{EN}} = \frac{2ml^3}{(r^2+r_+^2)^{\frac{d_3}{2}}} - \frac{2\Lambda(r^2+r_+^2)}{d_1 d_2} + \begin{cases} \frac{8\beta^2(r^2+r_+^2)}{d_1 d_2} + \frac{-8\beta^2\left(\int r(r^2+r_+^2)^{\frac{d_3}{2}}[\Gamma + \ln(\frac{\beta^2(r^2+r_+^2)^{d_2}}{2l^2q^2}(1-\Gamma))]dr\right)}{d_2(r^2+r_+^2)^{\frac{d-3}{2}}} & \text{LNEF} \\ -\frac{\beta^2(r^2+r_+^2)}{d_1 d_2} + \frac{4lq\beta\left(\int(\sqrt{-L_W}+\frac{1}{\sqrt{-L_W}})\frac{r}{\sqrt{r^2+r_+^2}}dr\right)}{d_2(r^2+r_+^2)^{\frac{d_3}{2}}} & \text{ENEF} \end{cases}, \tag{24}$$

$$f_{\mathrm{GB}} = \frac{(r^2+r_+^2)}{2d_3 d_4 \alpha_2}(1-\Psi^{1/2}), \tag{25}$$

$$f_{\mathrm{TOL}} = \frac{(r^2+r_+^2)}{d_3 d_4 \alpha_2}(1-\Psi^{1/3}), \tag{26}$$

where

$$\Psi = 1 + \frac{2\chi d_3 d_4 \alpha_2}{d_1 d_2}\left(\Lambda - \frac{d_1 d_2 l^3 m}{(r^2+r_+^2)^{d_1/2}} + \mathcal{W}_1\right), \tag{27}$$

with

$$\mathcal{W}_1 = \begin{cases} 4\beta^2\left\{\ln\left(\frac{\beta^2(r^2+r_+^2)^{d_2}}{2l^2q^2}[1-\Gamma]\right) - \frac{(2d_2+1)}{d_1}\Gamma\right\} + \frac{16d_2^2l^2q^2}{(r^2+r_+^2)^{d_2}d_1d_3}\mathcal{F} & \text{LNEF} \\ \beta^2\left[\frac{1}{2} + \frac{2d_1ql}{\beta(r^2+r_+^2)^{\frac{d_1}{2}}}\int\left(\sqrt{-L_W}+\frac{1}{\sqrt{-L_W}}\right)\frac{r}{\sqrt{r^2+r_+^2}}dr\right] & \text{ENEF} \end{cases}, \tag{28}$$

in which $L_W = \mathrm{Lambert}W(-\frac{16q^2l^2}{\beta^2(r^2+r_+^2)^{d_2}})$, $\mathcal{F} = {}_2F_1([\frac{1}{2}, \frac{d_3}{2d_2}], [\frac{3d_2-1}{2d_2}], \frac{4l^2q^2}{\beta^2(r^2+r_+^2)^{d_2}})$, and $\Gamma = \sqrt{1-\frac{4q^2l^2}{\beta^2(r^2+r_+^2)^{d_2}}}$. Since we suppose that $r_+ \geq \rho_0$, the solutions (electromagnetic field and metric functions) are real for $r \geq 0$. In addition, the function $f(r)$ given in Eqs. (24)–(26) is positive in the whole spacetime and is zero at $r=0$.

Although the Kretschmann scalar does not diverge in the range $0 \leq r < \infty$, one can show that there is a conical singularity at $r=0$. One can investigate the conic geometry by using the *circumference/radius* ratio. Using the Taylor expansion, in the vicinity of $r=0$, we find

$$f(r) = f(r)|_{r=0} + \left(\frac{df(r)}{dr}\bigg|_{r=0}\right)r + \frac{1}{2}\left(\frac{d^2f(r)}{dr^2}\bigg|_{r=0}\right)r^2 + O(r^3) + \cdots, \tag{29}$$

where

$$f(r)|_{r=0} = \frac{df(r)}{dr}\bigg|_{r=0} = 0,$$

and it is a matter of calculation to show that, regardless of gravity branches (EN, GB, and TOL), we will have the relation

$$\frac{d^2f(r)}{dr^2}\bigg|_{r=0} = -\frac{2\Lambda}{d_2} + \frac{2}{d_2}E_0 + \frac{2r_+}{d_1d_2}E_0' \neq 0, \tag{30}$$

where $E_0 = E(r)|_{r=0}$, in which $E(r)$ denotes the electromagnetic part of metric functions [third term of Eq. (24) and $\mathcal{W}_1$ in Eq. (27)], and $E_0' = \frac{dE(r)}{dr}|_{r=0}$. With employing obtained results, one can show that

$$\lim_{r\longrightarrow 0^+}\frac{1}{r}\sqrt{\frac{g_{\phi\phi}}{g_{rr}}} = \lim_{r\longrightarrow 0^+}\frac{\sqrt{r^2+r_+^2}\,lf(r)}{r^2} = \frac{lr_+}{2}\frac{d^2f(r)}{dr^2}\bigg|_{r=0} \neq 1, \tag{31}$$

which confirms that as the radius r tends to zero the limit of the *circumference/radius* ratio is not 2π and therefore the spacetime has a conical singularity at $r=0$. This canonical singularity may be removed if one identifies the coordinate ϕ with the period

$$\mathrm{Period}_\phi = 2\pi\left(\lim_{r\longrightarrow 0}\frac{1}{r}\sqrt{\frac{g_{\phi\phi}}{g_{rr}}}\right)^{-1} = 2\pi(1-4\mu), \tag{32}$$

where μ is given by

$$\mu = \frac{1}{4}\left[1 - \frac{2}{lr_+}\left(\frac{d^2f(r)}{dr^2}\bigg|_{r=0}\right)^{-1}\right]. \tag{33}$$

In other words, the near-origin limit of the metric (22) describes a locally flat spacetime that has a conical singularity at $r=0$ with a deficit angle $\delta\phi = 8\pi\mu$. Using the Vilenkin procedure, one can interpret μ as the mass per unit volume of the magnetic brane [42]. It is evident from Eqs. (30) and (33) that the deficit angle is

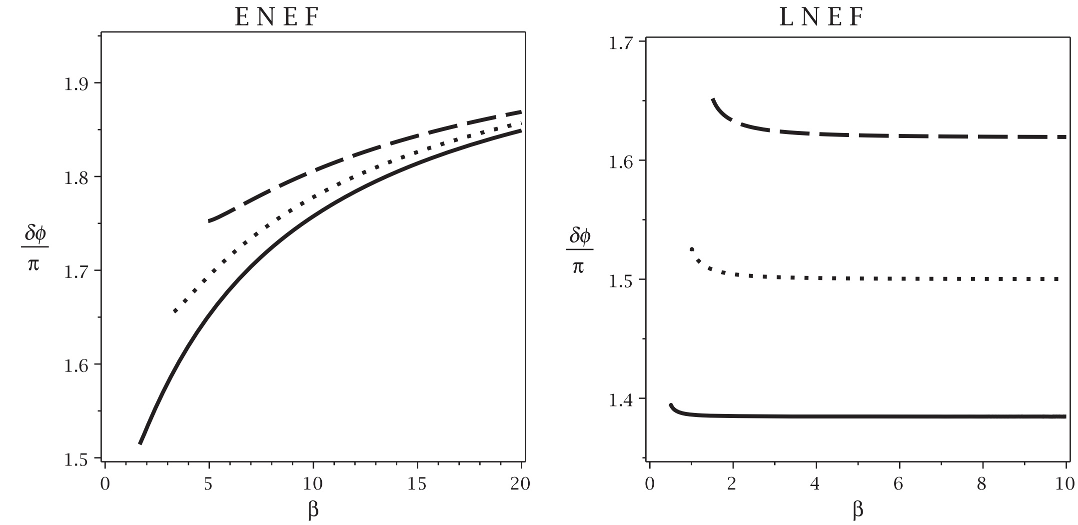

FIG. 4. $\delta\phi/\pi$ vs β for $d=4$, $l=1$ and $r_+=2$. Left panel (ENEF): $q=1$ (continuous line), $q=2$ (doted line), and $q=3$ (dashed line). Right panel (LNEF): $q=1$ (continuous line), $q=2$ (doted line), and $q=3$ (dashed line).

independent of the Lovelock coefficients and is only a function of the cosmological constant and electromagnetic field.

It is obvious that the nonlinearity of electrodynamics can change the value of deficit angle $\delta\phi$. To investigate the effects of nonlinearity, r_+, q, and dimensionality, we plot $\delta\phi$ vs β and r_+ (Figs. 4–8). Figures 4 and 5 show that, for the ENEF branch, the deficit angle is an increasing function of nonlinearity parameter, while for the LNEF branch, it is a decreasing function of β. In addition, figures of the deficit angle show that there is a minimum for nonlinearity parameter $\beta_{\min}$ in which for $\beta \leq \beta_{\min}$ the obtained values for deficit angle are not real (see Figs. 4–5). Besides, one finds $\beta_{\min}$ increases as the charge parameter of magnetic branes increases, whereas for an increasing value of r_+, $\beta_{\min}$ decreases (see Figs. 4 and 5).

The figures of the deficit angle versus r_+ (see Figs. 6 and 7) show that there is also a minimum $r_{+_{\min}}$ in which for $r_+ \geq r_{+_{\min}}$ the deficit angle is real. For large values of β, the deficit angle is an increasing function of r_+ (see Figs. 6 and 7). These figures show that there is an extremum $r_{+_{\mathrm{ext}}}$ that for $r_{+_{\min}} \leq r_+ \leq r_{+_{\mathrm{ext}}}$, deficit angle is a decreasing function of r_+ whereas for $r_+ \geq r_{+_{\mathrm{ext}}}$ the deficit angle is an increasing function of β (see Figs. 6 and 7).

Considering the fact that obtained results are magnetic branes in arbitrary dimensions, studying the effect of dimensionality on the deficit angle is another important issue. Figures 7 and 8 show that for fixed values of

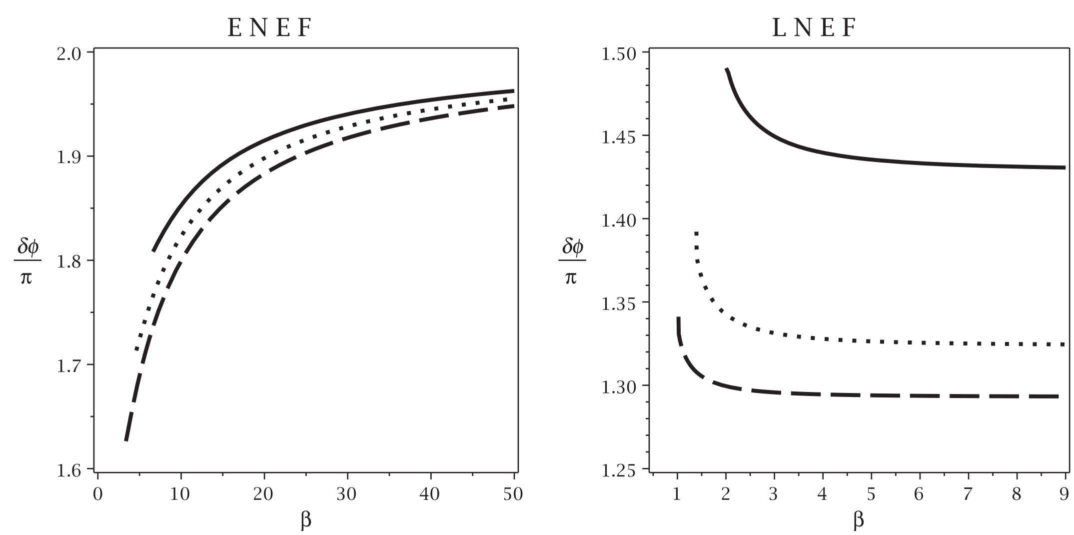

FIG. 5. $\delta\phi/\pi$ vs β for $d=4$, $l=1$, and $q=1$. Left panel (ENEF): $r_+=1$ (continuous line), $r_+=1.2$ (doted line), and $r_+=1.4$ (dashed line). Right panel (LNEF): $r_+=1$ (continuous line), $r_+=1.2$ (doted line), and $r_+=1.4$ (dashed line).

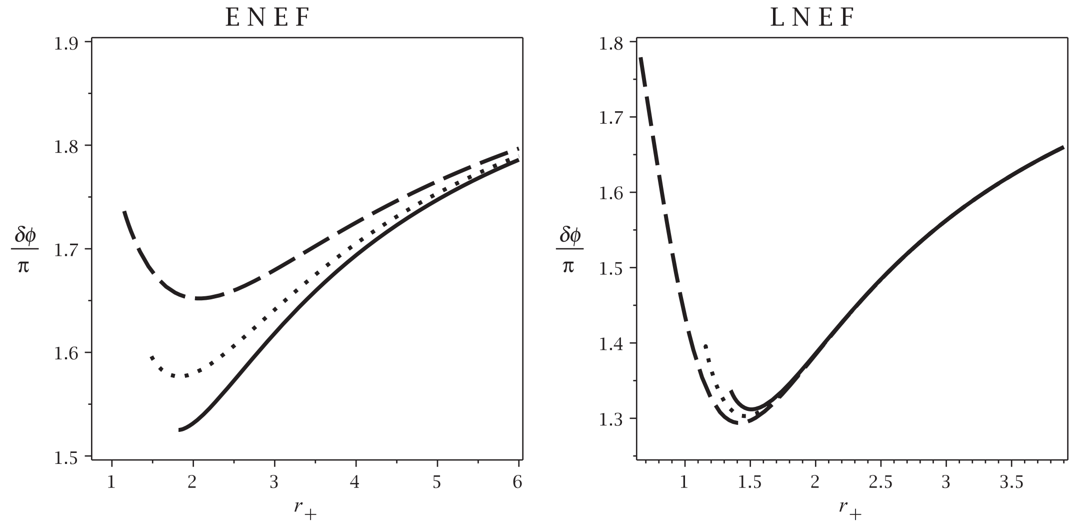

FIG. 6. $\delta\phi/\pi$ vs r_+ for $d = 4$, $l = 2$ and $q = 1$. Left panel (ENEF): $\beta = 2$ (continuous line), $\beta = 3$ (doted line), and $\beta = 5$ (dashed line). Right panel (LNEF): $\beta = 1.1$ (continuous line), $\beta = 1.5$ (doted line), and $\beta = 5$ (dashed line).

metric parameters the deficit angle is an increasing function of d. Also, as one can see, $\beta_{\min}$ is a decreasing function of dimensionality, and for higher dimensions $\beta_{\min}$ goes to zero (see Fig. 8). Also, numerical analysis confirms that $r_{+_{\min}}$ is an increasing function of dimensionality (see Fig. 7).

III. CLASS OF SPINNING SOLUTIONS

In this section, we generalize the static spacetime to the case of rotating solutions. As we know, the rotation group in d dimensions is $SO(d-1)$ with $[(d-1)/2]$ independent rotation parameters, in which $[x]$ denotes the integer part of x. The rotating magnetic solutions with $k \le [(d-1)/2]$ rotation parameters may be written as

$$\begin{aligned} ds^2 = & -\frac{r^2+r_+^2}{l^2}\left(\Xi dt - \sum_{i=1}^{k} a_i d\phi^i\right)^2 \\ & + f(r)\left(\sqrt{\Xi^2-1}dt - \frac{\Xi}{\sqrt{\Xi^2-1}}\sum_{i=1}^{k} a_i d\phi^i\right)^2 \\ & + \frac{r^2 dr^2}{(r^2+r_+^2)f(r)} + \frac{r^2+r_+^2}{l^2(\Xi^2-1)}\sum_{i<j}^{k}(a_i d\phi_j - a_j d\phi_i)^2 \\ & + \frac{r^2+r_+^2}{l^2}dX^2, \end{aligned} \tag{34}$$

where $\Xi = \sqrt{1+\sum_i^k a_i^2/l^2}$, dX^2 is the Euclidean metric on the $(d-k-2)$-dimensional submanifold with volume V_{d-k-2}, and $f(r)$ is the same as $f(r)$ given in Eqs. (24)–(26)

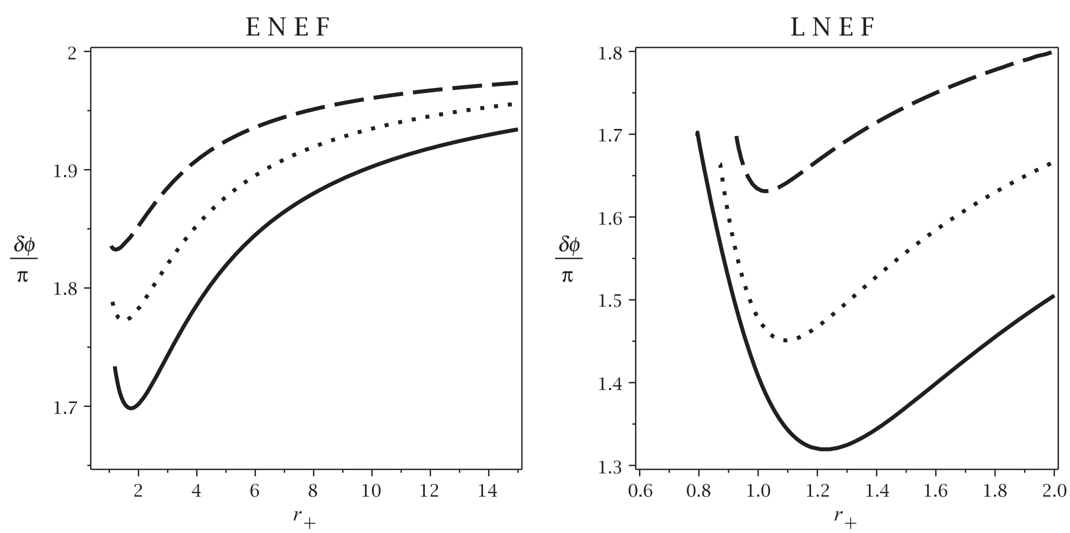

FIG. 7. $\delta\phi/\pi$ vs r_+ for $\beta = 4$, $l = 1$, and $q = 1$. Left panel (ENEF): $d = 5$ (continuous line), $d = 7$ (doted line), and $d = 11$ (dashed line). Right panel (LNEF): $d = 5$ (continuous line), $d = 7$ (doted line), and $d = 11$ (dashed line).

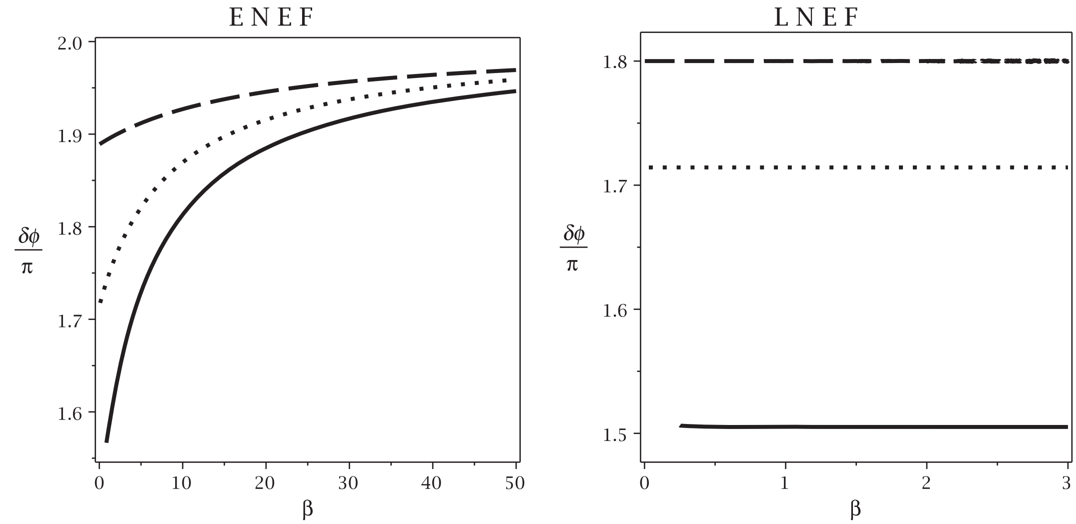

FIG. 8. $\delta\phi/\pi$ vs β for $r_+ = 2$, $l = 1$ and $q = 1$. Left panel (ENEF): $d = 5$ (continuous line), $d = 7$ (doted line), and $d = 11$ (dashed line). Right panel (LNEF): $d = 5$ (continuous line), $d = 7$ (doted line), and $d = 11$ (dashed line).

for various gravity. We should note that the nonvanishing components of electromagnetic field are

$$F_{rt} = -\frac{(\Xi^2-1)}{\Xi a_i} F_{r\phi_i} = -\frac{(\Xi^2-1)}{\Xi a_i} \times \begin{cases} \frac{2ql^2}{(r^2+r_+^2)^{d_2/2}} \exp\left(-\frac{L_W}{2}\right), & \text{ENEF} \\ \frac{\beta^2(r^2+r_+^2)^{d_2/2}}{q}(1-\Gamma), & \text{LNEF} \end{cases}. \tag{35}$$

Again, we should note that, although this rotating spacetime has no curvature singularity and horizon, it has a conical singularity at $r = 0$.

A. Conserved quantities

Here, we calculate the angular momentum and mass density of the magnetic solutions. To obtain finite conserved quantities for the asymptotically anti-de Sitter (AdS) solutions, one may use the counterterm method [43]. Here, for the asymptotically AdS solutions of the Lovelock gravity with flat boundary, $\hat{R}_{abcd}(\gamma) = 0$ (our solutions), the finite energy-momentum tensor is [44]

$$T^{ab} = \frac{1}{8\pi}\left[(K^{ab} - K\gamma^{ab}) + 2\alpha_2(3J^{ab} - J\gamma^{ab}) + 3\alpha_3(5P^{ab} - P\gamma^{ab}) + \frac{d_2}{l_{\mathrm{eff}}}\gamma^{ab}\right], \tag{36}$$

where l_{eff} is a function of l and α, and when α goes to zero (Einstein solutions), l_{eff} reduces to l. In Eq. (36), K^{ab} is the extrinsic curvature of the boundary, K is its trace, γ^{ab} is the induced metric of the boundary, and J and P are, respectively, traces of J^{ab} and P^{ab}, where

$$J_{ab} = \frac{1}{3}(K_{cd}K^{cd}K_{ab} + 2KK_{ac}K^c_b - 2K_{ac}K^{cd}K_{db} - K^2K_{ab}), \tag{37}$$

and

$$\begin{aligned} P_{ab} = \frac{1}{5}\{&[K^4 - 6K^2K^{cd}K_{cd} + 8KK_{cd}K^d_eK^{ec} \\ &- 6K_{cd}K^{de}K_{ef}K^{fc} + 3(K_{cd}K^{cd})^2]K_{ab} \\ &- (4K^3 - 12KK_{ed}K^{ed} + 8K_{de}K^e_fK^{fd})K_{ac}K^c_b \\ &- 24KK_{ac}K^{cd}K_{de}K^e_b \\ &+ (12K^2 - 12K_{ef}K^{ef})K_{ac}K^{cd}K_{db} \\ &+ 24K_{ac}K^{cd}K_{de}K^{ef}K_{bf}\}. \end{aligned} \tag{38}$$

To compute the conserved charges, one can write the boundary metric in Arnowitt–Deser–Misner form,

$$\gamma^{ab}dx^a dx^b = -N^2dt^2 + \sigma_{ij}(d\varphi^i + V^idt)(d\varphi^j + V^jdt), \tag{39}$$

where the coordinates φ^i are the angular variables parametrizing the hypersurface of constant r around the origin and N and V^i are the lapse and shift functions, respectively. The quasilocal conserved quantities associated with the stress tensors of Eq. (36) are

$$\mathcal{Q}(\xi) = \int_{\mathcal{B}} d^{d_2}\varphi\sqrt{\sigma}T_{ab}n^a\xi^b, \tag{40}$$

where σ is the determinant of the metric σ_{ij}, n^a is the timelike unit normal vector to the boundary $\mathcal{B}$, and ξ is a Killing vector field. The rotating magnetic spacetime (34) has two conserved quantities that are associated with the Killing vectors $\xi = \partial/\partial t$ and $\zeta_i = \partial/\partial\phi^i$. The total mass and angular momentum of the magnetic brane solutions per unit volume V_{d-k-2} are given by

$$M = \int_{\mathcal{B}} d^{d_2}x\sqrt{\sigma}T_{ab}n^a\xi^b = \frac{(2\pi)^k}{4}[d_1(\Xi^2-1)+1]m, \tag{41}$$

$$J_i = \int_{\mathcal{B}} d^{d_2}x\sqrt{\sigma}T_{ab}n^a\zeta_i^b = \frac{(2\pi)^k}{4}\Xi d_1 m a_i, \tag{42}$$

where the mass parameter m comes from the fact that $\lim_{r\to 0}f(r)=0$. Our last step will be devoted to calculate the electric charge of the magnetic solutions. To do so, we should consider the projections of the electromagnetic field tensor on a special hypersurface. The electric charge per unit volume V_{d-k-2} can be found by calculating the flux of the electromagnetic field at infinity, yielding

$$Q = \frac{(2\pi)^k}{2}ql\sqrt{\Xi^2-1}, \tag{43}$$

which shows that the electric charge is proportional to the magnitude of the rotation parameters and is zero for the static solutions ($\Xi=1$). This is due to the fact that the electric field, F_{tr}, vanishes for the static solutions. In addition, since the asymptotic behavior of the electromagnetic field is the same as that of the Maxwell theory, the nonlinearity does not affect the total electric charge.

IV. NED AS A CORRECTION

It is arguable that, instead of considering nonlinear theories of the Maxwell field, one can use the method in which the nonlinearity is playing as a correction term. In other words, one is free to consider nonlinearity as a perturbation to linear theory and construct a new nonlinear theory. This treatment is justified with the following reasons. First, to find experimental results for nonlinear electromagnetic fields, one should consider its weak nonlinearity and not strong. This is due to the fact that the Maxwell theory has acceptable consequences in most domains and the perturbed nonlinear theory of electrodynamics may increase the Maxwell accuracy. On the other hand, to avoid the complexity of nonlinear theories and obtaining interesting solutions, it is logical to consider the dominant nonlinearity terms and use them in order to study a nonlinear theory. As for BI types of nonlinear electrodynamics for large values of nonlinearity parameter, they have same structure with a little differences in some factors. One can show that the first and second leading-order terms are, respectively, the Maxwell Lagrangian and quadratic power of the Maxwell invariant. Therefore, in this section, we consider following Lagrangian as a source and study the effects of additional correction to the Maxwell theory (MC) as nonlinear electromagnetic field on solutions:

$$L(F) = -F + \eta F^2 + O(\eta^2). \tag{44}$$

One may follow the procedure of previous sections with the mentioned Lagrangian (44) and the metric (34) to obtain

$$\begin{aligned} F_{rt} &= -\frac{(\Xi^2-1)}{\Xi a_i}F_{r\phi_i} \\ &= -\frac{(\Xi^2-1)}{\Xi a_i}\left(\frac{q}{(r^2+r_+^2)^{\frac{d_4}{2}}} - \frac{4q^3\eta}{l^2(r^2+r_+^2)^{\frac{3d_2}{2}}}\right). \end{aligned} \tag{45}$$

Inserting Eq. (45) in the gravitational field equations, we find the following metric functions for the EN, the GB, and the TOL gravities in the presence of the Lagrangian (44),

$$\begin{aligned} f_{\mathrm{EN}} &= \frac{2Ml^3}{(r^2+r_+^2)^{d_3/2}} - \frac{2(r^2+r_+^2)}{d_1 d_2} \\ &\quad\times\left(\Lambda - \frac{4d_1 l^2 q^2}{d_3(r^2+r_+^2)^{d_2}} + \frac{32 d_1 l^4 q^4 \eta}{(3d-7)(r^2+r_+^2)^{2d_2}}\right), \end{aligned} \tag{46}$$

$$f_{\mathrm{GB}} = \frac{(r^2+r_+^2)}{2d_3 d_4 \alpha_2}(1-\Psi^{\frac{1}{2}}), \tag{47}$$

$$f_{\mathrm{TOL}} = \frac{(r^2+r_+^2)}{d_3 d_4 \alpha_2}(1-\Psi^{\frac{1}{3}}), \tag{48}$$

where

$$\begin{aligned} \Psi &= 1 + \frac{2\chi d_3 d_4 \alpha_2}{d_1 d_2}\left(\Lambda - \frac{d_1 d_2 l^3 M}{(r^2+r_+^2)^{d_1/2}} - \frac{4d_1 l^2 q^2}{d_3(r^2+r_+^2)^{d_2}}\right. \\ &\quad\left. + \frac{32 d_1 l^4 q^4 \eta}{(3d_2-1)(r^2+r_+^2)^{2d_2}}\right), \end{aligned} \tag{49}$$

and $\chi=4$ and 3 for the GB theory and the TOL gravity, respectively.

We should note that, regardless of various coefficients, one can obtain these solutions, directly, by suitable series expansions of Eqs. (24)–(26). In addition, in agreement with Eqs. (41), (42), and (43), independent calculations show that the conserved charges do not depend on the nonlinearity parameter of BI-type NED theories.

Here, we are in position to study the deficit angle. To do so, we employ the method that was mentioned in previous sections and plot various appropriate graphs. It is a matter of calculation to show that the second-order derivation of the metric with respect to the radial coordinate will be in the form

$$\left.\frac{d^2 f(r)}{dr^2}\right|_{r=0} = -\frac{2\Lambda}{d_2} - \frac{8l^2q^2}{d_2 r_+^{2d_2}} + \frac{64 l^4 q^4}{d_2 r_+^{4d_2}}\eta + O(\eta^2) \tag{50}$$

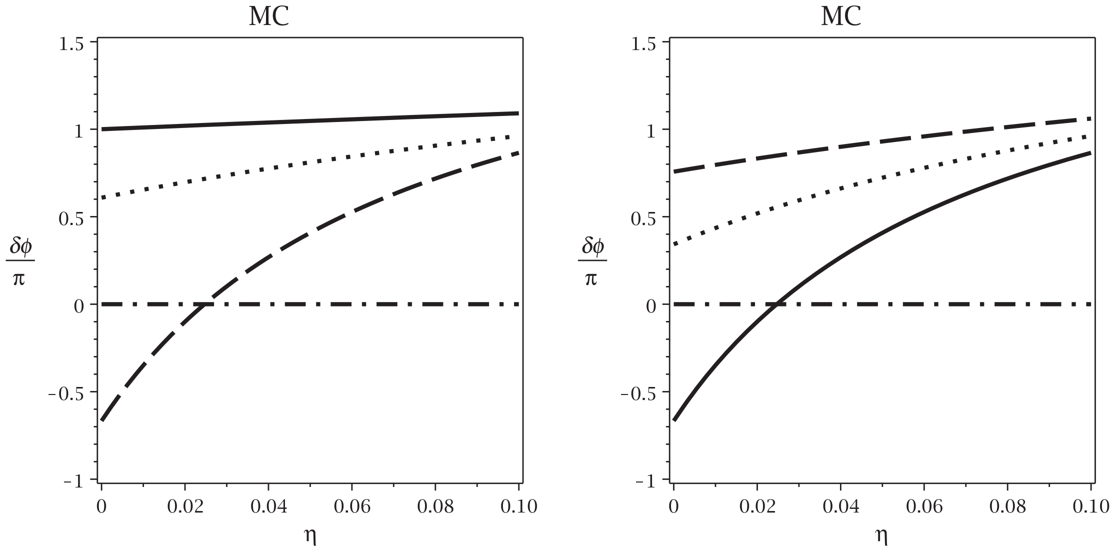

FIG. 9. $\delta\phi/\pi$ vs η for $l = 1$ and $d = 4$. Left panel: $r_+ = 2$, $q = 2$ (continuous line), $q = 2.5$ (doted line), and $q = 3$ (dashed line). Right panel: $q = 3$, $r_+ = 2$ (continuous line), $r_+ = 2.1$ (doted line), and $r_+ = 2.2$ (dashed line).

for all mentioned gravity branches, where it is confirmed that the deficit angle does not depend on the Lovelock coefficients.

Studying the effects of the charge parameter show that, for very small values of q and $\eta = 0$, the calculated deficit angle is nonzero and is a decreasing function of charge (see the left panel in Fig. 9). As charge increases, for a certain range of the correction parameter, the deficit angle is negative, and there is a η_0 where calculated deficit angle is zero. This η_0 is an increasing function of charge (see the left panel in Fig. 9). As for the effects of r_+, plotted graphs have similar behavior as the charge, whereas the effects of r_+ are exactly opposite of the effects of charge (see the right panel in Fig. 9).

Considering different values of the correction parameter, the deficit angle vs r_+ shows that calculated deficit angles have different behaviors. For small values of nonlinearity, three different behaviors are seen for different regions of r_+ in which these regions are specified with $r_{+_{\mathrm{Div1}}}$ and $r_{+_{\mathrm{Div2}}}$ (see the left panel in Fig. 10). For $0 < r_+ < r_{+_{\mathrm{Div1}}}$, the deficit angle is a decreasing function of r_+, and in $r_+ = r_{+_{\mathrm{Div1}}}$, there is a divergency. In this region, calculated deficit angles are positive and real valued, and in the case of $r_{+_{\mathrm{Div1}}} < r_+ < r_{+_{\mathrm{Div2}}}$ for a calculated deficit angle, first it is a decreasing and then increasing function of r_+, and for $r_+ = r_{+_{\mathrm{Div2}}}$, the second divergency happens. Next, for $r_+ > r_{+_{\mathrm{Div2}}}$, one finds that the deficit angle is an increasing function of r_+, but there exists a region in which calculated

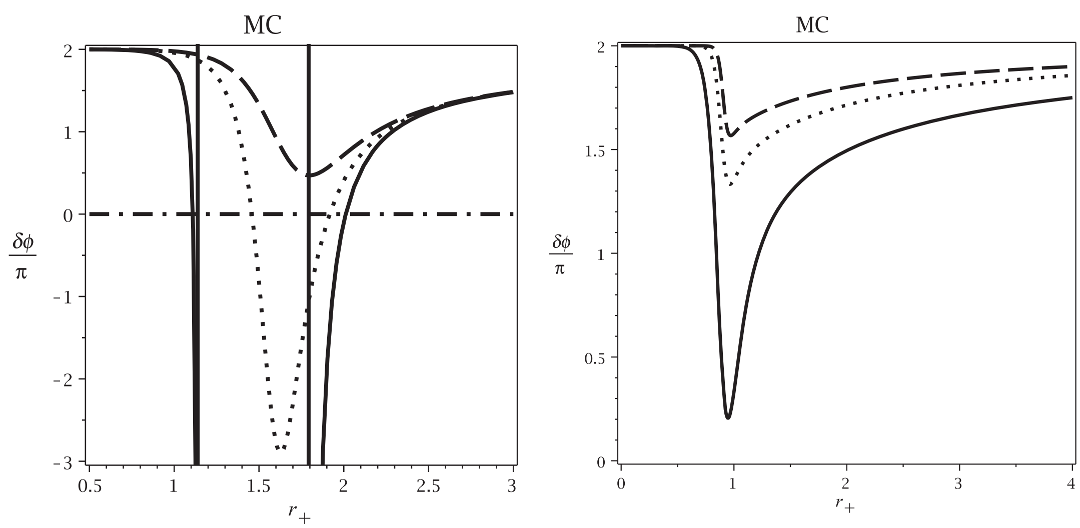

FIG. 10. $\delta\phi/\pi$ vs r_+ for $l = 1$. Left panel: $q = 3$, $d = 4$, $\eta = 0.02$ (continuous line), $\eta = 0.05$ (doted line), and $\eta = 0.08$ (dashed line). Right panel: $q = 1$, $\eta = 0.05$, $d = 5$ (continuous line), $d = 8$ (doted line), and $d = 11$ (dashed line).

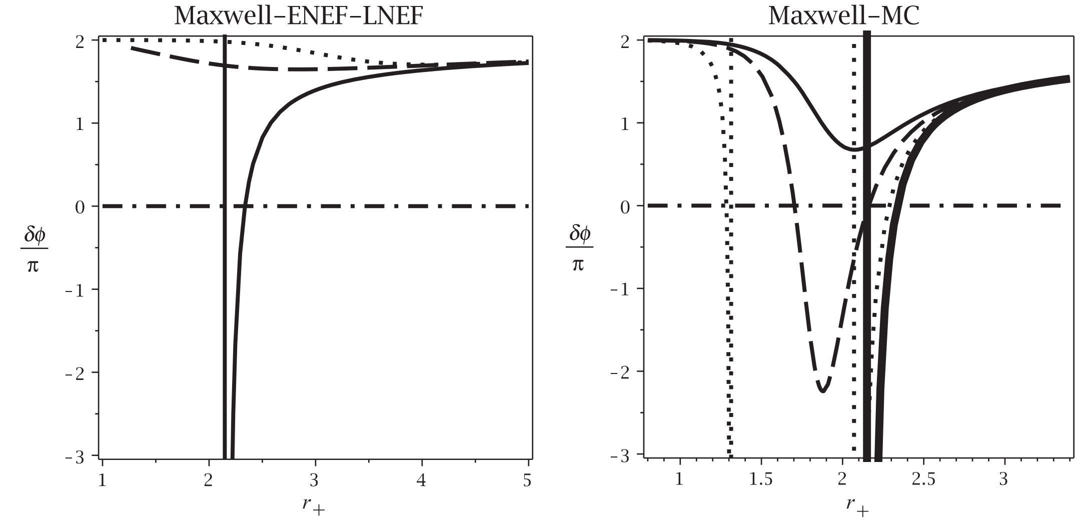

FIG. 11. $\delta\phi/\pi$ vs r_+ for $l = 1$, $q = 4$ and $d = 4$. Left panel: Maxwell case (continuous line), ENEF case for $\beta = 5$ (doted line), and LNEF case for $\beta = 5$ (dashed line), respectively. Right panel: Maxwell case (bold line), MC case for $\eta = 0.02$ (doted line), $\eta = 0.05$ (dashed line), and $\eta = 0.08$ (continuous line).

deficit angles are negative, and for an r_{+_0}, the deficit angle is zero (see the left panel in Fig. 10).

Next, for larger values of nonlinearity, there are regions identified with specific values naming r_{+_1}, $r_{+_{\mathrm{ext}}}$, and r_{+_2}. For $0 < r_+ < r_{+_1}$, deficit angles for different values of the nonlinearity parameter are almost the same. In other words, calculated values of the deficit angle are almost independent of the variation of the nonlinearity parameter because its effect is so small. r_{ext} is an extremum in which for $r_{+_1} \leq r_+ \leq r_{+_{\mathrm{ext}}}$ the deficit angle decreases where r_+ increases, while for $r_{+_2} \geq r_+ \geq r_{+_{\mathrm{ext}}}$, the deficit angle is an increasing function of r_+ (see Fig. 11). Finally, for large values of r_+ ($r_{+_2} \leq r_+$), similar behavior as for the case of small values of r_+ ($0 < r_+ < r_{+_1}$) is observed. Calculated values of the deficit angle are almost independent of the nonlinearity parameter and are almost the same. r_{+_1}, r_{+_2}, $r_{+_{\mathrm{ext}}}$, and the related deficit angle to this extremum are increasing functions of the nonlinearity parameter. As for the effects of dimensions, it is evident from plotted graphs that $r_{+_{\mathrm{ext}}}$ (and the related deficit angle) is a decreasing (increasing) function of dimensions (see the right panel in Fig. 10). These figures indicate that there exist regions in which calculated values of the deficit angle for different dimensions lead to almost the same result, and it is almost independent of dimensions.

Here, we present a geometric interpretation for the negative deficit angle. Considering a two-dimensional plain, we can cut a segment of a certain angular size and then sew together the edges to obtain a conical surface. The deleted segment from the plan is known as a deficit angle with positive values. Now, we imagine a new situation when a segment is added to a new plane to obtain a flat surface with a saddlelike cone (for more details, one can see Fig. 2 in Ref. [45]). This added segment is corresponding to a negative deficit angle (or surplus angle) [45,46]. We should mention that, although the deleted segment is bounded by the value of 2π, the added segment is unbounded. Therefore, we conclude that the range of deficit angles is from $-\infty$ to 2π.

V. CLOSING REMARKS

In this paper, we supposed that the geometry and matter field of spacetime come from the Lovelock gravity and NED. At first, we considered a suitable static metric to find horizonless magnetic solutions. We found that, for having the real electromagnetic field, we should consider a lower bound (ρ_0) for the coordinate ρ. We discussed the geometric properties of EN, GB, and TOL solutions and found that, although these solutions have no curvature singularity, there is a conical singularity at $r = 0$ with a deficit angle $\delta\phi = 8\pi\mu$, where one can interpret μ as the mass per unit volume of the magnetic brane. In addition, we found that both the NED and the Lovelock gravity do not affect the asymptotic behavior of the solutions, and in other words, obtained solutions are asymptotically AdS. We obtained the deficit angle of the conical geometry and investigated the effects of Lovelock gravity and NED. At first, calculated values for the deficit angle showed that it is independent of the GB and the TOL parameters. In other words, we found that the Lovelock parameters do not affect the deficit angle. This result comes from the fact that the value of second derivatives of the metric function does not depend on the Lovelock coefficients, which is the consequence of geometric properties of the $t =$ constant and $r =$ constant hypersurface (this hypersurface is a Ricci flat manifold). This behavior is similar to the property of Ricci-flat black holes in higher orders of the Lovelock gravity, in which their horizons and conserved quantities of the black hole do not depend on the Lovelock parameters.

We also investigated the effects of nonlinear electrodynamics. Although both ENEF and LNEF branches are BI type, they have different nature. We found that there is a minimum value for the nonlinearity parameter where for $\beta \leq \beta_{\min}$ the deficit angle was not real. This is because of the behavior of the Lambert function that is present in the ENEF branch and the logarithmic function that appears in the LNEF branch. We also showed that, considering higher-dimensional solutions, $\beta_{\min}$ may change, and for certain dimensions, the deficit angle is real for arbitrary β ($\beta_{\min} < 0$). We found that the deficit angle is an increasing function of the nonlinearity parameter in ENEF, whereas for LNEF, it showed the opposite behavior. We also saw that increasing the charge parameter leads to increasing $\beta_{\min}$, while for increasing r_+, the value of $\beta_{\min}$ decreased.

Looking at the behavior of the deficit angle vs r_+, we found that there is an $r_{+_{\min}}$, where for $r_+ \geq r_{+_{\min}}$, the deficit angle is real valued. Moreover, we found that for small values of the nonlinearity parameter the deficit angle is only an increasing function of r_+, whereas for an increasing value of β, there will be $r_{+_{\rm ext}}$ in which for $r_{+_{\min}} \leq r_+ \leq r_{+_{\rm ext}}$ the deficit angle is a decreasing function of r_+, and for $r_+ \geq r_{+_{\rm ext}}$ it increases as r_+ increases.

The next step was devoted to introducing spinning magnetic branes, which are horizonless. We found that for rotating magnetic branes there is an electric field in addition to the magnetic one. We employed the Gauss law and the counterterm method to calculate the electric charge, finite mass, and angular momentum of rotating magnetic brane solutions. We found that the electric charge is proportional to the rotation parameters, and it vanishes for the static solutions ($\Xi = 1$). We should note that vanishing the electric charge for $\Xi = 1$ is due to the fact that the electric field, F_{tr}, vanishes for the static solutions.

As one can see for the weak nonlinearity power, the obtained deficit angle for different theories of nonlinearity has different values compared to the Maxwell theory. One may argue that, for large values of β, the obtained values for the deficit angle should lead to those of the Maxwell theory and support this statement with fact that for large values of β these two electromagnetic fields become Maxwell theory. This idea is an acceptable one, when we are only dealing with the electromagnetic fields. But in calculation of the deficit angle, we are using the second derivation of the metric function. Because of different structures of nonlinear theories (logarithmic and exponential ones), it is most likely that this property of these two nonlinear electrodynamics (for large values of β, they lead to the Maxwell theory) is not preserved, and therefore the obtained values are different. In other words, one may expect to see different values for the deficit angle even for large values of the nonlinearity parameter, and they are not necessarily the same as Maxwell ones. It means that, although these two types of nonlinear theory are BI type and for $\beta \longrightarrow \infty$ they lead to the same result, they are completely different theories with their different characteristics and properties.

In addition, we found that the plotted graph for the Maxwell theory presents a divergency that is due to root(s) of f''. While for considering nonlinear theories, the divergency vanishes, and calculated values of the deficit angle and plotted graphs showed no divergency. In other words, in the process of going from a linear theory (Maxwell) to a nonlinear theory (logarithmic form or exponential one), calculated values of the deficit angle will be divergence free, and it has smooth behavior. Therefore, considering nonlinear theories changes properties of solutions and solves the problems regarding the linear theory that is the primary motivation of considering nonlinear electrodynamics. It is notable that considering nonlinear theories puts some restriction on values. In other words, there is a region in which the calculated values of the deficit angle are not real. But this region is not where the divergency of the Maxwell theory exists. In other words, by considering a suitable value of the nonlinearity parameter, one can cover regions in which the Maxwell theory has divergency.

Later, we investigated the effects of nonlinearity as a correction. We found that, despite other two nonlinear theories (logarithmic and exponential forms), deficit angle of MC theory is always a real value function, and there is no region in which the deficit angle is imaginary. Plotted graphs of this theory also showed that variation of the nonlinearity parameter is only effective in a region ($r_{+_1} \leq r_+ \leq r_{+_2}$), and in other regions ($r_{+_1} \geq r_+$, $r_+ \geq r_{+_2}$), it is almost independent of this variation. The same behavior was seen for the effects of dimensions as well. It was shown that for small and large values of r_+ the effect of the nonlinearity part decreases rapidly (almost vanishes) and the structure of magnetic branes (conelike) is similar to the Maxwell theory. On the other hand, for small values of the correction parameter, not only did it not solve the divergency of the Maxwell field, but it also added another divergency to it. In other words, two divergencies in the case of the very weak correction parameter were seen in MC theory. This shows the fact that this theory of nonlinearity and its deficit angle are quite sensitive to the modification of the correction parameter. This sensitivity is stronger even for small values of the correction parameter. Although for some regions the calculated values of the deficit angle are almost the same as the one for the Maxwell theory, there is an effective range in which nonlinearity (correction) will be dominant and has the most contribution in the deficit angle.

Other interesting results were the existence of the negative, vanishing and divergences values for the deficit angle. The positive deficit angle is representing a conelike structure for the object, whereas the negative deficit angle is denoted as an extra angle that is known as a surplus angle

[45,46]. This extra angle changes the shape of the object into a saddlelike cone.

Finally, it is worthwhile to think about the physical properties of deficit angle as well as the surplus one. In addition, one may investigate the possible wormhole solutions [47] of the mentioned models. These works are under examination.

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