PHYSICAL REVIEW D **91,** 076010 (2015)

Transition electromagnetic form factor and current conservation in the Bethe-Salpeter approach

J. Carbonell[1] and V. A. Karmanov[2]

[1]*Institut de Physique Nucléaire, Université Paris-Sud, IN2P3-CNRS, 91406 Orsay Cedex, France*
[2]*Lebedev Physical Institute, Leninsky Prospekt 53, 119991 Moscow, Russia*
(Received 10 March 2015; published 27 April 2015)

The transition form factor for electrodisintegration of a two-body bound system is calculated in the Bethe–Salpeter framework. For the initial (bound) and the final (scattering) states, we use our solutions of the Bethe–Salpeter equation in Minkowski space which were first obtained recently. The gauge invariance, which manifests itself in the conservation of the transition electromagnetic current $J \cdot q = 0$, is studied numerically. It results from a cancellation between the plane wave and the final state interaction contributions. This cancellation takes place only if the initial bound state Bethe–Salpeter amplitude, the final scattering state, and the operator of electromagnetic current are strictly consistent with each other, that is if they are found in the same dynamical framework. A reliable result for the transition form factor can be obtained in this case only.

DOI: 10.1103/PhysRevD.91.076010 PACS numbers: 03.65.Pm, 03.65.Ge, 11.10.St

I. INTRODUCTION

Computing the electromagnetic (EM) form factors in the Bethe-Salpeter (BS) approach [1] requires the solutions of the BS equation in Minkowski space.

The main reason is that the Wick rotation [2], which allows going from Minkowski to Euclidean space in the BS equation, cannot be performed in the integral expression of the EM form factor (see e.g. Ref. [3]). However, in contrast to the Euclidean case, finding the Minkowski space solution is complicated by the many singularities in the integrand of the BS equation and in the amplitude itself. In the recent years, and using different independent methods, these difficulties have been overcome, and an important progress was achieved.

In one of these methods [4], the kernel of the BS equation is approximately represented in a separable form. This allows one to considerably advance analytically and therefore simplifies finding the solution.

In the method developed in Refs. [5,6], the BS amplitude is represented as an integral over a weight function g—the so-called Nakanishi transform [7]—which satisfies a nonsingular equation. A modification of this method, based on the light-front projection of the BS amplitude, was developed in Refs. [8–12] and used to find the bound state Minkowski BS amplitude. The elastic EM form factor was also calculated in Ref. [13]. An equation for the Nakanishi function for the scattering states was derived [14].

Another method [15,16] is based on the direct solution in Minkowski space of the BS equation after an appropriate treatment of singularities. The scattering problem was there solved and the off-mass shell scattering amplitude first computed. It allows one to calculate the electrodisintegration of the bound system, i.e. the form factor of the transition bound to scattering state. Once reduced to the mass shell, this amplitude reproduces the phase shifts.

An important contribution to this form factor, incorporating the final state interaction (FSI), is given by the Feynman graph shown in Fig. 1 (left panel). The right and left vertices in this graph are just the Minkowski BS amplitudes for the bound (left) and scattering (right) states. If both vertices correspond to a bound state (case of the elastic form factor), the Nakanishi transform allows one to calculate the four-dimensional (4D) Feynman integral corresponding to Fig. 1 (left panel), with an integrand containing three singular propagators, analytically [13]. Then the nonsingular integral with the weight functions g is safely calculated numerically. For the scattering state, though the Nakanishi transform also exists [14], the corresponding weight function g at positive energies is not yet computed. Note, however, that very recently g was found in the zero-energy limit that allowed one to calculate the scattering length [17]. Without using the Nakanishi representation, the scattering state vertex can be obtained only numerically [15], and therefore the singular 4D Feynman integral corresponding to Fig. 1 (left panel) must be computed numerically as well. This calculation, providing the transition electromagnetic current and the form factor, requires, however, some care to take properly into account the pole singularities of the propagators.

The aim of this paper is to give the detail of the first results presented in Refs. [18,19] and analyze the conservation of the calculated electromagnetic current in the inelastic transition. We will see that this current is indeed conserved, as it should be from general principles [20]. However, this conservation is due to a rather delicate cancellation between the plane wave (PW) contribution (right panel of Fig. 1) and the final state interaction (left panel of Fig. 1) which requires a strict consistency between,

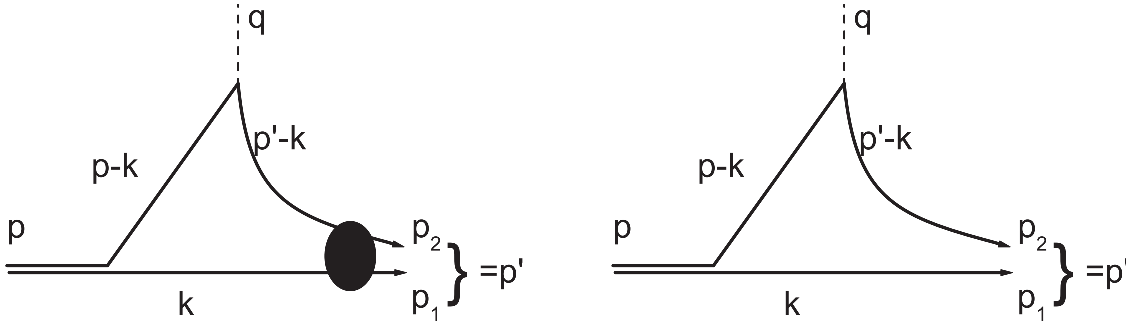

FIG. 1. Left panel: Feynman graph for contribution of FSI to the transition EM form factor. Right panel: PW contribution to the transition EM form factor.

on one hand, the bound and scattering state solutions and, on the other hand, the electromagnetic current operator. It thus provides a strong test for all these quantities simultaneously.

The need for an internal consistency between states, currents, and the dynamical equation to ensure the gauge invariance was extensively discussed in Ref. [20] in the framework of the BS and the Gross spectator equations. Our numerical results are in agreement with this general expectation. We will show that, if this consistency and hence the gauge invariance is violated, a consequence of that is not only the appearance of a nonconserved part in the current—which anyway drops out in the cross section—but that the current as a whole is not valid at all. In other words, the transition form factors extracted from the conserved part of the nonconserved current are also deficient.

To illustrate our treatment of the singularities, we will restrict to the spinless particles. The generalization to the fermion case is straightforward since the fermion and scalar propagators have the same singularities.

The paper is organized as follows. In Sec. II we discuss the decomposition of the transition current in the form factors without assuming the current conservation. In Secs. III and IV, the FSI and PW contributions in the conserved part of the current are calculated. Section V is devoted to the discussion of the current conservation. In particular, the FSI and PW contributions of the nonconserved part of the current have been calculated, and we have shown that they cancel each other. Some selected numerical results are presented in Sec. VI. Section VII contains the concluding remarks. The cumbersome details of the calculations are given in the Appendices A, B, and C.

II. TRANSITION FORM FACTOR

In the case of spinless particles, and without supposing the current conservation, the general form of the electromagnetic current involves two form factors:

$$J_\mu = (p_\mu + p'_\mu)F_1(Q^2) + (p'_\mu - p_\mu)F_2(Q^2). \quad (1)$$

The decomposition (1), together with the scalar character of the constituents, implies that the initial and final states have total zero angular momenta, i.e. that they are composed of S-waves only.

To study the current conservation, it is convenient to redefine the form factors by introducing the linear combinations F and F',

$$\begin{matrix} F = F_1 \\ F' = F_1 - \frac{Q^2}{Q_c^2} F_2 \end{matrix} \Longleftrightarrow \begin{matrix} F_1 = F \\ F_2 = \frac{Q_c^2}{Q^2}(F - F'), \end{matrix}$$

with

$$\begin{aligned} q_\mu &= p'_\mu - p_\mu \\ Q^2 &= -q^2 = -(p' - p)^2 \\ Q_c^2 &= M'^2 - M^2. \end{aligned} \quad (2)$$

M is the initial bound state mass, and M' is the invariant mass of the final scattering state. In terms of them, the current (1) can be rewritten in the form

$$J_\mu = \left[(p_\mu + p'_\mu) + (p'_\mu - p_\mu)\frac{Q_c^2}{Q^2}\right]F(Q^2) - (p'_\mu - p_\mu)\frac{Q_c^2}{Q^2}F'(Q^2). \quad (3)$$

Since

$$q \cdot J = Q_c^2 F'(Q^2), \quad (4)$$

the current conservation $q \cdot J = 0$ is equivalent to $F'(Q^2) \equiv 0$.

Notice that in the elastic case the form factor F' is absent since the term $\sim (p'_\mu - p_\mu)F'(Q^2)$ in (3) is forbidden by the symmetry between initial and final states.

Notice also that the form factor $F'(Q^2)$, even if it is not zero, does not contribute to the electrodisintegration amplitude A. Indeed, this amplitude is given by

$$A \sim \frac{J_\mu \bar{u}(k')\gamma^\mu u(k)}{Q^2}.$$

It contains the electron spinors $u(k)$ and $\bar{u}(k')$. Substituting here the current (3) and using the Dirac equation, we see that the term containing $F'(Q^2)$ drops out since

$$(p'_\mu - p_\mu)\bar{u}(k')\gamma^\mu u(k) = \bar{u}(k')(\not{k} - \not{k}')u(k) = 0.$$

Below we will calculate each of these form factors—F and F'—as a sum of FSI (left panel in Fig. 1) and PW (right panel in Fig. 1) contributions, in the form

$$F_{\rm inel}(Q^2) = F_{\rm fsi}(Q^2) + F_{\rm pw}(Q^2)$$
$$F'_{\rm inel}(Q^2) = F'_{\rm fsi}(Q^2) + F'_{\rm pw}(Q^2). \tag{5}$$

We will check that the full current is conserved; that is, for any Q^2, the contributions to $F'_{\rm inel}(Q^2)$ of the FSI and PW cancel each other,

$$F'_{\rm inel}(Q^2) = F'_{\rm fsi}(Q^2) + F'_{\rm pw}(Q^2) = 0, \tag{6}$$

provided the bound and scattering states are solutions of the BS equation with the one-boson exchange kernel. In this case, the EM current corresponding to the interaction of a photon with a constituent is free. We are, however, interested in a quantitative measure of the accuracy of this cancellation in a real calculation. This is the reason for introducing in (3) the nonconserved part—proportional to $(p'_\mu - p_\mu)F'(Q^2)$—and the value of the form factor $F'(Q^2)$ will give us this measure.

We will calculate separately the FSI and PW contributions to the form factor $F'(Q^2)$ and see with what accuracy they cancel each other in the sum (6).

III. FINAL STATE INTERACTION

We start with the FSI contribution. It is obtained by applying the Feynman rules to the left panel graph of Fig. 1 and has the form (following the convention of Ref. [21])

$$J_{\mu,\rm fsi} = i\int \frac{d^4k}{(2\pi)^4}\frac{(p_\mu + p'_\mu - 2k_\mu)\Gamma_i(\frac{1}{2}p - k, p)\Gamma_f(\frac{1}{2}p' - k, p')}{(k^2 - m^2 + i\epsilon)[(p-k)^2 - m^2 + i\epsilon][(p'-k)^2 - m^2 + i\epsilon]}. \tag{7}$$

Here Γ_i is the initial (bound state) vertex, and Γ_f is the final vertex (half-off-shell scattering BS amplitude). As mentioned, both vertex functions were found numerically by solving the S-wave BS equation in Ref. [15]. More precisely the function Γ_f is related to the scattering wave solution F_0 by Eq. (B29) from Appendix B 4.

The integrals of the type (7) are usually calculated by applying to the product of propagators the Feynman parametrization and then performing the Wick rotation. However, besides the product of propagators, expression (7) contains the initial (Γ_i) and final (Γ_f) BS amplitudes which are known numerically. Therefore, the Feynman parametrization cannot be applied, and we should calculate this 4D singular integral numerically, though after some transformations.

It is convenient to carry out this calculations in the system of reference where $p'_0 = p_0$ (i.e. $q_0 = 0$) and $\vec{p}$ and $\vec{p}'$ are collinear; i.e. they either are parallel or antiparallel to each other, depending on the kinematical conditions. In the elastic case, it coincides with the Breit frame $\vec{p} + \vec{p}' = 0$, and one has of course $|\vec{p}| = |\vec{p}'|$, $p'_0 = p_0$. In the inelastic case, in the frame with $p'_0 = p_0$, we have $|\vec{p}| \neq |\vec{p}'|$. Some useful kinematical relations valid in this reference system are given in Appendix A.

In this reference frame, we take the zero component of the current (3) and get the relation

$$J_0 = 2p_0 F(Q^2). \tag{8}$$

That is

$$F_{\rm fsi}(Q^2) = i\int \frac{dk_0 d^3k}{(2\pi)^4}\frac{(p_0 - k_0)}{p_0}\frac{\Gamma_i(\frac{1}{2}p - k, p)\Gamma_f(\frac{1}{2}p' - k, p')}{(k_0^2 - \varepsilon^2_{\vec{k}} + i\epsilon)[(p_0 - k_0)^2 - \varepsilon^2_{\vec{p}-\vec{k}} + i\epsilon][(p_0 - k_0)^2 - \varepsilon^2_{\vec{p}'-\vec{k}} + i\epsilon]} \tag{9}$$

with $\varepsilon_{\vec{q}} = \sqrt{m^2 + \vec{q}^2}$ and similar expressions for $\varepsilon_{\vec{p}-\vec{p}}$ and $\varepsilon_{\vec{p}'-\vec{k}}$ obtained using (A4) and (A5).

As detailed in Appendix B 4, in case of initial and final S-waves, all kinematical variables as well as the arguments of the vertex functions Γ appearing in (9) can be expressed in terms of $|\vec{p}|$, $|\vec{p}'|$ and the integration variables $(k_0, z, |\vec{k}|)$ with $z = \hat{k}\cdot\hat{p}$. To lighten the writing, we will denote hereafter abusively $p = |\vec{p}|$, $p' = |\vec{p}'|$, and $k = |\vec{k}|$.

After a trivial integration over the azimutal angle, the integration measure in (9) becomes

$$\frac{dk_0 d^3k}{(2\pi)^4} = \frac{dk_0 dz k^2 dk}{(2\pi)^3}.$$

Let us introduce the notations, making explicit only the dependence on the integration variables,

$$f(k_0,z,k) = \frac{G(k_0,z,k)}{(k_0^2 - \varepsilon_{\vec k}^2 + i\epsilon)[(p_0-k_0)^2 - \varepsilon_{\vec p-\vec k}^2 + i\epsilon][(p_0-k_0)^2 - \varepsilon_{\vec p'-\vec k}^2 + i\epsilon]}, \tag{10}$$

where

$$G(k_0,z,k) = \frac{(p_0-k_0)}{p_0}\Gamma_i\left(\frac{1}{2}p-k,p\right)\Gamma_f\left(\frac{1}{2}p'-k,p'\right). \tag{11}$$

Each pole singularity in (10) is represented as a sum of its principal value and a delta-function, and therefore the function f takes the form

$$\begin{aligned} f(k_0,z,k) &= G(k_0,z,k)\left[\mathrm{PV}\frac{1}{k_0^2-\varepsilon_{\vec k}^2} - i\pi\delta(k_0^2-\varepsilon_{\vec k}^2)\right]\left[\mathrm{PV}\frac{1}{(p_0-k_0)^2-\varepsilon_{\vec p-\vec k}^2} - i\pi\delta\left((p_0-k_0)^2-\varepsilon_{\vec p-\vec k}^2\right)\right] \\ &\quad\times\left[\mathrm{PV}\frac{1}{(p_0-k_0)^2-\varepsilon_{\vec p'-\vec k}^2} - i\pi\delta\left((p_0-k_0)^2-\varepsilon_{\vec p'-\vec k}^2\right)\right] \\ &\equiv f_3+f_2+f_1, \end{aligned} \tag{12}$$

where f_3 is the contribution of the product of three principal values and no delta-functions (one single term), f_2 is the contribution of the product of two principal values and one delta-function (three terms), and f_1 is the contribution of the product of one principal value and two delta-functions (also three terms). The product of three delta-functions does not contribute since their arguments cannot be zero simultaneously.

These functions f_i have the following explicit form:

$$f_3(k_0,z,k) = G(k_0,k)\mathrm{PV}\frac{1}{k_0^2-\varepsilon_{\vec k}^2}\mathrm{PV}\frac{1}{(p_0-k_0)^2-\varepsilon_{\vec p-\vec k}^2}\mathrm{PV}\frac{1}{(p_0-k_0)^2-\varepsilon_{\vec p'-\vec k}^2} \tag{13}$$

$$\begin{aligned} f_2(k_0,z,k) &= -i\pi\delta(k_0^2-\varepsilon_{\vec k}^2)G(k_0,z,k)\mathrm{PV}\frac{1}{(p_0-k_0)^2-\varepsilon_{\vec p-\vec k}^2}\mathrm{PV}\frac{1}{(p_0-k_0)^2-\varepsilon_{\vec p'-\vec k}^2} \\ &\quad - i\pi\delta((p_0-k_0)^2-\varepsilon_{\vec p-\vec k}^2)G(k_0,z,k)\mathrm{PV}\frac{1}{k_0^2-\varepsilon_{\vec k}^2}\mathrm{PV}\frac{1}{(p_0-k_0)^2-\varepsilon_{\vec p'-\vec k}^2} \\ &\quad - i\pi\delta((p_0-k_0)^2-\varepsilon_{\vec p'-\vec k}^2)G(k_0,z,k)\mathrm{PV}\frac{1}{k_0^2-\varepsilon_{\vec k}^2}\mathrm{PV}\frac{1}{(p_0-k_0)^2-\varepsilon_{\vec p-\vec k}^2} \end{aligned} \tag{14}$$

$$\begin{aligned} f_1(k_0,z,k) &= -\pi^2\delta(k_0^2-\varepsilon_{\vec k}^2)\delta((p_0-k_0)^2-\varepsilon_{\vec p-\vec k}^2)G(k_0,z,k)\mathrm{PV}\frac{1}{(p_0-k_0)^2-\varepsilon_{\vec p'-\vec k}^2} \\ &\quad - \pi^2\delta(k_0^2-\varepsilon_{\vec k}^2)\delta((p_0-k_0)^2-\varepsilon_{\vec p'-\vec k}^2)G(k_0,z,k)\mathrm{PV}\frac{1}{(p_0-k_0)^2-\varepsilon_{\vec p-\vec k}^2} \\ &\quad - \pi^2\delta((p_0-k_0)^2-\varepsilon_{\vec p-\vec k}^2)\delta((p_0-k_0)^2-\varepsilon_{\vec p'-\vec k}^2)G(k_0,z,k)\mathrm{PV}\frac{1}{k_0^2-\varepsilon_{\vec k}^2}. \end{aligned} \tag{15}$$

The index of f_i ($i=1,2,3$) denotes the number of the principal value products that involves.

Our task now is to calculate the 4D integral (9), rewritten as

$$F_{\rm fsi}(Q^2) = \frac{i}{(2\pi)^3}\int dk_0 dz k^2 dk\{f_3(k_0,z,k) + f_2(k_0,z,k) + f_1(k_0,z,k)\}$$

with f_i given by Eqs. (13)–(15). Part of this integration is calculated analytically, and the remaining part, once transformed into a nonsingular integrand, is calculated numerically.

For calculating the singular principal value integrals in f_3, we will use the subtraction technique. That is, we subtract and add to f_3 an appropriately chosen singular function h_3 which in variable k_0 has the same poles as f_3 and has no any other singularities:

$$f_3 = (f_3 - h_3) + h_3.$$

In the difference $(f_3 - h_3)$, the pole singularities cancel each other, and the result is a smooth function, whereas in the additional term h_3 the integral over dk_0 is calculated analytically.

After this calculation, there still remains a singular expression in variable $\vec{k}$. It is, however, logarithmic and can be treated by using standard numerical techniques, like variable change or by simply increasing the number of integration points. The details of all these calculations are given in Appendix B.

In the integrals containing the functions f_2 and f_1, the integration over k_0 is easily performed analytically by means of the delta-functions. After that, and a trivial azimuthal integration, the result is reduced to two- and one-dimensional numerical integrations, respectively.

The final result for the FSI contribution (9) reads

$$F_{\rm fsi}(Q^2) = \frac{i}{(2\pi)^3}\int dk_0 dz k^2 dk\{f_3(k_0,z,k) + f_2(k_0,z,k) + f_1(k_0,z,k)\} \equiv F_3(Q^2) + F_2(Q^2) + F_1(Q^2), \quad (16)$$

where $F_i(Q^2)$ are defined in Appendix B by Eqs. (B9), (B13), and (B26).

IV. PLANE WAVE CONTRIBUTION

This contribution is displayed in the right panel in Fig. 1. According to the Feynman rules, it reads

$$J_{\mu,pw} = -\int \frac{(p+p'-2k)_\mu \Gamma_i(\frac{p}{2}-k,p)}{(p-k)^2 - m^2 + i\epsilon}\delta^{(4)}\left(k - p_s - \frac{p'}{2}\right)d^4k. \quad (17)$$

The delta-function follows from the 4-momenta conservation in Fig. 1, right panel:

$$\delta^{(4)}(k-p_1) = \delta^{(4)}\left(k - p_s - \frac{p'}{2}\right).$$

We have introduced the total p' and relative p_s 4-momentum of the final (noninteracting) particles

$$2p_s = p_1 - p_2$$
$$p' = p_1 + p_2.$$

The spatial part of p_s in the rest frame of the final system $\vec{p}' = 0$ determines the invariant final state mass $M' = 2\sqrt{m^2 + \vec{p}_s^{\,2}}$. One could calculate the integral over d^4k by means of the delta-function. It is, however, interesting to keep this delta-function and carry out the integration later, once the S-wave is extracted from the final state.

Like in the case of FSI, the form factor can be found by applying Eq. (8) to the $J_{0,\rm pw}$ component of Eq. (17), in the system of reference where $q_0 = 0$. That is

$$F_{\rm pw} = -\int \frac{(p_0 - k_0)}{p_0}\frac{\Gamma_i(\frac{p}{2}-k,p)}{[(p-k)^2 - m^2 + i\epsilon]} \times \int \frac{d\Omega_{\hat{p}_s}}{4\pi}\delta^{(4)}\left(k - p_s - \frac{p'}{2}\right)d^4k. \quad (18)$$

We have introduced here the additional integration over $\frac{d\Omega_{\hat{p}_s}}{4\pi}$ in the rest frame $\vec{p}' = 0$ of the final state. We remind the reader that in the FSI contribution, calculated in the previous section, we decomposed the final state BS amplitude Γ_f in partial waves and took into account the S-wave only. The delta-function in (18) replaces now the final BS amplitude Γ_f. Averaging this delta-function over the solid angle $\hat{p}_s$ in the rest frame $\vec{p}' = 0$ allows one to select the partial S-wave in the plane wave. This is the meaning of the integral over $\frac{d\Omega_{\hat{p}_s}}{4\pi}$ in (18).

The integration over $d\Omega_{\hat{p}_s}$ and part of the integration over d^4k in (18) are done analytically in Appendix C. The final result reads

$$F_{\rm pw} = -\int_{k_-}^{k_+}\frac{(p_0 - k_0)}{p_0}\Gamma_i\left(\frac{p}{2}-k,p\right) \times \frac{1}{(p-k)^2 - m^2 + i\epsilon}\frac{M'k}{2\varepsilon_k p_s p'}dk, \quad (19)$$

where the integration limits $k_\mp$ are defined in (B15) and $p_s = \sqrt{M'^2/4 - m^2}$. In expression (19) one must insert $k_0 = \sqrt{m^2 + k^2}$ and in the scalar product $k\cdot p = k_0 p_0 - zkp$ take the value $z = z_0$ given by Eq. (B14). Variables p_0

and p are the components of the initial 4-momentum. The value p' is the spatial part of the total final state 4-momentum in the frame where $p'_0 = p_0$. All these components are expressed in terms of the momentum transfer Q^2 in Eqs. (A8) and (A9) from Appendix A.

Let us specify the arguments of $\Gamma_i(\frac{p}{2} - k, p)$ in (19). They are defined analogously to the case of FSI, Eq. (B27) in Appendix B 4. Namely, solving the BS equation in the rest frame $\vec{p} = 0$, we find $\Gamma_i(k_0, |\vec{k}|)$, where k_0 and $|\vec{k}|$ are also defined in the rest frame $\vec{p} = 0$. We should express them in the frame where $q_0 = 0$. These expressions are given in Appendix B 4. That is, we have to insert in (19) the function $\Gamma_i(\tilde{k}_0, |\tilde{\vec{k}}|)$ with arguments $\tilde{k}_0$ and $|\tilde{\vec{k}}|$ given by the first line of Eq. (B28) from the Appendix B 4,

$$\tilde{k}_0 = \frac{1}{2}M - \frac{1}{M}(k_0 p_0 - kpz),$$
$$|\tilde{\vec{k}}| = \sqrt{\frac{1}{M^2}(k_0 p_0 - kpz)^2 - k_0^2 + \vec{k}^2}$$

with $k_0 = \sqrt{m^2 + k^2}$ and z_0 defined in (B14).

To summarize these last two sections, we would like to emphasize that (i) the full PW contribution in the current is given by the simple equation (17), (ii) the expression (19) corresponds to the S-wave projection of the final plane wave state, and (iii) the full transition form factor—including both the FSI and PW contributions—is given by the sum (5), with $F_{\rm fsi}$ determined by Eq. (16) and $F_{\rm pw}$ by (19).

V. CURRENT CONSERVATION

As follows from Eq. (4), the conservation of the electromagnetic current $q \cdot J = 0$ implies $F'(Q^2) = 0$ for any value of Q^2; that is $F'(Q^2) \equiv 0$.

To ensure this conservation, all the contributions to the current, containing the interaction of a photon with a charged particle, must be taken into account. In other words, in an interacting system, the true current is, in general, not the free one.

For example, for the kernel given by the sum of ladder and cross-ladder, the full EM current should contain, in addition to the two (free) contributions displayed in Fig. 1, the cross-ladder FSI contribution shown in Fig. 2 and a similar cross-ladder contribution for the plane wave (we suppose the exchanged particle to be neutral). The latter contributions are not free; they contain the interaction of constituents. The sum of four contributions—Fig. 1 (left and right panels), Fig. 2 (cross-ladder with FSI), and the corresponding cross-ladder PW (not shown)—must be conserved.

In the case of the ladder kernel, the diagrams displayed in Fig. 1 provide the only contributions to the current. Therefore, the current determined by these two graphs has to be conserved if the initial and final BS amplitudes are

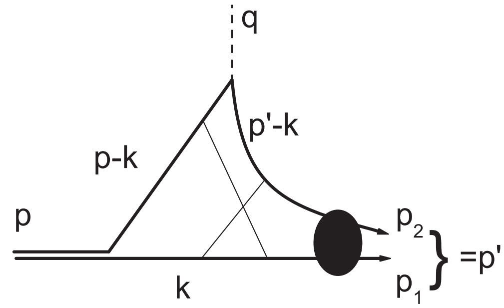

FIG. 2. Cross-ladder contribution to the transition EM form factor.

also obtained with the ladder kernel. At the same time, the expressions for the contributions (7) and (17) to the current in terms of the BS amplitudes are universal—they are the same for any BS amplitude (found with any kernel). The conservation of their sum is provided by the particular properties of the BS amplitudes determined by the ladder kernel. The current is not conserved, if in Eqs. (7) and (17) one substitutes other BS amplitudes (not the ladder ones). Therefore, the current conservation (if any) provides a very strong test for the solutions themselves. In this section we will calculate the form factor $F'(Q^2)$, and in the next section, we will check numerically whether it is identically zero or not. From (4) it follows that

$$F'(Q^2) = \frac{J \cdot q}{Q_c^2}. \tag{20}$$

In the expression (7) for $J_{\rm fsi}$, after multiplying it by q/Q_c^2, we consider, for a moment, only the factor

$$\begin{aligned}
&\frac{1}{Q_c^2}(p' - p) \cdot (p + p' - 2k) \\
&\quad = \frac{1}{Q_c^2}[M'^2 - M^2 - 2(p' - p) \cdot k]\Big|_{p'_0 = p_0} \\
&\quad = \left(1 - 2\frac{\sqrt{Q^2} zk}{Q_c^2}\right).
\end{aligned} \tag{21}$$

Instead of the function $G(k_0, z, k)$ defined in Eq. (11) and given by Eq. (B27) in Appendix B 4, we introduce the function

$$G'(k_0, z, k) = \left(1 - 2\frac{\sqrt{Q^2} zk}{Q_c^2}\right)\Gamma_i(\tilde{k}_0, \tilde{k})\Gamma_f(\tilde{k}'_0, \tilde{k}') \tag{22}$$

$G'(k_0, z, k)$ is obtained from $G(k_0, z, k)$, replacing the factor $(p_0 - k_0)/p_0$ in $G(k_0, z, k)$ by the factor (21)

$$\frac{(p_0 - k_0)}{p_0} \to \left(1 - 2\frac{\sqrt{Q^2} zk}{Q_c^2}\right), \tag{23}$$

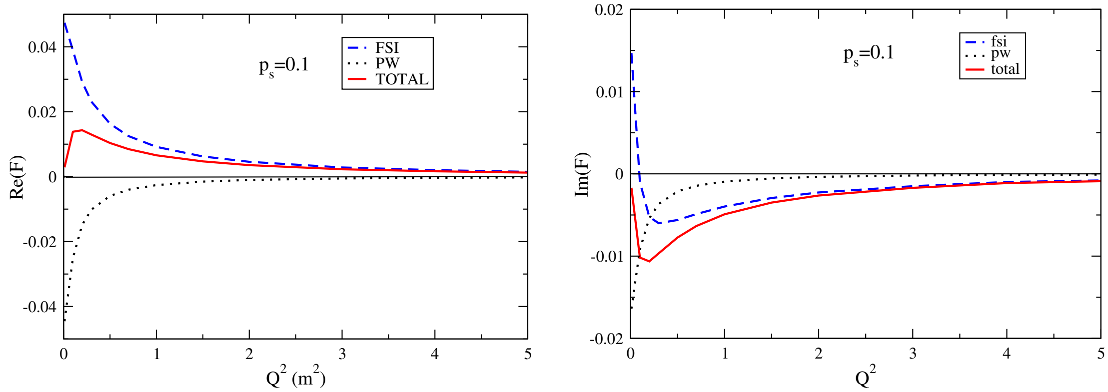

FIG. 3 (color online). Transition EM form factor $F(Q^2)$ as a function of Q^2. The initial (bound) state corresponds to the binding energy $B = 0.01$ m; the final (scattering) state corresponds to a relative momentum $p_s = 0.1$ m (final state mass $M' = 2.00998$ m). The FSI contribution is shown by the dashed curve, and the PW one is shown by a dotted curve. The full form factor is shown by the solid curve. The left panel is the real part of form factor, and the right panel is the imaginary part.

where as always z is the cosine of the angle between the integration variable $\vec{k}$ and the momentum $\vec{p}$ of the initial (bound state) system in the reference frame where $q_0 = 0$.

The FSI contribution $F'_{\rm fsi}(Q^2)$ to the full form factor

$$F'(Q^2) = F'_{\rm fsi}(Q^2) + F'_{\rm pw}(Q^2) \qquad (24)$$

is given by the same formulas as for $F_{\rm fsi}(Q^2)$, Eq. (16), with the replacement $G(k_0, z, k) \to G'(k_0, z, k)$.

The PW contribution $F'_{\rm pw}(Q^2)$ is calculated in a similar way. Namely $F'_{\rm pw}(Q^2)$ is given by Eq. (19) with the replacement (23). The value k_0 is the same used in Eq. (19), and z_0 is defined in Eq. (B14) from Appendix B 2.

To obtain $F'(Q^2)$ identically zero, the two contributions FSI and PW to the full form factor $F'(Q^2)$, Eq. (24), must cancel each other. Numerically this condition is never fulfilled exactly. The value of $F'(Q^2)$ will be rather compared to $F(Q^2)$, and the current conservation would manifest itself in the fact that $F'(Q^2) \ll F(Q^2)$ for any value of Q^2.

VI. NUMERICAL RESULTS

As an example, we have calculated the transition form factor for the initial (bound) state binding energy $B = 0.01$ m (initial state mass $M = 1.99$ m) and for two values of the final (scattering) state relative momentum $p_s = 0.1$ m and $p_s = 0.5$ m with corresponding final state masses $M' = 2\sqrt{m^2 + p_s^2}$ values $M' \approx 2.00998$ m and $M' \approx 2.236$ m.

In contrast to the elastic scattering, the inelastic transition form factor is complex. Its real and imaginary parts as a function of Q^2 for $p_s = 0.1$ m are shown in Fig. 3, at the left and right panels correspondingly. One can see that at relatively small momentum transfer $Q^2 < 1$ both contributions—FSI and PW—are important, and they considerably cancel each other.

The tail of the real part of the form factor for $Q^2 \geq 1$ and $p_s = 0.1$ m is shown in Fig. 4. In this momentum region, FSI dominates, especially when Q^2 increases.

This is a natural behavior for the kinematics corresponding to Fig. 4. Indeed, due to small binding energy ($B = 2$ m $- M = 0.01$ m), the constituents in the initial state have small relative momentum. In the scattering process, the photon transfers the large Q^2 value to one of the constituents only. However, since their relative energy in the final state is also small ($M' - 2$ m ≈ 0.01 m), both constituents have also small relative momentum. Therefore they move practically in the same direction, having both large momenta. Since the second constituent does not interact with the photon, it can obtain a large momentum only due to a strong interaction with the first constituent. This explains why in this kinematics the final state interaction (rescattering) determines the tail of the form factor and dominates over the plane wave.

The transition EM form factor for larger final relative momentum $p_s = 0.5$ m, final state mass $M' = 2.336$ m, and the same values of other parameters is shown in Fig. 5.

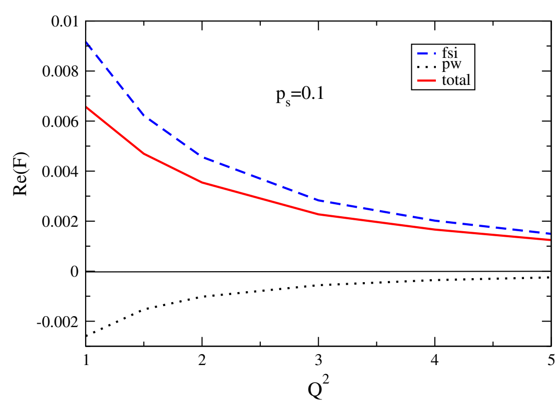

FIG. 4 (color online). The same as on the left panel of Fig. 3 for the tail of form factor $1 \leq Q^2 \leq 5$.

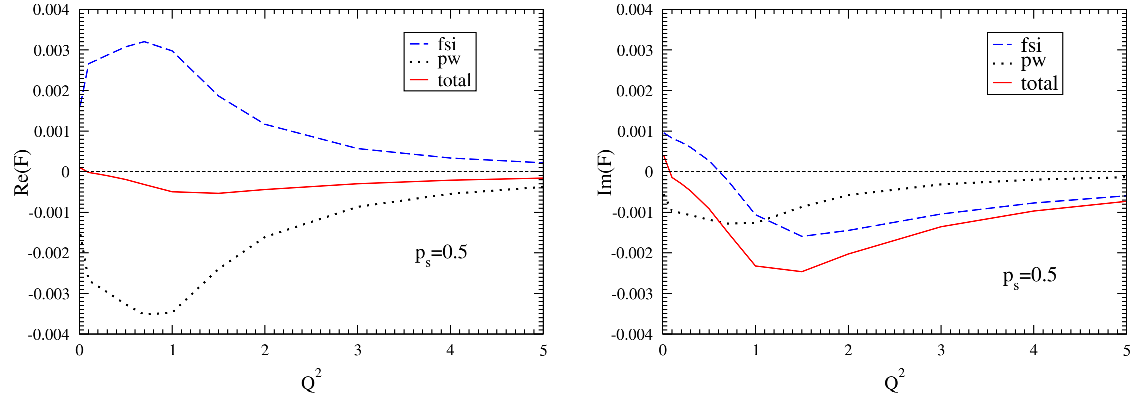

FIG. 5 (color online). The same as in Fig. 3 for the final relative momentum $p_s = 0.5$ m (final state mass: $M' = 2.236$ m).

For this larger value of the final state mass, the FSI contribution is still significant, but it does not dominate anymore. In the real part, FSI and PW contributions considerably cancel each other.

As mentioned above, the form factor $F'(Q^2)$—which vanishes if the current is conserved—is obtained from $F(Q^2)$ by the replacement (23) in the integrand. The corresponding numerical results for $p_s=0.1$ and $p_s = 0.5$ are shown in Figs. 6 and 7, respectively. We see that, in comparison with the $F(Q^2)$ results of Figs. 3–5, the value of $F'(Q^2)$ is indistinguishable from zero. This very small value is a result of a cancellation between FSI and PW contributions. We conclude that in the model considered the current is conserved.

Though the current conservation is natural, the cancellation of FSI and PW contributions is rather delicate and

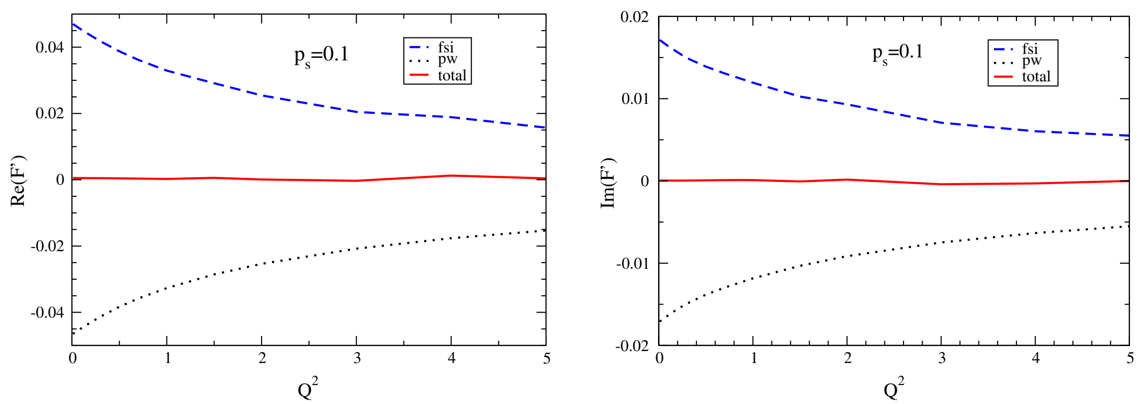

FIG. 6 (color online). Transition EM form factor $F'(Q^2)$ as a function of Q^2 for $p_s = 0.1$. Other parameters and notations are the same as in Fig. 3.

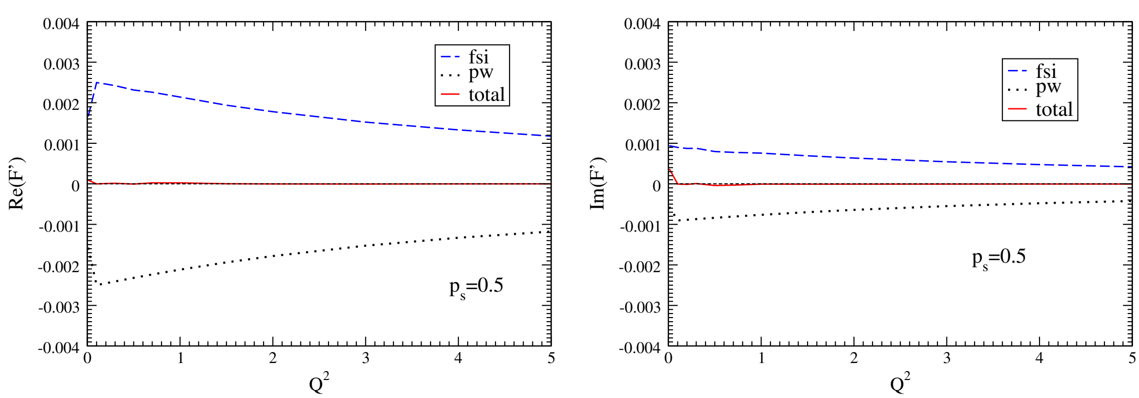

FIG. 7 (color online). Transition EM form factor $F'(Q^2)$ as a function of Q^2 for $p_s = 0.5$. Other parameters and notations are the same as in Fig. 3.

provides as a strong test of a calculation. Indeed, both FSI and PW contributions contain the same initial bound state BS amplitude. At the same time, the FSI contribution contains the scattering state BS amplitude, whereas the PW contribution does not. Their cancellation takes place provided both BS amplitudes—the bound and the scattering ones—as well as the current operator are consistent with each other, i.e. if they are correctly found in the same dynamics.

To illustrate how a violation of this consistency would affect the current conservation, we replaced the final BS amplitude, found for the one-boson exchange kernel, by an ad hoc function,

$$\Gamma_f(k_0, z, k) = \frac{1}{(k_0^2 + a^2)(k^2 + b^2)}, \tag{25}$$

without changing the initial BS amplitude and the current. The transition form factor $F(Q^2)$ calculated with the function (25) for $a^2 = 1.5$, $b^2 = 1$ is shown in Fig. 8. Apparently it has a typical behavior, and nothing indicates that it is a wrong result.

To see that we have displayed in Fig. 9 the transition form factor $F'(Q^2)$ calculated with the same function (25). We see in this figure that F' is different from zero and of the same order as form factor $F(Q^2)$. This means that the EM current calculated with the phenomenological FSI function (25) is not correct, and therefore the form factor $F(Q^2)$ extracted from this current—shown in Fig. 8—is also incorrect.

The study of the numerical stability as a function of the number of Gaussian integration points n_G shows that the sum $F'_{\rm tot} = F'_{\rm FSI} + F'_{\rm PW}$ decreases with n_G. For $n_G = 64$ it is 2 orders of magnitude smaller than each $F'_{\rm FSI}$ and $F'_{\rm PW}$ taken separately and also than the form factor F. This means that there exists a cancellation between $F'_{\rm FSI}$ and $F'_{\rm PW}$, hence the current conservation, with a numerical precision of about 1%. Increasing the value of n_G from 64 to 128 does not improve the result (does not reduce $F'_{\rm tot}$). Apparently, the precision of $F'_{\rm tot}$ is determined by the accuracy of the numeric solutions for the initial and final BS amplitudes.

The numerical results for $F(Q^2)$ and $F'(Q^2)$, calculated with $n_G = 64$, $p_s = 0.1$, are given in Table I. For $Q^2 = 1.5$ the value of $F'_{\rm tot}$ is 1 order of magnitude smaller than F and 2 orders of magnitude smaller than $F'_{\rm FSI}$ and $F'_{\rm PW}$. When Q^2 increases up to $Q^2 = 5$, the cancellation becomes worse and almost disappears, though $F'_{\rm tot}$ is still a few times smaller than $F'_{\rm FSI}$ and $F'_{\rm PW}$. This is related to the fact that

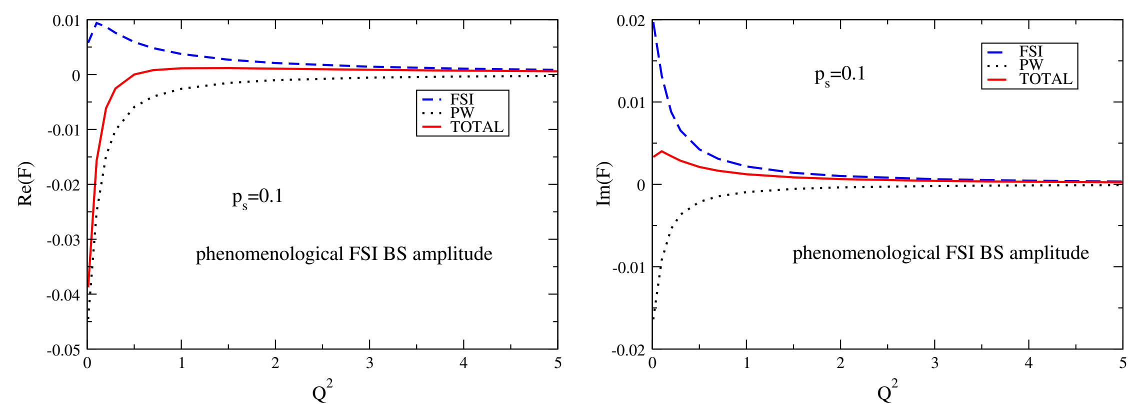

FIG. 8 (color online). Transition EM form factor $F(Q^2)$ as a function of Q^2 for $p_s = 0.1$ calculated with the "phenomenological" FSI function (25). The notations are the same as in Fig. 3.

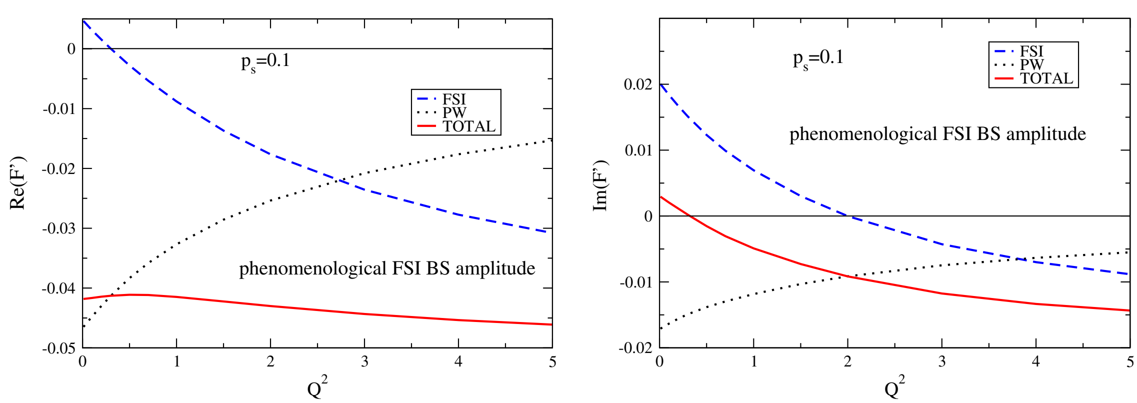

FIG. 9 (color online). Transition EM form factor $F'(Q^2)$ as a function of Q^2 for $p_s = 0.1$ calculated with the phenomenological FSI function (25). The notations are the same as in Fig. 3.

TABLE I. The value $F'(Q^2)$ for FSI, PW, and tot = FSI + PW vs Q^2 in comparison to $F(Q^2)$; $p_s = 0.1$.

Q^2	$F(Q^2)$	$F'_{\rm FSI}(Q^2)$	$F'_{\rm PW}(Q^2)$	$F'_{\rm tot}(Q^2)$
0.01	2.948–03 − 1.571–03	4.703–02 + i1.732–02	−4.648–02 − i1.709–02	5.490–04 + i2.304–04
0.1	1.391–02 − i1.008–02	4.541–02 + i1.641–02	−4.478–02 − i1.633–02	6.268–04 + i8.139–05
0.5	1.047–02 − 7.653–03	3.904–02 + i1.401–02	−3.833–02 − i1.383–02	7.089–04 + i1.848–04
1.	6.640–03 − i4.841–03	3.321–02 + i1.181–02	−3.268–02 − i1.184–02	5.303–04 − i3.634–05
2.	3.573–03 − i2.460–03	2.639–02 + i8.478–03	−2.537–02 − i9.150–03	1.018–03 − i6.726–04
3.	2.362–03 − i1.589–03	2.234–02 + i7.062–03	−2.079–02 − i7.482–02	1.552–03 − i4.204–04
4.	1.718–03 − i1.109–02	1.897–02 + i6.511–03	−1.763–02 − i6.339–02	1.341–03 + i1.721–04
5.	1.446–03 − i8.313–04	1.993–02 + i6.093–03	−1.531–02 − i5.506–02	4.614–03 + i5.873–04

the BS amplitudes were computed in a finite domain of variables k_0, k which, at large values of Q^2, is not enough to ensure enough accurate results for the BS solution.

VII. CONCLUSION

We have presented the first results of the transition electromagnetic form factor for the electrodisintegration of a two-body bound system described by the Bethe–Salpeter equation in Minkowski space. Calculations have been performed in a self-consistent way. The initial (bound state) and final (scattering state) BS amplitudes were found by solving the equation with the method developed in our previous works [15] and an one-boson exchange kernel.

We have shown that, provided the bound and scattering state Bethe–Salpeter amplitudes as well as the operator of EM current are consistent with each other, the electromagnetic current is conserved. If this consistency is destroyed, the conservation is violated. This violation has two consequences.

First, the decomposition of the current in form factors obtains an additional contribution [second term in Eq. (3)] which does not satisfy the equality $J\cdot q = 0$. However, the appearance of this term itself does not make any influence on observables—it does not contribute in the scattering amplitude—due to conservation of the electromagnetic current of the incident electron.

Second, and most important, is the fact that the nonconservation of the calculated current makes it physically meaningless. One cannot extract from a deficient current a reliable transition form factor. It is thus mandatory, in practical calculations, like e.g. in the deuteron electrodisintegration, to check the current conservation. If the form factor, responsible for nonconservation of the current, turns out to be comparable with the physical ones, one can hardly trust the calculated physical form factors, either.

The widely used recipe, consisting of replacing the nonconserved current J_μ by the conserved combination $\tilde{J}_\mu = J_\mu - q_\mu (J\cdot q)/q^2$, hides the problem but does not solve it. This combination $\tilde{J}_\mu$ satisfies tautologically the current conservation for any J_μ, not only for the correct one, and thus offers no any guarantee to the result. With an incorrect current, one cannot find the correct transition form factor, neither from J_μ nor from $\tilde{J}_\mu$.

The current conservation appears as a numerically subtle phenomenon since it manifests itself as a cancellation of large contributions: FSI and PW. To see it unambiguously, the solution of the BS equation should be found with high enough precision.

ACKNOWLEDGMENTS

Part of this work was done at the Institut de Physique Nucléaire d'Orsay (IPNO) during a visit of one of the authors (V. A. K.) funded by the French Embassy in Moscow. V. A. K. is grateful to IPN Orsay for the kind hospitality during his visit.

APPENDIX A: KINEMATICS

As mentioned in Sec. II, it is convenient to carry out these calculations in the system of reference where $p'_0 = p_0$, i.e. $q_0 = 0$. In the elastic case, it coincides with the Breit frame $\vec{p} + \vec{p}' = 0$ and $|\vec{p}| = |\vec{p}'|$, $p'_0 = p_0$.

This system exists in the inelastic case $M' \neq M$, too. Indeed, one can easily check that in the reaction $e + d \to e' + (np)$ the momentum transfer $q^2 = (p - p')^2 = (p_0 - p'_0)^2 - (\vec{p} - \vec{p}')^2$ is always negative. In particular, its maximal value [reached at the minimal value of $s = (M' + m_e)^2$, with m_e the electron mass] is still negative:

$$q^2 \leq -\frac{m_e(M'^2 - M^2)}{M' + m_e} < 0.$$

Therefore, one can find a reference frame where this negative value contains only spatial components, $q^2 = (p - p')^2 = -(\vec{p} - \vec{p}')^2$, so that $p'_0 = p_0$. Moving this frame, without changing the value $q_0 = 0$, one can also make the vectors $\vec{p}$ and $\vec{p}'$ collinear, i.e. either parallel or antiparallel, so that $Q^2 = -q^2 = (\vec{p} - \vec{p}')^2$ is equal either to $(p - p')^2$ (if $\vec{p}$ and $\vec{p}'$ are parallel) or $(p + p')^2$ (if $\vec{p}$ and $\vec{p}'$ are antiparallel), depending on the Q^2 value. We denote $p = |\vec{p}|$ and $p' = |\vec{p}'|$. We will specify below when one should take the plus or minus sign.

In case of an elastic collision $M' = M$, in the Breit frame with $q_0 = 0$ (i.e. $p'_0 = p_0$) and with the initial and final momenta satisfying $\vec{p} + \vec{p}' = 0$ and $|\vec{p}'| = |\vec{p}|$, the scattered system moves in the opposite direction than the incoming one, and this is the only possibility to get a nonzero momentum transfer.

In the inelastic case $M' > M$, still in the reference frame with $p'_0 = p_0$ and with the collinear momenta $\vec{p}$, $\vec{p}'$ (now with $|\vec{p}'| \neq |\vec{p}|$ but $|\vec{p}'| < |\vec{p}|$), there exists a critical momentum transfer Q_c^2. If the momentum transfer Q^2 is smaller than Q_c^2, it is not enough to change the direction of initial momentum $\vec{p}$ into the opposite one. In this kinematics, the final momentum $\vec{p}'$ after collision remains parallel to $\vec{p}$, though with p' smaller than p. However, for $Q^2 > Q_c^2$ the final momentum $\vec{p}'$ changes its direction relative to $\vec{p}$ like in the elastic collision. That is, when Q^2 increases, the final momentum $\vec{p}'$, being first parallel to $\vec{p}$, vanishes and appears again in a direction opposite to $\vec{p}$. When it crosses zero $\vec{p}' = 0$ (provided $p'_0 = p_0$), the corresponding momentum transfer is $Q_c^2 = p^2$. We get in this case

$$p'_0 = p_0 \to \sqrt{p'^2 + M'^2} = \sqrt{p^2 + M^2}$$
$$\to M' = \sqrt{p^2 + M^2}.$$

From the last equality, we find the critical value $Q_c^2 = p^2$ for which $p' = 0$,

$$Q_c^2 = M'^2 - M^2. \tag{A1}$$

In the frame where we perform the calculations ($q_0 = 0$), the kinematical relations

$$(\vec{p}' - \vec{p})^2 = (p' - \sigma p)^2 \tag{A2}$$

$$\vec{k} \cdot \vec{p}' = \sigma k p' z \tag{A3}$$

$$(\vec{p} - \vec{k})^2 = p^2 - 2zpk + k^2 \tag{A4}$$

$$(\vec{p}' - \vec{k})^2 = p'^2 - 2\sigma z p' k + k^2 \tag{A5}$$

$$\sqrt{Q^2} = p - \sigma p' \tag{A6}$$

hold, where we have introduced the "sign" variable σ depending on Q^2 and Q_c^2,

$$\sigma(Q^2, Q_c^2) = \begin{cases} +1 & \text{if } Q^2 < Q_c^2 \\ -1 & \text{if } Q^2 > Q_c^2, \end{cases} \tag{A7}$$

and denote hereafter (abusively) $k = |\vec{k}|$.

From the requirement $\sqrt{M'^2 + p'^2} = \sqrt{M^2 + p^2}$, we find the relation between p and p',

$$p' = \sqrt{M^2 - M'^2 + p^2}$$
$$p = \sqrt{M'^2 - M^2 + p'^2},$$

and so

$$p' = \frac{|Q_c^2 - Q^2|}{2\sqrt{Q^2}}$$
$$p = \frac{Q_c^2 + Q^2}{2\sqrt{Q^2}} \tag{A8}$$

$$p_0 = p'_0 = \frac{\sqrt{((M' - M)^2 + Q^2)((M' + M)^2 + Q^2)}}{2\sqrt{Q^2}}. \tag{A9}$$

APPENDIX B: CALCULATING THE FSI CONTRIBUTION TO THE TRANSITION FORM FACTOR

The contribution of FSI to the transition form factor is given in Eq. (16) as a sum of three terms $F_{3,2,1}$. In their turn, $F_{3,2,1}$ are obtained by integrating over $dk_0 d^3k$ the three functions $f_{3,2,1}$ defined in (13)–(15). We detail in what follows the calculation of these three contributions.

1. F_3 contribution

Let us first consider the F_3 contribution

$$F_3(Q^2) = \frac{i}{(2\pi)^3} \int_0^\infty k^2 dk \int_{-1}^{+1} dz \int_{-\infty}^{+\infty} dk_0 f_3(k_0, z, k)$$

with f_3 given by Eq. (13).

As a function of k_0, f_3 contains six poles in the k_0-variable at the points

$$k_0 = +\varepsilon_{\vec{k}}$$
$$k_0 = -\varepsilon_{\vec{k}}$$
$$k_0 = p_0 + \varepsilon_{\vec{p}-\vec{k}}$$
$$k_0 = p_0 - \varepsilon_{\vec{p}-\vec{k}}$$
$$k_0 = p_0 + \varepsilon_{\vec{p}'-\vec{k}}$$
$$k_0 = p_0 - \varepsilon_{\vec{p}'-\vec{k}}$$

and eventually other singularities resulting from $G(k_0, z, k)$. We subtract and add to f_3 a function h_3 depending on the same variables and having poles only in the variable k_0. That is

$$f_3 = f_3 - h_3 + h_3 = \bar{f}_3 + h_3 \tag{B1}$$

with

$$h_3(k_0,z,k) = \frac{g_1(z,k)}{k_0-\varepsilon_{\vec{k}}} + \frac{g_2(z,k)}{k_0+\varepsilon_{\vec{k}}} + \frac{g_3(z,k)}{k_0-p_0-\varepsilon_{\vec{p}-\vec{k}}} + \frac{g_4(z,k)}{k_0-p_0+\varepsilon_{\vec{p}-\vec{k}}} + \frac{g_5(z,k)}{k_0-p'_0-\varepsilon_{\vec{p}'-\vec{k}}} + \frac{g_6(z,k)}{k_0-p'_0+\varepsilon_{\vec{p}'-\vec{k}}}. \tag{B2}$$

The coefficients g_i, independent of k_0, are determined by imposing that the difference

$$\bar{f}_3 \equiv f_3 - h_3 \tag{B3}$$

is regular in k_0. For instance

$$\lim_{k_0\to\varepsilon_k}(k_0-\varepsilon_k)\bar{f}_3 = 0 \Longleftrightarrow \lim_{k_0\to\varepsilon_k}(k_0-\varepsilon_k)f_3 = g_1(z,k),$$

and similarly for other poles.

This gives

$$\begin{aligned}
g_1(z,k) &= +\frac{G(+\varepsilon_k,z,k)}{2\varepsilon_k[(p_0-\varepsilon_k)^2-\epsilon^2_{p-k}][(p_0-\varepsilon_k)^2-\epsilon^2_{p'-k}]}\\
g_2(z,k) &= -\frac{G(-\varepsilon_k,z,k)}{2\varepsilon_k[(p_0+\varepsilon_k)^2-\epsilon^2_{p-k}][(p_0+\varepsilon_k)^2-\epsilon^2_{p'-k}]}\\
g_3(z,k) &= +\frac{G(p_0+\varepsilon_{p-k},z,k)}{2\varepsilon_{p-k}[(p_0+\varepsilon_{p-k})^2-\varepsilon^2_k](\varepsilon^2_{p-k}-\varepsilon^2_{p'-k})}\\
g_4(z,k) &= -\frac{G(p_0-\varepsilon_{p-k},z,k)}{2\varepsilon_{p-k}[(p_0-\varepsilon_{p-k})^2-\varepsilon^2_k](\varepsilon^2_{p-k}-\varepsilon^2_{p'-k})}\\
g_5(z,k) &= -\frac{G(p_0+\varepsilon_{p'-k},z,k)}{2\varepsilon_{p'-k}[(p_0+\varepsilon_{p'-k})^2-\varepsilon^2_k](\varepsilon^2_{p-k}-\varepsilon^2_{p'-k})}\\
g_6(z,k) &= +\frac{G(p_0-\varepsilon_{p'-k},z,k)}{2\varepsilon_{p'-k}[(p_0-\varepsilon_{p'-k})^2-\varepsilon^2_k](\varepsilon^2_{p-k}-\varepsilon^2_{p'-k})},
\end{aligned} \tag{B4}$$

and the function $\bar{f}_3$ obtains the form

$$\begin{aligned}
\bar{f}_3(k_0,z,k) = {} & \frac{G(k_0,z,k)}{[k_0-\varepsilon_k][k_0+\varepsilon_k][k_0-p_0-\varepsilon_{p-k}][k_0-p_0+\varepsilon_{p-k}][k_0-p_0-\varepsilon_{p'-k}][k_0-p_0+\varepsilon_{p'-k}]}\\
& -\frac{g_1(z,k)}{k_0-\varepsilon_k} - \frac{g_2(z,k)}{k_0+\varepsilon_k} - \frac{g_3(z,k)}{k_0-p_0-\varepsilon_{p-k}} - \frac{g_4(z,k)}{k_0-p_0+\varepsilon_{p-k}} - \frac{g_5(z,k)}{k_0-p_0-\varepsilon_{p'-k}} - \frac{g_6(z,k)}{k_0-p_0+\varepsilon_{p'-k}}.
\end{aligned} \tag{B5}$$

The principal value (PV) integral over k_0 of the remaining integrand h_3 in (B1) vanish in the full integration domain:

$$\mathrm{PV}\int_{-\infty}^{\infty} h_3 dk_0 = 0. \tag{B6}$$

However, in the numerical solution, we restrict the integration domain to a finite interval $k_0 \in [-L,+L]$. The integral (B6) is no longer zero, and a finite volume correction must be taken into account. The integral over the finite domain $[-L,+L]$ of the function h_3 given in (B2) is analytic and reads

$$\begin{aligned}
f_{3,fv}(z,k) \equiv \mathrm{PV}\int_{-L}^{+L} dk_0 h_3(k_0,k,z) = {} & g_1(k,z)\log\left|\frac{L-\varepsilon_k}{L+\varepsilon_k}\right| + g_2(k,z)\log\left|\frac{L+\varepsilon_k}{L-\varepsilon_k}\right|\\
& + g_3(k,z)\log\left|\frac{L-p_0-\varepsilon_{p-k}}{L+p_0+\varepsilon_{p-k}}\right| + g_4(k,z)\log\left|\frac{L-p_0+\varepsilon_{p-k}}{L+p_0-\varepsilon_{p-k}}\right|\\
& + g_5(k,z)\log\left|\frac{L-p_0-\varepsilon_{p'-k}}{L+p_0+\varepsilon_{p'-k}}\right| + g_6(k,z)\log\left|\frac{L-p_0+\varepsilon_{p'-k}}{L+p_0-\varepsilon_{p'-k}}\right|.
\end{aligned} \tag{B7}$$

For the reasons that will become clear latter, we will include the above finite volume contributions,

$$F_{3,fv}(Q^2) = \frac{i}{(2\pi)^3}\int_0^\infty k^2 dk \int_{-1}^1 dz f_{3,fv}(z,k), \tag{B8}$$

in the F_2 contribution, to be discussed in the next section.

The $F_3(Q^2)$ will thus be given by the three-dimensional integral

$$F_3(Q^2) = \frac{i}{(2\pi)^3}\int_0^\infty k^2 dk \int_{-1}^1 dz \int_{-L}^{L} dk_0 \bar{f}_3(k_0,z,k) \tag{B9}$$

with $\bar{f}_3(k_0,z,k)$ defined in (B5).

The integrand in (B9) is by construction a smooth function in k_0. Concerning the integration over the variables k and z, further inspection is needed. For $M' > 2m$ the value $p_0 - \varepsilon_{\vec{k}} - \varepsilon_{\vec{p}'-\vec{k}} = p_0' - \varepsilon_{\vec{k}} - \varepsilon_{\vec{p}'-\vec{k}}$ in the denominator of (B5) can vanish. Indeed, a state with mass $M' > 2m$ can decay in two particles with masses m, and in the moving frame, this implies $p_0' = \varepsilon_{\vec{k}} + \varepsilon_{\vec{p}'-\vec{k}}$. The g_1 and g_6 terms in Eq. (B5) are therefore singular in variable $\vec{k}$. These two singularities cancel each other since an expansion of $\bar{f}_3$ in the variable $\varepsilon_{\vec{k}}$ near $\varepsilon_{\vec{k}} = p_0 - \varepsilon_{\vec{p}'-\vec{k}}$ shows that the result is proportional to $\sim\mathcal{O}((\varepsilon_{\vec{p}'-\vec{k}} - p_0')^0)$. As a consequence, the integrand $\bar{f}_3$ has no additional singularities in k, z and can be safely integrated numerically by standard methods.

2. F_2 contribution

Let us now calculate the contribution

$$F_2(Q^2) = \frac{i}{(2\pi)^3}\int_0^\infty k^2 dk \int_{-1}^{+1} dz \int_{-\infty}^{+\infty} dk_0 f_2(k_0, z, k) \tag{B10}$$

with f_2 given by Eq. (14).

The integration over the k_0-variable can be performed analytically by means of the delta-functions. The result—denoted $\tilde{f}_2$—is expressed in terms of functions g_i defined in (B4) and reads

$$\tilde{f}_2(z,k) = \int_{-\infty}^{+\infty} dk_0 f_2(k_0, z, k) = -i\pi\{g_1(z,k) - g_2(z,k) + g_3(z,k) - g_4(z,k) - g_5(z,k) - g_6(z,k)\}. \tag{B11}$$

As one can see, $\tilde{f}_2(z,k)$ has a similar structure and depends on the same variables as the finite volume corrections $f_{3,fv}(z,k)$ described above in (B7). It is thus natural to include both contributions in the same integrand,

$$\bar{f}_2(z,k) = \tilde{f}_2(z,k) + f_{3,fv}(z,k) = \sum_{i=1}^{6} c_i(k) g_i(k),$$

by introducing the coefficients

$$\begin{aligned}
c_1(k) &= -i\pi + \log\left|\frac{L-\varepsilon_k}{L+\varepsilon_k}\right| \\
c_2(k) &= +i\pi + \log\left|\frac{L+\varepsilon_k}{L-\varepsilon_k}\right| \\
c_3(k) &= -i\pi + \log\left|\frac{L-p_0-\varepsilon_{p-k}}{L+p_0+\varepsilon_{p-k}}\right| \\
c_4(k) &= +i\pi + \log\left|\frac{L-p_0+\varepsilon_{p-k}}{L+p_0-\varepsilon_{p-k}}\right| \\
c_5(k) &= +i\pi + \log\left|\frac{L-p_0-\varepsilon_{p'-k}}{L+p_0+\varepsilon_{p'-k}}\right| \\
c_6(k) &= +i\pi + \log\left|\frac{L-p_0+\varepsilon_{p'-k}}{L+p_0-\varepsilon_{p'-k}}\right|.
\end{aligned} \tag{B12}$$

The F_2 contribution is then given by the two-dimensional integral

$$F_2(Q^2) = \frac{i}{(2\pi)^3}\int_0^\infty k^2 dk \int_{-1}^{+1} dz \bar{f}_2(z,k). \tag{B13}$$

The integral (B13) over the z variable requires some care since both g_1 and g_6 can have pole singularities in z. In contrast to the function $\bar{f}_3$, Eq. (B5), these singularities in $\bar{f}_2$ do not cancel each other. However, they can be integrated analytically over z, so the pole singularities turn into the log ones.

Let us first consider the g_1 term:

$$g_1(z,k) = \frac{G(+\varepsilon_k, z, k)}{2\varepsilon_k[(p_0-\varepsilon_k)^2 - \epsilon_{p-k}^2][(p_0-\varepsilon_k)^2 - \epsilon_{p'-k}^2]}.$$

The denominator vanishes if

$$(p_0-\varepsilon_k)^2 - \epsilon_{p'-k}^2 = 0 \Longleftrightarrow 2p_0\varepsilon_k - M'^2 = 2\vec{p}'\cdot\vec{k} = 2\sigma p' kz$$

with $\sigma = \pm 1$ being the sign function defined in (A7). A singularity in the z-variable would exist at $z = z_0$ given by

$$z_0(k) = \sigma\frac{2p_0\varepsilon_k - M'^2}{2p'k} \tag{B14}$$

provided $|z_0| \le 1$, that is for k in the interval $k_- \le k \le k_+$ with

$$k_\mp = \frac{1}{2}\left|p' \mp p_0\sqrt{1-\left(\frac{2m}{M'}\right)^2}\right|. \tag{B15}$$

Notice, in particular, that this singularity exists only in the inelastic case since $2m < M'$. It is a moving singularity, depending on the value of the second argument k, as well as on the momentum transfer Q^2 and the parameter Q_c^2.

To properly account for this singularity, we split the k-integration interval in three domains,

$$[0,+\infty] = [0,k_-]\cup[k_-,k_+]\cup[k_+,+\infty],$$

as well as the corresponding integral (B13)

$$F_2(Q^2) = I_1 + I_2 + I_3$$

with

$$I_1 = \frac{i}{(2\pi)^3} \int_0^{k_-} k^2 dk \int_{-1}^{+1} dz \bar{f}_2(z,k)$$
$$I_2 = \frac{i}{(2\pi)^3} \int_{k_-}^{k_+} k^2 dk \int_{-1}^{+1} dz \bar{f}_2(z,k)$$
$$I_3 = \frac{i}{(2\pi)^3} \int_{k_+}^{+\infty} k^2 dk \int_{-1}^{+1} dz \bar{f}_2(z,k):$$

(i) The integrals over $[0, k_-]$ and $[k_+, +\infty]$ have a smooth integrand in both variables, and the contributions I_1 and I_3 can be computed by standard methods.

(ii) The integral over $[k_-, k_+]$ has a singularity at $z = z_0$. The integrand is regularized by using the usual subtraction procedure

$$g_1(z,k) = \left[g_1(z,k) - \frac{g_1'(k)}{z - z_0}\right] + \frac{g_1'(k)}{z - z_0} \tag{B16}$$

with

$$g_1'(k) = \lim_{z\to z_0}(z - z_0)g_1(z,k) = \mathrm{Res}[g_1(z,k)]_{z=z_0}.$$

To compute this quantity, we take z from

$$\begin{aligned}\epsilon^2_{p'-k} &= m^2 + k^2 + p'^2 - 2\sigma p' kz \\ &= \epsilon_k^2 + p_0^2 - M'^2 - 2\sigma p' kz \\ \Leftrightarrow z &= \sigma \frac{\epsilon_k^2 + p_0^2 - M'^2 - \epsilon^2_{p'-k}}{2p'k},\end{aligned}$$

and together with (B14) we have

$$\begin{aligned}z - z_0 &= \sigma \frac{\epsilon_k^2 + p_0^2 - \epsilon^2_{p'-k} - 2p_0\epsilon_k}{2p'k} \\ &= \sigma \frac{(p_0 - \epsilon_k)^2 - \epsilon^2_{p'-k}}{2p'k},\end{aligned}$$

and so

$$(z - z_0)g_1(z,k) = \sigma \frac{G(+\epsilon_k, z, k)}{4p'k\epsilon_k[(p_0 - \epsilon_k)^2 - \epsilon^2_{p-k}]}.$$

We get in this way the residue

$$g_1'(k) = \frac{\sigma}{4p'k\epsilon_k}\left\{\frac{G(\epsilon_k, z, k)}{(p_0 - \epsilon_k + \epsilon_{p-k})(p_0 - \epsilon_k - \epsilon_{p-k})}\right\}_{z=z_0}. \tag{B17}$$

The g_6 term

$$g_6(z,k) = \frac{G(p_0 - \epsilon_{p'-k}, z, k)}{2\epsilon_{p'-k}(p_0 - \epsilon_{p'-k} + \epsilon_k)(p_0 - \epsilon_{p'-k} - \epsilon_k)(\epsilon_{p-k} + \epsilon_{p'-k})(\epsilon_{p-k} - \epsilon_{p'-k})}$$

has the same singularity at $z = z_0$ as g_2 and has been treated in the same way by subtraction,

$$g_6(z,k) = \left[g_6(z,k) - \frac{g_6'(k)}{z - z_0}\right] + \frac{g_6'(k)}{z - z_0}.$$

By a similar calculation, we find

$$g_6'(k) = -g_1'(k).$$

Finally, once the singular terms g_2 and g_6 are regularized, the I_2 contribution is given by two integrals, corresponding to the two terms in the subtraction (B16),

$$I_2 = I_2' + I_2'' \tag{B18}$$

$$I_2' = \frac{i}{(2\pi)^3} \int_{k_1}^{k_2} k^2 dk \int_{-1}^{+1} dz \bar{f}_2'(z,k) \tag{B19}$$

$$I_4'' = \frac{i}{(2\pi)^3} \int_{k_1}^{k_2} k^2 dk c_1'(k) g_1'(k) \log\left|\frac{1 - z_0}{1 + z_0}\right|, \tag{B20}$$

where

$$\bar{f}'_2 = \bar{f}_2 - c'_1(k)\frac{g'_1(k)}{z - z_0} \quad \text{(B21)}$$

is a regular integrand, g'_1 the residue (B17), and $c'_1(k) \equiv c_1(k) - c_6(k)$ with c_i given in (B12).

3. F_1 contribution

The F_1 contribution is given by

$$F_1(Q^2) = \frac{i}{(2\pi)^3}\int dk_0 dz k^2 dk f_1(k_0, z, k). \quad \text{(B22)}$$

According to definition (15) of f_1, this function contains three contributions; each of them contains the product of two delta-functions. It turns out that the nonzero contribution results from the second term only.

We substitute (15) in (B22), integrate this term over k_0 analytically, and find

$$\tilde{f}_1(z,k) = \int dk_0 f_1$$
$$= -\frac{\pi^2}{4\varepsilon_{\vec{k}}\varepsilon_{\vec{p}'-\vec{k}}}\frac{G(\varepsilon_{\vec{k}}, z, k)}{(\varepsilon^2_{\vec{p}'-\vec{k}} - \varepsilon^2_{\vec{p}-\vec{k}})}\delta(p_0 - \varepsilon_{\vec{k}} - \varepsilon_{\vec{p}'-\vec{k}}) \quad \text{(B23)}$$

to be integrated then over z and k:

$$F_1(Q^2) = \frac{i}{(2\pi)^3}\int dz k^2 dk \tilde{f}_1(z,k). \quad \text{(B24)}$$

In general, the denominator $(\varepsilon^2_{\vec{p}'-\vec{k}} - \varepsilon^2_{\vec{p}-\vec{k}})$ in (B23) can vanish. However, one can check that if the argument of the delta function $(p_0 - \varepsilon_{\vec{k}} - \varepsilon_{\vec{p}'-\vec{k}})$ is zero—providing a nonzero contribution—the denominator $(\varepsilon^2_{\vec{p}'-\vec{k}} - \varepsilon^2_{\vec{p}-\vec{k}})$ is not zero. Therefore, there is no singularity from this denominator.

Concerning the first and the third terms in (15), the arguments of the δ-functions in them vs k_0 can also cross zero. However, in the first term, they cannot be zero simultaneously. In the third term, the arguments of the δ-functions can be zero simultaneously. That is, after integration over k_0, we obtain the delta-function $\sim\delta(E^2_{\vec{p}-\vec{k}} - E^2_{\vec{p}'-\vec{k}})$ which could contribute. However, a more careful analysis shows that its contribution is in fact zero. For this aim we represent the delta-function as

$$\delta(x) = \frac{\epsilon}{\pi(x^2 + \epsilon^2)}$$

and integrate over both dz and dk. Taking after that the limit $\epsilon \to 0$, we find a zero result. Care must be taken, however, to not take this limit too early, i.e. before integration, since we will get in this way a wrong nonzero contribution.

After integrating analytically over dz in (B24) by means of the delta-function, we obtain

$$\bar{f}_1(k) = \int_{-1}^{1}\tilde{f}_1 dz$$
$$= \pi^2\theta(1 - |z_0|)\frac{1}{2p'k\varepsilon_{\vec{k}}}\frac{G(k_0 = \varepsilon_{\vec{k}}, k)}{(\varepsilon^2_{\vec{p}-\vec{k}} - \varepsilon^2_{\vec{p}'-\vec{k}})}\Bigg|_{z=z_0}, \quad \text{(B25)}$$

where the value of z_0 is given by (B14).

Finally, the integration over k is reduced, due to the theta-function $\theta(1 - |z_0|)$, to the interval $k \in [k_-, k_+]$ with $k_\mp$ given (B15). That is

$$F_1(Q^2) = \frac{i}{(2\pi)^3}\int_{k_-}^{k_+} k^2 dk \bar{f}_1(k). \quad \text{(B26)}$$

As a test, we carry out an independent calculation for $\Gamma_i = \Gamma_f = 1$, using Feynman parametrization for the 4D integrals. In this way, we find the imaginary part which coincides with the contribution (B26).

4. About the function $G(k_0,z,k)$

An important quantity used in our formalism, which contains all the information about the initial and final state vertex amplitudes, is the function $G(k_0, z, k)$, defined in Eq. (11):

$$G(k_0, z, k) = \frac{(p_0 - k_0)}{p_0}\Gamma_i\left(\frac{p}{2} - k, p\right)\Gamma_f\left(\frac{p'}{2} - k, p'\right).$$

Some useful relations concerning this function are specified in what follows.

The initial state amplitude $\Gamma_i(k_0, k)$—we denote $|\vec{k}| = k$—is computed in the reference frame where $\vec{p} = 0$. In an arbitrary frame, $\Gamma_i(k_0, k)$ is written as $\Gamma_i(\tilde{k}_0, \tilde{k})$ where

$$\tilde{k}_0 = \frac{k\cdot p}{M}$$
$$\tilde{k} = \sqrt{\frac{(k\cdot p)^2}{M^2} - k^2}.$$

The arguments of $\Gamma_i(\frac{p}{2} - k, p)$ are obtained from these relations by the shift $k \to \frac{p}{2} - k$. The same happens for the final state vertex amplitude $\Gamma_f(\frac{p'}{2} - k, p')$. The function $G(k_0, z, k)$ should be therefore understood as

$$G(k_0, z, k) = \frac{(p_0 - k_0)}{p_0}\Gamma_i(\tilde{k}_0, \tilde{k})\Gamma_f(\tilde{k}'_0, \tilde{k}') \quad \text{(B27)}$$

with, after performing the shift $k \to \frac{p}{2} - k$, the arguments given by

$$\tilde{k}_0 = \frac{M}{2} - \frac{k\cdot p}{M}$$
$$\tilde{k} = \sqrt{\frac{(k\cdot p)^2}{M^2} - k^2}.$$

Written in more detail,

$$\begin{aligned}
\tilde{k}_0 &= \frac{M}{2} - \frac{k_0 p_0 - kpz}{M}\\
\tilde{k} &= \sqrt{\frac{(k_0 p_0 - kpz)^2}{M^2} - k_0^2 + \vec{k}^2}\\
\tilde{k}'_0 &= \frac{M'}{2} - \frac{k_0 p_0 - \vec{k}\cdot\vec{p}'}{M'}\\
\tilde{k}' &= \sqrt{\frac{(k_0 p_0 - \vec{k}\cdot\vec{p}')^2}{M'^2} - k_0^2 + \vec{k}^2}.
\end{aligned} \tag{B28}$$

We remind the reader that the sign in the scalar product $\vec{k}\cdot\vec{p}'$ is given by (A3) and the values of p, p' and p_0 are defined by Eqs. (A8) and (A9). They all depend on the value of Q^2.

The initial (bound state) solution Γ_i is normalized so that the elastic EM form factor at $Q^2 = 0$ is 1. There is no any uncertainty in the normalization of the scattering state solution Γ_f determined by the inhomogeneous BS equation. One should also take into account that the solution found in Ref. [15] was the partial wave amplitude F_0 related to the full amplitude by

$$\Gamma_f = 16\pi \sum_{l=0}^{\infty} F_l P_l(\cos\theta)$$

and that for the S-wave the function Γ_f in (B27) is related to our solution F_0 obtained in Ref. [15] by

$$\Gamma_f = 16\pi F_0. \tag{B29}$$

APPENDIX C: CALCULATING THE PW CONTRIBUTION TO THE TRANSITION FORM FACTOR

We carry out here the part of the integration over $d\Omega_{\vec{p}_s}$ and d^4k in the form factor (18) that can be done analytically. Integrating first over $d\Omega_{\vec{p}_s}$ in the frame $\vec{p}' = 0$, $p'_0 = M' = 2\sqrt{m^2 + \vec{p}_s^2}$, we get

$$\begin{aligned}
\int \delta^{(4)}\left(k - p_s - \frac{p'}{2}\right)\frac{d\Omega_{\vec{p}_s}}{4\pi} &= \int \delta\left(k_0 - \frac{M'}{2}\right)\frac{1}{p_s^2}\delta(|\vec{k}| - p_s)\delta^{(2)}(\Omega_{\vec{k}} - \Omega_{\vec{p}_s})\frac{d\Omega_{\vec{p}_s}}{4\pi}\\
&= \frac{1}{4\pi p_s^2}\delta\left(k_0 - \frac{M'}{2}\right)\delta(|\vec{k}| - |\vec{p}_s|).
\end{aligned}$$

In an arbitrary frame, k_0 and $|\vec{k}|$ are rewritten as

$$k_0 \to \frac{k\cdot p'}{M'}, \qquad |\vec{k}| = \sqrt{k_0^2 - k^2} \to \sqrt{\frac{(k\cdot p')^2}{M'^2} - k^2}.$$

Explicitly, in the frame where $p'_0 = p_0$,

$$\begin{aligned}
\frac{k\cdot p'}{M'} &= \frac{k_0 p_0 - \sigma z|\vec{k}|p'}{M'}\\
\sqrt{\frac{(k\cdot p')^2}{M'^2} - k^2} &= \sqrt{\frac{(k_0 p_0 - \sigma z|\vec{k}|p')^2}{M'^2} - k_0^2 + |\vec{k}|^2},
\end{aligned}$$

where the sign σ is defined in (A7).

After these transformations, the form factor Eq. (18) obtains the form

$$F_{pw} = -\int \frac{(p_0 - k_0)}{p_0}\frac{\Gamma_i(\frac{p}{2} - k, p)}{[(p-k)^2 - m^2 + i\epsilon]}\delta\left(\frac{(k_0 p_0 - \sigma zkp'}{M'} - \frac{M'}{2}\right)\delta\left(\sqrt{\frac{(k_0 p_0 - \sigma kp')^2}{M'^2} - k_0^2 + k^2} - p_s\right)\frac{dk_0 k^2 dk dz}{2p_s^2}. \tag{C1}$$

Equation (C1) contains two delta-functions and integration over three variables k_0, k, and z. By means of the delta-functions, we integrate over the variables k_0 and z. Denote the arguments of the first and second delta-functions by $\arg_1$ and $\arg_2$ respectively. Then the conditions that the arguments of both deltas equal to zero give the system of equations $\arg_1 = 0, \arg_2 = 0$ which we solve relative to k_0 and z. We find that $k_0 = \varepsilon_k = \sqrt{m^2 + k^2}$ and $z = z_0$ with the value of z_0 given by (B14). Remember that $|z_0| \leq 1$ if $k \in [k_-, k_+]$ with $k_\mp$ given by (B15).

Calculating an integral containing the delta-function, one should divide the result by the derivative, over the integration variable, of the argument of the delta-function. Since the integral (C1) contains two delta-functions, we have to divide the result by the product $d_1 d_2$ of derivatives $d_1 = \arg_1'$, $d_2 = \arg_2'$ of the arguments of both delta-functions. Each derivative depends on the order of calculation (first over k_0, then over z or in the opposite order), though the final result, which is determined by their product, is the same. That is

$$d_1 d_2 = \arg'_{1,k_0} \arg'_{2,z} = \arg'_{1,z} \arg'_{2,k_0} = \frac{k \varepsilon_k p'}{M' p_s},$$

provided $\arg_1 = \arg_2 = 0$.

Finally, the integral over z is reduced to

$$\int_{-1}^{1} \ldots \delta(z - z_0) dz dk \sim \int \ldots \theta(1 - |z_0|) dk = \int_{k_-}^{k_+} \ldots dk.$$

Thus, after integration over k_0 and z, we obtain the integral (19) over k in the limits determined by the condition $|z_0| \leq 1$ and given by Eq. (B15).

[1] E. E. Salpeter and H. Bethe, Phys. Rev. **84**, 1232 (1951).
[2] G. C. Wick, Phys. Rev. **96**, 1124 (1954).
[3] J. Carbonell and V. A. Karmanov, Few-Body Syst. **49**, 205 (2011).
[4] S. S. Bondarenko, V. V. Burov, and E. P. Rogochaya, Few Body Syst. **49**, 121 (2011); Phys. Lett. B **705**, 264 (2011); JETP Lett. **94**, 738 (2012).
[5] K. Kusaka and A. G. Williams, Phys. Rev. D **51**, 7026 (1995).
[6] K. Kusaka, K. Simpson, and A. G. Williams, Phys. Rev. D **56**, 5071 (1997).
[7] N. Nakanishi, Phys. Rev. **130**, 1230 (1963).
[8] V. A. Karmanov and J. Carbonell, Eur. Phys. J. A **27**, 1 (2006).
[9] J. Carbonell and V. A. Karmanov, Eur. Phys. J. A **27**, 11 (2006).
[10] V. Sauli, J. Phys. G **35**, 035005 (2008).
[11] T. Frederico, G. Salmè, and M. Viviani, Phys. Rev. D **89**, 016010 (2014).
[12] G. Salmè, T. Frederico, and M. Viviani, Few-Body Syst. **55**, 693 (2014); arXiv:1402.1656.
[13] J. Carbonell, V. A. Karmanov, and M. Mangin-Brinet, Eur. Phys. J. A **39**, 53 (2009).
[14] T. Frederico, G. Salmè, and M. Viviani, Phys. Rev. D **85**, 036009 (2012).
[15] J. Carbonell and V. A. Karmanov, Phys. Lett. B **727**, 319 (2013); Phys. Rev. D **90**, 056002 (2014).
[16] V. A. Karmanov, Few-Body Syst. **55** (2014) 545.
[17] T. Frederico, G. Salmè, and M. Viviani, arXiv:1504.01624.
[18] J. Carbonell and V. A. Karmanov, Few-Body Syst. **55**, 687 (2014).
[19] V. A. Karmanov and J. Carbonell, Workshop LC2014, Raleigh, NC, 2014 (to be published), arXiv:1412.0590.
[20] J. Adams, Jr., J. W. V. Orden, and F. Gross, Nucl. Phys. **A640**, 391 (1998).
[21] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (Dover, New York, 1980).