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Stability and antievaporation of the Schwarzschild–de Sitter black holes in bigravity

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We study the stability under the perturbation and the related antievaporation of the Nariai space-time in bigravity. If we impose a specific condition for the solutions and parameters, we obtain asymptotically de Sitter space-time, and show the existence of the Nariai space-time as a background solution. Considering the perturbation around the Nariai space-time up to the first order, we investigate the behavior of the black hole horizon. We show that the antievaporation does not occur on the classical level in the bigravity.

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I. INTRODUCTION

One can find that there exist many reasons and motivations to consider alternative theories of gravity to general relativity. Some theories are motivated by the modifications in the infrared regime and they mainly aim to resolve a question about the dark energy. For instance, $F(R)$ gravity [1,2] can explain the accelerating expansion of the current Universe without the cosmological constant and avoid the hierarchy problem. Others are motivated by the modifications in the ultraviolet regime and they are often associated with the effects of quantum gravity [3,4]. Higher curvature theories typified by curvature-squared and the Gauss-Bonnet terms are induced from quantum corrections and also the corrections from the string theory. Naturally, challenges to the theory beyond general relativity themselves are important because there is no fundamental reason to choose the Einstein-Hilbert action or Einstein's equation over many kinds of alternatives.

Recently, much attention has been paid to bimetric theory or what we call bigravity, which includes two independent metric tensor fields, $g_{\mu\nu}$ and $f_{\mu\nu}$ [5–7]. Bigravity contains the massive spin-2 propagating mode in addition to the ordinary massless spin-2 mode corresponding to the graviton. This theory has been successfully constructed as the generalization of de Rham-Gabadadze-Tolley massive gravity in recent years [8,9]. Some people expect that the new degrees of freedom introduced by another metric can solve remaining problems in cosmology, that is, dark energy [10–21] and dark matter [22–25] problems. Interactions between two metric tensors produce the effective cosmological constant; furthermore, the massive spin-2 fields and matter fields coupled with the metric $f_{\mu\nu}$ can be candidates of dark matter.

When we intend to view the bigravity to be an alternative theory of gravity, it is also interesting that we apply this theory to other phenomena in cosmology or astrophysics and find the differences from general relativity. In our work, we focus on the nature of black holes. It is well known that the horizon radius of the black hole in the vacuum usually decreases by the Hawking radiation, which is called the black hole evaporation. However, Bousso and Hawking have observed a phenomenon where the black hole radius increases by the quantum correction for the specific Nariai black hole [26]. This phenomenon is called antievaporation of black holes. Note that the Schwarzschild–de Sitter black holes and the Nariai black holes can be primordial ones; thus, they are not expected to appear at the final stage of star collapse. As we have introduced, some modifications of gravitational action are inspired by the quantum gravity, and modified actions should be regarded as low-energy effective ones of the quantum gravity. Therefore, it is natural to expect that such effective theories can describe the very early Universe, and it is also reasonable to study the properties of primordial black holes, the Nariai black holes, in modified gravities.

We should also note that the usual evaporation of the black holes occurs by the quantum corrections, and the antievaporation could occur also by the quantum corrections from the matters in general relativity. Whether the usual evaporation or the antievaporation occurs depends on what kind of black hole we consider. The antievaporation is a unique phenomenon in the Nariai black hole. The point to understand the difference between the antievaporation and the ordinary evaporation could be the isometry of the background space-time, which we refer to later.

In $F(R)$ gravity theories, however, it has been shown that the antievaporation may occur even on the classical level [27–29], which might be remarkable. Realization of the antievaporation without the quantum corrections could be due to the modification of the field equations because the behavior of perturbations depends on the equations of motion. The equations for the $F(R)$ gravity are indeed different from those for general relativity, but at present it is not so clear what could be essential for the antievaporation on the classical level. Then it might be interesting if the antievaporation in the classical level might be a general phenomenon in the modified gravity. In this paper, we consider the possibility of the antievaporation in bigravity on the classical level because the contribution from the interaction between two metric tensors is not so trivial. We give a classical analysis in the stability of the Nariai black hole in the bigravity and study if the antievaporation could occur or not on the classical level, which may clarify what could be necessary for the antievaporation to occur.

This paper is organized as follows: First, we explain about the Nariai black hole. The Nariai black hole is defined as a subset of the Schwarzschild–de Sitter black hole where the radii of the cosmological and black hole horizons are degenerate. Second, we give a brief review about the bigravity. In order to show the existence of the Nariai black hole as an exact solution in the bigravity, we specify some parameters and solutions; then we give a proof that the asymptotically de Sitter solutions can be realized. Finally, we consider the perturbations around the black hole, evaluate their stability, and investigate if the antievaporation could occur on the classical level.

II. ANTIEVAPORATION OF THE NARIAI BLACK HOLES

A. Nariai space-time and its property

At first, we introduce the Nariai space-time as a family of the Schwarzschild–de Sitter space-time. The Schwarzschild–de Sitter solution is expressed in the following form:

$$ds^2 = -V(r)dt^2 + V(r)^{-1}dr^2 + r^2 d\Omega^2, \tag{1}$$

where the function $V(r)$ is defined by

$$V(r) = 1 - \frac{2\mu}{r} - \frac{\Lambda}{3}r^2. \tag{2}$$

Here, μ is a mass parameter and Λ is a positive cosmological constant. For $0 < \mu < \frac{1}{3}\Lambda^{-1/2}$, $V(r)$ has two positive roots r_c and r_b, corresponding to the cosmological and black hole horizon, respectively. In the limit $\mu \to \frac{1}{3}\Lambda^{-1/2}$, the radius of the black hole horizon coincides with that of the cosmological horizon. Here, the coordinate system in Eq. (1) becomes inappropriate because $V(r) \to 0$ between the two horizons. Then it is useful to introduce a new coordinate system as follows:

$$t = \frac{1}{\epsilon\sqrt{\Lambda}}\psi, \qquad r = \frac{1}{\sqrt{\Lambda}}\left(1 - \epsilon\cos\chi - \frac{1}{6}\epsilon^2\right), \tag{3}$$

where ϵ is the parameter defined as $9\mu^2\Lambda = 1 - 3\epsilon^2$, and $\epsilon \to 0$ corresponds to the degeneracy of the two horizons.

In the above coordinate, the black hole horizon corresponds to $\chi = 0$, the cosmological horizon corresponds to $\chi = \pi$, and the metric takes the following form:

$$ds^2 = -\frac{1}{\Lambda}\left(1 + \frac{2}{3}\epsilon\cos\chi\right)\sin^2\chi d\psi^2 + \frac{1}{\Lambda}\left(1 - \frac{2}{3}\epsilon\cos\chi\right)d\chi^2 + \frac{1}{\Lambda}(1 - 2\epsilon\cos\chi)d\Omega^2. \tag{4}$$

In the degenerate case, $\epsilon = 0$, the metric is given by

$$ds^2 = \frac{1}{\Lambda}(-\sin^2\chi d\psi^2 + d\chi^2) + \frac{1}{\Lambda}d\Omega^2, \tag{5}$$

and this space-time is called the Nariai black hole. Note that the topology of the spacelike sections of the Schwarzschild–de Sitter space-time (and the Nariai space-time) is $S^1 \times S^2$ while that of the ordinary black hole solution is S^2 in four dimensions. In this coordinate system, the radius of the two sphere, r, varies along the one-sphere coordinate, χ; the minimal two sphere corresponds to the black hole horizon and the maximal one corresponds to the cosmological horizon.

B. Trace anomaly and antievaporation

In this section, we give a brief review of the antievaporation in general relativity, following the paper by Bousso and Hawking [26]. First of all, we begin with the Hawking radiation from the black holes. It is well known that there is radiation by the quantum effects of matter fields around the black hole horizon, which is called the Hawking radiation. For the massless scalar field as the radiation around the black hole horizon, we consider the following action:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{2}\sum_{i=1}^{n}(\nabla f_i)^2\right], \tag{6}$$

where f_i are N scalar fields that carry the quantum radiation. The quantum corrections by the scalar field lead to the trace anomaly of the energy-momentum tensor although the trace of the energy-momentum tensor should classically vanish, $T^\mu{}_\mu = 0$. When we reduce the four-dimensional space-time to the two-dimensional one in a spherically symmetric way by assuming the metric in the following form,

$$ds^2 = \sum_{\mu,\nu=t,r} g_{\mu\nu}dx^\mu dx^\nu + e^{-2\phi}d\Omega^2, \tag{7}$$

the trace anomaly can be expressed by the following effective action [30,31]:

$$S_{\rm eff} = -\frac{1}{48\pi G}\int d^2x\sqrt{-g} \times \left[\frac{1}{2}R\frac{1}{\Box}R - 6(\nabla\phi)^2\frac{1}{\Box}R - \omega\phi R\right]. \tag{8}$$

Here, ω is the redundant parameter corresponding to the renormalization scheme.

Next, we investigate the effective action with the trace anomaly in two-dimensional space-time. We can render the effective action (8) local by introducing the scalar field Z [32], and integrate out the classical solution, $f_i = 0$. Then we obtain the following expression:

$$S = \frac{1}{16\pi G}\int d^2x\sqrt{-g}\Big[\Big(e^{-2\phi} + \frac{\kappa}{2}(Z+\omega\phi)\Big)R - \frac{\kappa}{4}(\nabla Z)^2 + 2 + 2e^{-2\phi}(\nabla\phi)^2 - 2e^{-2\phi}\Lambda\Big], \tag{9}$$

where $\kappa \equiv 2N/3$. We now consider the large N limit, $\kappa \gg 1$, where the quantum fluctuations of the metric are dominated by the contribution from the N scalar fields. We also assume that the quantum correction itself should be small, that is, $b \equiv \kappa\Lambda \ll 1$. Then we consider the perturbation around the Nariai space-time in general relativity. According to the topology, $S^1 \times S^2$, we make a spherically symmetric metric ansatz as follows:

$$g_{\mu\nu}dx^\mu dx^\nu = e^{2\rho(t,x)}(-dt^2 + dx^2) + e^{-2\phi(t,x)}d\Omega^2. \tag{10}$$

Here, the two-dimensional metric, corresponding to t and x components, is written in the conformal gauge and x is the coordinate system on the one sphere and has the period of 2π. In this coordinate, the Nariai solution is expressed as follows:

$$g_{\mu\nu}dx^\mu dx^\nu = \frac{1}{\Lambda\cos^2 t}(-dt^2 + dx^2) + \frac{1}{\Lambda}d\Omega^2. \tag{11}$$

One can obtain the equations of motion for ρ, ϕ, and Z by substituting the anstaz (10) into action (9), and then find the following solution:

$$e^{2\rho} = \frac{1}{\Lambda_1}\frac{1}{\cos^2 t}, \qquad e^{2\phi} = \Lambda_2, \tag{12}$$

where

$$\frac{1}{\Lambda_1} = \frac{1}{\Lambda}\Big(1 - \frac{\omega b}{4}\Big), \qquad \Lambda_2 = \Lambda\Big(1 - \frac{b}{2}\Big), \tag{13}$$

until the first order of b.

Finally, we perturb this solution so that the two-sphere radius, $e^{-\phi}$, varies along the one-sphere coordinate, x. We assume the perturbation in the following form:

$$e^{2\phi} = \Lambda_2[1 + 2\epsilon\sigma(t)\cos x], \qquad |\epsilon| \ll 1. \tag{14}$$

We now trace the time evolution of the black hole horizon. The condition for a horizon is $(\nabla\delta\varphi_1)^2 = 0$, which is required so that the gradient of the two-sphere size is null. Here, perturbation ansatz (10) yields

$$\delta\dot{\phi} = \epsilon\dot{\sigma}\cos x, \qquad \delta\phi' = -\epsilon\sigma\sin x. \tag{15}$$

From the above conditions, the locations of the black hole horizon x_b and cosmological horizon x_c are found as follows:

$$x_b = \arctan\left|\frac{\dot{\sigma}}{\sigma}\right|, \qquad x_c = \pi - x_b. \tag{16}$$

Therefore, the radius of the black hole horizon, r_b, is given by

$$r_b^{-2}(t) = e^{2\phi(t,x_b)} = \Lambda_2\{1 + 2\epsilon\delta(t)\}, \tag{17}$$

where we define the perturbation for the horizon $\delta(t)$,

$$\delta(t) \equiv \sigma(t)\cos x_b = \sigma\left\{1 + \Big(\frac{\dot{\sigma}}{\sigma}\Big)^2\right\}^{-1/2}. \tag{18}$$

For the classical case, $\kappa = 0$, one can find the analytical solution for $\sigma(t)$,

$$\sigma(t) = \frac{\sigma_0}{\cos t}. \tag{19}$$

When we substitute this solution into the horizon perturbation Eq. (18), we obtain

$$\delta(t) = \sigma_0 = \text{const.} \tag{20}$$

This result implies that the black hole size remains to be that of the initial perturbation, which is just a static Schwarzschild–de Sitter black hole of nearly maximal mass.

In contrast, for the semiclassical case, $\kappa > 0$, one cannot find the analytic solution because the quantum corrections from matter field lead to the modification of the equation for $\sigma(t)$. However, one can solve the equation of motion as a power series in t for the early Universe. The horizon perturbation is given by

$$\delta(t) \approx \sigma_0\Big(1 - \frac{1}{2}bt^2\Big). \tag{21}$$

This result implies that the black hole perturbation shrinks from its initial value, and the size of the black hole horizon increases at least initially. This phenomenon is called antievaporation. Note that if we do not include the quantum correction, the quantum correction induces the usual evaporation for the black holes like the Schwarzschild one but the correction sometimes generates the antievaporation.

We should note that, in the case of $F(R)$ gravity, the antievaporation occurs even on the classical level, that is, without quantum corrections [27,28]. The field equations in $F(R)$ gravity are complicated as we will see later, which may generate the antievaporation on the classical level. Because the equations in the bigravity are also pretty complicated, we may expect that the antievaporation could occur on the classical level, and, therefore, it could be interesting to investigate the antievaporation in the bigravity even on the classical level.

III. NARIAI BLACK HOLES IN BIGRAVITY

In this section, we give a brief review of the bigravity and show that the Nariai space-time is an exact solution in this theory. The action of the bigravity [9] is given by

$$\begin{aligned} S_{\mathrm{bigravity}} &= M_g^2 \int d^4x \sqrt{-\det(g)} R(g) \\ &\quad + M_f^2 \int d^4x \sqrt{-\det(f)} R(f) \\ &\quad - 2m_0^2 M_{\mathrm{eff}}^2 \int d^4x \sqrt{-\det(g)} \sum_{n=0}^{4} \beta_n e_n(\sqrt{g^{-1}f}). \end{aligned} \tag{22}$$

Here, g and f are dynamical variables and rank-two tensor fields that have properties as metrics, $R(g)$ and $R(f)$ are the Ricci scalars for $g_{\mu\nu}$ and $f_{\mu\nu}$, respectively, M_g and M_f are the two Planck mass scales for $g_{\mu\nu}$ and $f_{\mu\nu}$ as well, and the scale M_{eff} is the effective Planck mass scale defined by

$$\frac{1}{M_{\mathrm{eff}}^2} = \frac{1}{M_g^2} + \frac{1}{M_f^2}. \tag{23}$$

The quantities β_ns and m_0 are free parameters; the former defines the form of interactions and the latter expresses the mass of the massive spin-2 field. The matrix $\sqrt{g^{-1}f}$ is defined by the square root of $g^{\mu\rho}f_{\rho\nu}$, that is,

$$(\sqrt{g^{-1}f})^{\mu}{}_{\rho}(\sqrt{g^{-1}f})^{\rho}{}_{\nu} = g^{\mu\rho}f_{\rho\nu}. \tag{24}$$

For a general matrix $\mathbf{X}$, $e_n(\mathbf{X})$s are polynomials of the eigenvalues of X:

$$\begin{aligned} e_0(\mathbf{X}) &= 1, \qquad e_1(\mathbf{X}) = [\mathbf{X}], \\ e_2(\mathbf{X}) &= \frac{1}{2}([\mathbf{X}]^2 - [\mathbf{X}^2]), \\ e_3(\mathbf{X}) &= \frac{1}{6}([\mathbf{X}]^3 - 3[\mathbf{X}][\mathbf{X}^2] + 2[\mathbf{X}^3]), \\ e_4(\mathbf{X}) &= \frac{1}{24}([\mathbf{X}]^4 - 6[\mathbf{X}]^2[\mathbf{X}^2] + 3[\mathbf{X}^2]^2 \\ &\quad + 8[\mathbf{X}][\mathbf{X}^3] - 6[\mathbf{X}^4]) \\ &= \det(\mathbf{X}), \\ e_k(\mathbf{X}) &= 0 \quad \text{for } k > 4, \end{aligned} \tag{25}$$

where the square brackets denote the traces of the matrices, that is, $[X] = X^{\mu}_{\mu}$. For conventional notation, we explicitly denote the determinant of matrix A as $\det(A)$, and $\sqrt{A}$ represents a matrix that is the square root of A.

Now we consider the variation of the action (22) with respect to $g_{\mu\nu}$. The obtained equation of motion for $g_{\mu\nu}$ is given by

$$\begin{aligned} 0 &= R_{\mu\nu}(g) - \frac{1}{2}R(g)g_{\mu\nu} + \frac{1}{2}\left(\frac{m_0 M_{\mathrm{eff}}}{M_g}\right)^2 \sum_{n=0}^{3}(-1)^n \beta_n \\ &\quad \times \{g_{\mu\lambda}Y^{\lambda}_{(n)\nu}(\sqrt{g^{-1}f}) + g_{\nu\lambda}Y^{\lambda}_{(n)\mu}(\sqrt{g^{-1}f})\}. \end{aligned} \tag{26}$$

Here, for a matrix $\mathbf{X}$, $Y_n(\mathbf{X})$s are defined by

$$Y^{\lambda}_{(n)\nu}(\mathbf{X}) = \sum_{r=0}^{n}(-1)^r (X^{n-r})^{\lambda}{}_{\nu} e_r(\mathbf{X}), \tag{27}$$

or explicitly

$$\begin{aligned} Y_0(\mathbf{X}) &= \mathbf{1}, \qquad Y_1(\mathbf{X}) = \mathbf{X} - \mathbf{1}[\mathbf{X}], \\ Y_2(\mathbf{X}) &= \mathbf{X}^2 - \mathbf{X}[\mathbf{X}] + \frac{1}{2}\mathbf{1}([\mathbf{X}]^2 - [\mathbf{X}^2]), \\ Y_3(\mathbf{X}) &= \mathbf{X}^3 - \mathbf{X}^2[\mathbf{X}] + \frac{1}{2}\mathbf{X}([\mathbf{X}]^2 - [\mathbf{X}^2]) \\ &\quad - \frac{1}{6}\mathbf{1}([\mathbf{X}]^3 - 3[\mathbf{X}][\mathbf{X}^2] + 2[\mathbf{X}^3]). \end{aligned} \tag{28}$$

We also obtain the equation of motion for $f_{\mu\nu}$,

$$\begin{aligned} 0 &= R_{\mu\nu}(f) - \frac{1}{2}R(f)f_{\mu\nu} + \frac{1}{2}\left(\frac{m_0 M_{\mathrm{eff}}}{M_f}\right)^2 \sum_{n=0}^{3}(-1)^n \beta_{4-n} \\ &\quad \times \{f_{\mu\lambda}Y^{\lambda}_{(n)\nu}(\sqrt{f^{-1}g}) + f_{\nu\lambda}Y^{\lambda}_{(n)\mu}(\sqrt{f^{-1}g})\}. \end{aligned} \tag{29}$$

In this case, we do not consider the energy-momentum tensor for the ordinary matter fields. The constraints for the conservation law appear if the minimal couplings to the matter are introduced, and we find

$$\begin{aligned} 0 = \nabla^{\mu}_{(g)}\Bigg[\sum_{n=0}^{3}(-1)^n \beta_n \{g_{\mu\lambda}Y^{\lambda}_{(n)\nu}(\sqrt{g^{-1}f}) \\ + g_{\nu\lambda}Y^{\lambda}_{(n)\mu}(\sqrt{g^{-1}f})\}\Bigg], \end{aligned} \tag{30}$$

$$\begin{aligned} 0 = \nabla^{\mu}_{(f)}\Bigg[\sum_{n=0}^{3}(-1)^n \beta_{4-n} \{f_{\mu\lambda}Y^{\lambda}_{(n)\nu}(\sqrt{f^{-1}g}) \\ + f_{\nu\lambda}Y^{\lambda}_{(n)\mu}(\sqrt{f^{-1}g})\}\Bigg]. \end{aligned} \tag{31}$$

Here, $\nabla_{(g)}$ and $\nabla_{(f)}$ are covariant derivatives that are defined in terms of $g_{\mu\nu}$ and $f_{\mu\nu}$, respectively.

In order to discuss the antievaporation in the bigravity, we need to confirm that the asymptotically de Sitter solutions are realized in this theory. However, it is not so easy to investigate this problem for all combinations of the included parameters; thus, we impose specific assumptions to make discussion simpler. One of the authors considered a particular class of solutions where the two metric tensors are proportional to each other [33],

$$f_{\mu\nu} = C^2 g_{\mu\nu}. \tag{32}$$

This proportional relation leads to the Einstein equation with a cosmological constant because $\sqrt{g^{-1}f}$ and $\sqrt{f^{-1}g}$ turn out to be proportional to unity. The two equations of motion are given by

$$0 = R_{\mu\nu}(g) - \frac{1}{2}R(g)g_{\mu\nu} + \Lambda_g(C)g_{\mu\nu}, \tag{33}$$

$$0 = R_{\mu\nu}(f) - \frac{1}{2}R(f)f_{\mu\nu} + \Lambda_f(C)f_{\mu\nu}. \tag{34}$$

Note that the dynamics of two metric tensors $g_{\mu\nu}$ and $f_{\mu\nu}$ are separated from each other and the constraints derived from the preservation of energy-momentum tensor are automatically satisfied. And we have not assumed any symmetries in the space-time; thus, we can impose the spherical symmetry to the solutions later.

Furthermore, we consider a specific parametrization for the interacting parameter β_ns,

$$\beta_0 = 6 - 4\alpha_3 + \alpha_4, \quad \beta_1 = -3 + 3\alpha_3 - \alpha_4,$$
$$\beta_2 = 1 - 2\alpha_3 + \alpha_4, \quad \beta_3 = \alpha_3 - \alpha_4, \quad \beta_4 = \alpha_4. \tag{35}$$

This combination of two parameters, α_3 and α_4, is required by the existence of the solution corresponding to the flat space-time in massive gravity, which is often used in bigravity. With an assumption $M_g = M_f$, it has been shown that the de Sitter solution can be realized in some parameter regions (Fig. 1). For instance, the minimal model $(\alpha_3, \alpha_4) = (1, 1)$ has only asymptotically flat solutions although the next-to-minimal models $(\alpha_3, \alpha_4) = (1, -1), (-1, 1), (-1, -1)$ have asymptotically de Sitter solutions. Therefore, the Schwarzschild–de Sitter black hole solutions are realized in the bigravity, and we can obtain the Nariai black hole solution by the limit $\mu \to \frac{1}{3}\Lambda^{-1/2}$. Note that it has recently been implied that the de Sitter solution is an attractor with homothetic relation, which may support us in considering our setting [24,25].

Under the assumption that $f_{\mu\nu} = C^2 g_{\mu\nu}$, the dynamics of two metric tensors $g_{\mu\nu}$ and $f_{\mu\nu}$ are separated from each other, described by the Einstein equations. Here, this property is just for the background solution and perturbations can be independent; degrees of freedom of bigravity do not descend to that of general relativity. When we consider the perturbations from a background space-time, the interaction terms give nontrivial contribution to the evolution compared with the case of general relativity. Therefore, it could be important to analyze the stability of perturbation even on the classical level, and we need to investigate if the antievaporation could be realized on the classical level.

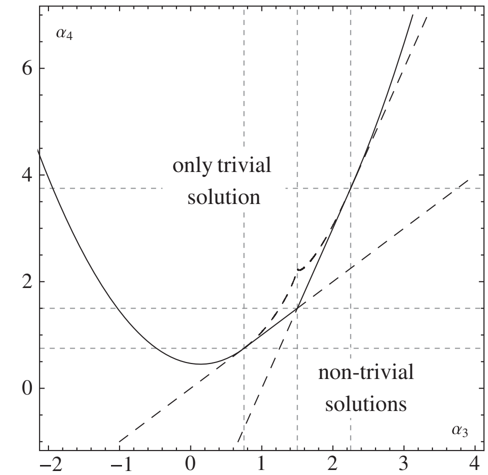

FIG. 1. Classification of the parameters α_3 and α_4 is given [33]. We obtain only flat solutions in the region "only trivial solution," although asymptotically nonflat (de Sitter and/or anti–de Sitter) solutions are realized in "nontrivial solutions."

IV. STABILITY OF THE SCHWARZSCHILD–DE SITTER BLACK HOLE

A. Background solution

We now consider the perturbation from the Nariai black hole in the bigravity. According to the topology, $S^1 \times S^2$, we make a spherically symmetric metric ansatz as follows:

$$g_{\mu\nu}dx^\mu dx^\nu = e^{2\rho_1(t,x)}(-dt^2 + dx^2) + e^{-2\varphi_1(t,x)}(d\theta^2 + \sin^2\theta d\phi^2), \tag{36}$$

$$f_{\mu\nu}dx^\mu dx^\nu = e^{2\rho_2(t,x)}(-dt^2 + dx^2) + e^{-2\varphi_2(t,x)}(d\theta^2 + \sin^2\theta d\phi^2). \tag{37}$$

Here, a two-dimensional metric, corresponding to t and x components, is written in the conformal gauge and x is the coordinate on the one sphere and has the period of 2π. We should also note that the black hole and cosmological horizons are located at same place [34], respectively.

Under the above ansatz, we can calculate each component of the Einstein tensor $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$,

$$
\begin{aligned}
G_{tt} &= \dot{\varphi}^2 - 2\dot{\rho}\dot{\varphi} - 2\rho'\varphi' + 2\varphi'' - 3\varphi'^2 + e^{2(\varphi+\rho)},\\
G_{tx} &= 2\dot{\varphi}' - 2\varphi'\dot{\varphi} - 2\rho'\dot{\varphi} - 2\dot{\rho}\varphi',\\
G_{xx} &= \varphi'^2 - 2\dot{\rho}\dot{\varphi} - 2\rho'\varphi' + 2\ddot{\varphi} - 3\dot{\varphi}^2 - e^{2(\varphi+\rho)},\\
G_{\theta\theta} &= e^{-2(\rho+\varphi)}(-\ddot{\rho} + \rho'' + \ddot{\varphi} - \varphi'' - \dot{\varphi}^2 + \varphi'^2),\\
G_{\phi\phi} &= e^{-2(\rho+\varphi)}(-\ddot{\rho} + \rho'' + \ddot{\varphi} - \varphi'' - \dot{\varphi}^2 + \varphi'^2)\sin^2\theta. \qquad (38)
\end{aligned}
$$

Here, $\dot{} \equiv \partial/\partial t$ and $' \equiv \partial/\partial x$, and sub indices are omitted for simplicity.

Furthermore, we need to calculate the interaction terms in Eqs. (26) and (29). The interaction terms in each equation of motion are written in terms of $\sqrt{g^{-1}f}$ and $\sqrt{f^{-1}g}$. Defining $\mathbf{A} = \sqrt{g^{-1}f}$ and $\mathbf{B} = \sqrt{f^{-1}g}$ for convention, these two matrices are expressed as follows:

$$\mathbf{A} = \mathrm{diag}(e^{-\zeta}, e^{-\zeta}, e^{\xi}, e^{\xi}), \qquad (39)$$

$$\mathbf{B} = \mathrm{diag}(e^{\zeta}, e^{\zeta}, e^{-\xi}, e^{-\xi}), \qquad (40)$$

where we define $\zeta \equiv \rho_1 - \rho_2, \xi \equiv \varphi_1 - \varphi_2$.

After a short calculation, we obtain Y_ns that take the following forms:

$$
\begin{aligned}
Y_0(\mathbf{A}) &= \mathbf{1},\\
Y_1(\mathbf{A}) &= \mathrm{diag}(-e^{-\zeta} - 2e^{\xi}, -e^{-\zeta} - 2e^{\xi}, -2e^{-\zeta} - e^{\xi}, -2e^{-\zeta} - e^{\xi}),\\
Y_2(\mathbf{A}) &= \mathrm{diag}(2e^{-\zeta+\xi} + e^{2\xi}, 2e^{-\zeta+\xi} + e^{2\xi}, e^{-2\zeta} + 2e^{-\zeta+\xi}, e^{-2\zeta} + 2e^{-\zeta+\xi}),\\
Y_3(\mathbf{A}) &= \mathrm{diag}(-e^{-\zeta+2\xi}, -e^{-\zeta+2\xi}, -e^{-2\zeta+\xi}, -e^{-2\zeta+\xi}) \qquad (41)
\end{aligned}
$$

$$
\begin{aligned}
Y_0(\mathbf{B}) &= \mathbf{1},\\
Y_1(\mathbf{B}) &= \mathrm{diag}(-e^{\zeta} - 2e^{-\xi}, -e^{\zeta} - 2e^{-\xi}, -2e^{\zeta} - e^{-\xi}, -2e^{\zeta} - e^{-\xi}),\\
Y_2(\mathbf{B}) &= \mathrm{diag}(2e^{\zeta-\xi} + e^{-2\xi}, 2e^{\zeta-\xi} + e^{-2\xi}, e^{2\zeta} + 2e^{\zeta-\xi}, e^{2\zeta} + 2e^{\zeta-\xi}),\\
Y_3(\mathbf{B}) &= \mathrm{diag}(-e^{\zeta-2\xi}, -e^{\zeta-2\xi}, -e^{2\zeta-\xi}, -e^{2\zeta-\xi}). \qquad (42)
\end{aligned}
$$

Next, we consider the equations of motion. When we choose $M_g = M_f$, the effective Planck mass scale is given by

$$M_{\mathrm{eff}}^2 = \frac{1}{2}M_g^2 = \frac{1}{2}M_f^2. \qquad (43)$$

And, we do not restrict the combinations of parameters in the interaction terms but consider a general case, that is, the case in which there are independent five parameters β_ns. Two equations of motion for $g_{\mu\nu}$ and $f_{\mu\nu}$ take the following form:

$$
\begin{aligned}
0 = R_{\mu\nu}(g) - \frac{1}{2}R(g)g_{\mu\nu} + \frac{1}{2}m_0^2[\beta_0 Y^{\lambda}_{(0)\nu}(\mathbf{A}) - \beta_1 Y^{\lambda}_{(1)\nu}(\mathbf{A})\\
+ \beta_2 Y^{\lambda}_{(2)\nu}(\mathbf{A}) - \beta_3 Y^{\lambda}_{(3)\nu}(\mathbf{A})]g_{\mu\lambda}, \qquad (44)
\end{aligned}
$$

$$
\begin{aligned}
0 = R_{\mu\nu}(f) - \frac{1}{2}R(f)f_{\mu\nu} + \frac{1}{2}m_0^2[\beta_4 Y^{\lambda}_{(0)\nu}(\mathbf{B}) - \beta_3 Y^{\lambda}_{(1)\nu}(\mathbf{B})\\
+ \beta_2 Y^{\lambda}_{(2)\nu}(\mathbf{B}) - \beta_1 Y^{\lambda}_{(3)\nu}(\mathbf{B})]f_{\mu\lambda}. \qquad (45)
\end{aligned}
$$

As we have discussed, one can show that we obtain an asymptotically de Sitter solution for the specific combinations of parameters under the ansatz (32). Furthermore, when we impose the spherical symmetry on the solutions, the Schwarzschild–de Sitter space-time can be solutions in our setting. So we consider the condition to obtain the Nariai space-time as a background solution.

In the coordinate system of Eqs. (36)–(37), the Nariai solutions are expressed as follows:

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = \frac{1}{\Lambda\cos^2 t}(-dt^2 + dx^2) + \frac{1}{\Lambda}d\Omega^2, \qquad (46)$$

$$f_{\mu\nu}dx^{\mu}dx^{\nu} = \frac{C^2}{\Lambda\cos^2 t}(-dt^2 + dx^2) + \frac{C^2}{\Lambda}d\Omega^2. \qquad (47)$$

Therefore, the corresponding $\rho(t,x)$s and $\varphi(t,x)$s in the Nariai solutions are given by

$$
\begin{aligned}
e^{2\rho_1(t,x)} &= \frac{1}{\Lambda\cos^2 t}, \qquad e^{-2\varphi_1(t,x)} = \frac{1}{\Lambda},\\
e^{2\rho_2(t,x)} &= \frac{C^2}{\Lambda\cos^2 t}, \qquad e^{-2\varphi_2(t,x)} = \frac{C^2}{\Lambda}, \qquad (48)
\end{aligned}
$$

that is,

$$
\begin{aligned}
\rho_1 &= -\frac{1}{2}\log\Lambda - \log(\cos t),\\
\varphi_1 &= \frac{1}{2}\log\Lambda,\\
\rho_2 &= \log C - \frac{1}{2}\log\Lambda - \log(\cos t),\\
\varphi_2 &= -\log C + \frac{1}{2}\log\Lambda,\\
\zeta &= -\log C, \qquad \xi = \log C. \qquad (49)
\end{aligned}
$$

Now we substitute these solutions into the equations of motion. Substituting (49) into (38) and (41)–(42), we find that the Einstein tensors and the interaction terms take the following forms:

$$\begin{aligned} G_{tt}(g) &= G_{tt}(f) = \frac{1}{\cos^2 t}, \\ G_{tx}(g) &= G_{tx}(f) = 0, \\ G_{xx}(g) &= G_{xx}(f) = -\frac{1}{\cos^2 t}, \\ G_{\theta\theta}(g) &= G_{\theta\theta}(f) = -1, \\ G_{\phi\phi}(g) &= G_{\phi\phi}(f) = -\sin^2\theta, \end{aligned} \tag{50}$$

$$\begin{aligned} &Y_0(\mathbf{A}) = \mathbf{1}, \qquad Y_1(\mathbf{A}) = -3C\mathbf{1}, \\ &Y_2(\mathbf{A}) = 3C^2\mathbf{1}, \qquad Y_3(\mathbf{A}) = -C^3\mathbf{1}, \end{aligned} \tag{51}$$

$$\begin{aligned} &Y_0(\mathbf{B}) = \mathbf{1}, \qquad Y_1(\mathbf{B}) = -3C^{-1}\mathbf{1}, \\ &Y_2(\mathbf{B}) = 3C^{-2}\mathbf{1}, \qquad Y_3(\mathbf{B}) = -C^{-3}\mathbf{1}. \end{aligned} \tag{52}$$

And we find that two equations of motion are given by

$$0 = 1 - \frac{1}{2\Lambda} m_0^2 [\beta_0 + 3\beta_1 C + 3\beta_2 C^2 + \beta_3 C^3], \tag{53}$$

$$0 = 1 - \frac{C^2}{2\Lambda} m_0^2 [\beta_4 + 3\beta_3 C^{-1} + 3\beta_2 C^{-2} + \beta_1 C^{-3}]. \tag{54}$$

Here, one can identify the two cosmological constants as follows:

$$\Lambda_g(C) = \Lambda = \frac{1}{2} m_0^2 [\beta_0 + 3\beta_1 C + 3\beta_2 C^2 + \beta_3 C^3], \tag{55}$$

$$\Lambda_f(C) = \frac{1}{2} m_0^2 [\beta_4 + 3\beta_3 C^{-1} + 3\beta_2 C^{-2} + \beta_1 C^{-3}]. \tag{56}$$

Then we obtain the quartic equation of C,

$$\Lambda_g(C) = C^2 \Lambda_f(C), \tag{57}$$

by using the two equations of motion.

When one chooses the interacting parameters β_ns, Eq. (57) is determined and the consistent C can be given as a solution that reproduces the positive cosmological constants in Eqs. (55)–(56). Therefore, in order to obtain the Nariai solutions, all we have to do is find the suitable interacting parameters. In the following discussion, we assume that the β_ns are chosen to realize the asymptotically de Sitter space-time.

B. Perturbations

Next, we define the perturbations as follows:

$$\begin{aligned} &\rho_1 \equiv \bar{\rho}_1 + \delta\rho_1(t,x), \qquad \varphi_1 \equiv \bar{\varphi}_1 + \delta\varphi_1(t,x), \\ &\rho_2 \equiv \bar{\rho}_2 + \delta\rho_2(t,x), \qquad \varphi_2 \equiv \bar{\varphi}_2 + \delta\varphi_2(t,x). \end{aligned} \tag{58}$$

Here, $\bar{\rho}$s and $\bar{\varphi}$s correspond to the unperturbed Nariai space-time and $\delta\rho$s and $\delta\varphi$s are the perturbations. Note that these perturbations are not general, but keep the space-time isometry to be $S^1 \times S^2$. By substituting the above expressions into (36)–(37), we find the metric perturbations of $g_{\mu\nu}$ and $f_{\mu\nu}$ in the first order,

$$\begin{aligned} g_{\mu\nu} &= \mathrm{diag}(-e^{2\bar{\rho}_1}, e^{2\bar{\rho}_1}, e^{-2\bar{\varphi}_1}, e^{-2\bar{\varphi}_1}\sin^2\theta) \\ &\quad + \mathrm{diag}(-2e^{2\bar{\rho}_1}\delta\rho_1, 2e^{2\bar{\rho}_1}\delta\rho_1, \\ &\quad - 2e^{-2\bar{\varphi}_1}\delta\varphi_1, -2e^{-2\bar{\varphi}_1}\delta\varphi_1\sin^2\theta) \\ &= \bar{g}_{\mu\nu} + \delta g_{\mu\nu}, \end{aligned} \tag{59}$$

$$\begin{aligned} f_{\mu\nu} &= \mathrm{diag}(-e^{2\bar{\rho}_2}, e^{2\bar{\rho}_2}, e^{-2\bar{\varphi}_2}, e^{-2\bar{\varphi}_2}\sin^2\theta) \\ &\quad + \mathrm{diag}(-2e^{2\bar{\rho}_2}\delta\rho_2, 2e^{2\bar{\rho}_2}\delta\rho_2, \\ &\quad - 2e^{-2\bar{\varphi}_2}\delta\varphi_2, -2e^{-2\bar{\varphi}_2}\delta\varphi_2\sin^2\theta) \\ &= \bar{f}_{\mu\nu} + \delta f_{\mu\nu}, \end{aligned} \tag{60}$$

where we define

$$\begin{aligned} \delta g_{\mu\nu} &= \mathrm{diag}\bigg(-\frac{2}{\Lambda\cos^2 t}\delta\rho_1, \frac{2}{\Lambda\cos^2 t}\delta\rho_1, \\ &\qquad -\frac{2}{\Lambda}\delta\varphi_1, -\frac{2}{\Lambda}\delta\varphi_1\sin^2\theta\bigg), \end{aligned} \tag{61}$$

$$\begin{aligned} \delta f_{\mu\nu} &= \mathrm{diag}\bigg(-\frac{2C^2}{\Lambda\cos^2 t}\delta\rho_1, \frac{2C^2}{\Lambda\cos^2 t}\delta\rho_1, \\ &\qquad -\frac{2C^2}{\Lambda}\delta\varphi_1, -\frac{2C^2}{\Lambda}\delta\varphi_1\sin^2\theta\bigg). \end{aligned} \tag{62}$$

We now evaluate the equations of motion for the perturbation. At first, we calculate the perturbation of the Einstein tensor in the first order. When we substitute the metric perturbations into (38), we obtain the deviations of $G_{\mu\nu}(g)$,

$$G_{tt}(g) = \frac{1}{\cos^2 t} + 2\delta\varphi_1'' - 2\tan t\,\delta\dot{\varphi}_1 + \frac{2}{\cos^2 t}(\delta\varphi_1 + \delta\rho_1), \tag{63}$$

$$G_{tx}(g) = 2\delta\dot{\varphi}_1' - 2\tan t\,\delta\varphi_1', \tag{64}$$

$$\begin{aligned} G_{xx}(g) &= -\frac{1}{\cos^2 t} + 2\delta\ddot{\varphi}_1 \\ &\quad - 2\tan t\,\delta\dot{\varphi}_1 - \frac{2}{\cos^2 t}(\delta\varphi_1 + \delta\rho_1), \end{aligned} \tag{65}$$

$$G_{\theta\theta}(g) = -1 + 2(\delta\rho_1 + \delta\varphi_1) + \cos^2 t(-\delta\ddot{\rho}_1 + \delta\rho_1'' + \delta\ddot{\varphi}_1 - \delta\varphi_1''), \quad (66)$$

$$G_{\phi\phi}(g) = -\sin^2\theta + \sin^2\theta\{2(\delta\rho_1 + \delta\varphi_1) + \cos^2 t(-\delta\ddot{\rho}_1 + \delta\rho_1'' + \delta\ddot{\varphi}_1 - \delta\varphi_1'')\}. \quad (67)$$

Then we define the deviations of the Einstein tensor from the Nariai space-time. Note that the deviations of the Einstein tensor for $f_{\mu\nu}$ are obtained by changing $\rho_1 \to \rho_2$ and $\varphi_1 \to \varphi_2$, because $\rho_1 + \varphi_1 = \rho_2 + \varphi_2$ and $\log C$ is constant. Compared with Eq. (50), we find the deviations of the Einstein tensor are given by

$$\delta G_{tt} = 2\delta\varphi'' - 2\tan t\delta\dot{\varphi} + \frac{2}{\cos^2 t}(\delta\varphi + \delta\rho), \quad (68)$$

$$\delta G_{tx} = 2\delta\dot{\varphi}' - 2\tan t\delta\varphi', \quad (69)$$

$$\delta G_{xx} = 2\delta\ddot{\varphi} - 2\tan t\delta\dot{\varphi} - \frac{2}{\cos^2 t}(\delta\varphi + \delta\rho), \quad (70)$$

$$\delta G_{\theta\theta} = 2(\delta\rho + \delta\varphi) + \cos^2 t(-\delta\ddot{\rho} + \delta\rho'' + \delta\ddot{\varphi} - \delta\varphi''), \quad (71)$$

$$\delta G_{\phi\phi} = \sin^2\theta\{2(\delta\rho + \delta\varphi) + \cos^2 t(-\delta\ddot{\rho} + \delta\rho'' + \delta\ddot{\varphi} - \delta\varphi'')\}. \quad (72)$$

Next, we evaluate the interaction terms. We define the deviation of ζ and ξ as follows:

$$\zeta = -\log C + \delta\rho_1 - \delta\rho_2 \equiv \bar{\zeta} + \delta\zeta, \quad (73)$$

$$\xi = \log C + \delta\varphi_1 - \delta\varphi_2 \equiv \bar{\xi} + \delta\xi. \quad (74)$$

Then we can calculate the deviations of the interaction terms Y_ns from the Nariai space-time, and they are given by

$$\delta Y_0(\mathbf{A}) = \mathbf{0}, \quad \delta Y_1(\mathbf{A}) = -C^{-1}\mathbf{Z}, \quad \delta Y_2(\mathbf{A}) = 2C^{-2}\mathbf{Z}, \quad \delta Y_3(\mathbf{A}) = -C^{-3}\mathbf{Z}, \quad (75)$$

$$\delta Y_0(\mathbf{B}) = \mathbf{0}, \quad \delta Y_1(\mathbf{B}) = C\mathbf{Z}, \quad \delta Y_2(\mathbf{B}) = -2C^2\mathbf{Z}, \quad \delta Y_3(\mathbf{B}) = C^3\mathbf{Z}, \quad (76)$$

where we define the tensor $\mathbf{Z}$ as follows,

$$\mathbf{Z} = \mathrm{diag}(\delta\zeta - 2\delta\xi, \delta\zeta - 2\delta\xi, 2\delta\zeta - \delta\xi, 2\delta\zeta - \delta\xi). \quad (77)$$

Finally, we consider the equations for the perturbations. For convention, we express the equations of motion as follows:

$$G_{\mu\nu}(g) + I^\lambda{}_\nu(\mathbf{A})g_{\mu\lambda} = 0, \quad (78)$$

$$G_{\mu\nu}(f) + I^\lambda{}_\nu(\mathbf{B})f_{\mu\lambda} = 0, \quad (79)$$

where $I^\lambda{}_\nu$s are the sum of Y_ns. When we consider the perturbation up to first order, the above equations are divided by background part and deviation part, and the equations for the deviation take the following forms:

$$\delta G_{\mu\nu}(g) + \delta I^\lambda{}_\nu(\mathbf{A})g_{\mu\lambda} + I^\lambda{}_\nu(\mathbf{B})\delta g_{\mu\lambda} = 0, \quad (80)$$

$$\delta G_{\mu\nu}(f) + \delta I^\lambda{}_\nu(\mathbf{B})f_{\mu\lambda} + I^\lambda{}_\nu(\mathbf{A})\delta f_{\mu\lambda} = 0. \quad (81)$$

Here, we define

$$I(\mathbf{A}) = \frac{1}{2}m_0^2[\beta_0 + 3\beta_1 C + 3\beta_2 C^2 + \beta_3 C^3]\mathbf{1} = \Lambda\mathbf{1}, \quad (82)$$

$$I(\mathbf{B}) = \frac{1}{2}m_0^2[\beta_4 + 3\beta_3 C^{-1} + 3\beta_2 C^{-2} + \beta_1 C^{-3}]\mathbf{1} = \frac{\Lambda}{C^2}\mathbf{1}, \quad (83)$$

$$\delta I(\mathbf{A}) = -\frac{1}{2}m_0^2[\beta_1 C + 2\beta_2 C^2 + \beta_3 C^3]\mathbf{Z} = -C_1\mathbf{Z}, \quad (84)$$

$$\delta I(\mathbf{B}) = \frac{1}{2}m_0^2[\beta_3 C^{-1} + 2\beta_2 C^{-2} + \beta_1 C^{-3}]\mathbf{Z} = C^{-4}C_1\mathbf{Z}, \quad (85)$$

$$C_1 \equiv \frac{1}{2}m_0^2[\beta_4 + 3\beta_3 C^{-1} + 3\beta_2 C^{-2} + \beta_1 C^{-3}]. \quad (86)$$

The explicit expressions of the above equations (80)–(81) are given in Appendix B.

C. Evolution of black hole horizon

In order to describe the evolution of black holes due to the perturbations, we need to know where the horizons are located for $g_{\mu\nu}$ and $f_{\mu\nu}$. In the following, we consider the black hole horizon for $g_{\mu\nu}$ at first. Let us specify the form of perturbations according to the original procedure by Hawking and Bousso:

$$e^{2\varphi_1} = \Lambda\{1 + 2\epsilon\sigma_1(t)\cos x\}, \qquad |\epsilon| \ll 1, \quad (87)$$

that is,

$$\delta\varphi_1 \equiv \epsilon\sigma_1(t)\cos x. \quad (88)$$

Substituting the above form of perturbation into the (t, x) component of (80), we obtain

$$\dot{\sigma_1} = \sigma_1 \tan t. \quad (89)$$

With the boundary condition, $\dot{\sigma_1} = 0$ at $t = 0$, the solution is

$$\sigma_1(t) = \frac{\sigma_g}{\cos t}. \tag{90}$$

Then we find the horizon perturbation Eq. (18) as follows:

$$\delta(t) = \sigma_g = \text{const}. \tag{91}$$

This result means that no antievaporation takes place as well as in the classical case of general relativity. Furthermore, if we define the same form of perturbation for $\delta\varphi_2$ as that for $\delta\varphi_1$, we obtain the same results because the equations have the same form as that of φ_1. Then one can find that antievaporation does not occur for two metrics $g_{\mu\nu}$ and $f_{\mu\nu}$ on the classical level.

Note that the perturbation is consistent with Eq. (80). We can solve the (t, x) component of (80) without fixing the form of perturbations we obtain,

$$\delta\varphi_1 = \frac{h(x)}{\cos t} + k(t). \tag{92}$$

Here, $h(x)$ and $k(t)$ are determined by the other components of the equation.

V. DIFFERENCE FROM GENERAL RELATIVITY

In the previous section, we found that the antievaporation is not realized in bigravity on the classical level, which is not changed from the result in general relativity. In this section, we focus on the problem of how we can identify the difference between the case in general relativity and in bigravity.

When we substitute the perturbations into the (t, t) and (x, x) components of (80), we obtain

$$\delta\zeta - 2\delta\xi = 0. \tag{93}$$

Thus, the contributions from the interaction terms in (t, t), (x, x), and (t, x) components vanish, and the equations are the same as that in general relativity. The deviations of the interaction terms (84) and (86) exactly vanish if $\delta\zeta = 0$ or $\delta\xi = 0$. When we define the perturbation for $f_{\mu\nu}$ as

$$e^{2\varphi_2} = \frac{\Lambda}{C^2}\{1 + 2\epsilon\sigma_2(t)\cos x\}, \qquad \sigma_2(t) = \frac{\sigma_f}{\cos t}, \tag{94}$$

$\delta\xi$ vanishes in the case where the amplitude of the perturbations is identical, $\sigma_g = \sigma_f$. This means that the two sets of metric perturbations are proportional to each other and the relation between the perturbations is not changed from the background, $\delta f_{\mu\nu} = C^2\delta g_{\mu\nu}$. In this case, whole metrics including the perturbations are proportional and they do not lead to difference from general relativity. Therefore, we cannot distinguish bigravity theory from general relativity.

Note that, regarding the perturbation, one can introduce the different forms between $\delta\varphi_1$ and $\delta\varphi_2$. For instance, we may assume the following form:

$$\delta\varphi_1 = \epsilon\frac{\sigma_g}{\cos t}\cos(x + \alpha), \tag{95}$$

$$\delta\varphi_2 = \epsilon\frac{\sigma_f}{\cos t}\cos(x + \beta), \tag{96}$$

which is consistent with Eqs. (80)–(81). In this case, the amplitude and phase can take independent values and these are different from general relativity.

VI. SUMMARY AND DISCUSSION

We have studied the possibility of the antievaporation on the classical level in the bigravity. For the assumption $f_{\mu\nu} = C^2 g_{\mu\nu}$ and particular parameters β_ns and Planck mass scales $M_g = M_f$, we obtained the asymptotically de Sitter space-time. When we considered the perturbations around the Nariai space-time, the size of the black hole horizon does not increase. And we have found that the antievaporation does not take place on the classical level although the equations of motion are different from general relativity. In contrast to our result, it has been shown that "bi-Schwarzschild" solutions are classically unstable [35]. As we stressed in Sec. IV.B, the perturbations that we considered are not generally spherically symmetric but the specific ones to keep the background space-time isometry. Thus, the stability of the biNariai solution in our work may relate to the symmetry of the space-time.

When we assume the perturbation (88), Eq. (90) is derived from the (t, x) component of Eq. (80). However, the nondiagonal components of Eq. (80) take the forms identical with those in general relativity because the interaction terms do not modify the nondiagonal components. Note that this outcome depends on the special configuration of background solutions, that is, simultaneously diagonalized metrics. In this condition, the nondiagonal components of Y_ns vanish; as a result, the size of the black hole horizon does not increase as well as the case in general relativity. In the $F(R)$ gravity, however, the antievaporation can occur on the classical level because the equations of motion are modified in a different way. The general equation of motion in $F(R)$ gravity without matter fields is given by

$$0 = R_{\mu\nu}F_R(R) - \frac{1}{2}g_{\mu\nu}F(R) + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)F_R(R), \tag{97}$$

where $F_R = \partial F/\partial R$. This equation is apparently different from Eqs. (26) and (29), and the difference from general relativity is not given by the addition to the Einstein tensors but by nonlinear couplings of the curvature. In fact, the field equation of $\delta\varphi$ corresponding to (t, x) components of Eq. (80) is modified; then this modification makes it an open possibility to realize the antievaporation on the classical level [28].

We may expect that the antievaporation could occur if we include the quantum correction of matter fields as in the case of general relativity. The explicit calculation could be pretty complicated, but an interesting problem could be to study whether we need to introduce the quantum corrections only for one of the two metrics $g_{\mu\nu}$ and $f_{\mu\nu}$ or both of the metrics. In bigravity, we may assume the two kinds of matter fields $\Psi_g(x)$ and $\Psi_f(x)$ that are coupled to $g_{\mu\nu}$ and $f_{\mu\nu}$, respectively. Thus, there are potentially quantum radiations and the corrections from the two kinds of matter. For instance, if we found that the antievaporation occurs by including the quantum corrections from the matters only coupled with $f_{\mu\nu}$, black hole radius could increase even though we do not include the quantum corrections from the matters coupled with $g_{\mu\nu}$.

There could be another way to realize the antievaporation by modification to $F(R)$ bigravity theory [36–39]. This theory modifies the kinetic terms of bigravity, from the Ricci scalar to the function of it. In $F(R)$ bigravity, we find similar problem to introducing the quantum corrections. That is, we need to study if the modification is required for only one metric or both metrics.

It is interesting that our approach may be generalized to the case of other background solutions. As we mentioned above, we took the background $\bar{g}_{\mu\nu}$ and $\bar{f}_{\mu\nu}$ as the Nariai space-time, and these metrics are diagonalized because of the proportional relation between two metrics $f_{\mu\nu} = C^2 g_{\mu\nu}$. In general, however, two metrics cannot be simultaneously diagonalized because we have only one set of diffeomorphisms for two independent metrics in the bigravity [40]. If we remove the assumption $f_{\mu\nu} = C^2 g_{\mu\nu}$ and we can find the nondiagonal solution for $g_{\mu\nu}$ and/or $f_{\mu\nu}$, the interaction terms do modify the nondiagonal components for the equations of perturbations, and these modifications lead to nontrivial contributions. From the point of view of specifying the difference from general relativity, nondiagonal components of metric are of great interest. For instance, nondiagonal solutions even for the spherically symmetric space-time are permitted because of one set of diffeomorphisms for two metrics. Therefore, if we can detect the phenomena that stem from such solutions in the cosmological and astrophysical observation, it leads us to the possibility to distinguish or restrict the bigravity theory.

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APPENDIX A: GEOMETRICAL QUANTITIES

The connections and curvature in the conformal gauge are given as follows:

$$\begin{aligned}
&\Gamma^t{}_{tt} = \Gamma^t{}_{xx} = \dot{\rho}, \qquad \Gamma^t{}_{tx} = \rho', \\
&\Gamma^t{}_{\theta\theta} = -\dot{\varphi} e^{-2(\rho+\varphi)}, \qquad \Gamma^t{}_{\phi\phi} = -\dot{\varphi} e^{-2(\rho+\varphi)} \sin^2\theta, \\
&\Gamma^x{}_{tx} = \dot{\rho}, \qquad \Gamma^x{}_{xx} = \Gamma^x{}_{tt} = \rho', \\
&\Gamma^x{}_{\theta\theta} = \varphi' e^{-2(\rho+\varphi)}, \qquad \Gamma^x{}_{\phi\phi} = \varphi' e^{-2(\rho+\varphi)} \sin^2\theta, \\
&\Gamma^\theta{}_{t\theta} = -\dot{\varphi}, \qquad \Gamma^\theta{}_{x\theta} = -\varphi', \qquad \Gamma^\theta{}_{\phi\phi} = -\sin\theta\cos\theta, \\
&\Gamma^\phi{}_{t\phi} = -\dot{\varphi}, \qquad \Gamma^\phi{}_{x\phi} = -\varphi', \qquad \Gamma^\phi{}_{\theta\phi} = \cot\theta, \\
&R_{tt} = -\ddot{\rho} + 2\ddot{\varphi} + \rho'' - 2\dot{\varphi}^2 - 2\dot{\rho}\dot{\varphi} - 2\rho'\varphi', \\
&R_{xx} = \ddot{\rho} + 2\varphi'' - \rho'' - 2\varphi'^2 - 2\dot{\rho}\dot{\varphi} - 2\rho'\varphi', \\
&R_{tx} = 2\dot{\varphi}' - 2\varphi'\dot{\varphi} - 2\rho'\dot{\varphi} - 2\dot{\rho}\varphi', \\
&R_{\theta\theta} = 1 + e^{-2(\rho+\varphi)}(-\ddot{\varphi} + \varphi'' + 2\dot{\varphi}^2 - 2\varphi'^2), \\
&R_{\phi\phi} = \{1 + e^{-2(\rho+\varphi)}(-\ddot{\varphi} + \varphi'' + 2\dot{\varphi}^2 - 2\varphi'^2)\}\sin^2\theta, \\
&R = (2\ddot{\rho} - 2\rho'' - 4\ddot{\varphi} + 4\varphi'' + 6\dot{\varphi}^2 - 6\varphi'^2)e^{-2\rho} + 2e^{2\varphi}.
\end{aligned}$$

Here, $\dot{} \equiv \partial/\partial t$ and $' \equiv \partial/\partial x$.

APPENDIX B: PERTURBATIONS

The equations for the perturbations are given as follows:

(i) (t, t) component of (80),

$$0 = \delta\varphi_1'' - \tan t\,\delta\dot{\varphi}_1 + \frac{1}{\cos^2 t}\delta\varphi_1 + \frac{C_1}{2\Lambda\cos^2 t}(\delta\zeta - 2\delta\xi). \tag{B1}$$

(ii) (t, x) component of (80),

$$0 = \delta\dot{\varphi}_1' - \tan t\,\delta\varphi_1'. \tag{B2}$$

(iii) (x, x) component of (80),

$$0 = \delta\ddot{\varphi}_1 - \tan t\,\delta\dot{\varphi}_1 - \frac{1}{\cos^2 t}\delta\varphi_1 - \frac{C_1}{2\Lambda\cos^2 t}(\delta\zeta - 2\delta\xi). \tag{B3}$$

(iv) (θ, θ), (ϕ, ϕ) component of (80),

$$0 = 2\delta\rho_1 + \cos^2 t(-\delta\ddot{\rho}_1 + \delta\rho_1'' + \delta\ddot{\varphi}_1 - \delta\varphi_1'') - \frac{C_1}{\Lambda}(2\delta\zeta - \delta\xi). \tag{B4}$$

(v) (t, t) component of (81),

$$0 = \delta\varphi_2'' - \tan t\delta\dot{\varphi}_2 + \frac{1}{\cos^2 t}\delta\varphi_2 - \frac{C_1}{2C^2\Lambda\cos^2 t}(\delta\zeta - 2\delta\xi). \tag{B5}$$

(vi) (t, x) component of (81),

$$0 = \delta\dot{\varphi}_2' - \tan t\delta\varphi_2'. \tag{B6}$$

(vii) (x, x) component of (81),

$$0 = \delta\ddot{\varphi}_2 - \tan t\delta\dot{\varphi}_2 - \frac{1}{\cos^2 t}\delta\varphi_2 + \frac{C_1}{2C^2\Lambda\cos^2 t}(\delta\zeta - 2\delta\xi). \tag{B7}$$

(viii) (θ, θ), (ϕ, ϕ) component of (81),

$$0 = 2\delta\rho_2 + \cos^2 t(-\delta\ddot{\rho}_2 + \delta\rho_2'' + \delta\ddot{\varphi}_2 - \delta\varphi_2'') + \frac{C_1}{C^2\Lambda}(2\delta\zeta - \delta\xi). \tag{B8}$$

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