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# Planck-scale traces from the interference pattern of two Bose-Einstein condensates

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We analyze the possible effects arising from the Planck scale regime upon the interference pattern of two noninteracting Bose-Einstein condensates. We start with the analysis of the free expansion of a condensate, and have taken into account the effects produced by a deformed dispersion relation, suggested in several quantum-gravity models. The analysis of the condensate free expansion, in particular, the *modified* free velocity expansion, suggests in a natural way, a modified uncertainty principle that could lead to *new* phenomenological implications related to the quantum structure of space-time. Finally, we analyze the corresponding separation between the interference fringes after the two condensates overlap, in order to explore the sensitivity of the system to possible signals caused by the Planck scale regime.

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## I. INTRODUCTION

Recently, the use of many-body systems as theoretical tools in searching some possible Planck scale manifestations has become a very interesting line of research [1–6]. In particular, due to its quantum properties, and also to its high experimental precision, Bose-Einstein condensates become an excellent tool in the search for traces from Planck-scale physics, and has produced several interesting works in this direction [4–11], and references therein.

First of all, in Refs. [1,2], for instance, it was argued that a modified uncertainty principle could be used to explore some properties of the center of mass motion of macroscopic bodies, which could lead to observable manifestations of Planck scale physics in low energy earth-based experiments. However, in Ref. [3], it was suggested that the extrapolation of Planck scale quantization to macroscopic bodies is *incorrect*, due to the fact that these possible manifestations would be more weak for macroscopic bodies than for its constituents. This last conclusion comes from the fact that the corrections caused by the quantum structure of space-time, on the properties associated with the center of mass motion of the macroscopic body, seems to be suppressed by the number of particles $N$, composing the system. In other words, as it was argued in Ref. [3], this simple analysis suggests that the possible signals arising from Planck scale quantization are more weak for macroscopic bodies than for its own constituents.

Nevertheless, the argument exposed in Ref. [3] seems to be not a generic criterion, at least for some properties associated with Bose-Einstein condensates. For instance, in Refs. [5,6] it was demonstrated that the corrections arising from the quantum structure of space-time, characterized by a deformed dispersion relation, on some relevant properties associated with a Bose-Einstein condensate scale as a nontrivial function of the number of particles.

As it was mentioned, the use of Bose-Einstein condensates opens an alternative scenario in searching some possible Planck scale signals, through a deformed dispersion relation in low-energy earth-based experiments. In fact, the analysis of some relevant properties associated with a homogeneous condensate, i.e., a condensate in a box, for instance, the corresponding ground state energy, and consequently the pressure and the speed of sound [4], present corrections caused by the quantum structure of space-time, which scales as a nontrivial function of the number of particles. Additionally, it is quite remarkable that the inclusion of a trapping potential improves the sensitivity to Planck scale signals, compared to a condensate in a box [6]. These facts suggest that the properties associated with many-body systems, in particular some properties associated with a Bose-Einstein condensate could be used, in principle, to obtain representative bounds on the deformation parameters [4,8–10] or to explore the sensitivity for these systems to Planck scale signals [5–7,12]. Thus, it is quite interesting to explore the sensitivity to Planck scale signals on certain

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properties of the condensate, in which the corrections caused by the quantum structure of space-time can be amplified, instead of being suppressed.

On another matter, it is generally accepted that the dispersion relation between the energy $\epsilon$ and the modulus of momentum $p$ of microscopic particles should be modified due to the quantum structure of space-time [13–16].

Such modifications are a general feature of quantum-gravity models, for instance, loop quantum gravity [17,18] or noncommutative geometries [19–21]. Further, it is also of great relevance in the general context of Lorentz-symmetry breaking [22]. Quite generally, a deformed dispersion relation can be written in ordinary units as follows:

$$\epsilon^2 = p^2c^2 + m^2c^4 + f(p, m, c, M_p), \quad (1)$$

where $\epsilon$ is the corresponding single particle energy, $p$ the momentum, $m$ the mass of the particle, $c$ the speed of light and $f(p, m, c, M_p)$ a model dependent function of the Planck mass $M_p(\simeq 2.18 \times 10^{-8}$ Kg).

We must mention that if we take the limit where the Planck mass is removed then we recover the usual relativistic form of the dispersion relation, assuming also that $mc^2$ is still the rest energy. In other words, the function $f(p, m, c, M_p) \to 0$ when the momentum $p \to 0$ and the Planck mass $M_p \to \infty$ [16].

Initially, attempts to constrain the functional form of the function $f(p, m, c, M_p)$ were in analyses of astrophysical context where particles are in the ultrarelativistic regime, $p \gg mc$ [23].

In this limit the deformation in the dispersion relation can be parametrized quite generally, regardless of the explicit model under study. Due to the extremely large value of $M_p$ a series expansion of the function $f(p, m, c, M_p)$ in inverse powers of the Planck mass might prove useful along with the leading term in $1/M_p$ as a first approximation. Thus, according to [15,16] the function $f(p, m, c, M_p)$ takes the following form:

$$f(p, m, c, M_p) \approx \frac{\eta_1}{2M_p} p^2 + \frac{\eta_2}{2M_p} mcp + \frac{\eta_3}{2M_p} m^2c^2, \quad (2)$$

where $\eta_1, \eta_2, \eta_3$ are real parameters associated with the expansion. As mentioned above, analysis in astrophysical data could be sensitive to the leading order deformation with $|\eta_1| \lesssim 1$ [23–27].

Finally, the form of the deformed dispersion relation was recently constrained in the nonrelativistic limit $p \ll mc$ [15,16] based on ultraprecise cold-atom-recoil-frequency experiments. In this scenario the form of the deformed dispersion relation can be expressed in ordinary units as follows [15,16]:

$$\epsilon \simeq mc^2 + \frac{p^2}{2m} + \frac{1}{2M_p}\left(\xi_1 mcp + \xi_2 p^2 + \xi_3 \frac{p^3}{mc}\right). \quad (3)$$

The three parameters $\xi_1$, $\xi_2$, and $\xi_3$, are model dependent [14,15], and should take positive or negative values close to 1. There is some evidence within the formalism of loop quantum gravity [15,16,28,29] that indicates a nonzero value for the three parameters, $\xi_1, \xi_2, \xi_3$, and particularly [28,30] that produces a linear-momentum term in the nonrelativistic limit. Unfortunately, as it is usual in a possible quantum gravity phenomenology, the possible bounds associated with the deformation parameters open a wide range of possible magnitudes, which is translated to a significant challenge.

Indeed, the most difficult aspect in searching experimental hints relevant for the quantum-gravity problem is the smallness of the involved effects [31,32]. If these kinds of deformations are characterized by some Planck scale, then the quantum gravity effects become very small for a single particle [14,15]. It is precisely in this direction that some many-body properties associated with Bose-Einstein condensates could be helpful to improve the sensitivity of possible effects caused by the quantum structure of space-time.

It is noteworthy to mention that one of the more interesting phenomena related to Bose-Einstein condensates is the interference pattern when two condensates overlap [33,34]. The interference pattern is a manifestation of the wave (quantum) nature of these many-body systems, and could be produced even when the two condensates are initially completely decoupled. Then, after switching off the corresponding traps, this allows the systems to expand, overlap, and eventually produce interference fringes. Such an interference pattern was observed in the experiment mentioned in Ref. [34], among others, where interference fringes with a period of $\sim 15 \times 10^{-6}$ m were observed after switching off the trapping potential and letting the condensates expand for 40 ms and overlap. Indeed, several experiments associated with the interference pattern of condensates in different situations has been made, see for instance [35–37] and references therein. Let us remark that when the trapping potential is turned off, the free velocity expansion of the cloud corresponds, approximately, to the velocity predicted by the Heisenberg's uncertainty principle [33,34].

In this aim, we explore the free velocity expansion of the condensate and consequently, the corresponding interference pattern when two of these systems overlap, assuming that the single particle energy spectrum is given by Eq. (3), taking into account only the leading order deformation, i.e., setting $\xi_2 = \xi_3 = 0$. Additionally, we are not interested here in the relative phase between the two condensates, which is a nontrivial topic and also deserves deeper analysis. Thus, we restrict ourselves on the analysis of the free expansion of the condensate together with the separation of the interference fringes when two of these systems overlap.

## II. ANOMALOUS DISPERSION RELATION AND FREE EXPANSION OF THE CONDENSATE

In order to explore the properties of the condensate under free expansion, let us propose the following *modified* energy associated with the system:

$$E(\psi) = \int d\mathbf{r}\left[\frac{\hbar^2}{2m}|\nabla\psi(\mathbf{r})|^2 + V(\mathbf{r})|\psi(\mathbf{r})|^2 + \frac{1}{2}U_0|\psi(\mathbf{r})|^4 + \hbar\alpha|\psi(\mathbf{r})|\sqrt{|\nabla|^2}|\psi(\mathbf{r})|\right], \quad (4)$$

where $\psi$ is the wave function of the condensate or the so-called order parameter, $V(r) = m\omega_0^2 r^2/2$ is the external potential, which we will assume for simplicity as an isotropic harmonic oscillator. The term $U_0 = \frac{4\pi\hbar^2}{m}a$ depicts the interatomic potential, being $a$ the s-wave scattering length i.e., only two-body interactions are taken into account. Notice also that we have included in the total energy of the cloud, the leading order modification in the deformed dispersion relation (3), through the *linear* operator $|\sqrt{|\nabla|^2}|$, as in Ref. [6], where $\alpha = \xi_1\frac{mc}{2M_p}$, assuming as mentioned above that $\xi_2 = \xi_3 = 0$. If we set $\alpha = 0$, we recover the usual expression associated with the total energy of the cloud [33].

An accurate expression for the total energy of the cloud can be obtained employing, as usual, an *ansatz* of the form [33]

$$\psi(\mathbf{r}) = \frac{N^{1/2}}{\pi^{3/4}R^{3/2}}\exp(-r^2/2R^2)\exp(i\phi(r)), \quad (5)$$

where $N$ is the corresponding number of particles and $R$ is a characteristic length, which is interpreted as the radius of the system.

Notice that Eq. (5) corresponds to the solution of the Schrödinger equation associated with noninteracting systems, where the phase $\phi$ can be associated with particle currents [33]. Thus, by inserting the *ansatz* (5) in the energy functional (4) we are able to obtain the corresponding energy,

$$E = E_F + E_R, \quad (6)$$

where $E_F$ is the kinetic energy associated with particle currents,

$$E_F = \frac{\hbar^2}{2m}\int d\mathbf{r}|\psi(\mathbf{r})|^2(\nabla\phi)^2. \quad (7)$$

Additionally, $E_R$ can be interpreted as the energy associated with an effective potential, which is equal to the total energy of the condensate when the phase $\phi$ does not vary in space. The term $E_R$ contains the contributions of the ground state energy ($E_0$), the harmonic oscillator potential ($E_P$), and the contributions due to the interactions among the particles within the condensate ($E_I$). Notice that we have inserted also the contribution $E_\alpha$ caused by the deformation parameter $\alpha$:

$$E_R = E_0 + E_P + E_I + E_\alpha, \quad (8)$$

where

$$E_0 = \frac{\hbar^2}{2m}\int d\mathbf{r}\left(\frac{d|\psi(\mathbf{r})|}{dr}\right)^2, \quad (9)$$

$$E_P = \frac{1}{2}m\omega_0^2\int d\mathbf{r}r^2|\psi(\mathbf{r})|^2, \quad (10)$$

$$E_I = \frac{1}{2}U_0\int d\mathbf{r}|\psi(\mathbf{r})|^4, \quad (11)$$

$$E_\alpha = \hbar\alpha\int d\mathbf{r}\left(\frac{d|\psi(\mathbf{r})|^2}{dr}\right). \quad (12)$$

Consequently, $E_R$ can be written as follows:

$$E_R = \frac{3}{4}\frac{\hbar^2}{mR^2}N + \frac{3}{4}m{\omega_0}^2R^2N + \frac{U_0}{2(2\pi)^{3/2}R^3}N^2 - \alpha\frac{2\hbar}{\sqrt{\pi}R}N, \quad (13)$$

where we have used the trial function (5) together with Eqs. (9)–(12) in order to obtain the above expression.

The equilibrium radius of the system, let us say $R_0$, can be obtained by minimizing the total energy (6). Additionally, the contribution of the kinetic energy (7) is positive definite, and is zero when the phase $\phi$ is constant [33].

However, when the radius $R$ differs from its equilibrium condition, after the external potential $V(r) = m\omega_0^2r^2/2$ is turned off at, let us say $t = 0$, there is a force that changes $R$ and produces an expansion of the cloud. In order to determine an equation for the dynamics of the system, we must deduce the corresponding kinetic energy $E_F$ in function of time, through its dependence on the radius $R$. Changing $R$ from its initial value to a new value $\tilde{R}$ amounts to a uniform dilation of the system, since the new density distribution $|\psi(\mathbf{r})|^2 = n(\mathbf{r})$ may be obtained from the old one by changing the radial coordinate of each atom by a factor $\tilde{R}/R$, see Ref. [33] for details. Thus, the velocity of a particle can be expressed as follows:

$$v(r) = r\frac{\dot{R}}{R}, \quad (14)$$

where the dot stands for derivative with respect to time. Consequently, the kinetic energy ($E_F$) is given by

$$E_F = \frac{mN}{2R^2}\frac{\int d\mathbf{r}n(\mathbf{r})r^2}{\int d\mathbf{r}n(\mathbf{r})}\dot{R}^2, \tag{15}$$

where the ratio between the integrals is a mean-square radius of the condensate [33].

Then, it is straightforward to obtain the kinetic energy $E_F$ by using the *ansatz* Eq. (5), with the result $E_F = 3\dot{R}^2Nm/4$. Moreover, assuming that the energy is conserved at any time, we obtain the following energy conservation condition associated with our system:

$$\frac{3m\dot{R}^2}{4} + \frac{3\hbar^2}{4mR^2} + \frac{U_0}{2(2\pi)^{3/2}R^3}N - \alpha\frac{2\hbar}{\sqrt{\pi}R} = \frac{3\hbar^2}{4mR_0^2} + \frac{U_0}{2(2\pi)^{3/2}R_0^3}N - \alpha\frac{2\hbar}{\sqrt{\pi}R_0}, \tag{16}$$

where $R_0$ is the radius of the condensate at time $t = 0$, which is approximately equal to the oscillator length $a_{ho} = (\hbar/m\omega_0)^{1/2}$ in the noninteracting case. $R$ is a function of time which corresponds to the radius at time $t$. Equation (16) must be solved numerically, even in the case $\alpha = 0$. However, if we neglect interparticle interactions, i.e., setting $U_0 = 0$ then, we are able to obtain an analytical solution for the above equation, with the result

$$\frac{1}{\beta^2}\sqrt{\beta^2R^2 + \frac{2\hbar\alpha}{\sqrt{\pi}}R - \frac{3\hbar^2}{4m}} - \frac{\hbar\alpha}{\sqrt{\pi}\beta^3}\ln\left[\frac{\beta^2R + \frac{\hbar\alpha}{\sqrt{\pi}}}{\beta^2R_0 + \frac{\hbar\alpha}{\sqrt{\pi}}} + \left(\left(\frac{\beta^2R + \frac{\hbar\alpha}{\sqrt{\pi}}}{\beta^2R_0 + \frac{\hbar\alpha}{\sqrt{\pi}}}\right)^2 - 1\right)^{1/2}\right] = \sqrt{\frac{4}{3m}}t, \tag{17}$$

where we have defined

$$\beta^2 = \frac{3\hbar^2}{4mR_0^2} - \frac{2\hbar\alpha}{\sqrt{\pi}R_0}. \tag{18}$$

A rough approximation for the *modified* width of the packet which is valid for large expansion times and $\alpha \ll 1$ renders the following solution:

$$R_\alpha^2(t) = R_0^2 + \left[\frac{\hbar^2}{m^2R_0^2} - \alpha\frac{8}{3\sqrt{\pi}}\frac{\hbar}{mR_0}\right]t^2 + \cdots, \tag{19}$$

which also is equivalent when $\alpha \ll 1$ for $R_\alpha \gg R_0$. If we set $\alpha = 0$ then, we recover the usual solution [33]

$$R^2(t) = R_0^2 + \left(\frac{\hbar}{mR_0}\right)^2 t^2. \tag{20}$$

Notice that in the usual case, $\alpha = 0$, $v_0 = \frac{\hbar}{mR_0}$ is defined as the velocity expansion of the condensate, corresponding to the velocity predicted by the Heisenberg's uncertainty principle for a particle confined within a distance $R_0$ [33]. Thus, in the usual case $\alpha = 0$, the width of the cloud at time $t$ can be written in its usual form $R^2(t) = R_0^2 + (v_0t)^2$.

Here it is appropriate to analyze the role played by the interactions among the components of the system compared to the corrections caused by Planck scale physics. Notice that Eq. (16) must be solved numerically when interactions are taken into account even in the case $\alpha = 0$. However, the asymptotic behavior for $t \to \infty$ can be calculated by using the energy conservation condition Eq. (16).

In this context, we obtain from the energy conservation condition Eq. (16) that the final velocity, $t \to \infty$, is given by

$$(v_\infty^2)^\alpha \approx \left(\frac{\hbar}{mR_0}\right)^2 + \frac{4\sqrt{\pi}\hbar^2N}{\sqrt{18}m^2R_0^3}a - \alpha\frac{8\hbar}{3\sqrt{\pi}mR_0}, \tag{21}$$

if we set $\alpha = 0$ the usual result is recovered. Additionally, in this scenario the initial radius $R_0$ must be corrected due to interactions. If we assume that the initial radius corresponds to the result for an isotropic trap [33]

$$R_0 = \left(\frac{2}{\pi}\right)^{1/10}\left(\frac{Na}{a_{ho}}\right)^{1/5}a_{ho}, \tag{22}$$

then, when $t \to \infty$ the system expands according to the following expression:

$$\left(\frac{R_\alpha(t)}{R_0}\right)^2 \approx \left[\frac{2}{3}\omega_0^2 - \alpha\left(\frac{m\omega_0^6}{\hbar}\right)^{1/5}\frac{1}{(Na)^{3/5}}\right]t^2, \tag{23}$$

which is valid when interactions dominate the final velocity. Consequently, when $t \to \infty$, the condition to be fulfilled for the contributions of Planck scale regime reads

$$\omega_0 \sim 10^{31}\left(\frac{m^2}{Na}\right)^{3/4}, \tag{24}$$

assuming $\xi_1 \sim 1$. In this scenario, a mass of order $10^{-25}$ kg, $N \sim 10^6$ and $a \sim 10^{-6}$ m, lead to frequencies of order $\omega_0 \sim 10^{10}$ Hz, i.e., 4 orders of magnitude bigger that typical frequencies of MHz. Conversely, if $\omega_0 \sim 10^2$ Hz together with $N \sim 10^4$ then, $a \sim 10^{-12}$ m. In other words if the interactions dominate the final velocity then, higher frequencies are required. In other case, for small frequencies the system must evolve deeper in the linear regime, i.e., almost in the noninteracting case. However, the evolution of the system under free expansion must be analyzed from the numerical point of view at any time. This topic will be presented elsewhere [38].

In the case when the interactions are neglected we are able to define from Eq. (19), the *square modified velocity expansion* $(v_0^\alpha)^2$ as follows:

$$(v_0^\alpha)^2 = \frac{\hbar^2}{m^2 R_0^2} - \alpha \frac{8}{3\sqrt{\pi}} \frac{\hbar}{m R_0}, \quad (25)$$

which is well defined, since the deformation parameter $\alpha$ has dimensions of velocity. The modification caused by $\alpha$ is quite small, then the following expansion is justified:

$$(v_0^\alpha) = \frac{\hbar}{m R_0} - \frac{4}{3\sqrt{\pi}}\alpha + O(\alpha^2). \quad (26)$$

Here, let us remark that the presence of the deformation parameter $\alpha$ suggests a modification to the Heisenberg's uncertainty principle, which appears in a natural way, just by looking up to the predicted *modified* velocity $(v_0^\alpha)$. If we define a *new* deformation parameter $\alpha' = \alpha \frac{4m}{3\sqrt{\pi}}$, together with $R_0 = x$, then the resulting *modified* uncertainty principle seems to be

$$\Delta x \Delta p \geq \frac{\hbar}{2} - \alpha' x + O(\alpha^2). \quad (27)$$

Notice that the leading order modification obtained from the analysis of the free expansion of the condensate is apparently linear in the position which, as far as we know, has not been reported in the literature, see for instance Refs. [39–41] and references therein. If so, this fact would open some new phenomenological implications concerning the quantum structure of space-time. Additionally, it is clear that the parameters $\xi_2$ and $\xi_3$ also contribute to the functional form of the modified uncertainty principle. This scenario is a nontrivial topic which also deserves deeper investigation, which we will present in [38].

In order to obtain a bound for the deformation parameter $\xi_1$, let us appeal to the measurements of the kinetic energy of an atom recoiling due to absorption of a photon using an interferometric technique called "contrast interferometry," as it was reported in the experiment (Ref. [42]) for a sodium Bose-Einstein condensate.

First, notice that the quantity $h/m$ is related to the velocity $v_0$ by the de Broglie equation

$$\frac{h}{m} = \lambda v_0, \quad (28)$$

where $\lambda$ is the corresponding wavelength. Consequently, the photon recoil frequency $(\omega_r)$ can be related to the velocity $v_0$ through the quantity $h/m$ as follows:

$$\omega_r = \frac{\hbar}{2m} k^2, \quad (29)$$

where $k$ is the wave vector of the photon absorbed by the atom, whose value is accurately accessible [43]. In the experiment reported in Ref. [42], a measurement of the photon recoil frequency leads to $\omega_r = 2\pi \times 24.9973\ \text{kHz}\ (1 \pm 6.7 \times 10^{-6})$.

Finally, let us add that the form of the energy dispersion relation (3), was constrained by using high precision atom-recoil frequency measurements [15,16]. In such a scenario, bounds for the deformation parameters of order $\xi_1 \sim -1.8 \pm 2.1$ and $-3.8 \times 10^9 < \xi_2 < 1.5 \times 10^9$ were obtained.

However, in order to analyze an alternative procedure compared to those used in Refs. [15,16], i.e., by using the *modified* free expansion velocity of the condensate Eq. (26), we are able to obtain the following *modified* de Broglie equation associated with our system:

$$\frac{2\pi\hbar}{m} = R_0 \left( v_0 - \alpha \frac{8}{3\sqrt{\pi}} \right). \quad (30)$$

Consequently, the *modified* photon recoil frequency $\omega_r^{(\alpha)}$ is given by

$$\omega_r^{(\alpha)} = \frac{R_0}{4\pi} \left( v_0 - \alpha \frac{8}{3\sqrt{\pi}} \right) k^2, \quad (31)$$

where we have assumed that the wave vector $\vec{k}$ of the photon absorbed by an atom is independent of the deformation parameter $\alpha$.

Therefore, the relative shift $(\omega_r^{(\alpha)} - \omega_r)/\omega_r \equiv \Delta\omega_r^{(\alpha)}/\omega_r$ caused by the deformation parameter $\alpha$ is given by the following expression:

$$\frac{\Delta\omega_r^{(\alpha)}}{\omega_r} = \alpha \frac{4 R_0 m}{\pi^{3/2}\hbar}. \quad (32)$$

The value $\omega_r = 2\pi \times 24.9973\ \text{kHz} \quad (1 \pm 6.7 \times 10^{-6})$ obtained in Ref. [42] together with Eq. (32) allows us to obtain a bound for the deformation parameter $\xi_1$, under typical laboratory conditions. In such a case we are able to obtain an upper bound up to $|\xi_1| \sim 1$, by using the relative shift Eq. (32) through its dependence on the *modified* velocity expansion Eq. (26), which is compatible with the upper bound reported in Refs. [15,16].

## III. INTERFERENCE PATTERN OF TWO CONDENSATES AND PLANCK SCALE SIGNALS

Finally, let us analyze the interference pattern of two overlapping Bose-Einstein condensates, in order to explore some possible Planck-scale signals in such a phenomenon. If there is coherence between two condensates, the state may be described by a single condensate wave function, which has the following form:

$$\psi_{1,2}(\mathbf{r}, t) = \sqrt{N_1}\psi_1(\mathbf{r}, t) + \sqrt{N_2}\psi_2(\mathbf{r}, t), \quad (33)$$

where $N_1$ and $N_2$ correspond to the number of particles within each cloud. After the free expansion, the two condensates overlap and interfere. If the effects of interactions are neglected in the overlapping region, the particle density at any point is given by

$$n_{1,2}(\mathbf{r},t) = |\psi_{1,2}(\mathbf{r},t)|^2 = N_1|\psi_1(\mathbf{r},t)|^2 + N_2|\psi_2(\mathbf{r},t)|^2 + 2\sqrt{N_1N_2}\mathrm{Re}[\psi_1(\mathbf{r},t)\psi_2^*(\mathbf{r},t)]. \quad (34)$$

The third right-hand term of expression (34) corresponds to an interference pattern, caused by the overlap of the two condensates. In order to obtain the corrections caused by the deformation parameter $\alpha$, on the properties of the interference pattern of two condensates, let us appeal as usual to the following time dependent condensate wave functions [33]:

$$\psi_1(\mathbf{r},t) = \frac{e^{i\phi_1}}{(\pi R_\alpha^2(t))^{3/4}}\exp\left[-\frac{(\mathbf{r}-\mathbf{d}/2)^2(1-i\hbar t/mR_0^2)}{2R_\alpha^2(t)}\right], \quad (35)$$

$$\psi_2(\mathbf{r},t) = \frac{e^{i\phi_2}}{(\pi R_\alpha^2(t))^{3/4}}\exp\left[-\frac{(\mathbf{r}+\mathbf{d}/2)^2(1-i\hbar t/mR_0^2)}{2R_\alpha^2(t)}\right], \quad (36)$$

where $\phi_1$ and $\phi_2$ are the initial phases for each condensate, $R_0$ is the initial radius of the cloud, which is approximately equal to the oscillator length $a_{ho} = (\hbar/m\omega_0)^{1/2}$. Additionally, $R_\alpha(t)$ is the width of a packet at time $t$, given by Eq. (19). If we set $\alpha = 0$ in Eqs. (35) and (36), then we recover the usual expressions [33].

The interference term in Eq. (34) thus is given by

$$\mathrm{Re}[\psi_1(\mathbf{r},t)\psi_2^*(\mathbf{r},t)] = \frac{e^{-\frac{r^2}{R_\alpha^2(t)}}e^{-\frac{d^2}{4R_\alpha^2(t)}}}{[\pi R_\alpha^2(t)]^{3/2}} \times \cos\left(\frac{\hbar}{m}\frac{\mathbf{r}\cdot\mathbf{d}}{R_0^2R_\alpha^2(t)}t + \phi\right). \quad (37)$$

Notice that the phase shift $\phi = \phi_1 - \phi_2$ is measurable, although the individual phases $\phi_1$ and $\phi_2$ are not [44]. Here the prefactor $\exp(-r^2/R_\alpha{}^2(t))$ depends slowly on $r$ but the cosine function can give rise to rapid spatial variations. We can notice also from Eq. (37) that planes of constant phase are perpendicular to the vector between the centers of the two clouds. The positions of the maxima depend on the relative phase of the two condensates, and if we take $\mathbf{d}$ to lie in the $z$ direction, the distance between maxima is given by

$$z_{(\alpha)} = 2\pi\frac{mR_\alpha^2(t)R_0^2}{\hbar td}. \quad (38)$$

If the expansion time is sufficiently large, i.e., the cloud has expanded to a size much greater than $R_0$ then, as mentioned before, $R_\alpha^2(t)$ is given approximately by Eq. (19). Therefore, the distance between maxima associated with the interference fringes is given by the following expression:

$$z_{(\alpha)} \approx 2\pi\left(\frac{\hbar}{md} - \frac{8\alpha R_0}{3\sqrt{\pi}d}\right)t. \quad (39)$$

When $\alpha = 0$, we recover the usual result [33,34]. In the usual case, $\alpha = 0$, the separation between maxima is typically of order $10^{-6}$ m [34]. From relation (39), we are able to obtain the sensitivity of our system to Planck scale signals upon the fringes separation. Under typical laboratory conditions, i.e., $\omega_0 \sim 10$ Hz and a typical mass of order $m \sim 10^{-26}$ kg, $d = 40 \times 10^{-6}$ m, together with a free expansion time of order $t = 40 \times 10^{-3}$ s, the correction caused by the deformation parameter $\alpha$ can be inferred up to $|\xi_1| \times 10^{-11}$ m, i.e., 5 orders of magnitude smaller than the typical distance between the maxima reported in Ref. [34], when $|\xi_1| \sim 1$.

In order to obtain a more accurate description for the possible measurement of the contributions caused by the quantum structure of space-time, let us analyze the experimental scenario in this context. If the contributions of Planck scale physics could eventually be measured, this implies that the usual term in Eq. (39) must be known more accurately than the size of the correction associated with the deformation parameter $\alpha$.

Unfortunately, the corresponding experimental error associated with the interference fringes separation is not reported in the literature, at least, in the literature known by the authors. In this attempt, let us analyze the error propagation in the measure of the fringes separation, when $\alpha = 0$, in order to obtain experimental conditions that could allow one to detect possible signals arising from the Planck scale regime. In other words, in the most unfavorable case this entails

$$\Delta z_{(\alpha=0)} = \left[\left|\frac{\partial z_{(\alpha=0)}}{\partial m}\right|\Delta m + \left|\frac{\partial z_{(\alpha=0)}}{\partial h}\right|\Delta h + \left|\frac{\partial z_{(\alpha=0)}}{\partial d}\right|\Delta d\right]t, \quad (40)$$

where $\Delta z_{(\alpha=0)}$ depicts the experimental error as usual, and so on. Notice that for simplicity, we have reabsorbed the $2\pi$ factor in the definition of the Planck constant $h$. Additionally, the expansion time $t$ can be interpreted here, without lost of generality, as an evolution parameter. The above expression leads to the following error associated with the fringes separation $\Delta z_{(\alpha=0)}$ in the usual case $\alpha = 0$,

$$\Delta z_{(\alpha=0)} = z_{(\alpha=0)}\left[\frac{md\Delta h + hd\Delta m + hm\Delta d}{mhd}\right]t, \quad (41)$$

where $z_{(\alpha=0)}$ is the usual value when $\alpha = 0$. The corresponding uncertainties $\Delta m = 0.17$ ppb in atomic mass units for $^{23}$Na, and $\Delta h = 20$ ppb in SI units reported in the experiments [45] and [46] respectively, can be used to calculate $\Delta z_{(\alpha=0)}$. Unfortunately, as far as we know, there is not uncertainty reported for the corresponding initial separation $d$. In this situation we obtain an error for the fringes separation of order $\Delta z_{(\alpha=0)} \sim (10^{-18} + 1.5 \times 10^{-2}\Delta d)$ for the usual case $\alpha = 0$ with $t = 40 \times 10^{-3}$ s. As mentioned above, the order of magnitude associated with $\alpha$ can be inferred up to $|\xi_1| \times 10^{-11}$ m. This fact implies, in principle, that uncertainties for the distance $d$ of order $\Delta d \sim 6.67 \times 10^{-10}$ m for expansion times of order $10^{-3}$ s are necessary (assuming $|\xi_1| \sim 1$) to obtain a possible detection of Planck scale signals under typical laboratory conditions. However, large expansion times up to 4 s can be achieved, for instance, in interference free fall experiments [37]. In these circumstances, we obtain $\Delta d \sim 2.5 \times 10^{-9}$ m. In other words, large expansion times implies better precision in knowing the initial separation $d$. Notice also that $\Delta z_{(\alpha=0)}$ and $\Delta d$ are basically the same order of magnitude in the cases described above.

In the same spirit, we are capable to calculate the corresponding experimental error ($\Delta z_\alpha$) associated with the deformation parameter $\alpha$, i.e., the second term in Eq. (39) assuming that $M_p$ and $c$ are constants, together with $\xi_1 \sim -1.8 \pm 2.1$ [15,16] and $R_0 = \sqrt{\frac{\hbar}{m\omega_0}}$.

We assume also that the uncertainty corresponding to typical frequencies is of order $\omega_0 = 21 \pm 4$ MHz in the case of magneto-optical traps [47]. Under these conditions, the corresponding uncertainty can be inferred here up to $\Delta z_\alpha \sim 10^{-14}$ m for $t = 40 \times 10^{-3}$ s, assuming $\Delta d \sim 10^{-10}$ m. Conversely, we obtain $\Delta z_\alpha \sim 10^{-11}$ m for $t = 4$ s, assuming $\Delta d \sim 10^{-9}$ m. The corresponding errors $\Delta z_{\alpha=0}$ and $\Delta z_\alpha$ can be used as a criterion to discriminate how precise the eventual measurement of the correction term caused by $\alpha$ with respect to the usual term is. In fact the above results indicate that if the corrections caused by $\alpha$ wants to be measured, then better precision is needed, compared with the usual term.

The uncertainties obtained for $\Delta z_{\alpha=0}$ can be also used as a criteria to optimize the value of $d$. For instance, assuming that $\Delta z_{(\alpha=0)} \sim 10^{-10}$ m, corresponding to expansion times of order of $40 \times 10^{-3}$ s, together with the corrections caused by $\alpha$ in Eq. (39), this leads to initial separations of order $d \sim 6.8 \times 10^{-8}$ m. Conversely, if $\Delta z_{(\alpha=0)} \sim 10^{-9}$ m for $t = 4$ s, this implies $d \sim 1.15 \times 10^{-10}$ m. In other words, according to our results, an optimal value for the initial separation $d$ seems to be between $10^{-8}$ and $10^{-10}$ m.

Finally, let us analyze the relative shift on the fringes separation caused by the deformation term $\alpha$. The relative shift can be expressed as follows:

$$\frac{z_{(\alpha)} - z_{(\alpha=0)}}{z_{(\alpha=0)}} = -\frac{4}{3\sqrt{\pi}}\xi_1\sqrt{\frac{m^3c^2}{\hbar\omega_0 M_p^2}}, \tag{42}$$

where $z_{(\alpha)}$ is given by expression Eq. (39) and $z_{(\alpha=0)}$ is the usual result, setting $\alpha = 0$. We notice that the relative shift is apparently independent of the initial separation $d$. The relative shift in Eq. (42) can be inferred under typical laboratory conditions up to $\xi_1 \times 10^{-6}$ for $\omega_0 \sim 10^3$ Hz, which is approximately the same order of magnitude as the usual fringes separation when $\alpha = 0$, and apparently impossible to be measured. However, let us mention that such a shift can be improved for small $\omega_0$. For instance, if $\omega_0 \sim 10$ Hz, then the relative shift is of order $\xi_1 \times 10^{-4}$, for a typical mass $m$ of order $10^{-26}$ kg, i.e., 2 orders of magnitude bigger than the typical fringes separation, which is notable.

## IV. CONCLUSIONS

We have analyzed the free expansion of a condensate, and also its properties when two of these systems overlap, assuming as a fundamental fact a deformed dispersion relation. We have proved that the free velocity expansion is corrected as a consequence of a possible quantum structure of space-time. Additionally, the predicted *modified velocity expansion* endows in a natural way a modification in the Heisenberg's uncertainty principle, which indeed opens the possibility to explore some phenomenological consequences in other systems and clearly deserves deeper investigation.

We have explored possible traces arising from Planck scale physics upon the properties associated with the interference fringes when two condensates overlap, and also we have analyzed the experimental scenario under typical laboratory conditions. Here it is important to mention that the contribution caused by interactions among the constituents of the system are expected to be also bigger, evidently, than the contributions caused by the deformation parameter $\alpha$. However, as was recently reported in experiment, Ref. [37], the nonlinear evolution of the condensate occurs at very short times (< 30 ms). This fact suggests that possible Planck scale signals could be measured in principle, for times larger than 30 ms, in which the system operates deeper in the linear regime i.e., almost in the noninteracting case. In fact, free fall experiments could account for Planck scale signals in this context, in which expansion times of order 4 s can be achieved [37]. Nevertheless, the scenario presented in this paper must be extended to more realistic situations, in which the contribution caused by the interactions among the constituents of the condensate could be representative, together with the presence of a gravitational field. This is a not a trivial topic that we will present elsewhere [38]

Finally, we must add that the possible detection of these corrections could be out of the current technology. However, it is remarkable that an adequate choice of the initial conditions in the free expansion of the condensates opens the possibility of planning specific scenarios that could be used to obtain possible traces or signals caused by the quantum structure of space-time in low-energy earth-based experiments.

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