

Unbiased constraints on the clumpiness of the Universe from standard candles

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We perform unbiased tests for the clumpiness of the Universe by confronting the Zel'dovich-Kantowski-Dyer-Roeder luminosity distance, which describes the effect of local inhomogeneities on the propagation of light with the observational one estimated from measurements of standard candles, i.e., type Ia supernovae (SNe Ia) and gamma-ray bursts (GRBs). Methodologically, we first determine the light-curve fitting parameters which account for distance estimation in SNe Ia observations and the luminosity/energy relations which are responsible for distance estimation of GRBs in the global fit to reconstruct the Hubble diagrams in the context of a clumpy Universe. Subsequently, these Hubble diagrams allow us to achieve unbiased constraints on the matter density parameter Ω_m , as well as the clumpiness parameter η which quantifies the fraction of homogeneously distributed matter within a given light cone. At a 1σ confidence level, the constraints are $\Omega_m = 0.34 \pm 0.02$ and $\eta = 1.00^{+0.00}_{-0.02}$ from the joint analysis. The results suggest that the Universe full of Friedman-Lemaître-Robertson-Walker fluid is favored by observations of standard candles with very high statistical significance. On the other hand, they may also indicate that the Zel'dovich-Kantowski-Dyer-Roeder approximation is a sufficiently accurate form to describe the effects of local homogeneity on the expanding Universe.

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I. INTRODUCTION

The standard physical model of cosmology is based on the solution of general relativity describing a spatially homogeneous and isotropic spacetime, known as the Friedmann-Lemaître-Robertson-Walker (FLRW) solution. It is assumed that the geometry of our Universe is smooth on large scales. One of the major tasks in modern cosmology is to precisely determine the parameters which characterize the postulated model by fitting the observational data. The cornerstone of observational evidence that supports the FLRW model is the existence of highly isotropic cosmic microwave background radiation (CMBR). It could be inferred that the spacetime should be exactly FLRW when the background radiation appears to be exactly isotropic to a given family of observers [1]. Therefore, we can prove the Universe to be FLRW just from our own observations of the CMBR by taking the Copernican principle into consideration. Moreover, this result could be extended to the case of an almost isotropic background radiation, which hints at an almost FLRW spacetime [2]. Although this simple solution of Einstein field equation provides an excellent description for the Universe on large scales, it also makes clear that we need to understand the departures from a spatially homogeneous model when interpreting observational data. Indeed, departures from perfect homogeneity change the distance-redshift relation. However, in practice, cosmological observations

are usually fitted just using relationships derived from homogeneous models.

The fact that matter is not continuously distributed can imprint most cosmological observations probing quantities related to light propagation (as discussed in detail in Ref. [3]), in particular regarding the propagation of light with narrow beams, such as the redshift, the angular diameter distance, the luminosity distance, and the image distortion. The importance of quantifying the effects of inhomogeneities on light propagation was first pointed out by Zel'dovich [4] and Kantowski [5]. They designed an “empty beam” approximation by arguing that photons should mostly propagate in vacuum. Later, this was generalized by Dyer and Roeder as the “partially filled beam” approach [6,7]. More generally, the early work of Ref. [4] stimulated many studies on this issue [8–20]. In this framework, the proportion of clumped matter with respect to the homogeneous fluid is characterized by the clumpiness or smoothness parameter. In addition, they arrived at an equation for the angular diameter distance which, via the Etherington relation, connects to the observable luminosity distance. We refer to it here as the Zel'dovich-Kantowski-Dyer-Roeder (ZKDR) luminosity distance.

Since the 1960s, a rich literature has formed which concerns the ZKDR approach and its cosmological implications. Phenomenologies and investigations involving many different physical aspects were performed, such as analytical or approximate expressions [21–23], critical redshift for the angular diameter distance [24], gravitational lensing [25,26], and accelerated expanding Universe

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models driven by particle creation [27]. Recently, some quantitative analysis from such compact radio sources as standard rulers [28,29], and such type Ia supernovae (SNe Ia) or gamma-ray bursts (GRBs) as standard candles [30–35] were also performed. To be specific, in Ref. [30], constraints on the dark energy and smoothness parameter from the so-called gold SN Ia sample released by the High- z Supernova team [36] and the first year results of the Supernova Legacy Survey (SNLS), which is a planned five-year project [37], were examined. The results suggested that SNe Ia data alone was incapable of constraining the smoothness parameter although the gold SN Ia provided a little more stringent constraint since this sample extended to appreciably higher redshifts. Later, Busti *et al.* [32] performed an updated investigation where the statistical analysis was based on the 557 SNe Ia Union2 compilation data [38] and 59 Hymnium GRBs [39], and almost the same conclusion was achieved. More recently, this issue was also studied by using Union2.1 SN Ia [40] plus nine long GRBs in $1.55 \leq z \leq 3.57$ [41] and the constrained value of the smoothness parameter indicated a clumped Universe [33]. On the other hand, as concluded in their work, this result may be an indication that the ZKDR approximation is not a precise form of describing the effects of clumpiness in the expanding Universe.

However, in these previous analysis, all distances of SNe Ia and GRBs applied to test the inhomogeneity of the Universe were derived from a global fit in the context of standard dark energy scenarios where the clumpiness has vanished, i.e., the flat Λ cold dark matter (Λ CDM) or w CDM model. That is, the light-curve fitting parameters accounting for the distance estimation in SNe Ia observations (e.g., α and β in the most widely used SALT2 training method [42]) are left as free parameters (on the same weight as cosmological parameters) and are determined by fitting the distances of SNe Ia, which is a linear combination of light-curve fitting parameters and observed quantities, to the model-predicted ones in the context of the standard Λ CDM or w CDM scenario. Therefore, HDs constructed in this way are somewhat model dependent. Moreover, cosmological implications on nonstandard dark energy scenarios or a Universe with homogeneity taken into consideration derived from these HDs are model biased [43]. It has been shown that this kind of bias cannot be neglected and may be significant in the era of precision cosmology [44,45]. Certainly, this kind of bias also hides in the GRB cosmology where luminosity relations being responsible for distance estimation of GRB are calibrated with the model-dependent HDs of low-redshift SNe Ia [46,47].

In this paper, we first reconstruct Hubble diagrams for the latest SNe Ia and for long GRB observations by calibrating the light-curve fitting parameters and luminosity relations, respectively, in the context of an inhomogeneous Universe with the cosmological constant. These Hubble

diagrams can lead to unbiased tests for the matter density parameter Ω_m as well as the clumpiness parameter η . For the joint light-curve analysis of the SDSS-II and the SNLS (JLA SN Ia) in the range of $0.01 \leq z \leq 1.23$ [48], the constraints are $\Omega_m = 0.29^{+0.07}_{-0.05}$ and $\eta = 0.76^{+0.24}_{-0.65}$, slightly indicating a clumped Universe. For the long GRBs in the range of $1.48 \leq z \leq 8.20$ [49], the best fits are $\Omega_m = 0.42 \pm 0.06$ and $\eta = 1.00^{+0.00}_{-0.12}$, strongly supporting a homogeneous Universe. For the combination of these two probes, the constraints are $\Omega_m = 0.34 \pm 0.02$ and $\eta = 1.00^{+0.00}_{-0.02}$, also favoring a Universe full of FLRW fluid with a very high confidence level. We suggest that the matter density parameter Ω_m is mainly determined by the SNe Ia observations while the clumpiness parameter η is primarily constrained from the observed GRB events. Moreover, it is also shown that larger scales are explored; the test more strongly implies a homogeneous Universe. These reasonable results may be an indication that the ZKDR approximation remains to be a precise description for the luminosity distance-redshift relation in a locally inhomogeneous Universe with the cosmological constant.

II. THE ZKDR LUMINOSITY DISTANCE

For most cosmological models, angular or apparent size distance, which is proportional to the square root of the cross-sectional area $A(z)$, is related to the luminosity distance by $d_A(z) = d_L(z)/(1+z)^2$. In the model only including dark matter and dark energy, the luminosity distance $d_L(z)$, which accounts for a partially depleted mass density in the observing beam but neglects lensing by external masses, is obtained by integrating the second-order differential equation for $A(z)$ of an observing beam from the source at redshift z to the observer at $z = 0$ [21,50]:

$$(1+z)^2 E(z) \frac{d}{dz} \left[(1+z)^2 E(z) \frac{d}{dz} \sqrt{A(z)} \right] + \frac{3}{2} \eta \Omega_m (1+z)^5 \sqrt{A(z)} = 0, \quad (1)$$

where $E(z)$ is the reduced Hubble parameter at redshift z :

$$E(z) = \frac{H(z)}{H_0} = (1+z) \sqrt{1 + \Omega_m z + \Omega_\Lambda [(1+z)^{-2} - 1]}, \quad (2)$$

and the phenomenological parameter $\eta = 1 - \rho_{cl}/\rho$ is the so-called clumpiness or smoothness parameter, which quantifies the amount of matter in clumps relative to the amount of matter uniformly distributed. The required boundary conditions for Eq. (1) are

$$\sqrt{A}|_{z=0} = 0, \quad \left. \frac{d\sqrt{A}}{dz} \right|_{z=0} = -\sqrt{\delta\Omega} \frac{c}{H_0}, \quad (3)$$

where $\delta\Omega$ is the solid angle of the beam. By using an approximate change of variables

$$h(A, z) \equiv (1+z)\sqrt{\frac{A}{\delta\Omega}}, \quad (4)$$

$$\zeta(z) = \frac{\Omega_m}{1-\Omega_m}(1+z)^3 + 1, \quad (5)$$

Eq. (1) can be transformed into a hypergeometric equation,

$$(1-\zeta)\zeta\frac{d^2h}{d\zeta^2} + \left(\frac{1}{2} - \frac{7}{6}\zeta\right)\frac{dh}{d\zeta} + \frac{\nu(\nu+1)}{36} = 0. \quad (6)$$

The resulting luminosity distance is then given by

$$d_L(z) = (1+z)h(\zeta(0)). \quad (7)$$

Expressed in terms of hypergeometric functions, Eq. (7) becomes

$$\begin{aligned} d_L(z; \Omega_m, \nu) = & \frac{c}{H_0 \Omega_m^{1/3} (1+2\nu)} [1 + \Omega_m z(3+3z+z^2)]^{\nu/6} \left\{ {}_2F_1\left(-\frac{\nu}{6}, \frac{3-\nu}{6}; \frac{5-2\nu}{6}; \frac{1-\Omega_m}{1+\Omega_m z(3+3z+z^2)}\right) \right. \\ & \times {}_2F_1\left(\frac{1+\nu}{6}, \frac{4+\nu}{6}; \frac{7+2\nu}{6}; 1-\Omega_m\right) \\ & - [1 + \Omega_m z(3+3z+z^2)]^{-(1+2\nu)/6} {}_2F_1\left(-\frac{\nu}{6}, \frac{3-\nu}{6}; \frac{5-2\nu}{6}; 1-\Omega_m\right) \\ & \left. \times {}_2F_1\left(\frac{1+\nu}{6}, \frac{4+\nu}{6}; \frac{7+2\nu}{6}; \frac{1-\Omega_m}{1+\Omega_m z(3+3z+z^2)}\right) \right\}. \quad (8) \end{aligned}$$

The parameter ν presented in Eqs. (6) and (8) corresponds to the clumpiness parameter η by

$$\eta = \frac{1}{6}(3+\nu)(2-\nu). \quad (9)$$

The range for ν is $0 \leq \nu \leq 2$, where $\nu = 0$ ($\eta = 1$) is related to a FLRW fluid, while $\nu = 2$ ($\eta = 0$) to a totally clumped case.

Actually, the ZKDR approach has been criticized by several authors (e.g., a few detailed comments gathered in Ref. [33]). However, so far, confrontations of the ZKDR luminosity distance with observations have not led to conclusive results in the sense of totally excluding this model. Moreover, we should keep in mind that most previous tests in this field were somewhat dependent on the standard dark energy model (the flat Λ CDM or w CDM). Therefore, it is necessary to clarify the validity and the scope of the ZKDR luminosity distance in describing the Universe in a model-unbiased way. Here, we follow the simplest treatment, where η is assumed to be a constant.

III. SAMPLES AND RESULTS

We carry out analysis by using the latest observations of standard candles, including the joint light-curve analysis of the SDSS-II and SNLS supernova samples [48]—which is referred to as JLA SN Ia in the literature—and the long gamma-ray bursts reported in Ref. [49]. Descriptions for the samples, methodology, and results are presented in this section.

A. Type Ia supernovae

The cosmic acceleration was discovered 16 years ago by measuring accurate distances to distant SNe Ia [51–53]. The reason for the acceleration remains uncertain and a large experimental effort in observational cosmology has been driven to reveal the mechanism of this ostensibly counterintuitive phenomenon. By precisely mapping the distance-redshift relation up to redshift $z \approx 1$, SNe Ia remain, at this stage, the most promising probe of the late-time history of the Universe. Because of the variability of the large spectra features, distance estimation for SNe Ia is based on the empirical observation that these events form a homogeneous class whose remaining variability is reasonably well captured by two parameters [54]. One of them characterizes the stretching of the light curve (X_1 in what follows), and the other describes the color at maximum brightness (C in what follows).

With the assumption that SNe Ia at all redshifts with the identical color, shape, and galactic environment have, on average, the same intrinsic luminosity, the distance estimator (distance modulus: $\mu = 5 \log[(d_L/\text{Mpc})] + 25$) used in most cosmological analysis is quantified by a linear model,

$$\mu_B(\alpha, \beta; M) = m_B^* - M + \alpha \times X_1 - \beta \times C, \quad (10)$$

where m_B^* is the observed peak magnitude in the rest-frame B band, and α and β are nuisance parameters which characterize the stretch-luminosity and color-luminosity relationships, corresponding to the well-known broader-brighter and

bluer-brighter relationships, respectively. The value of M is another nuisance parameter representing the absolute magnitude of a fiducial SNe Ia. In general, α and β are left as free parameters (on the same weight as cosmological parameters) that are determined in the global fit in the context of the standard dark energy scenario to construct the Hubble diagram for SNe Ia. It should be noted that cosmological implications derived from this Hubble diagram for other nonstandard models, which are different from the standard Λ CDM (or w CDM) scenario used to carry out the global fit, are model biased.

In order to achieve model-unbiased constraints on the clumpiness of the Universe, we should fit the light-curve fitting parameters (α and β) and the model parameters (Ω_m and ν) simultaneously to construct a Hubble diagram of SNe Ia in an inhomogeneity-allowed scenario by confronting the distances estimated from SNe Ia observations via Eq. (10) with the ones predicted from the ZKDR luminosity distance model,

$$\mu_{\text{mod}}(z; \theta_1, \mu_0) = 5 \log_{10}[D_L(z; \theta_1)] + \mu_0. \quad (11)$$

Here D_L is the Hubble-constant free luminosity distance, θ_1 represents the model parameter vector (Ω_m, ν), and $\mu_0 = 5 \log_{10}[c/H_0] + 25$. For the latest JLA SN Ia, the standard χ^2 function is given by

$$\chi^2(\mu_0, M; \theta_1, \theta_2) = \sum_{i=1}^{740} \frac{[\mu_{\text{mod}}(z_i; \theta_1, \mu_0) - \mu_{B,i}(\theta_2; M)]^2}{\sigma_{\mu,i}^2}, \quad (12)$$

where θ_2 denotes the vector of light-curve fitting parameters (α, β) and $\sigma_{\mu,i}$ is the error on the distance modulus for the i th SNe Ia. It should be noted that we take only the statistical uncertainties into account and they are also dependent on the light-curve fitting parameters. In order to marginalize over the nuisance parameters, H_0 and M , we expand the χ^2 function with respect to $\tilde{\mu}_0 = \mu_0 + M$ as [55–57]

$$\chi^2(\theta_1, \theta_2; \tilde{\mu}_0) = A - 2\tilde{\mu}_0 B + \tilde{\mu}_0^2 C, \quad (13)$$

where

$$A(\theta_1, \theta_2) = \sum_{i=1}^{740} \frac{[\mu_{\text{mod}}(z_i; \theta_1, \mu_0 = 0) - \mu_{B,i}(\theta_2; M = 0)]^2}{\sigma_{\mu,i}^2}, \quad (14)$$

$$B(\theta_1, \theta_2) = \sum_{i=1}^{740} \frac{[\mu_{\text{mod}}(z_i; \theta_1, \mu_0 = 0) - \mu_{B,i}(\theta_2; M = 0)]}{\sigma_{\mu,i}^2}, \quad (15)$$

$$C(\theta_2) = \sum_{i=1}^{740} \frac{1}{\sigma_{\mu,i}^2}. \quad (16)$$

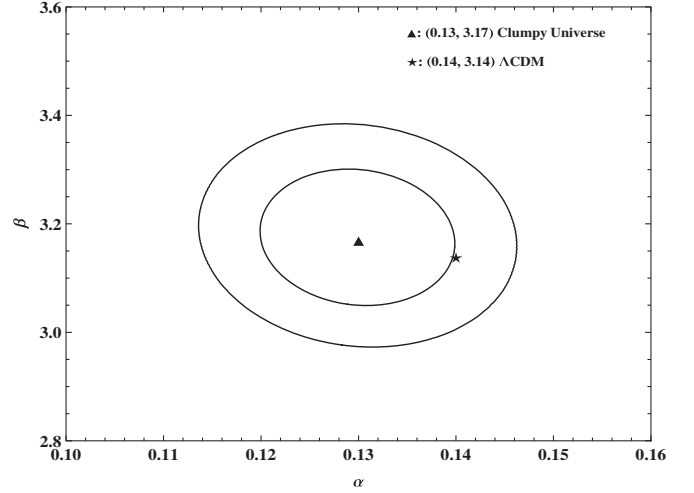


FIG. 1. Constraints on the light-curve fitting parameters, α and β , from the global fit in the context of a clumpy Universe. The triangle and star represent the best fits when the ZKDR approximation and the standard Λ CDM framework are considered, respectively.

Equation (13) has a minimum at $\tilde{\mu}_0 = B/C$, and it is

$$\tilde{\chi}^2(\theta_1, \theta_2) = A - \frac{B^2}{C}. \quad (17)$$

Therefore, we can minimize $\tilde{\chi}^2(\theta_1, \theta_2)$ to get rid of the dependence on nuisance parameters.

The constraint on the light-curve fitting parameters vector is presented in Fig. 1. The best fit value is $(\alpha, \beta) = (0.13, 3.17)$, which is marginally compatible with the result estimated in the flat Λ CDM at a 1σ confidence level. By applying a minimization of $\tilde{\chi}^2$, we can get an estimation for $\tilde{\mu}_0$ which is a combination of H_0 and M . Here, we break the degeneracy by fixing $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and obtain $M = -19.08$. With the constraint on (α, β) and estimation of M , an indicative Hubble diagram in the framework of the ZKDR luminosity distance model is constructed and shown in Fig. 2. Moreover, results for confidence regions constrained in the (Ω_m, ν) plane are presented in Fig. 3 and Table II. We suggest that the clumpiness parameter η is poorly constrained, being bounded on the interval $0.16 \leq \eta \leq 1.00$ within a 1σ confidence level. However, a tighter constraint is obtained for the matter density parameter Ω_m , being restricted on the interval $0.25 \leq \Omega_m \leq 0.37 (1\sigma)$. These are very similar to what was obtained in previous analyses [29,32], but quite different from the results included in Ref. [33]. That is, our unbiased tests slightly indicate an inhomogeneity and the standard FLRW cosmology is consistent with SNe Ia observations within a 1σ confidence level.

B. Long gamma-ray bursts

GRBs, which are the most intensive explosions observed in the Universe and thus are visible across much larger

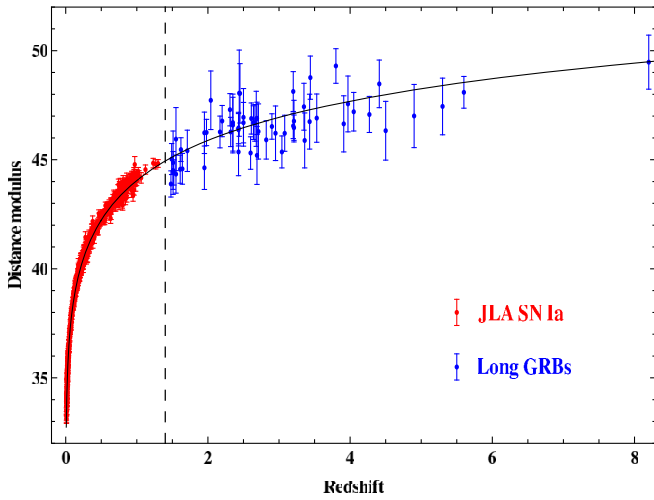


FIG. 2 (color online). Hubble diagram of the standard candles constructed from the global fit in the context of a clumpy Universe. The distance modulus redshift relation of the best-fit ZKDR approximation for a fixed $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is shown as the solid line.

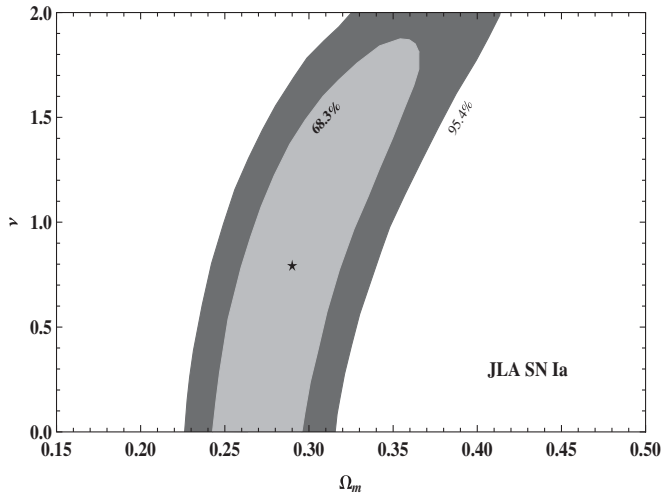


FIG. 3. Confidence regions in the (Ω_m, ν) plane for the model with a ZKDR luminosity distance constrained from the JLA SNIa.

distances than SNe Ia, are deemed as a potential probe to explore the Universe at higher redshift, a redshift of at least 6 and up to even $z = 10$ [58–61]. Specifically, relations between the luminosity/energy and the measurable

properties of the prompt gamma-ray emission imply that GRBs may be appropriate candidates for cosmological standard candles. In the past few years, several empirical luminosity relations have been statistically inferred from observations. For instance, several two-variable relations—the relation between spectral lag and luminosity ($\tau_{\text{lag}} - L$) [62], the relation between variability and luminosity ($V - L$) [63,64], the relation between peak spectral energy and luminosity ($E_{\text{peak}} - L$) [65,66], the relation between peak spectral energy and collimation-corrected energy ($E_{\text{peak}} - E_\gamma$) [67], the relation between the minimum raising time in the GRB light curve and luminosity ($\tau_{\text{RT}} - L$) [68], and the relation between peak spectral energy and isotropic energy ($E_{\text{peak}} - E_{\gamma, \text{iso}}$) [69]—have been successfully deduced from observations. Meanwhile, a few multivariable relations have also been obtained, such as the connection between E_{iso} , E_{peak} , and the break time of the optical afterglow light curves (t_b) [70], the correlation between the luminosity, E_{peak} , and the rest-frame “high-signal” time scale ($T_{0.45}$) [71]. Moreover, these luminosity relations have been proposed to calibrate GRBs as distance indicators (see, e.g., Refs. [68,72] for reviews).

In particular, in Refs. [32,33], distances of GRBs used to constrain the clumpiness of the Universe are obtained by calibrating their luminosity relations with low-redshift SNe Ia [39,41,46]. However, it is necessary to make clear that distances of SNe Ia quoted to calibrate luminosity relations are estimated from a global fit in the frame of a standard dark energy model. In other words, the distances of GRBs given in Refs. [39,41,46] are still somewhat dependent on the standard dark energy model and thus subsequent tests for the inhomogeneity of the Universe derived from them are model biased. In this work, we construct the Hubble diagram of 116 long GRBs [49] in the framework of an inhomogeneous Universe by calibrating their luminosity/energy relations in the global fit where the context of the ZKDR luminosity distance model is considered. This Hubble diagram can then lead to an unbiased examination of the clumpiness of the Universe. In Ref. [49], six luminosity correlations ($\tau_{\text{lag}} - L$, $V - L$, $E_{\text{peak}} - L$, $E_{\text{peak}} - E_\gamma$, $\tau_{\text{RT}} - L$, $E_{\text{peak}} - E_{\gamma, \text{iso}}$) have been derived from the latest observations of 116 long GRBs. In their work, it was also found that the intrinsic scatter of the $V - L$ correlation was too large to infer an inherent correlation between these two quantities using the currently observed

TABLE I. Summary of the constraints on luminosity relations of GRBs from the global fit in the context of a clumpy Universe.

Luminosity relation	$a(1\sigma)$	$b(1\sigma)$	σ_{int}	$N(z_{\text{GRB}} > 1.4)$
$\tau_{\text{lag}} - L$	52.60 ± 0.04	-0.76 ± 0.06	0.12	26
$E_{\text{peak}} - L$	52.10 ± 0.04	1.38 ± 0.12	0.16	62
$E_{\text{peak}} - E_\gamma$	50.36 ± 0.07	1.56 ± 0.20	0.01	12
$\tau_{\text{RT}} - L$	52.95 ± 0.05	-1.03 ± 0.13	0.16	36

GRB events. What is more, the luminosity correlations $E_{\text{peak}} - E_\gamma$ and $E_{\text{peak}} - E_{\gamma,\text{iso}}$ mirror almost the same physics; we should include one of them to avoid strong correlation among the luminosity correlations. Therefore, we choose the $E_{\text{peak}} - E_\gamma$ correlation, which has a smaller intrinsic scatter, and then use the rest of the four correlations for the following analysis. The same as previous works that derived cosmological implications from GRBs, we use only the subsample at $z > 1.4$ for the complementary redshift range to the SN Ia.

The remaining four luminosity correlations involved in this paper are

$$\log \frac{L}{1 \text{ erg s}^{-1}} = a_1 + b_1 \log \left[\frac{\tau_{\text{lag}}(1+z)^{-1}}{0.1 \text{ s}} \right], \quad (18)$$

$$\log \frac{L}{1 \text{ erg s}^{-1}} = a_2 + b_2 \log \left[\frac{E_{\text{peak}}(1+z)}{300 \text{ keV}} \right], \quad (19)$$

$$\log \frac{E_\gamma}{1 \text{ erg}} = a_3 + b_3 \log \left[\frac{E_{\text{peak}}(1+z)}{300 \text{ keV}} \right], \quad (20)$$

$$\log \frac{L}{1 \text{ erg s}^{-1}} = a_4 + b_4 \log \left[\frac{\tau_{\text{RT}}(1+z)^{-1}}{0.1 \text{ s}} \right], \quad (21)$$

where a and b are the intercept and the slope of the relation, respectively. In these correlations, the isotropic peak luminosity L is given by

$$L = 4\pi d_L^2 P_{\text{bolo}}, \quad (22)$$

where P_{bolo} is the bolometric flux of gamma rays in the burst. The isotropic energy released in a burst is

$$E_{\gamma,\text{iso}} = 4\pi d_L^2 S_{\text{bolo}}(1+z)^{-1}, \quad (23)$$

where S_{bolo} is the bolometric fluence of gamma rays in the burst at redshift z . The total collimation-corrected energy can be calculated by

$$E_\gamma = E_{\gamma,\text{iso}}(1 - \cos \theta_{\text{jet}}), \quad (24)$$

where θ_{jet} is the opening angle of the jet.

In order to completely avoid any circularity and obtain model-unbiased constraints on the clumpiness of the Universe from GRBs [65,68], we separately calibrate each

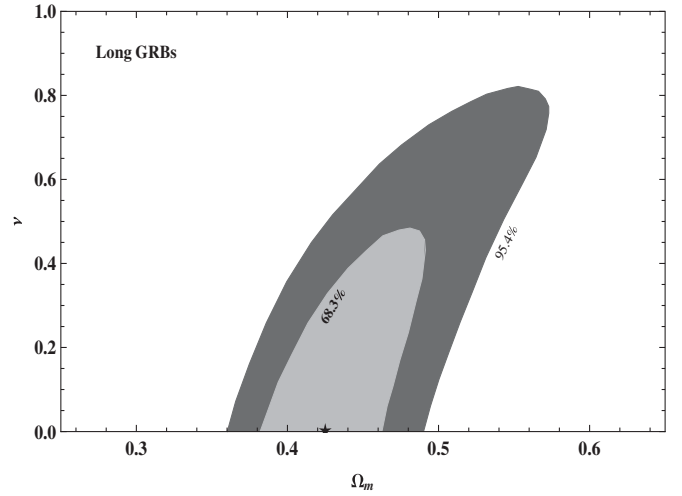


FIG. 4. Confidence regions in the (Ω_m, ν) plane for the model with a ZKDR luminosity distance constrained from the long GRBs.

luminosity relation, Eqs. (18)–(21), by carrying out a similar simultaneous global fitting route presented in the above subsection. Results are shown in Table I. Here, σ_{int} is the systematic error and it can be estimated by finding the value such that an χ^2 fit to each relation calibration curve produces a value of reduced χ^2 of unity [68]. This quantity accounts the extra scatter of the luminosity relations. In this global fitting route, we marginalize the nuisance parameter Hubble constant by fixing $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Following the method about uncertainty calculation and distance estimation from calibrated luminosity relations [46,68], as shown in Fig. 2, we construct a Hubble diagram of GRBs in the context of the ZKDR luminosity distance scenario. In addition, results concerning the constraints on model parameters are presented in Fig. 4 and Table II. It is suggested that a Universe composed only by homogeneously distributed matter is strongly favored by GRB observations. This is greatly different from what was obtained in previous works [32,33].

Finally, we perform a joint analysis from the combination of JLA SN Ia and long GRBs. Results are displayed in Fig. 5 and Table II. Within a 1σ confidence level, the matter density parameter is restricted in the interval $0.32 \leq \Omega_m \leq 0.36$ and the smoothness parameter is bounded in the interval $0.98 \leq \eta \leq 1.00$. It is shown that the constraint on the matter density parameter is mainly dependent on SNe Ia

TABLE II. Summary of the unbiased constraints on model parameters in the ZKDR luminosity distance from observations of standard candles.

Sample	$\Omega_m(1\sigma)$	$\nu(1\sigma)$	$\eta(1\sigma)$
JLA SN Ia	$0.25 \leq \Omega_m \leq 0.37$	$0.00 \leq \nu \leq 1.80$	$0.16 \leq \eta \leq 1.00$
Long GRBs	$0.38 \leq \Omega_m \leq 0.49$	$0.00 \leq \nu \leq 0.48$	$0.88 \leq \eta \leq 1.00$
Joint analysis	$0.32 \leq \Omega_m \leq 0.36$	$0.00 \leq \nu \leq 0.12$	$0.98 \leq \eta \leq 1.00$

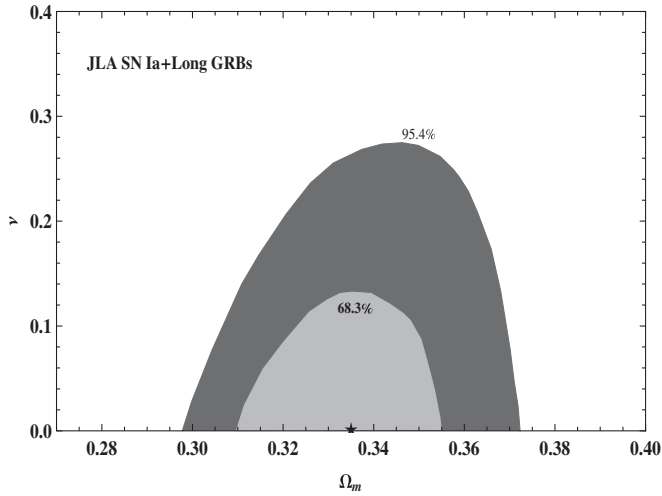


FIG. 5. Confidence regions in the (Ω_m, η) plane for the model with a ZKDR luminosity distance constrained from the combination of JLA SN Ia and long GRBs.

observations while the estimation of the smoothness parameter is basically determined by the long GRBs. The fact that high redshift GRBs prefer a homogeneous Universe with a great significance of probability can be understood as follows: they explore much larger scales of the Universe and should contribute to diminishing the corresponding space parameter. That is, since the Universe is more homogeneous on larger scales (a higher redshift), a higher value of the smoothness parameter η is favored. In addition, it should be noted that, although large redshift GRBs are very important for the tests of the clumpiness parameter, there are only four GRBs at redshift larger than 5.

IV. CONCLUSIONS AND DISCUSSIONS

In the era of precision cosmology, where one aims at determining the cosmological parameters at the percent level, distance estimations for standard candles and rulers with increasing accuracy are expected to provide powerful constraints on dark energy or other fundamental dynamical parameters. However, it is necessary to be aware of the physical hypothesis underlying these probes when we proceed with such a program. As far as we know, the Universe is effectively inhomogeneous at least in the small-scale domain. Furthermore, notice that even the large-scale homogeneity also has been challenged [73]. In this topic, the method based on the ZKDR luminosity distance is a simple alternative and is usually applied to quantitatively assessing the influences of the clumpiness on the light propagation. In the past few years, there has been a rich literature concerning the constraints on the smoothness parameter from observations of standard candles [30,32–34,50]. However, we should keep in mind that distances of SNe Ia applied to test the inhomogeneity were estimated from the global

fit in the context of a standard homogeneous dark energy model, i.e., the flat Λ CDM or w CDM model. Therefore, in these previous analyses, constraints on the smoothness parameter from the distance modulus of SNe Ia were somewhat model biased. Meanwhile, results obtained from GRBs suffered the same problem since the distances of them were determined by calibrating luminosity relations with low-redshift SNe Ia.

In this paper, we first construct Hubble diagrams for SNe Ia and GRBs by calibrating the light-curve fitting parameters and the luminosity relations, respectively, in the global fit where the context of the ZKDR luminosity distance model is taken into account. And then, these Hubble diagrams can lead to unbiased tests for the inhomogeneity of the Universe. For the JLA SN Ia, as shown in Fig. 3, constraint on the smoothness parameter is not stringent and slightly implies a locally inhomogeneous background, while the matter density parameter is well constrained, being bounded in the interval $0.25 \leq \Omega_m \leq 0.37(1\sigma)$. For the long GRBs, as shown in Fig. 4, the Universe with matter uniformly distributed is favored with a high confidence level. This is completely different from what was obtained in Ref. [33]. Finally, we perform a joint analysis which provides good constraints on both model parameters. At a 1σ confidence level, the intervals are $0.32 \leq \Omega_m \leq 0.36$ and $0.98 \leq \eta \leq 1.00$. It is suggested that the constraint on the matter density parameter is mainly based on the observations of low-redshift SNe Ia, while the test for the clumpiness parameter is primarily determined from the observations of high-redshift GRBs. Just as expected, the investigation of the inhomogeneity was very sensitive to the scales explored by the observations; i.e., the Universe should be more homogeneous on larger scales. These also may be an indication that the ZKDR approximation remains to be a precise description for the luminosity distance-redshift relation in a locally inhomogeneous Universe with the cosmological constant.

Frankly, it should be pointed out that constraints on the model parameters from low-redshift SNe Ia and high-redshift GRBs are somewhat inconsistent. This inconsistency may imply that the assumption with the smoothness parameter η being a constant is not accurate enough to fit the practical observations. That is, the smoothness parameter η might evolve with cosmic time (or redshift). Moreover, the intrinsic scatters in GRB observations may also lead to this tension. Therefore, in the near future, a more precise and larger sample of high-redshift GRB data (even some other distance measurements with new methods, e.g., extremely luminous active galactic nuclei readily observed over a range of distances from ~ 10 Mpc to $z > 7$ [74–76]) and a plausible extension of the ZKDR approach are expected to perform more accurate tests for the inhomogeneity and contribution of matter in the Universe.

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