PHYSICAL REVIEW D **91,** 083534 (2015)

Emergent universe scenario and the low CMB multipoles

Pedro Labraña*

Departamento de Física, Universidad del Bío-Bío, Casilla 5-C Concepción, Chile and Departament d'Estructura i Constituents de la Matèria, Institut de Ciències del Cosmos, Universitat de Barcelona, Diagonal 647, 08028 Barcelona, Spain

(Received 24 December 2013; published 24 April 2015)

In this work we study superinflation in the context of the emergent universe (EU) scenario. The existence of a superinflating phase before the onset of slow-roll inflation arises in any emergent universe model. We found that the superinflationary period in the EU scenario produces a suppression of the CMB anisotropies at large scale which could be responsible for the observed lack of power at large angular scales of the CMB.

DOI: 10.1103/PhysRevD.91.083534 PACS numbers: 98.80.Cq, 98.70.Vc

I. INTRODUCTION

Cosmological inflation has become an integral part of the standard model of the Universe. Apart from being capable of removing the shortcomings of the standard cosmology, it gives important clues for large scale structure formation [1–4] (see [5] for a review).

The scheme of inflation is based on the idea that there was an early phase, before the big bang, in which the Universe evolved through a nearly exponential expansion during a short period of time at high-energy scales. During this phase, the Universe was dominated by a potential $V(\phi)$ of a scalar field, which is called the inflaton ϕ. The idea of cosmological inflation was first developed by Guth [1] and later refined simultaneously by Linde [3,4] and Albrecht and Steinhardt [2] in a version known as slow-roll inflation.

In the standard inflationary universe, quantum fluctuations of the inflaton field give rise to a curvature perturbation that is constant for modes outside the horizon. This curvature perturbation is then the seed for structure formation in the Universe. The quantum fluctuations of the inflaton could be calculated using the semiclassical theory of quantum fields in curved spacetime; see [6–10]. In particular, for a very flat inflaton potential, the inflaton can be taken to be massless and the quantum fluctuation becomes $\delta\phi(k) = H_k/(2\pi)$ at the moment of crossing the horizon, where H is the Hubble parameter during inflation and k is the comoving momentum related with the fluctuation. The notation on the right-hand side emphasizes the following property: Modes of different momenta exit the horizon at different times, with the larger k being the later. Once a given mode exits the horizon, its amplitude freezes. This perturbation then reenters the horizon during the postinflationary era and becomes the seed for the structure formation, leaving its imprint on the CMB.

The standard slow-roll inflation predicts a slightly red-tilted power spectrum of the primordial perturbation. This means that large scale (small k) perturbation has more power than small scale perturbation. This red tilt arises from the fact that the Hubble parameter slowly decreases with time as the scalar field rolls downwards to its potential.

The recent cosmological observations are entirely consistent with the simplest slow-roll inflationary models [11–13]. Probing that the observed universe is almost flat, the bispectral non-Gaussianity parameter f_{NL} measured is consistent with zero. The scalar perturbation spectral index n_s is less than one, which is a measure of the tilt discussed above ($n_s < 1$ red tilt, $n_s > 1$ blue tilt and $n_s = 1$ scale invariant spectrum). All these results are predicted by the simplest slow-roll inflationary models.

However, there are intriguing observations on cosmic microwave background radiation, suggesting a lack of power at large angular scales (very low multipoles, $l < 40$). These results were first obtained by COBE [14] and WMAP [11] and now are confirmed by Planck [12]. Although these results are well within our cosmic variance and statistically their significance is still low, the power deficit is not insignificant. The Planck Collaboration reported a power deficit in the low multipoles CMB power spectrum of order 5%–10% (with respect to the Planck best-fit ΛCDM model [15,16]) with statistical significance $2.5 \sim 3\sigma$.

This situation is interesting because the very low l modes in the CMB spectrum at present time correspond to very large wavelength modes. Since these modes have been superhorizon sized between inflation and now, they have not been contaminated by the later evolution of the Universe. For this reason, we could attribute the new feature observed in the spectrum at low l to physics at the very earliest Universe, perhaps before slow-roll inflation [17].

There are different approaches developed in order to explain this problem. One possibility is to consider the hypothesis of the *"small universe"* with a compact topology. In this case, perturbations on scales exceeding the fundamental cell size are suppressed; see [18–24]. Another possibility is hyperspherical topology corresponding to a closed universe [25,26], or consider an anisotropic universe [27–29]. The topological approach has some problems with

*plabrana@ubiobio.cl

the S-statistic and the matched circle test [30]. On the other hand, Planck searches yield no detection of the compact topology [31].

Another approach is to introduce a cutoff in the primordial power spectrum [32–41]. This cutoff is normally introduced by hand but linked to the spatial curvature scale [26], string physics [37,39,40], the bouncing universe scheme [40,41], or a fast-rolling stage in the evolution of the inflaton field [34]. This approach is interesting since it has been claimed that from the observed angular power spectrum, it is possible to deconvolve the primordial power spectrum for a given set of cosmological parameters. The most prominent feature of the recovered primordial power spectrum is a sharp, infrared cutoff on the horizon scale [42,43].

In this respect, it has been suggested that the low-l power could be related to a period of superinflation, previous to the standard slow-roll inflationary regimen [44–46], where the superinflationary period is characterized by the condition $\dot{H} > 0$. In particular, in Ref. [44], this possibility was studied in the context of bouncing universes and was discussed regarding the emergent universe scheme.

It is interesting to note, see [44], that a superinflationary period is related to any mechanism which attempts to solve the cosmological singularity problem [47–51] in a semiclassical spacetime description. There are two ways to avoid the singularity problem in this context. One possibility is to consider a nonsingular bounce [41,52–70]; the other possibility is to consider the emergent universe scenario [71–73].

The emergent universe (EU) refers to models in which the universe emerges from an Einstein static state, inflates, and is then submitted into a hot big bang era. Such models are appealing since they provide specific examples of nonsingular (geodesically complete) inflationary universes. These models have been studied in different contexts during recent years; see [71–92].

In this paper, we study the period of superinflation in the context of the EU scenario. The existence of a superinflating phase before the onset of slow-roll inflation arises in any EU scenario [44] and has not been studied thus far. As an arena to explore the superinflation phase, we consider the EU model developed in Ref. [71,72]. This EU model is based on general relativity and considered a closed universe dominated by a scalar field minimally coupled to gravity. We found that the superinflationary period in the EU scenario produces a suppression of the CMB anisotropies at large scale which could be responsible of the observed lack of power at large angular scales of the CMB [11,12,14].

The paper is organized as follows. In Sec. II we present the principal characteristics of the EU scenario and the superinflationary regimen. In Sec. III we calculate the primordial perturbation generated during the superinflationary phase in the context of the EU scenario. In Sec. IV we estimate the effective power spectrum generated by the EU model which takes into account the early period of superinflation and the subsequent period of standard slow roll. In Sec. V we compute the theoretical CMB power spectrum of this model by using CLASS code [93,94]. In Sec. VI we summarize our results.

II. THE EMERGENT UNIVERSE SCENARIO

In the emergent universe scenario, the Universe is initially in a past-eternal classical Einstein static state which eventually evolves into a subsequent inflationary phase; see [71–78]. During the past-eternal static regime, it is assumed that the scalar field is rolling on the asymptotically flat part of the scalar potential with a constant velocity, providing the conditions for a static universe. But once the scalar field exceeds some value, the scalar potential slowly drops from its original value. The overall effect of this is to distort the equilibrium behavior breaking the static solution. If the potential has a suitable form in this region, slow-roll inflation will occur.

In the EU scenario, the evolution of the scale factor could be modeled by the following expression (see [71,72]):

$$a(t) \simeq a_0 + Ae^{H_0 t}, \tag{1}$$

where a_0, A, H_0 are positive constants. This universe is past asymptotic to an Einstein static state, since $a(t) \to a_0$ as $t \to -\infty$. Thus, a_0 is identified with the radius of the Einstein static universe. At late times, on the other hand, $a(t) \to Ae^{H_0 t}$ and the model approache a de Sitter expansion phase.

For example, in Ref. [72], the scalar potential has been reconstructed from the evolution Eq. (1), where we can note that this reconstructed potential exhibits the same shape of the effective potential Fig. 1.

We can note that a generic characteristic of the EU scenario is the existence of a superinflation phase where $\dot{H} > 0$ before slow-roll inflation. In this model the evolution

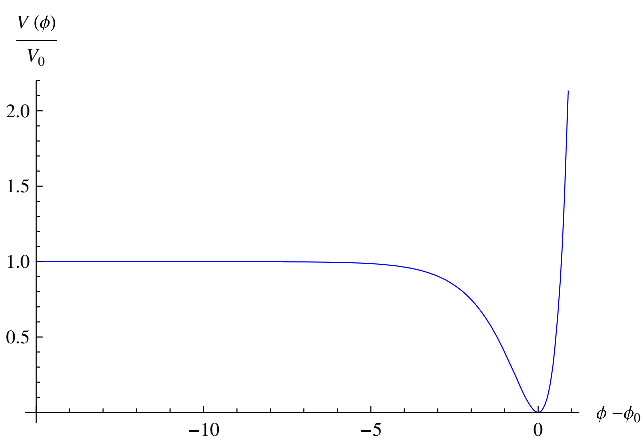

FIG. 1 (color online). Schematic representation of a potential for a standard emergent universe scenario.

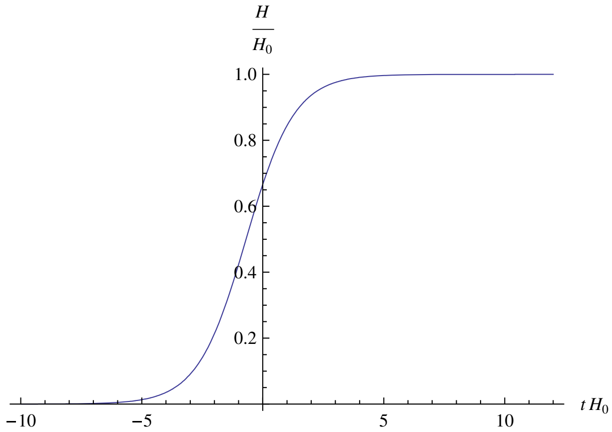

FIG. 2 (color online). Evolution of H as a function of the cosmological time.

described in Eq. (1) corresponds precisely to the superinflationary phase of the evolution of the EU, which asymptotically approaches the de Sitter expansion phase.

Figure 2 shows a generic evolution of H as a function of the cosmological time obtained from Eq. (1). We can note that H increases with time from zero (the static regimen) to the constant value H_0 (the onset of slow-roll inflation).

The prediction of a superinflating phase before the onset of slow-roll inflation arises in any EU scenario since the behavior of H as a function of the cosmological time depicted in Fig. 2 is general for all the EU models.

As was recently mentioned in the context of bouncing universes [44] and early in Ref. [45], a superinflation period could modify the spectral tilt at low l by making it blue-tilt before the conventional slow-roll inflation in which a decreasing H yields a red-tilted spectrum. This possibility will be studied in the next section for the EU scenario.

The mechanism which generates this superinflationary period depends on the particular EU model under consideration, but it is a generic characteristic of the EU scenario. For example, in the models in [71,72], it is considered a closed universe. Then, in this case, the spatial curvature is responsible for the superinflationary period.

III. THE PRIMORDIAL PERTURBATION IN EU

The scalar perturbations to the FRW geometry, in the longitudinal gauge, can be written as follows:

$$ds^2 = (1+2\Phi)dt^2 - (1-2\Phi)a(t)^2 d\vec{x}^2, \qquad (2)$$

where Φ is the Newtonian gravitational potential.

Normally in the emergent universe scenario, the Universe is positively curved; see for example [71,72]. In this first approach to the problem, we have neglected the contributions of the space curvature to the primordial perturbation, but we have included a first approach to this point in Appendix A; see also [95].

The equation for the perturbations in momentum space is given by

$$v_k'' + k^2 v_k - \frac{z''}{z} v_k = 0, \qquad (3)$$

where we have used the Mukhanov variable [6,96],

$$v_k = a\left(\delta\phi_k + \frac{\phi'}{h}\Phi_k\right), \qquad (4)$$

where $\delta\phi_k$ are the perturbations in the inflaton field and $'$ is derivative with respect to the conformal time $\eta = \int dt/a$ and we have used units $M_p = (8\pi G)^{-1/2} = 1$. Also, we have defined

$$z = \frac{a\phi'}{h}, \qquad (5)$$

$$h = \frac{a'}{a}. \qquad (6)$$

In order to solve Eq. (3), we consider the evolution of the scale factor $a(t)$ given in Eq. (1) written in the conformal time,

$$a(\eta) = \frac{a_0}{1 - e^{a_0 H_0 \eta}}, \qquad \eta < 0. \qquad (7)$$

From Eq. (3) and using Eq. (7), we obtain

$$v_k'' - (a_0 H_0)^2 e^{a_0 H_0 \eta}\left(\frac{(1 + e^{a_0 H_0 \eta})}{(-1 + e^{a_0 H_0 \eta})^2}\right) v_k + k^2 v_k = 0, \qquad (8)$$

where we have considered $z''/z \approx a''/a$. This equation is solved by

$$\begin{aligned} v_k(\eta) = {}& \frac{1}{\sqrt{2k}}\left[\frac{e^{-ik\eta}}{1 - e^{a_0 H_0 \eta}}\right] \\ & \times {}_2F_1\left(-1 - \frac{ik}{a_0 H_0} - \sqrt{1 - \left(\frac{k}{a_0 H_0}\right)^2},\right. \\ & -1 - \frac{ik}{a_0 H_0} + \sqrt{1 - \left(\frac{k}{a_0 H_0}\right)^2}; \\ & \left. 1 - 2\frac{ik}{a_0 H_0}; e^{a_0 H_0 \eta}\right), \end{aligned} \qquad (9)$$

where $2F_1$ is the hypergeometric function.

In the solution (9) we have considered (and appropriately normalized) the solution of Eq. (8) such that in the short wavelength limit, the normalized positive frequency modes correspond to the minimal quantum fluctuations,

$$v_k \approx \frac{e^{-ik\eta}}{\sqrt{2k}}, \qquad aH \ll k. \tag{10}$$

Following [34], we consider the spectrum of $Q \equiv v/a$ which becomes constant at late time,

$$P_Q = \frac{k^3}{2\pi^2}|Q|^2 \longrightarrow \frac{H_0^2}{\pi^2}\frac{\chi^2\Gamma[x_1]\Gamma[x_1^*]}{\Gamma[x_2]\Gamma[x_2^*]\Gamma[x_3]\Gamma[x_3^*]}, \tag{11}$$

where we have defined

$$x_1 = 1 - 2i\chi \tag{12}$$

$$x_2 = 2 - i\chi - \sqrt{1-\chi^2} \tag{13}$$

$$x_3 = 2 - i\chi + \sqrt{1-\chi^2} \tag{14}$$

$$\chi = \frac{k}{a_0 H_0}. \tag{15}$$

We can note that in the short wavelength limit($k \gg H_0$), we recovered the standard result of a nearly scale-invariant spectrum,

$$P_Q \to \left(\frac{H_0}{2\pi}\right)^2. \tag{16}$$

At superhorizon scales, the two modes Q_k and Φ_k are related by a k-independent rescaling so that the spectrum given by Eq. (11) directly translates into the spectrum of Φ. In Fig. (3) we have plotted the spectrum of P_Q as a function of χ obtained from the analytical calculation Eq. (11), solid line. We can note that there is a suppression of the long wave modes as we expected given the superinflationary regimen.

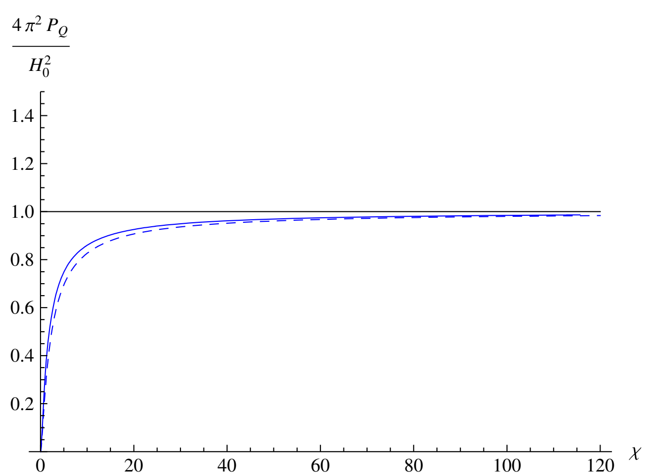

FIG. 3 (color online). Power spectrum for Q. The analytical computation Eq. (11), solid line. The approximate spectrum, dashed line Eq. (19).

IV. THE EFFECTIVE POWER SPECTRUM

In the last section we study the spectrum generated during the superinflationary regimen. From this result and by following Ref. [41], we can estimate an effective power spectrum generated by the EU model which takes into account the early period of superinflation and the subsequent period of standard slow roll. In this respect, from the result of previous section, we can note that the scale invariance of the spectrum is the result of inflationary evolution after the superinflationary stage. Then, if the inflationary period is generated by the standard slow-roll conditions, we can expect the usual slight red-tilted spectrum during this period. From Eq. (11) and by following Ref. [41], we could model this situation by considering the following scalar spectrum:

$$\mathcal{P}_\Phi = Ak^{n_s-1}\frac{\chi^2\Gamma[x_1]\Gamma[x_1^*]}{\Gamma[x_2]\Gamma[x_2^*]\Gamma[x_3]\Gamma[x_3^*]}. \tag{17}$$

For small k, this spectrum reproduces the behavior of spectrum Eq. (11), i.e., suppression of the long wave modes. On the other hand, when $k \gtrsim H_0 a_0$, the spectrum reproduces the usual slightly red spectrum of slow-roll inflation.

Another possibility is to consider a cutoff in the spectrum. This cutoff is related to the transition from the superinflationary regimen to slow-roll inflation which occurs when the mode $k_{\max}$ exits the horizon, after which we would get the usual red-tilted power law spectrum. In this case, we can write

$$\mathcal{P}_\Phi = \begin{cases} \bar{A}\frac{\chi^2\Gamma[x_1]\Gamma[x_1^*]}{\Gamma[x_2]\Gamma[x_2^*]\Gamma[x_3]\Gamma[x_3^*]}, & k < k_{\max} \\ A(\frac{k}{k_{\max}})^{n_s-1}, & k > k_{\max} \end{cases}. \tag{18}$$

The constants A and $\bar{A}$ are chosen in order to match the superinflationary power spectrum Eq. (11) with the slow-roll inflationary power spectrum at $k = k_{\max}$.

V. CMB ANISOTROPIES

We compute the theoretical CMB power spectrum of this model by using CLASS code [93,94]. In this first approximation, in the context of the EU scenario, for the study of the consequences of the superinflationary phase to the CMB power spectrum, we have simplified the code for computing the C_l by considering an approximation of the spectrums Eqs. (17), and (18). In particular, we approximate the spectrum of the superinflation phase Eq. (11) with the following expression (see for example [44]),

$$P_Q \sim \frac{H_0^2}{4\pi^2}\frac{\chi^2}{(1+\chi)^2}, \tag{19}$$

which is obtained by considering the evolution of the scale factor given by Eq. (1) and the Hubble crossing condition,

$$k = aH \Rightarrow k = H_0 A e^{H_0 t}. \qquad (20)$$

In Fig. 3 we have plotted the spectrum Eq. (19), where we can compare it with the analytic calculation Eq. (11). We can note that the approximated spectrum reproduces very well the spectrum obtained by the analytic calculation.

In order to show the suppression at large scales coming from the EU scenario, we have plotted in Fig. 4 the temperature power spectrum obtained from the pure power law (dashed line), from the EU scenario (solid line), and from the EU scenario with a cutoff (dotted line). The points show the Planck data. We can note that compared to the standard power-law model, the C_l spectrum in the emergent universe scenario is suppressed at large scales.

In these examples, we have considered the following values for the parameters in the case of the EU scenario in Eqs. (17) and (19): $A = 2.07 \times 10^{-9}$, $n_s = 0.9603$, and $a_0 H_0 = 0.0002\ \mathrm{Mpc}^{-1}$. In the case of the EU scenario with the cutoff Eqs. (18) and (19), we consider $A = 2.42 \times 10^{-9}$, $n_s = 0.967$, $a_0 H_0 = 0.0003\ \mathrm{Mpc}^{-1}$, and $k_{\max} = 0.0015\ \mathrm{Mpc}^{-1}$. At this moment we are not doing a best-fit calculation of theses parameters, just showing two particular cases and how they produce a suppression of the spectrum at large scales.

We have to mention that there is a tuning between the turn-around from superinflation to slow-roll inflation and when the comoving scale corresponding to the current horizon crosses the Hubble radius during inflation. This is typical for trying to account for the suppression of the low multipoles due to preinflationary dynamics; see for example [34,38,39,97–99]. In particular, in this work we consider that this scale, which is determined by the combination $(a_0 H_0)$, is a free parameter. However, there is the possibility of linking this scale with the stability conditions of the ES solution. In this case, we could obtain bounds on this scale which could be related with the current horizon scale, similar to the case where stability conditions of the ES solution impose an upper bound on the cosmological constant [91]. We expect to return to this point in future work.

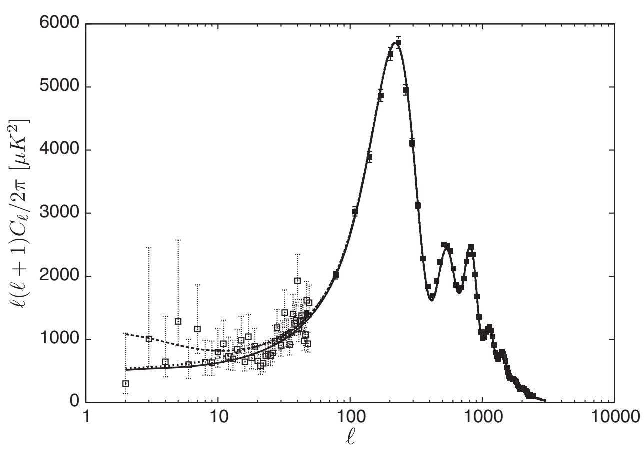

FIG. 4. Temperature power spectrum for the pure power law (dashed line), EU scenario (solid line), and EU scenario with a cutoff (points line). The points show the Planck data.

VI. CONCLUSIONS

In recent cosmological observations, there are intriguing results on the cosmic microwave background radiation suggesting a lack of power at large angular scales. This situation is interesting because it may give us clues towards physics at the very early Universe, perhaps before slow-roll inflation. In this context, the EU models are an interesting arena to explore preinflationary physics and its possible implications for the CMB anomalies.

In this paper we study the primordial perturbations in the context of the EU scenario. In particular, we focus on the primordial perturbations generated during the superinflationary phase. We find that the superinflationary period in the EU scenario produces a suppression of the primordial perturbations at large scale which could be responsible for the observed lack of power at large angular scales of the CMB.

In particular, in this work we considered the EU model developed in Ref. [71,72] as an arena in which to explore the consequences of the superinflationary regimen. We calculated the primordial perturbations generated during the superinflationary phase and the effective power spectrum generated by the EU model, which take into account the early period of superinflation and the subsequent period of standard slow-roll inflation. By using the CLASS code, we compute the theoretical CMB power spectrum generated by this model and compare it with the standard results of slow-roll inflation and the Planck data.

In this first approach to the problem, we have neglected the contributions of the space curvature to the primordial perturbation (most of the EU models consider closed universes), but we have included a first approach to this point in Appendix A.

In this model, the scale of transition from superinflation to slow-roll inflations is determined by the combination $(a_0 H_0)$, which is considered a free parameter. Preliminary results show that the global behavior (suppression of the CMB anisotropies at large scale and agreement with Planck data at large l) is not particulary sensitive to the election of this combination once the scale $a_0 H_0$ is smaller than a typical scale of order $\approx 0.002\ \mathrm{Mpc}^{-1}$. However, there is the possibility of linking this scale with the stability conditions of the ES solution. In this first approach to the problem, we have not considered a best-fit calculation of the parameters of the model. We expect to return to these points in the near future by including a best-fit calculation and by considering bounds on the free parameters of the model that come from stability conditions of the ES solution; see [91].

In summary, in this work we show that the superinflation phase, which is a characteristic shared by all EU models, could be responsible for part of the anomaly in the low multipoles of the CMB, in particular, for the observed lack of power at large angular scales.

ACKNOWLEDGMENTS

P. L. has been partially supported by FONDECYT grant No. 11090410, Mecesup UBB0704, and Universidad del Bío-Bío through Grants No. GI121407/VBC and No. 141407 3/R. We are grateful to I. Duran for collaboration with the CLASS code. We thank J. Garriga and D. Espriu for useful discussion and A. Cid for reading and providing comments on the manuscript.

APPENDIX

In this appendix we study the period of superinflation in the specific EU model developed in Ref. [71,72]. This EU model is based on general relativity and considered a closed universe dominated by a scalar field minimally coupled to gravity.

The energy density, ρ, and the pressure, P, are expressed by the following equations,

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{A1}$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{A2}$$

where $V(\phi)$ is the scalar potential show in Fig. 1. The Friedmann and the Raychaudhuri field equations become

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) - \frac{1}{a^2}, \tag{A3}$$

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V(\phi)}{\partial \phi}. \tag{A4}$$

The other nonindependent equation is

$$\dot{H} = -4\pi G\dot{\phi}^2 + \frac{1}{a^2}. \tag{A5}$$

During the past-eternal static regime, it is assumed that the scalar field is rolling on the asymptotically flat part of the scalar potential with a constant velocity, providing the conditions for a static universe. But once the scalar field exceeds some value, the scalar potential slowly droops from its original value. The overall effect of this is to distort the equilibrium behavior breaking the static solution. If the potential has a suitable form in this region, superinflation and slow-roll inflation will occur. In this case, the evolution of the scale factor could be modeled by Eq. (1); see Ref. [72] and the discussion in Sec. II.

From Eq. (A5) we can note that, in this model, the mechanism which generated the superinflationary period is the effect of the curvature of the closed universe.

The scalar perturbations to the closed FRW geometry, in the longitudinal gauge, can be written as follows,

$$ds^2 = (1+2\Phi)dt^2 - (1-2\Phi)a(t)^2\left[\frac{dr^2}{1-r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{A6}$$

where Φ is the Newtonian gravitational potential.

The equation for the perturbations in momentum space is similar to the one discussed in Sec. III and it is given by

$$v_k'' + k^2 v_k - \frac{z''}{z}v_k = 0, \tag{A7}$$

where we have considered that the scalar potential is nearly constant, and we have neglected terms proportional to $dV/d\phi$. Also, we have used the Mukhanov variable [6,96]

$$v_k = a\left(\delta\phi_k + \frac{\phi'}{h}\Phi_k\right), \tag{A8}$$

where $\delta\phi_k$ is the perturbation in the inflaton field and $'$ is derivative with respect to the conformal time $\eta = \int dt/a$, and we have used units $M_p = (8\pi G)^{-1/2} = 1$. Also, we have defined

$$z = \frac{a\phi'}{h}\left(1 + \frac{3}{\Delta}\right)^{1/2}, \tag{A9}$$

$$h = \frac{a'}{a}. \tag{A10}$$

Here, the Laplacian should be understood as a c number, representing the corresponding eigenvalue [96].

In order to solve Eq. (A7), we consider the evolution of the scale factor $a(t)$ given in Eq. (1) written in the conformal time Eq. (7). Then, from Eq. (A7) and by using Eq. (7), we obtain the following equation,

$$v_k'' - (a_0H_0)^2 e^{a_0H_0\eta}\left(\frac{(1+e^{a_0H_0\eta})}{(-1+e^{a_0H_0\eta})^2}\right)v_k + k^2 v_k = 0, \tag{A11}$$

where we have considered the eigenvalues of the Laplacian operator for a closed space (see [100]):

$$\Delta v_k = -k^2 v_k \tag{A12}$$

$$= -(\beta^2 - 1)v_k. \tag{A13}$$

In this case $\beta = 3, 4, 5, \ldots$. The modes $\beta = 1, 2$ are pure gauge modes and are not included in the spectrum [100]. Also, we have considered $z''/z \approx a''/a$.

Equations (A11) are solved by Eq. (9), where k is now defined in Eqs. (A12) and (A13). From this solution and by following a similar procedure as in Sec. III, we found the spectrum of primordial perturbations. We can note that the qualitative behavior of these primordial perturbations is similar to that discussed in Sec. III: a flat spectrum for short wavelengths and a suppression of the long wave modes, as we expect given the superinflation regimen.

[1] A. Guth, Phys. Rev. D **23**, 347 (1981).
[2] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
[3] A. D. Linde, Phys. Lett. **108B**, 389 (1982).
[4] A. D. Linde, Phys. Lett. **129B**, 177 (1983).
[5] A. D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, Academic, Chur, Switzerland, 1990).
[6] V. F. Mukhanov and G. Chibisov, JETP Lett. **33**, 532 (1981); Sov. Phys. JETP **56**, 258 (1982); V. F. Mukhanov, JETP Lett. **41**, 493 (1985); V. F. Mukhanov, H. Feldman, and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992).
[7] S. Hawking, Phys. Lett. **115B**, 295 (1982).
[8] A, H. Guth and S. Pi, Phys. Rev. Lett. **49**, 1110 (1982).
[9] A. A. Starobinsky, Phys. Lett. **117B**, 175 (1982).
[10] J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, Phys. Rev. D **28**, 679 (1983).
[11] E. Komatsu *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **192**, 18 (2011); D. Larson, J. Dunkley, G. Hinshaw, E. Komatsu, M. R. Nolta, C. L. Bennett, B. Gold, M. Halpern *et al.*, Astrophys. J. Suppl. Ser. **192**, 16 (2011); N. Jarosik *et al.*, Astrophys. J. Suppl. Ser. **192**, 14 (2011); E. Komatsu *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **180**, 330 (2009); D. N. Spergel *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **170**, 377 (2007).
[12] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **571**, A1 (2014).
[13] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **571**, A22 (2014).
[14] G. F. Smoot *et al.*, Astrophys. J. **396**, L1 (1992).
[15] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **571**, A15 (2014).
[16] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **571**, A23 (2014).
[17] M. Bastero-Gil, K. Freese, and L. Mersini-Houghton, Phys. Rev. D **68**, 123514 (2003).
[18] I. Y. Sokolov, JETP Lett. **57**, 617 (1993).
[19] L. Z. Fang, Mod. Phys. Lett. A **08**, 2615 (1993).
[20] D. Stevens, D. Scott, and J. Silk, Phys. Rev. Lett. **71**, 20 (1993).
[21] A. de Oliveira-Costa and G. F. Smoot, Astrophys. J. **448**, 477 (1995).
[22] G. Rocha, L. Cayón, R. Bowen, A. Canavezes, J. Silk, A. J. Banday, and K. M. Górski, Mon. Not. R. Astron. Soc. **351**, 769 (2004).
[23] J. Levin, arXiv:astro-ph/0108043 (2001).
[24] A. de Oliveira-Costa, G. F. Smoot, and A. A. Starobinski, Astrophys. J. **468**, 457 (1996).
[25] J. P. Uzan, A. Riazuelo, R. Lehoucq, and J. Weeks, Phys. Rev. D **69**, 043003 (2004).
[26] G. Efstathiou, arXiv:astro-ph/0303623 (2003).
[27] L. Campanelli, P. Cea, and L. Tedesco, Phys. Rev. Lett. **97**, 131302 (2006); **97**, 209903(E) (2006).
[28] L. Campanelli, P. Cea, and L. Tedesco, Phys. Rev. D **76**, 063007 (2007).
[29] L. Campanelli, Phys. Rev. D **80**, 063006 (2009).
[30] A. de Oliveira-Costa, M. Tegmark, M. Zaldarriaga, and A. Hamilton, Phys. Rev. D **69**, 063516 (2004).
[31] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **571**, A26 (2014).
[32] S. L. Bridle, A. M. Lewis, J. Weller, and G. Efstathiou, Mon. Not. R. Astron. Soc. **342**, L72 (2003).
[33] J. P. Uzan *et al.*, arXiv:astro-ph/0302597.
[34] C. R. Contaldi, M. Peloso, L. Kofman, and A. D. Linde, J. Cosmol. Astropart. Phys. 07 (2003) 002.
[35] J. M. Cline, P. Crotty, and J. Lesgourgues, J. Cosmol. Astropart. Phys. 09 (2003) 010.
[36] B. Feng and X. Zhang, Phys. Lett. B **570**, 145 (2003).
[37] M. Bastero-Gil, K. Freese, and L. Mersini-Houghton, Phys. Rev. D **68**, 123514 (2003).
[38] E. Ramirez and D. J. Schwarz, Phys. Rev. D **85**, 103516 (2012).
[39] E. Dudas, N. Kitazawa, S. P. Patil, and A. Sagnotti, J. Cosmol. Astropart. Phys. 05 (2012) 012.
[40] Y.-S. Piao, B. Feng, and X.-m. Zhang, Phys. Rev. D **69**, 103520 (2004).
[41] Z.-G. Liu, Z.-K. Guo, and Y.-S. Piao, Phys. Rev. D **88**, 063539 (2013).
[42] A. Shafieloo and T. Souradeep, Phys. Rev. D **70**, 043523 (2004).
[43] D. K. Hazra, A. Shafieloo, and T. Souradeep, arXiv:1303.5336.
[44] T. Biswas and A. Mazumdar, Classical Quantum Gravity **31**, 025019 (2014).
[45] D.-H. Zhang and C.-Y. Sun, Chin. Phys. Lett. **21**, 1865 (2004).
[46] Z.-G. Liu, Z.-K. Guo, and Y.-S. Piao, Eur. Phys. J. C **74**, 3006 (2014).
[47] A. Borde and A. Vilenkin, Phys. Rev. Lett. **72**, 3305 (1994).
[48] A. Borde and A. Vilenkin, Phys. Rev. D **56**, 717 (1997).
[49] A. H. Guth, Ann. N.Y. Acad. Sci. **950**, 66 (2001).
[50] A. Borde, A. H. Guth, and A. Vilenkin, Phys. Rev. Lett. **90**, 151301 (2003).
[51] A. Vilenkin, arXiv:gr-qc/0204061.

[52] Y.-F. Cai, T. Qiu, Y.-S. Piao, M. Li, and X. Zhang, J. High Energy Phys. 10 (2007) 071.
[53] T. Biswas, A. Mazumdar, and W. Siegel, J. Cosmol. Astropart. Phys. 03 (2006) 009.
[54] P. Peter and N. Pinto-Neto, Phys. Rev. D **66**, 063509 (2002).
[55] M. Novello and S. E. P. Bergliaffa, Phys. Rep. **463**, 127 (2008).
[56] J. Khoury, B. A. Ovrut, P. J. Steinhardt, and N. Turok, Phys. Rev. D **64**, 123522 (2001).
[57] M. Gasperini and G. Veneziano, Phys. Rep. **373**, 1 (2003).
[58] Y. Shtanov and V. Sahni, Phys. Lett. B **557**, 1 (2003).
[59] R. Kallosh, L. Kofman, and A. D. Linde, Phys. Rev. D **64**, 123523 (2001).
[60] M. Lilley, L. Lorenz, and S. Clesse, J. Cosmol. Astropart. Phys. 06 (2011) 004.
[61] P. Singh, K. Vandersloot, and G. V. Vereshchagin, Phys. Rev. D **74**, 043510 (2006).
[62] T. J. Battefeld, S. P. Patil, and R. Brandenberger, Phys. Rev. D **70**, 066006 (2004).
[63] M. Gasperini, M. Giovannini, and G. Veneziano, Phys. Lett. B **569**, 113 (2003).
[64] J.-c. Hwang and H. Noh, Phys. Rev. D **65**, 124010 (2002).
[65] R. Brandenberger, Phys. Rev. D **80**, 043516 (2009).
[66] Y.-F. Cai, T. Qiu, Y.-S. Piao, M. Li, and X. Zhang, J. High Energy Phys. 10 (2007) 071.
[67] P. Creminelli and L. Senatore, J. Cosmol. Astropart. Phys. 11 (2007) 010.
[68] M. Bojowald, R. Maartens, and P. Singh, Phys. Rev. D **70**, 083517 (2004).
[69] Y. Shtanov and V. Sahni, Phys. Lett. B **557**, 1 (2003).
[70] P. Peter and N. Pinto-Neto, Phys. Rev. D **66**, 063509 (2002).
[71] G. F. R. Ellis and R. Maartens, Classical Quantum Gravity **21**, 223 (2004).
[72] G. F. R. Ellis, J. Murugan, and C. G. Tsagas, Classical Quantum Gravity **21**, 233 (2004).
[73] D. J. Mulryne, R. Tavakol, J. E. Lidsey, and G. F. R. Ellis, Phys. Rev. D **71**, 123512 (2005).
[74] S. Mukherjee, B. C. Paul, S. D. Maharaj, and A. Beesham, arXiv:gr-qc/0505103.
[75] S. Mukherjee, B. C. Paul, N. K. Dadhich, S. D. Maharaj, and A. Beesham, Classical Quantum Gravity **23**, 6927 (2006).
[76] A. Banerjee, T. Bandyopadhyay, and S. Chakraborty, Gravitation Cosmol. **13**, 290 (2007).
[77] N. J. Nunes, Phys. Rev. D **72**, 103510 (2005).
[78] J. E. Lidsey and D. J. Mulryne, Phys. Rev. D **73**, 083508 (2006).
[79] A. Beesham, S. V. Chervon, and S. D. Maharaj, Classical Quantum Gravity **26**, 075017 (2009).
[80] B. C. Paul and S. Ghose, arXiv:0809.4131.
[81] U. Debnath, Classical Quantum Gravity **25**, 205019 (2008).
[82] A. Banerjee, T. Bandyopadhyay, and S. Chakraborty, Gen. Relativ. Gravit. **40**, 1603 (2008).
[83] S. del Campo, R. Herrera, and P. Labraña, J. Cosmol. Astropart. Phys. 11 (2007) 030.
[84] S. del Campo, R. Herrera, and P. Labrana, J. Cosmol. Astropart. Phys. 07 (2009) 006.
[85] S. del Campo, E. I. Guendelman, A. B. Kaganovich, R. Herrera, and P. Labrana, Phys. Lett. B **699**, 211 (2011).
[86] S. del Campo, E. I. Guendelman, R. Herrera, and P. Labrana, J. Cosmol. Astropart. Phys. 06 (2010) 026.
[87] P. Labrana, Phys. Rev. D **86**, 083524 (2012).
[88] Y.-F. Cai, M. Li, and X. Zhang, Phys. Lett. B **718**, 248 (2012).
[89] P. Rudra, Mod. Phys. Lett. A **27**, 1250189 (2012).
[90] Z.-G. Liu and Y.-S. Piao, Phys. Lett. B **718**, 734 (2013).
[91] E. Guendelman and P. Labrana, Int. J. Mod. Phys. D **22**, 1330018 (2013).
[92] Y.-F. Cai, Y. Wan, and X. Zhang, Phys. Lett. B **731**, 217 (2014).
[93] J. Lesgourgues, arXiv:1104.2932.
[94] D. Blas, J. Lesgourgues, and T. Tram, J. Cosmol. Astropart. Phys. 07 (2011) 034.
[95] P. Labrana, A. Cid, and C. Rios (to be published).
[96] J. Garriga and V. F. Mukhanov, Phys. Lett. B **458**, 219 (1999).
[97] M. Cicoli, S. Downes, B. Dutta, F. G. Pedro, and A. Westphal, J. Cosmol. Astropart. Phys. 12 (2014) 030.
[98] D. J. Schwarz and E. Ramirez, arXiv:0912.4348.
[99] E. Ramirez, Phys. Rev. D **85**, 103517 (2012).
[100] L. F. Abbott and R. K. Schaefer, Astrophys. J. **308**, 546 (1986).