SU(3) in D decays: From 30% symmetry breaking to 10^{-4} precision

Michael Gronau

Physics Department, Technion-Israel Institute of Technology, Haifa 32000, Israel

Flavor SU(3) symmetry, including 30% first-order SU(3) breaking, has been shown to describe adequately a vast amount of data for charmed meson decays to two pseudoscalar mesons and to a vector and a pseudoscalar meson. We review a recent dramatic progress achieved by applying a high-order perturbation expansion in flavor SU(3) breaking and treating carefully isospin breaking. We identify a class of U-spin related D^0 decays to pairs involving charged pseudoscalar or vector mesons, for which high-precision nonlinear amplitude relations are predicted. Symmetry-breaking terms affecting these relations are fourth-order U-spin breaking and terms which are first order in isospin breaking and second order in U-spin breaking. The predicted relations are shown to hold within experimental errors at a precision varying between 10^{-3} and 10^{-4}, in agreement with estimates of high-order terms. Improved branching ratio measurements for $D^0 \to K^+\rho^-, K^{*+}\pi^-$ and for decay modes involving three other pairs of charged pseudoscalar and vector mesons could further sharpen two of these precision tests. We also obtain amplitude relations for D^0 decays to pairs of neutral pseudoscalar mesons, and relations for rate asymmetries between decays involving K_S^0 and K_L^0, which hold up to second-order U-spin breaking at a level of several percent.

I. INTRODUCTION

A useful tool for studying hadronic decay amplitudes of charmed mesons is approximate flavor SU(3) symmetry. First-order symmetry-breaking corrections in amplitudes, due to the light quark mass term in the Standard Model Lagrangian, are expected to be of order $(m_s - m_{u,d})/\Lambda_{\rm QCD} \sim f_K/f_\pi - 1 \sim 0.2$–$0.3$.

In the seventies, shortly after the discovery of charm, SU(3) group theory has been used to obtain amplitude relations for charmed meson decays into pairs involving two pseudoscalar mesons or a pseudoscalar and a vector meson [1]. This study was extended in the early nineties to include numerous first-order SU(3) breaking terms one of which was assumed to dominate over the others [2,3]. A diagrammatic approach [4], equivalent to SU(3) group theory, has been developed and applied to fit data for branching ratios as they have been accumulated [5]. Assumptions made in these studies about SU(3) breaking were often model-dependent. Other studies of these decay processes went much beyond SU(3) by assuming factorization of hadronic amplitudes [6]. A recent SU(3) fit to current data of charmed meson decays to two pseudoscalar mesons worked reasonably well when including two of the numerous first-order SU(3) breaking terms (of order 30%) available in a group theoretical approach [7].

While flavor SU(3), including 30% first-order symmetry breaking, has been shown to describe adequately a vast amount of data of hadronic decays of charmed mesons, this has not provided a precision test. For this matter one would hope to develop a perturbative expansion in reasonably small SU(3) breaking parameters, in which only high-order

symmetry-breaking terms survive certain relations among amplitudes. A small step in this direction was made in Ref. [8], searching for linear relations among amplitudes for two-body and quasi two-body charmed meson decays in which first-order SU(3) breaking terms cancel. Testing these relations, expected to hold within several percent, requires in most cases measuring relative strong phases between amplitudes which is a highly challenging task.

The purpose of this paper is to review, expand and improve new results obtained in recent work published in two short reports [9,10] applying a high-order perturbation expansion in flavor SU(3) breaking and treating carefully isospin breaking. We will identify a class of D^0 decays, for which high-precision nonlinear relations among magnitudes of amplitudes hold. The lowest-order symmetry-breaking terms affecting these relations will be shown to be fourth-order SU(3) breaking terms and terms which are first order in isospin breaking and second order in SU(3) breaking [11].

One major motivation for this work is searching for signals of new physics. Very precise relations as discussed here, which would fail at some high-order flavor symmetry breaking, could provide such signatures. For great convenience we will use U-spin, an SU(2) subgroup of flavor SU(3), rather than applying the full SU(3) group structure combining U-spin and isospin operators. Our U-spin expansion is, of course, consistent with SU(3) expansions in Refs. [3,7,8].

In Sec. II we identify sets of four two-particle final states in D^0 decays, each consisting of pairs involving charged pseudoscalar and vector mesons. In a given set two of these states and a linear combination of the other two form a U-spin triplet, playing an important role in $D^0 - \bar{D}^0$

mixing. Our discussion in Secs. III–VII studies in detail one of these sets denoted by $D^0 \to P^+P^-$ consisting of $D^0 \to \pi^+K^-, K^+\pi^-, K^+K^-, \pi^+\pi^-$. We derive U-spin symmetry relations for these processes in Sec. III and study first-order and arbitrary-order U-spin breaking corrections in Secs. IV and V, respectively. In Sec. VI we obtain a high-precision relation, obeyed up to fourth-order U-spin breaking, for ratios of amplitudes of the above four processes. Section VII investigates first-order isospin-breaking terms occurring in this relation, showing that they are also suppressed by factors associated with second-order U-spin breaking. Sections VIII, IX, and X discuss experimental tests in $D^0 \to P^+P^-$, $D^0 \to V^+P^-$ and $D^0 \to P^+V^-$, respectively, where $V^\pm$ denote charged vector mesons. In Sec. XI we study D^0 decays into pairs of neutral pseudoscalar mesons, deriving amplitude relations and $K_S^0 - K_L^0$ rate asymmetry relations which hold up to second-order U-spin breaking. A short conclusion is given in Sec. XII.

II. D^0 DECAYS TO TWO-BODY $U = 1$ STATES

An SU(2) subgroup of flavor SU(3) that is useful for studying charmed mesons is U-spin [12]. The quark pair (d, s) behaves like a doublet under this group while the u quark and the heavier c, b and t quarks are singlets. U-spin symmetry leads to interesting relations among amplitudes of hadronic D decays [13,14]. It also implies the vanishing of $D^0 - \bar{D}^0$ mixing up to second-order U-spin breaking [15], which underlies the vanishing of $D^0 - \bar{D}^0$ mixing within full flavor SU(3) [16]. This behavior of $D^0 - \bar{D}^0$ mixing under U-spin has been shown to follow from a cancellation up to second-order U-spin breaking of mixing contributions from intermediate U-spin triplet states. The high-order U-spin breaking perturbation expansion that will be studied in this paper works well, as we will show, for these two-body or quasi two-body D^0 decays to $U = 1$ states.

One class of $U = 1$ two-body states involves pairs of opposite charge pseudoscalar or vector mesons. Our study will focus on these final states for which a high-order U-spin breaking expansion is applicable. Another class of processes involves decay into pairs of neutral mesons. In this case two-body final states do not have well-defined values of U-spin. Instead, linear superpositions of final states have $U = 1$. Consequently, in these decays a U-spin expansion works well for certain linear combinations of decay amplitudes.

We start by classifying single meson states of positive or negative charge as doublets of U-spin. Since a pair of d and s quark and their antiquarks form two U-spin doublets, (d, s) and $(\bar{s}, -\bar{d})$, one has two doublets of pseudoscalar mesons

$$P^+ = \begin{pmatrix} K^+ \\ -\pi^+ \end{pmatrix} \equiv \begin{pmatrix} u\bar{s} \\ -u\bar{d} \end{pmatrix},$$

$$P^- = \begin{pmatrix} \pi^- \\ K^- \end{pmatrix} \equiv \begin{pmatrix} d\bar{u} \\ s\bar{u} \end{pmatrix}, \tag{1}$$

and two doublets of vector mesons

$$V^+ = \begin{pmatrix} K^{*+} \\ -\rho^+ \end{pmatrix}, \qquad V^- = \begin{pmatrix} \rho^- \\ K^{*-} \end{pmatrix}. \tag{2}$$

One can then form two-particle states of charge zero in four different forms, P^+P^-, V^+P^-, P^+V^- and V^+V^-.

In the next several sections we will study the four processes $D^0 \to P^+P^-, P^+P^- = \pi^+K^-, K^+\pi^-, \pi^+\pi^-, K^+K^-$, for which the most precise data exist. This discussion is also applicable to the other three sets of processes, $D^0 \to V^+P^-, P^+V^-$ and $D^0 \to V^+V^-$. For $D^0 \to V^+V^-$ one may, in principle, treat separately S, P and D-wave amplitudes, or amplitudes for longitudinal polarizations, and for transverse polarizations which are mutually parallel and perpendicular to each other. We will not discuss further these latter challenging decay modes for which no branching ratios have been measured [17].

III. U-SPIN SYMMETRY LIMIT

As a starting point we derive amplitude relations in the U-spin symmetry limit. The four possible two-particle states $|P^+P^-\rangle$ can be written in the form of three U-spin triplet states and one singlet state:

$$|K^+\pi^-\rangle = |1, 1\rangle,$$

$$|\pi^+K^-\rangle = -|1, -1\rangle,$$

$$\frac{1}{\sqrt{2}}|K^+K^- - \pi^+\pi^-\rangle = |1, 0\rangle, \tag{3}$$

$$\frac{1}{\sqrt{2}}|K^+K^- + \pi^+\pi^-\rangle = |0, 0\rangle. \tag{4}$$

The charm-changing weak Hamiltonian has a simple transformation property under U-spin. Its three pieces responsible for Cabibbo-favored (CF), singly Cabibbo-suppressed (SCS) and doubly Cabibbo-suppressed (DCS) decays transform, when normalized suitably, like three components of a U-spin triplet operator denoted $(U = 1, U_3 = -1, 0, +1)$:

$$H_W^{\text{CF}} = \frac{G_F}{\sqrt{2}}\cos^2\theta_C(\bar{s}c)(\bar{u}d) = -\cos^2\theta_C(1, -1),$$

$$H_W^{\text{SCS}} = \frac{G_F}{\sqrt{2}}\cos\theta_C\sin\theta_C[(\bar{s}c)(\bar{u}s) - (\bar{d}c)(\bar{u}d)]$$
$$= \sqrt{2}\cos\theta_C\sin\theta_C(1, 0),$$

$$H_W^{\text{DCS}} = -\frac{G_F}{\sqrt{2}}\sin^2\theta_C(\bar{d}c)(\bar{u}s) = -\sin^2\theta_C(1, +1). \tag{5}$$

We have suppressed the chiral structure of V-A operators, using $V_{ud} = V_{cs} = \cos\theta_C, V_{us} = -V_{cd} = \sin\theta_C$ for Cabibbo-Kobayashi-Maskawa (CKM) matrix elements.

Charmed meson decays are dominated by real CKM matrix elements associated with the first two quark families. For the most part we will be using a parametrization of the CKM matrix up to terms which are fourth order in $\lambda \equiv V_{us}$ [17]. In Sec. VI we will discuss the effect of higher-order terms in λ on four ratios of amplitudes studied in this section.

Virtual b quarks in penguin amplitudes may lead to tiny direct CP asymmetries in SCS decays of order $(\alpha_s(m_c^2)/\pi)(|V_{cb}V_{ub}|/|V_{cs}V_{us}|) \sim 10^{-4}$ [18], depending on the final state. No CP asymmetries of this order are expected in CF and DCS decays. An order of magnitude larger asymmetries may occur in SCS decays which are subject to potential penguin amplitude enhancement [19]. No CP asymmetries at these small levels have been measured so far. We will seek amplitude relations which hold at this high precision but not at higher accuracy. Errors in amplitudes are half the errors measured in decay rates. Thus we will neglect in our discussion direct CP asymmetries, assuming that branching ratios for $D^0 \to f$ are given by averages measured for decay processes and their CP conjugates,

$$\mathcal{B}(D^0 \to f) = \mathcal{B}(D^0 \to f)_{\text{CPav}}$$
$$\equiv \frac{1}{2}[\mathcal{B}(D^0 \to f) + \mathcal{B}(\bar{D}^0 \to \bar{f})]. \quad (6)$$

The D^0 is a U-spin singlet. Denoting hadronic matrix elements in the U-spin symmetry limit by a superscript (0) and defining a reduced matrix element, $A \equiv \langle 1, U_3|(1, U_3)|0, 0\rangle$, one obtains from Eqs. (3) and (5)

$$\frac{\langle \pi^+ K^-|H_W^{\text{CF}}|D^0\rangle^{(0)}}{\cos^2\theta_C} = A, \quad (7)$$

$$\frac{\langle K^+ K^- - \pi^+ \pi^-|H_W^{\text{SCS}}|D^0\rangle^{(0)}}{\cos\theta_C \sin\theta_C} = 2A, \quad (8)$$

$$\frac{\langle K^+ \pi^-|H_W^{\text{DCS}}|D^0\rangle^{(0)}}{-\sin^2\theta_C} = A. \quad (9)$$

Equation (4) leads to

$$\langle K^+ K^- + \pi^+ \pi^-|H_W^{\text{SCS}}|D^0\rangle^{(0)} \propto \langle 0, 0|(1, 0)|0, 0\rangle = 0. \quad (10)$$

Using (8), this implies

$$\frac{\langle K^+ K^-|H_W^{\text{SCS}}|D^0\rangle^{(0)}}{\cos\theta_C \sin\theta_C} = \frac{\langle \pi^+ \pi^-|H_W^{\text{SCS}}|D^0\rangle^{(0)}}{-\cos\theta_C \sin\theta_C} = A. \quad (11)$$

Thus, the four amplitudes in (7), (9), and (11) denoted by the decay final state, $A(f) \equiv \langle f|H_W|D^0\rangle$ have simple ratios in the U-spin symmetry limit [1],

$$A^{(0)}(\pi^+ K^-): A^{(0)}(K^+ K^-): A^{(0)}(\pi^+ \pi^-): A^{(0)}(K^+ \pi^-)$$
$$= 1: \tan\theta_C: -\tan\theta_C: -\tan^2\theta_C. \quad (12)$$

We note that a derivation of the two ratios, $A^0(K^+\pi^-)/A^0(\pi^+K^-) = -\tan^2\theta_C$ and $A^0(\pi^+\pi^-)/A^0(K^+K^-) = -1$, uses only symmetry under $d \leftrightarrow s$ reflection [14], implying

$$\langle \pi^+ K^-|(\bar{s}c)(\bar{u}d)|D^0\rangle = \langle K^+ \pi^-|(\bar{d}c)(\bar{u}s)|D^0\rangle,$$
$$\langle K^+ K^-|(\bar{s}c)(\bar{u}s) - (\bar{d}c)(\bar{u}d)|D^0\rangle = -\langle \pi^+ \pi^-|(\bar{s}c)(\bar{u}s)$$
$$- (\bar{d}c)(\bar{u}d)|D^0\rangle, \quad (13)$$

and does not require full SU(2) U-spin symmetry. This full symmetry is required for the relation between these two pairs of processes.

IV. FIRST-ORDER U-SPIN BREAKING

First-order U-spin breaking corrections to the amplitudes (7), (9), and (11) are obtained by multiplying the weak Hamiltonian or the final states by an $s - d$ spurion mass operator, $M_{\text{Ubrk}} \propto (\bar{s}s) - (\bar{d}d)$ [2]:

$$\langle f|H_{\text{eff}}|D^0\rangle^{(1)} = \langle f|H_{\text{eff}}M_{\text{Ubrk}}|D^0\rangle + \langle M_{\text{Ubrk}}f|H_{\text{eff}}|D^0\rangle. \quad (14)$$

While for CF and DCS decays one has simply

$$H_{\text{eff}}^{\text{CF,DCS}}M_{\text{Ubrk}} = H_W^{\text{CF,DCS}}M_{\text{Ubrk}}, \quad (15)$$

the effective Hamiltonian for SCS decays obtains at first order an additional nonperturbative $s + d$ penguin term P_{s+d} due to an $s - d$ mass difference [20]:

$$H_{\text{eff}}^{\text{SCS}}M_{\text{Ubrk}} = H_W^{\text{SCS}}M_{\text{Ubrk}} + P_{s+d}. \quad (16)$$

The $U = 0$ penguin amplitude in SCS decays interferes with opposite signs with the $U = 1$ tree amplitudes in $D^0 \to K^+ K^-$ and in $D^0 \to \pi^+ \pi^-$. This may potentially increase the first amplitude and decrease the second. This effect of the penguin amplitude has been pointed out very early in Refs. [21–23], and has been studied recently in Refs. [20,24,25] with its implication on CP asymmetries in these processes.

Equations (15) and (16) imply different first-order U-spin breaking corrections in $D^0 \to \pi^+ K^-$ and $D^0 \to K^+ \pi^-$, on the one hand, and in $D^0 \to K^+ K^-$ and $D^0 \to \pi^+ \pi^-$, on the other. We will show now that the corrections within each of these two pairs of processes have equal magnitudes and opposite signs when normalized by the corresponding U-spin symmetric amplitudes. We will first follow a full SU(2) U-spin argument presented in [9],

and then derive this result in a simpler manner using $d \leftrightarrow s$ reflection.

Consider first Eq. (14) for decays to the two U-spin triplet states, $|f_1\rangle = |\pi^+K^-\rangle = -|1,-1\rangle$ and $|f_2\rangle = |K^+\pi^-\rangle = |1,1\rangle$, to which (15) applies. H_W^{CF} and H_W^{DCS} transform like $(1,-1)$ and $(1,+1)$ while the $s-d$ spurion mass operator M_{Ubrk} behaves like $(1,0)$. Since the D^0 is a U-spin singlet only the triplet operators in the products $H_{eff}^{DCS,CF} M_{Ubrk} \propto (1,\pm1) \otimes (1,0)$ contribute to the triplet final states $\pm|1,\pm1\rangle$, and only the triplet states in $M_{Ubrk}|1,\pm1\rangle \propto (1,0)|1,\pm1\rangle$ obtain contributions from the triplet Hamiltonian operator. Thus the two terms in (14) involve coupling of the product $(1,\pm1) \otimes (1,0)$ into $(1,\pm1)$, where $\pm$ signs correspond to $K^+\pi^-$ and π^+K^- final states. Using a property of Clebsch-Gordan coefficients,

$$(1,1;n,0|1,1) = (-1)^n(1,-1;n,0|1,-1), \quad (17)$$

we therefore find

$$\frac{\langle \pi^+K^-|H_{eff}^{CF}|D^0\rangle^{(1)}}{\cos^2\theta_C} = \frac{\langle K^+\pi^-|H_{eff}^{DCS}|D^0\rangle^{(1)}}{\sin^2\theta_C}. \quad (18)$$

That is, first-order U-spin breaking terms contribute equally but with opposite signs to $D^0 \to \pi^+K^-$ and $D^0 \to K^+\pi^-$, when normalized by corresponding U-spin symmetric amplitudes given in Eqs. (7) and (9):

$$\frac{\langle \pi^+K^-|H_{eff}^{CF}|D^0\rangle^{(1)}}{\langle \pi^+K^-|H_W^{CF}|D^0\rangle^{(0)}} = -\frac{\langle K^+\pi^-|H_{eff}^{DCS}|D^0\rangle^{(1)}}{\langle K^+\pi^-|H_W^{DCS}|D^0\rangle^{(0)}} \equiv -\epsilon_1. \quad (19)$$

We denote by ϵ_1 the U-spin breaking term in $D^0 \to K^+\pi^-$ normalized by its U-spin invariant amplitude.

A simpler and rather immediate derivation of (19) may be obtained by applying $d \leftrightarrow s$ reflection to Eq. (14) for $f = \pi^+K^-$. Noting that the $s-d$ spurion mass operator M_{Ubrk} changes sign under this reflection, one has

$$\langle \pi^+K^-|(\bar{s}c)(\bar{u}d)M_{Ubrk}|D^0\rangle + \langle M_{Ubrk}(\pi^+K^-)|(\bar{s}c)(\bar{u}d)|D^0\rangle$$
$$= -\langle K^+\pi^-|(\bar{d}c)(\bar{u}s)M_{Ubrk}|D^0\rangle$$
$$- \langle M_{Ubrk}(K^+\pi^-)|(\bar{d}c)(\bar{u}s)|D^0\rangle. \quad (20)$$

This leads directly to (18) and (19).

This short argument applies also to first-order U-spin breaking in SCS decays, $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$, since the penguin operator P_{s+d} in (16) also changes sign under $d \leftrightarrow s$ reflection. Therefore,

$$\frac{\langle \pi^+\pi^-|H_{eff}^{SCS}|D^0\rangle^{(1)}}{\langle \pi^+\pi^-|H_W^{SCS}|D^0\rangle^{(0)}} = -\frac{\langle K^+K^-|H_{eff}^{SCS}|D^0\rangle^{(1)}}{\langle K^+K^-|H_W^{SCS}|D^0\rangle^{(0)}} \equiv -\epsilon_2, \quad (21)$$

where ϵ_2 denotes the U-spin breaking term in $D^0 \to K^+K^-$ normalized by its U-spin invariant amplitude. Furthermore,

the change in sign between first-order terms in $D^0 \to K^+K^-$ and $D \to \pi^+\pi^-$ applies separately to contributions of the two operators on the right-hand side of (16) representing tree and penguin amplitudes. This leads one to expect that the U-spin breaking parameter ϵ_2, involving both tree and penguin amplitudes, is larger than ϵ_1 which involves only tree amplitudes [20].

Combining the results (19) and (21) with the zeroth-order Eqs. (7), (9), and (11), one obtains the following first-order expressions for decay amplitudes:

$$A(D^0 \to \pi^+K^-) = \cos^2\theta_C A(1 - \epsilon_1),$$
$$A(D^0 \to K^+\pi^-) = -\sin^2\theta_C A(1 + \epsilon_1),$$
$$A(D^0 \to \pi^+\pi^-) = -\cos\theta_C \sin\theta_C A(1 - \epsilon_2),$$
$$A(D^0 \to K^+K^-) = \cos\theta_C \sin\theta_C A(1 + \epsilon_2). \quad (22)$$

V. U-SPIN BREAKING OF ARBITRARY ORDER

U-spin breaking of order n in decay amplitudes $\langle f|H_{eff}|D^0\rangle$ is obtained by introducing in the Hamiltonian or in the final state a total of n powers of the $s-d$ spurion mass operator, applying (16) to SCS decays. Generalizing the argument for a relative negative sign in first-order breaking, based on a change of sign of M_{Ubrk} and P_{s+d} under $d \leftrightarrow s$ reflection, we conclude that a negative relative sign applies to odd n and a positive sign to even n:

$$\frac{\langle \pi^+K^-|H_{eff}^{CF}|D^0\rangle^{(n)}}{\cos^2\theta_C} = (-1)^n \frac{\langle K^+\pi^-|H_{eff}^{DCS}|D^0\rangle^{(n)}}{-\sin^2\theta_C}. \quad (23)$$

$$\frac{\langle K^+K^-|H_W^{SCS}|D^0\rangle^{(n)}}{\cos\theta_C \sin\theta_C} = (-1)^n \frac{\langle \pi^+\pi^-|H_W^{SCS}|D^0\rangle^{(n)}}{-\cos\theta_C \sin\theta_C}. \quad (24)$$

Thus, we may sum up:

$$A(D^0 \to \pi^+K^-) = \cos^2\theta_C A[1 - \epsilon_1 + a_1\epsilon_1^2 - a_1'\epsilon_1^3 + \cdots],$$
$$A(D^0 \to K^+\pi^-) = -\sin^2\theta_C A[1 + \epsilon_1 + a_1\epsilon_1^2 + a_1'\epsilon_1^3 + \cdots],$$
$$A(D^0 \to \pi^+\pi^-) = -\cos\theta_C \sin\theta_C A[1 - \epsilon_2 + a_2\epsilon_2^2 - a_2'\epsilon_2^3 + \cdots],$$
$$A(D^0 \to K^+K^-) = \cos\theta_C \sin\theta_C A[1 + \epsilon_2 + a_2\epsilon_2^2 + a_2'\epsilon_2^3 + \cdots]. \quad (25)$$

While the complex U-spin breaking parameters $\epsilon_{1,2}$ and the nonperturbative coefficients $a_{1,2}, a_{1,2}', \ldots$ are not calculable from first principles, one expects the first two parameters to be around 0.2–0.3 (ϵ_2 being larger in magnitude than ϵ_1 for the above-mentioned argument) and the coefficients to be of order one, $|a_{1,2}| \sim |a_{1,2}'| \sim 1$.

Expanding magnitudes of amplitudes up to and including third order, we find

$$|1 \pm \epsilon + a\epsilon^2 \pm a'\epsilon^3| = 1 \pm \mathrm{Re}\epsilon + \frac{1}{2}(\mathrm{Im}\epsilon)^2 + \mathrm{Re}(a\epsilon^2)$$
$$\pm \mathrm{Re}(a'\epsilon^3) \mp \frac{1}{2}\mathrm{Re}\epsilon(\mathrm{Im}\epsilon)^2$$
$$\pm \mathrm{Im}\epsilon\,\mathrm{Im}(a\epsilon^2). \qquad (26)$$

In the next section we will use this expansion for studying ratios of magnitudes of decay amplitudes, identifying relations among ratios in which U-spin breaking terms up to third order cancel, thus being sensitive to tiny fourth-order U-spin breaking. We will then argue that there is no need to go beyond third order in (26) for showing that fourth-order terms do not cancel in these relations.

VI. HIGH-PRECISION RELATION AMONG RATIOS OF AMPLITUDES

We define four ratios of amplitudes:

$$R_1 \equiv \frac{|A(D^0 \to K^+\pi^-)|}{|A(D^0 \to \pi^+K^-)|\tan^2\theta_C},$$

$$R_2 \equiv \frac{|A(D^0 \to K^+K^-)|}{|A(D^0 \to \pi^+\pi^-)|},$$

$$R_3 \equiv \frac{|A(D^0 \to K^+K^-)| + |A(D^0 \to \pi^+\pi^-)|}{|A(D^0 \to \pi^+K^-)|\tan\theta_C + |A(D^0 \to K^+\pi^-)|\tan^{-1}\theta_C},$$

$$R_4 \equiv \sqrt{\frac{|A(D^0 \to K^+K^-)||A(D^0 \to \pi^+\pi^-)|}{|A(D^0 \to \pi^+K^-)||A(D^0 \to K^+\pi^-)|}}. \qquad (27)$$

These four ratios are not mutually independent. They obey a trivial identity,

$$R_4 = R_3\sqrt{\frac{1 - [(R_2-1)/(R_2+1)]^2}{1 - [(R_1-1)/(R_1+1)]^2}}. \qquad (28)$$

We will prove a nontrivial relation involving R_3 and R_4 and another nonlinear function of R_1 and R_2, that holds up to *maximal* fourth-order U-spin breaking.

Using (26) we start by expanding the two ratios R_1 and R_2 up to third-order U-spin breaking:

$$R_1 = 1 + 2[\mathrm{Re}\epsilon_1 + (\mathrm{Re}\epsilon_1)^2] + \mathcal{O}(\epsilon_1^3),$$
$$R_2 = 1 + 2[\mathrm{Re}\epsilon_2 + (\mathrm{Re}\epsilon_2)^2] + \mathcal{O}(\epsilon_2^3). \qquad (29)$$

These two ratios involve first-order corrections given by $2\mathrm{Re}\epsilon_1$ and $2\mathrm{Re}\epsilon_2$. Second-order corrections in these ratios are given by squares of these same real parts with no dependence on the unknown parameters a_1 and a_2. Thus measurements of R_1 and R_2 provide a way for calculating $\mathrm{Re}\epsilon_1$ and $\mathrm{Re}\epsilon_2$ up to third-order corrections. Solutions for $\mathrm{Re}\epsilon_1$ and $\mathrm{Re}\epsilon_2$ using Eqs. (29) should include the U-spin symmetry limit, requiring $\mathrm{Re}\epsilon_1 = 0$ and $\mathrm{Re}\epsilon_2 = 0$ for $R_1 = 1$ and $R_2 = 1$, respectively, rather than $\mathrm{Re}\epsilon_1 = -1$ and $\mathrm{Re}\epsilon_2 = -1$. This implies

$$\mathrm{Re}\epsilon_1 = \frac{1}{2}(\sqrt{2R_1 - 1} - 1) + \mathcal{O}(\epsilon_1^3),$$

$$\mathrm{Re}\epsilon_2 = \frac{1}{2}(\sqrt{2R_2 - 1} - 1) + \mathcal{O}(\epsilon_2^3). \qquad (30)$$

As we will see immediately, these two first-order U-spin breaking parameters do not determine only R_1 and R_2 but also the difference $R_3 - R_4$.

The two ratios R_3 and R_4, in which first- and third-order terms cancel [26,27], may be expanded up to fourth order:

$$R_3 = 1 + \frac{1}{2}[(\mathrm{Im}\epsilon_2)^2 - (\mathrm{Im}\epsilon_1)^2] + \mathrm{Re}[a_2\epsilon_2^2 - a_1\epsilon_1^2]$$
$$+ \mathcal{O}(\epsilon_1^4, \epsilon_2^4),$$

$$R_4 = 1 - \frac{1}{2}\mathrm{Re}(\epsilon_2^2 - \epsilon_1^2) + \mathrm{Re}(a_2\epsilon_2^2 - a_1\epsilon_1^2) + \mathcal{O}(\epsilon_1^4, \epsilon_2^4)$$

$$= 1 + \frac{1}{2}[(\mathrm{Im}\epsilon_2)^2 - (\mathrm{Im}\epsilon_1)^2] + \mathrm{Re}(a_2\epsilon_2^2 - a_1\epsilon_1^2)$$
$$- \frac{1}{2}[(\mathrm{Re}\epsilon_2)^2 - (\mathrm{Re}\epsilon_1)^2] + \mathcal{O}(\epsilon_1^4, \epsilon_2^4). \qquad (31)$$

These two ratios are noticed to differ by second-order U-spin breaking terms depending solely on $\mathrm{Re}\epsilon_1$ and $\mathrm{Re}\epsilon_2$:

$$R_3 - R_4 = \frac{1}{2}[(\mathrm{Re}\epsilon_2)^2 - (\mathrm{Re}\epsilon_1)^2] + \mathcal{O}(\epsilon_1^4, \epsilon_2^4). \qquad (32)$$

Using

$$(\mathrm{Re}\epsilon_i)^2 = \frac{1}{4}(\sqrt{2R_i - 1} - 1)^2 + 2\mathrm{Re}\epsilon_i\mathcal{O}(\epsilon_i^3)$$

$$= \frac{1}{4}(\sqrt{2R_i - 1} - 1)^2 + \mathcal{O}(\epsilon_i^4), \quad i = 1, 2, \qquad (33)$$

one obtains the following nonlinear relation among the four ratios of amplitudes, which holds up to tiny fourth-order U-spin breaking terms:

$$\Delta R \equiv R_3 - R_4 + \frac{1}{8}[(\sqrt{2R_1 - 1} - 1)^2 - (\sqrt{2R_2 - 1} - 1)^2]$$
$$= \mathcal{O}(\epsilon_1^4, \epsilon_2^4). \qquad (34)$$

This relation may also be obtained by expanding the identity (28) to first order in $[(R_{1,2} - 1)/(R_{1,2} + 1)]^2$ and applying (29) and (30). Fourth-order terms in $(\mathrm{Re}\epsilon_i)^2$ are proportional to $\mathrm{Re}\epsilon_i$. It can be easily shown that this is not the case for terms of this order occurring in R_3, R_4 and in their difference. Therefore, one concludes that fourth-order terms do not cancel in ΔR and in any higher-order expansion of the square root in (28). Thus, in hindsight, there is no way of obtaining a U-spin breaking relation of higher order than (34), and one does not need to go beyond third order in (26).

In our derivation of (34), we have used a parametrization of the CKM matrix up to terms which are fourth order in

$\lambda \equiv V_{us} = 0.2253 \pm 0.008$ [17]. We now study the effects of higher-order terms in λ on R_i and on this relation. Including λ^4 and λ^5 terms in the CKM matrix, one has [28]

$$V_{ud} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4,$$

$$V_{us} = \lambda,$$

$$V_{cd} = -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)],$$

$$V_{cs} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 - \frac{1}{2}A^2\lambda^4, \tag{35}$$

where $A = 0.82 \pm 0.02, \rho = 0.12 \pm 0.02, \eta = 0.36 \pm 0.02$ [17].

We define a small parameter ξ determining the effect of λ^4 and λ^5 terms on ratios of amplitudes:

$$1 + \xi \equiv \frac{1 - \frac{1}{2}A^2\lambda^4|1 - 2(\rho + i\eta)|}{1 - \frac{1}{2}A^2\lambda^4}. \tag{36}$$

Using the above values of λ, A, ρ, and η, we find $|\xi| \lesssim 10^{-4}$, while R_4 is unaffected by ξ, R_3, and the two quadratic terms in (34) obtain corrections suppressed by ξ and by first-order U-spin breaking. These corrections, respectively, $-\frac{1}{2}\xi(\text{Re}\epsilon_1 + \text{Re}\epsilon_2)$, $+\frac{1}{2}\xi\text{Re}\epsilon_1$ and $+\frac{1}{2}\xi\text{Re}\epsilon_2$, cancel each other. The remaining corrections in (34), suppressed by ξ and by second-order U-spin breaking, are much below the level of 10^{-4} and may be safely neglected relative to tiny fourth-order U-spin breaking terms of order 10^{-3} or 10^{-4}.

Taking $\epsilon_i \sim 0.2$ as a typical value for first-order U-spin breaking, the nonlinear relation $\Delta R = 0$ is expected to hold at a very high precision of order $\epsilon_i^4 \sim 10^{-3}$. We will confirm this prediction in Sec. VIII. At this high precision, one cannot ignore first-order isospin-breaking terms, which one would generally assume to be around $(m_d - m_u)\Lambda_{\text{QCD}} \sim 10^{-2}$. We will study these terms in the next section showing that, in fact, they are suppressed by both isospin-breaking parameters and by second-order U-spin breaking.

VII. FIRST-ORDER ISOSPIN BREAKING

First-order isospin breaking is introduced by multiplying the weak Hamiltonian by a $d - u$ spurion mass operator,

$$M_{\text{Ibrk}} \propto (\bar{d}d - \bar{u}u) = \frac{1}{2}(\bar{d}d + \bar{s}s) - \bar{u}u + \frac{1}{2}(\bar{d}d - \bar{s}s), \tag{37}$$

transforming like a combination of a U-spin singlet and triplet. Isospin-breaking contributions of the U-spin singlet operator in the four amplitudes (25) are identical when normalized by suitable CKM factors, and may be absorbed into the U-spin symmetric amplitude A. This is true also for $U = 0$ isospin-breaking electromagnetic interactions because the d and s quarks have identical charges.

Contributions of the triplet operator in (37) follow the signs of first-order U-spin breaking corrections. They are represented by two distinct first-order isospin-breaking parameters: δ_1 for U-spin triplet states π^+K^- and $K^+\pi^-$ and δ_2 for K^+K^- and $\pi^+\pi^-$, the two components of a U-spin singlet state. Thus

$$A(D^0 \to \pi^+K^-) = \cos^2\theta_C A(1 - \epsilon_1 + a_1\epsilon_1^2 - a_1'\epsilon_1^3 - \delta_1 + \cdots),$$

$$A(D^0 \to K^+\pi^-) = -\sin^2\theta_C A(1 + \epsilon_1 + a_1\epsilon_1^2 + a_1'\epsilon_1^3 + \delta_1 + \cdots),$$

$$A(D^0 \to \pi^+\pi^-) = -\cos\theta_C\sin\theta_C A(1 - \epsilon_2 + a_2\epsilon_2^2 - a_2'\epsilon_2^3 - \delta_2 + \cdots),$$

$$A(D^0 \to K^+K^-) = \cos\theta_C\sin\theta_C A(1 + \epsilon_2 + a_2\epsilon_2^2 + a_2'\epsilon_2^3 + \delta_2 + \cdots). \tag{38}$$

Instead of (26) we now expand:

$$|1 \pm \epsilon + a\epsilon^2 \pm a'\epsilon^3 \pm \delta| = 1 \pm \text{Re}\epsilon + \frac{1}{2}(\text{Im}\epsilon)^2 + \text{Re}(a\epsilon^2) \pm \text{Re}(a'\epsilon^3)$$

$$\mp \frac{1}{2}\text{Re}\epsilon(\text{Im}\epsilon)^2 \pm \text{Im}\epsilon\text{Im}(a\epsilon^2) \pm \text{Re}\delta + \text{Im}\delta\text{Im}\epsilon. \tag{39}$$

This characteristic amplitude expansion includes two new terms, $\pm\text{Re}\delta$ and $+\text{Im}\delta\text{Im}\epsilon$, the latter involving suppression by both isospin and U-spin breaking parameters. We will show that terms of this order do not affect the nonlinear relation (34).

The expansion of the four ratios of amplitudes now includes terms which are first order in isospin breaking and other terms suppressed by both isospin and U-spin breaking:

$$R_1 = 1 + 2[\text{Re}\epsilon_1 + (\text{Re}\epsilon_1)^2] + 2\text{Re}\delta_1 + 4\text{Re}\delta_1\text{Re}\epsilon_1 + \mathcal{O}(\epsilon_1^3) + \mathcal{O}(\delta_1\epsilon_1^2),$$

$$R_2 = 1 + 2[\text{Re}\epsilon_2 + (\text{Re}\epsilon_2)^2] + 2\text{Re}\delta_2 + 4\text{Re}\delta_2\text{Re}\epsilon_2 + \mathcal{O}(\epsilon_2^3) + \mathcal{O}(\delta_2\epsilon_2^2), \tag{40}$$

$$R_3 = 1 + \frac{1}{2}[(\mathrm{Im}\epsilon_2)^2 - (\mathrm{Im}\epsilon_1)^2] + \mathrm{Re}(a_2\epsilon_2^2 - a_1\epsilon_1^2) + (\mathrm{Im}\delta_2\mathrm{Im}\epsilon_2 - \mathrm{Im}\delta_1\mathrm{Im}\epsilon_1) + \mathcal{O}(\epsilon_{1,2}^4) + \mathcal{O}(\delta_{1,2}\epsilon_{1,2}^3),$$

$$R_4 = 1 + \frac{1}{2}[(\mathrm{Im}\epsilon_2)^2 - (\mathrm{Im}\epsilon_1)^2] + \mathrm{Re}(a_2\epsilon_2^2 - a_1\epsilon_1^2) + (\mathrm{Im}\delta_2\mathrm{Im}\epsilon_2 - \mathrm{Im}\delta_1\mathrm{Im}\epsilon_1)$$

$$- \frac{1}{2}[(\mathrm{Re}\epsilon_2)^2 - (\mathrm{Re}\epsilon_1)^2] - (\mathrm{Re}\delta_2\mathrm{Re}\epsilon_2 - \mathrm{Re}\delta_1\mathrm{Re}\epsilon_1) + \mathcal{O}(\epsilon_{1,2}^4) + \mathcal{O}(\delta_{1,2}\epsilon_{1,2}^3). \tag{41}$$

Equations (40) imply for $i = 1, 2$,

$$\mathrm{Re}\epsilon_i = \frac{1}{2}(\sqrt{2R_i - 1} - 1) - \mathrm{Re}\delta_i - 2\mathrm{Re}\delta_i\mathrm{Re}\epsilon_i$$
$$+ \mathcal{O}(\delta_i\epsilon_i) + \mathcal{O}(\epsilon_i^3), \tag{42}$$

or

$$(\mathrm{Re}\epsilon_i)^2 = \frac{1}{4}(\sqrt{2R_i - 1} - 1)^2 - 2\mathrm{Re}\delta_i\mathrm{Re}\epsilon_i$$
$$+ \mathcal{O}(\delta_i\epsilon_i^2) + \mathcal{O}(\epsilon_i^4). \tag{43}$$

Equations (41) lead to

$$R_3 - R_4 = \frac{1}{2}[(\mathrm{Re}\epsilon_2)^2 - \mathrm{Re}\epsilon_1)^2] + (\mathrm{Re}\delta_2\mathrm{Re}\epsilon_2 - \mathrm{Re}\delta_1\mathrm{Re}\epsilon_1)$$
$$+ \mathcal{O}(\epsilon_{1,2}^4) + \mathcal{O}(\delta_{1,2}\epsilon_{1,2}^3). \tag{44}$$

Consequently, isospin-breaking terms of the form $\mathrm{Re}\delta_2\mathrm{Re}\epsilon_2 - \mathrm{Re}\delta_1\mathrm{Re}\epsilon_1$ cancel in ΔR:

$$\Delta R \equiv R_3 - R_4 + \frac{1}{8}[(\sqrt{2R_1 - 1} - 1)^2 - (\sqrt{2R_2 - 1} - 1)^2]$$
$$= \mathcal{O}(\epsilon_1^4, \epsilon_2^4) + \mathcal{O}(\delta_1\epsilon_1^2, \delta_2\epsilon_2^2). \tag{45}$$

It is remarkable that isospin breaking modifies the nonlinear relation (34) by terms which are suppressed by both first-order isospin-breaking and second-order U-spin breaking factors. Taking $\delta_i \sim 10^{-2}$, $\epsilon_i \sim 0.2$–0.3, these terms are expected to be at most 10^{-3}, similar in magnitude to fourth-order U-spin breaking terms affecting this relation.

VIII. EXPERIMENTAL TESTS IN $D^0 \to P^+ P^-$

We will now apply current experimental data to study the hierarchy among U-spin breaking terms of increasing order. Our final goal is testing the predicted amplitude relation (45). Hadronic decay amplitudes (A) are obtained from measured branching ratios ($\mathcal{B}$) by eliminating phase space factors depending on final particles center-of-mass 3-momenta (p), and on the D meson mass and its lifetime (M_D and τ_D),

$$|A| = M_D\sqrt{\frac{8\pi\mathcal{B}}{\tau_D p}}. \tag{46}$$

In our calculation of amplitudes we will disregard a common factor $M_D\sqrt{8\pi/\tau_D}$ which cancels in the four ratios R_i. Values for measured branching ratios [17], center-of-mass momenta, and amplitudes defined in this manner are quoted in Tables I.

Note that all four amplitudes include a factor $\mathcal{B}_{\pi K}^{1/2}$ corresponding to the branching fraction of the Cabibbo-favored decay $D^0 \to \pi^+ K^-$. We have included no error in the amplitude for this process as the other three branching ratios (including errors) have been measured relative to this process [17]. The relative errors in the amplitudes of these three processes are all below the level of one percent. We will assume no correlation between these errors, which have been measured in three independent analyses for different final states. The high precision achieved recently by the CDF, LHCb and Belle collaborations in measuring the DCS amplitude is remarkable [29], as it required time-dependent separation between this highly suppressed decay and $D^0 - \bar{D}^0$ mixing followed by the CF decay.

Using values of amplitudes given in Tables I and $\tan\theta_C = 0.23125 \pm 0.00082$ [17] we calculate the four ratios R_i defined in Eq. (27),

$$R_1 = 1.115 \pm 0.012,$$
$$R_2 = 1.811 \pm 0.020,$$
$$R_3 = 1.052 \pm 0.008,$$
$$R_4 = 1.008 \pm 0.007. \tag{47}$$

TABLE I. Branching fractions and amplitudes for $D^0 \to P^+ P^-$ decays [17].

Decay mode	Branching fraction	p (GeV/c)	$|A| = \sqrt{\mathcal{B}/p}(\mathrm{GeV}/c)^{-1/2}$
$D^0 \to \pi^+ K^-$	$\mathcal{B}_{\pi K} = (3.88 \pm 0.05) \times 10^{-2}$	0.861	$1.078\mathcal{B}_{\pi K}^{1/2}$
$D^0 \to K^+ \pi^-$	$(3.56 \pm 0.06) \times 10^{-3}\mathcal{B}_{\pi K}$	0.861	$(0.06430 \pm 0.00054)\mathcal{B}_{\pi K}^{1/2}$
$D^0 \to \pi^+ \pi^-$	$(3.59 \pm 0.06) \times 10^{-2}\mathcal{B}_{\pi K}$	0.922	$(0.1973 \pm 0.0016)\mathcal{B}_{\pi K}^{1/2}$
$D^0 \to K^+ K^-$	$(10.10 \pm 0.16) \times 10^{-2}\mathcal{B}_{\pi K}$	0.791	$(0.3573 \pm 0.0028)\mathcal{B}_{\pi K}^{1/2}$

Errors in the ratios have been obtained by adding in quadrature errors in the relevant amplitudes.

We have seen that the first two ratios involve first-order U-spin breaking terms. These terms, depending on two distinct U-spin breaking parameters ϵ_1 and ϵ_2, are considerably larger in R_2 than in R_1. This has been anticipated in the discussion below Eq. (21). Using (42), in which we neglect first-order isospin breaking and third-order U-spin breaking, we calculate reasonably small U-spin breaking parameters,

$$\text{Re}\epsilon_1 = 0.054 \pm 0.005,$$
$$\text{Re}\epsilon_2 = 0.310 \pm 0.006. \tag{48}$$

The other two ratios, R_3 and R_4, given in (41) in terms of $\epsilon_{1,2}$ and coefficients $a_{1,2}$ of order one, deviate from one by second-order U-spin breaking terms (we neglect terms suppressed by both isospin and U-spin breaking and fourth-order terms in U-spin breaking),

$$R_3 - 1 = \frac{1}{2}[(\text{Im}\epsilon_2)^2 - (\text{Im}\epsilon_1)^2] + \text{Re}(a_2\epsilon_2^2 - a_1\epsilon_1^2)$$
$$= 0.052 \pm 0.08,$$

$$R_4 - 1 = \text{Re}\left[\left(a_2 - \frac{1}{2}\right)\epsilon_2^2 - \left(a_1 - \frac{1}{2}\right)\epsilon_1^2\right]$$
$$= 0.008 \pm 0.007. \tag{49}$$

The hierarchy between the first-order parameters in (48) and the second-order terms calculated in (49) confirms and justifies the perturbative approach we have applied in this study to U-spin breaking. Without having a method for calculating the nonperturbative coefficients a_i, the almost exact cancellation of second-order terms in R_4 seems to be accidental. In view of the small value of $\text{Re}\epsilon_1$ and the much larger value of $\text{Re}\epsilon_2$ this approximate cancellation seems to imply $a_2 \simeq 1/2$.

Having shown that second-order U-spin breaking terms are a few percent, one expects fourth-order terms to be of order 10^{-3}, comparable in magnitude or larger than terms which are first order in isospin breaking and second order in U-spin breaking. Let us now check this prediction in the relation (45) which contains on the right-hand side terms of these two kinds. Using the definitions of R_i in (27) and adding in quadrature errors in amplitudes [rather than errors in R_i given in (47)], we obtain

$$\Delta R = (-3.2 \pm 0.4) \times 10^{-3}. \tag{50}$$

This confirms our prediction.

IX. EXPERIMENTAL TEST IN $D^0 \to V^+P^-$

As mentioned, the discussion in Secs. III–VII applies also to three other classes of processes involving one or two charged vector mesons, $D^0 \to V^+P^-$, $D^0 \to P^+V^-$, and $D^0 \to V^+V^-$. In particular, a nonlinear amplitude relation

similar to (45) holds in each one of these classes with a precision depending on the size of U-spin breaking. In this section we summarize concisely the situation relevant to this question in $D^0 \to V^+P^-$, consisting of the four processes, $D^0 \to \rho^+K^-$, $D^0 \to K^{*+}\pi^-$, $D^0 \to \rho^+\pi^-$, and $D^0 \to K^{*+}K^-$. We denote first-order U-spin breaking and isospin-breaking parameters in these processes by $\epsilon'_{1,2}$ and $\delta'_{1,2}$, respectively, in analogy to $\epsilon_{1,2}$ and $\delta_{1,2}$ in $D^0 \to P^+P^-$.

Defining four ratios of amplitudes R'_i in analogy with (27),

$$R'_1 \equiv \frac{|A(D^0 \to K^{*+}\pi^-)|}{|A(D^0 \to \rho^+K^-)|\tan^2\theta_C},$$

$$R'_2 \equiv \frac{|A(D^0 \to K^{*+}K^-)|}{|A(D^0 \to \rho^+\pi^-)|},$$

$$R'_3 \equiv \frac{|A(D^0 \to K^{*+}K^-)| + |A(D^0 \to \rho^+\pi^-)|}{|A(D^0 \to \rho^+K^-)|\tan\theta_C + |A(D^0 \to K^{*+}\pi^-)|\tan^{-1}\theta_C},$$

$$R'_4 \equiv \sqrt{\frac{|A(D^0 \to K^{*+}K^-)||A(D^0 \to \rho^+\pi^-)|}{|A(D^0 \to \rho^+K^-)||A(D^0 \to K^{*+}\pi^-)|}}, \tag{51}$$

one obtains a sum rule analogous to (45):

$$\Delta R' \equiv R'_3 - R'_4 + \frac{1}{8}[(\sqrt{2R'_1 - 1} - 1)^2 - (\sqrt{2R'_2 - 1} - 1)^2]$$
$$= \mathcal{O}(\epsilon'^4_1, \epsilon'^4_2) + \mathcal{O}(\delta'_1\epsilon'^2_1, \delta'_2\epsilon'^2_2). \tag{52}$$

Magnitudes of amplitudes for the P-wave decays $D^0 \to V^+P^-$ are obtained from corresponding branching ratios using

$$|A| = M_D\sqrt{\frac{8\pi\mathcal{B}}{\tau_D p^3}}. \tag{53}$$

We will disregard again the factor $M_D\sqrt{8\pi/\tau_D}$ since we are only concerned with ratios of amplitudes. Values for measured branching ratios [17], center-of-mass momenta, and amplitudes defined in this manner are quoted in Table II. Relative errors in CF and SCS amplitudes are reasonably small, between two and three percent. In contrast, the relative error in the DCS amplitude $|A(D^0 \to K^{*+}\pi^-)|$, obtained by the CLEO and *BABAR* Collaborations through Dalitz plot analyses of $D^0 \to K_S\pi^+\pi^-$ [30], is quite large, $^{+26\%}_{-15\%}$. This large asymmetric error limits considerably the precision of R'_1, R'_3 and R'_4. Adding in quadrature errors in relevant amplitudes, measured independently for different three-body final states, we calculate

$$R'_1 = 0.971^{+0.257}_{-0.148},$$
$$R'_2 = 0.939 \pm 0.029,$$
$$R'_3 = 1.061^{+0.082}_{-0.140},$$
$$R'_4 = 1.061^{+0.083}_{-0.142}. \tag{54}$$

Expansions similar to (40) and (41) apply to these four ratios in terms of $\epsilon'_{1,2}$ and $\delta'_{1,2}$. The leading U-spin breaking corrections in R'_1 and R'_2 are first order, while in R'_3 and R'_4 they are second order. The measured values of the first two ratios imply

$$\mathrm{Re}\epsilon'_1 = \frac{1}{2}(\sqrt{2R'_1 - 1} - 1) = -0.015^{+0.118}_{-0.083},$$

$$\mathrm{Re}\epsilon'_2 = \frac{1}{2}(\sqrt{2R'_2 - 1} - 1) = -0.032 \pm 0.016. \quad (55)$$

We note that the numerical value of $\mathrm{Re}\epsilon'_2$ is smaller by about an order of magnitude than the value of $\mathrm{Re}\epsilon_2$ calculated in (48). That is, first-order U-spin breaking in SCS $D^0 \to V^+P^-$ decays is about an order of magnitude smaller than in corresponding $D^0 \to P^+P^-$ decays. This seems to imply that an enhancement of the U-spin breaking penguin amplitude suggested to occur in $D^0 \to P^+P^-$ [19] is not at work in $D^0 \to V^+P^-$. The other U-spin breaking parameter, $\mathrm{Re}\epsilon'_1$, does not involve a penguin amplitude. In spite of the current large error in this parameter the first of Eqs. (55) favors strongly $|\mathrm{Re}\epsilon'_1| \leq 0.1$, suggesting that a value close to that measured for $|\mathrm{Re}\epsilon_1|$ is not unlikely.

These values of $\mathrm{Re}\epsilon'_{1,2}$ imply that typical second-order U-spin breaking terms in $R'_{3,4}$ are around one percent. Confirming this prediction and obtaining a more precise value for $\mathrm{Re}\epsilon'_1$ requires a substantial improvement in the measurement of $\mathcal{B}(D^0 \to K^{*+}\pi^-)$. In the meantime we use the values measured for $|\mathrm{Re}\epsilon'_{1,2}|$ to argue that fourth-order U-spin breaking terms should be around 10^{-4}. This is also expected to be the magnitude of terms in $\Delta R'$ suppressed by both isospin breaking and by second-order U-spin breaking. Thus, due to smaller U-spin breaking parameters in $D^0 \to V^+P^-$ relative to $D^0 \to P^+P^-$ one predicts $\Delta R'$ to be about an order of magnitude smaller than ΔR. Using the amplitudes in Table II, we calculate

$$\Delta R' = (0.2^{+3.2}_{-5.5}) \times 10^{-4}, \quad (56)$$

where the error is dominated by the error in $|A(D^0 \to K^{*+}\pi^-)|$. This confirms our prediction. A more precise test could be achieved by improving the measurement of the branching ratio for this process.

X. EXPERIMENTAL TESTS IN $D^0 \to P^+V^-$

Current branching ratios and amplitudes for $D^0 \to P^+V^-$, consisting of $D^0 \to \pi^+K^{*-}, K^+\rho^-, \pi^+\rho^-, K^+K^{*-}$ are given in Table III. Reference [17] quotes no branching ratio measurement for the DCS decay $D^0 \to K^+\rho^-$. Three-body decays, $D^0 \to K^+\pi^-\pi^0$, involving $K^+\rho^-, K^{*+}\pi^-$ and other intermediate states, have been observed by Belle [31] and *BABAR* [32] with branching ratios $(3.18 \pm 0.29) \times 10^{-4}$ and $(2.97 \pm 0.19) \times 10^{-4}$, respectively. The $D^0 \to K^+\pi^-\pi^0$ events involve interference of DCS decays with $D^0 - \bar{D}^0$ mixing followed by CF decays. Evidence for $D^0 - \bar{D}^0$ mixing at 3.2 standard deviation was presented by *BABAR* [33], measuring the fraction of $K^+\rho^-$ in these events to be $(39.8 \pm 6.5)\%$. (No interference would have implied $\mathcal{B}(D^0 \to K^+\rho^-) \sim 1.2 \times 10^{-4}$.) More work is needed for resolving the effect of $D^0 - \bar{D}^0$ mixing on these events and for obtaining a solid measurement of $\mathcal{B}(D^0 \to K^+\rho^-)$.

We define ratios of amplitudes R''_i in analogy with (27),

$$R''_1 \equiv \frac{|A(D^0 \to K^+\rho^-)|}{|A(D^0 \to \pi^+K^{*-})|\tan^2\theta_C},$$

$$R''_2 \equiv \frac{|A(D^0 \to K^+K^{*-})|}{|A(D^0 \to \pi^+\rho^-)|},$$

$$R''_3 \equiv \frac{|A(D^0 \to K^+K^{*-})| + |A(D^0 \to \pi^+\rho^-)|}{|A(D^0 \to \pi^+K^{*-})|\tan\theta_C + |A(D^0 \to K^+\rho^-)|\tan^{-1}\theta_C},$$

$$R''_4 \equiv \sqrt{\frac{|A(D^0 \to K^+K^{*-})||A(D^0 \to \pi^+\rho^-)|}{|A(D^0 \to \pi^+K^{*-})||A(D^0 \to K^+\rho^-)|}}. \quad (57)$$

TABLE II. Branching fractions and amplitudes for $D^0 \to V^+P^-$ decays [17].

Decay mode	Branching fraction	p (GeV/c)	$|A| = \sqrt{\mathcal{B}/p^3}(\mathrm{GeV}/c)^{-3/2}$
$D^0 \to \rho^+K^-$	0.108 ± 0.007	0.675	0.593 ± 0.019
$D^0 \to K^{*+}\pi^-$	$(3.42^{+1.80}_{-1.02}) \times 10^{-4}$	0.711	$0.0308^{+0.0081}_{-0.0046}$
$D^0 \to \rho^+\pi^-$	$(9.8 \pm 0.4) \times 10^{-3}$	0.764	0.148 ± 0.003
$D^0 \to K^{*+}K^-$	$(4.38 \pm 0.21) \times 10^{-3}$	0.610	0.1389 ± 0.0033

TABLE III. Branching fractions and amplitudes for $D^0 \to P^+V^-$ decays [17].

Decay mode	Branching fraction	p (GeV/c)	$|A| = \sqrt{\mathcal{B}/p^3}(\mathrm{GeV}/c)^{-3/2}$
$D^0 \to \pi^+K^{*-}$	$(4.98^{+0.45}_{-0.51}) \times 10^{-2}$	0.711	$0.372^{+0.017}_{-0.019}$
$D^0 \to K^+\rho^-$	$\cdots$	0.675	$\cdots$
$D^0 \to \pi^+\rho^-$	$(4.96 \pm 0.24) \times 10^{-3}$	0.764	0.1055 ± 0.0026
$D^0 \to K^+K^{*-}$	$(1.56 \pm 0.12) \times 10^{-3}$	0.610	0.0829 ± 0.0032

We denote first-order U-spin breaking and isospin-breaking parameters in these amplitudes by $\epsilon''_{1,2}$ and $\delta''_{1,2}$, respectively.

In the absence of a solid measurement of $|A(D^0 \to K^+\rho^-)|$ one can only calculate R''_2. Neglecting isospin breaking and third-order U-spin breaking, one obtains

$$R''_2 = 1 + 2[\text{Re}\epsilon''_2 + (\text{Re}\epsilon''_2)^2] = 0.786 \pm 0.036, \quad (58)$$

which implies

$$\text{Re}\epsilon''_2 = -0.122 \pm 0.024. \quad (59)$$

That is, the magnitude of the U-spin breaking parameter in SCS $D^0 \to P^+V^-$ decays is intermediary between corresponding parameters in $D^0 \to P^+P^-$ ($\text{Re}\epsilon_2 = 0.310 \pm 0.006$) and $D^0 \to V^+P^-$ ($\text{Re}\epsilon'_2 = -0.032 \pm 0.016$). Namely, no significant U-spin breaking penguin enhancement applies to $D^0 \to P^+V^-$. One expects a similar or smaller magnitude for $\text{Re}\epsilon''_1$.

The two ratios R''_3 and R''_4 deviate from one by second-order U-spin breaking terms [see Eqs. (31)] which are expected to be at most a few percent. Using $R''_3 = 1 \pm 0.05$, where we include a conservative uncertainty of 5% due to second-order U-spin breaking corrections, one obtains the following prediction for $\mathcal{B}(D^0 \to K^+\rho^-)$ [34],

$$|A(D^0 \to K^+\rho^-)| = 0.0237 \pm 0.0025 \Rightarrow \mathcal{B}(D^0 \to K^+\rho^-)$$
$$= (1.73 \pm 0.36) \times 10^{-4}. \quad (60)$$

For comparison, assuming $R''_4 = 1 \pm 0.05$ implies a very similar prediction,

$$|A(D^0 \to K^+\rho^-)| = 0.0235 \pm 0.0028 \Rightarrow \mathcal{B}(D^0 \to K^+\rho^-)$$
$$= (1.70 \pm 0.40) \times 10^{-4}. \quad (61)$$

This value of $|A(D^0 \to K^+\rho^-)|$ would imply $\text{Re}\epsilon''_1 = 0.08 \pm 0.06$, comparable in magnitude to $\text{Re}\epsilon''_2$ and in agreement with expectation.

Taking for $|A(D^0 \to K^+\rho^-)|$ the value in (61), using the three measured amplitudes quoted in Table III, and assuming no error correlation between the four amplitudes, one obtains

$$\Delta R'' \equiv R''_3 - R''_4 + \frac{1}{8}[(\sqrt{2R''_1 - 1} - 1)^2 - (\sqrt{2R''_2 - 1} - 1)^2]$$
$$= (0.8^{+2.2}_{-5.6}) \times 10^{-4}. \quad (62)$$

This value, which is similar to (56), is in agreement with our prediction that, in view of the above values of $\text{Re}\epsilon''_{1,2}$, fourth-order U-spin breaking terms and isospin-breaking terms suppressed by second-order U-spin breaking should be of order 10^{-4}. The positive error in $\Delta R''$ is dominated by the uncertainty assumed in the unmeasured DCS

amplitude. One still awaits a solid measurement of $\mathcal{B}(D^0 \to K^+\rho^-)$ which would test the prediction (61). The larger negative error in $\Delta R''$, originating in errors on the three measured amplitudes in Table III, may be reduced by improving the corresponding branching ratio measurements.

XI. D^0 DECAYS TO PAIRS OF NEUTRAL PSEUDOSCALARS

In the U-spin symmetry limit one obtains simple amplitude relations for D^0 decays to pairs of light neutral pseudoscalar mesons [35]. We will go through the symmetry argument first, extending it to include U-spin breaking at arbitrary order in CF and DCS decays. We will then demonstrate a few amplitude relations which hold up to second-order U-spin breaking. Other relations of this kind have been studied in Ref. [8] in the framework of flavor SU(3).

In the symmetry limit one neglects $\eta - \eta'$ mixing which is due to first-order U-spin breaking represented by the spurion mass operator M_{Ubrk}. Thus, we will write amplitudes for $\eta = \eta_8$ in our discussion of the symmetry limit, while η_8 will be used explicitly when introducing U-spin breaking. When discussing amplitudes, rates and asymmetries for η_8 we will assume (as has been assumed in Ref. [8]) knowledge of amplitudes including a relative phase for η and η' and favored values of the mixing angle. Taking $\eta = \eta_8 \equiv (2s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{6}$, the following superpositions of single neutral particle states belong to a U-spin triplet,

$$|K^0\rangle \equiv |d\bar{s}\rangle = |1, 1\rangle,$$
$$|\bar{K}^0\rangle \equiv |s\bar{d}\rangle = -|1, -1\rangle,$$
$$\frac{1}{2}(\sqrt{3}|\eta\rangle - |\pi^0\rangle) \equiv |s\bar{s} - d\bar{d}\rangle/\sqrt{2} = |1, 0\rangle, \quad (63)$$

while the orthogonal U-spin singlet is

$$\frac{1}{2}(|\eta\rangle + \sqrt{3}|\pi^0\rangle) \equiv |s\bar{s} + d\bar{d} - 2u\bar{u}\rangle/\sqrt{6} = |0, 0\rangle. \quad (64)$$

Two-particle states in an S-wave are obtained by symmetrizing products of single particle states.

Symmetrized products of two $U = 1$ single-particle states consist of $U = 0$ and $U = 2$ (with $U_3 = \pm 1, 0$) states, while the product of two $U = 0$ states is pure $U = 0$. D^0 decay matrix elements of the $U = 1$ weak Hamiltonian (5) vanish for each one of these five states. This implies the following five U-spin symmetry relations: (For short notation, we denote amplitudes by their final states.)

$$\sqrt{3}A^{(0)}(K^0\eta) - A^{(0)}(K^0\pi^0) = 0,$$
$$\sqrt{3}A^{(0)}(\bar{K}^0\eta) - A^{(0)}(\bar{K}^0\pi^0) = 0,$$
$$A^{(0)}(K^0\bar{K}^0) = 0,$$
$$A^{(0)}(\eta\eta) + A^{(0)}(\pi^0\pi^0) = 0,$$
$$\sqrt{3}A^{(0)}(\eta\pi^0) + \sqrt{2}A^{(0)}(\pi^0\pi^0) = 0. \quad (65)$$

Hadronic matrix elements for symmetrized products of $U = 1, U_3 = \pm 1, 0$ and $U = 0$ states are then given by a single $U = 1$ amplitude $\mathcal{A}$:

$$\sqrt{3}A^{(0)}(K^0\eta) = A^{(0)}(K^0\pi^0) = -\sin^2\theta_C\mathcal{A}, \quad (66)$$

$$\sqrt{3}A^{(0)}(\bar{K}^0\eta) = A^{(0)}(\bar{K}^0\pi^0) = \cos^2\theta_C\mathcal{A}, \quad (67)$$

$$\sqrt{3}A^{(0)}(\eta\pi^0) = \sqrt{2}A^{(0)}(\eta\eta) = -\sqrt{2}A^{(0)}(\pi^0\pi^0)$$
$$= \sqrt{2}\cos\theta_C\sin\theta_C\mathcal{A}. \quad (68)$$

Note that since we symmetrized final states also for identical particles, corresponding amplitudes have been divided by $\sqrt{2}$ in order that their squares give decay rates.

The above amplitude expressions are analogous to the U-spin symmetry expressions (7), (9) and (11) for decays into pairs of charged pseudoscalar mesons. Some of these relations follow merely from $d \leftrightarrow s$ symmetry. Since the $U = 1$ and $U = 0$ superpositions of π^0 and η states in (63) and (64) are, respectively, antisymmetric and symmetric with respect to $d \leftrightarrow s$ reflection one has

$$\langle \bar{K}^0(\sqrt{3}\eta - \pi^0)|(\bar{s}c)(\bar{u}d)|D^0\rangle = -\langle K^0(\sqrt{3}\eta - \pi^0)|(\bar{d}c)(\bar{u}s)|D^0\rangle,$$
$$\langle \bar{K}^0(\eta + \sqrt{3}\pi^0)|(\bar{s}c)(\bar{u}d)|D^0\rangle = \langle K^0(\eta + \sqrt{3}\pi^0)|(\bar{d}c)(\bar{u}s)|D^0\rangle, \quad (69)$$

implying

$$\frac{\sqrt{3}A^{(0)}(\bar{K}^0\eta) - A^{(0)}(\bar{K}^0\pi^0)}{\cos^2\theta_C} = \frac{\sqrt{3}A^{(0)}(K^0\eta) - A^{(0)}(K^0\pi^0)}{\sin^2\theta_C} = 0,$$
$$\frac{A^{(0)}(\bar{K}^0\eta) + \sqrt{3}A^{(0)}(\bar{K}^0\pi^0)}{\cos^2\theta_C} = -\frac{A^{(0)}(K^0\eta) + \sqrt{3}A^{(0)}(K^0\pi^0)}{\sin^2\theta_C} = \frac{4}{\sqrt{3}}\mathcal{A}. \quad (70)$$

The right-hand sides follow using (66) and (67) based on the full SU(2) structure of U-spin.

U-spin breaking of order n in CF and DCS amplitudes is introduced by multiplying transition operators or final states by a total of n powers of the $s - d$ spurion mass operator M_{Ubrk} which changes sign under $d \leftrightarrow s$. Consequently, one has

$$\frac{\sqrt{3}A^{(n)}(\bar{K}^0\eta_8) - A^{(n)}(\bar{K}^0\pi^0)}{\cos^2\theta_C} = (-1)^n\frac{\sqrt{3}A^{(n)}(K^0\eta_8) - A^{(n)}(K^0\pi^0)}{\sin^2\theta_C},$$
$$\frac{A^{(n)}(\bar{K}^0\eta_8) + \sqrt{3}A^{(n)}(\bar{K}^0\pi^0)}{\cos^2\theta_C} = (-1)^n\frac{A^{(n)}(K^0\eta_8) + \sqrt{3}A^{(n)}(K^0\pi^0)}{-\sin^2\theta_C}. \quad (71)$$

Denoting first-order U-spin breaking parameters in these two pairs of processes by ϵ_0 and ϵ_0', one may expand the above four linear combinations of amplitudes to arbitrary order,

$$\sqrt{3}A(\bar{K}^0\eta_8) - A(\bar{K}^0\pi^0) = \cos^2\theta_C\mathcal{A}[\epsilon_0 - a_0\epsilon_0^2 + \cdots],$$
$$\sqrt{3}A(K^0\eta_8) - A(K^0\pi^0) = -\sin^2\theta_C\mathcal{A}[\epsilon_0 + a_0\epsilon_0^2 + \cdots], \quad (72)$$

$$\frac{\sqrt{3}}{4}[A(\bar{K}^0\eta_8) + \sqrt{3}A(\bar{K}^0\pi^0)] = \cos^2\theta_C\mathcal{A}[1 - \epsilon_0' + a_0'\epsilon_0'^2 + \cdots],$$

$$\frac{\sqrt{3}}{4}[A(K^0\eta_8) + \sqrt{3}A(K^0\pi^0)] = -\sin^2\theta_C\mathcal{A}[1 + \epsilon_0' + a_0'\epsilon_0'^2 + \cdots], \quad (73)$$

where $|a_0| \sim, |a_0'| \sim 1$.

Equations (72) imply a linear amplitude relation in which first-order U-spin breaking terms cancel,

$$[\sqrt{3}A(\bar{K}^0\eta_8) - A(\bar{K}^0\pi^0]\tan^2\theta_C + \sqrt{3}A(K^0\eta_8)$$
$$- A(K^0\pi^0) = 0. \quad (74)$$

This relation has also been obtained using a general first-order SU(3) breaking expansion [8], in which a dozen SU(3) breaking parameters contributing to these processes

cancel in this relation. We note that while it follows from Eqs. (73) that the linear relation

$$[A(\bar{K}^0\eta_8) + \sqrt{3}A(\bar{K}^0\pi^0)]\tan^2\theta_C$$
$$- [A(K^0\eta_8) + \sqrt{3}A(K^0\pi^0)] = \frac{4}{\sqrt{3}}\sin^2\theta_C\mathcal{A} \quad (75)$$

is also free of first-order U-spin breaking, the U-spin invariant amplitude $\mathcal{A}$ on the right-hand side is not necessarily invariant under the full flavor SU(3) group

when including other states. This is just like the amplitude A defined above (7) for decays to charged particles.

The four amplitudes for two body decays involving K^0 or $\bar{K}^0$ and π^0 or η_8 have the following first-order expansions:

$$A(\bar{K}^0\pi^0) = \cos^2\theta_C \mathcal{A}\left[1 - \epsilon'_0 - \frac{1}{4}\epsilon_0 + \mathcal{O}(\epsilon_0^2, \epsilon_0'^2)\right],$$

$$\sqrt{3}A(\bar{K}^0\eta_8) = \cos^2\theta_C \mathcal{A}\left[1 - \epsilon'_0 + \frac{3}{4}\epsilon_0 + \mathcal{O}(\epsilon_0^2, \epsilon_0'^2)\right],$$

$$A(K^0\pi^0) = -\sin^2\theta_C \mathcal{A}\left[1 + \epsilon'_0 - \frac{1}{4}\epsilon_0 + \mathcal{O}(\epsilon_0^2, \epsilon_0'^2)\right],$$

$$\sqrt{3}A(K^0\eta_8) = -\sin^2\theta_C \mathcal{A}\left[1 + \epsilon'_0 + \frac{3}{4}\epsilon_0 + \mathcal{O}(\epsilon_0^2, \epsilon_0'^2)\right]. \tag{76}$$

This implies that the two ratios of amplitudes in (66) and (67), which are equal in the U-spin symmetry limit, are also equal when including first-order U-spin breaking:

$$\frac{\sqrt{3}|A(K^0\eta_8)|}{|A(K^0\pi^0)|} = \frac{\sqrt{3}|A(\bar{K}^0\eta_8)|}{|A(\bar{K}^0\pi^0)|} = 1 + \mathrm{Re}\epsilon_0 + \mathcal{O}(\epsilon_0^2, \epsilon_0'^2), \tag{77}$$

$$\frac{|A(K^0\pi^0)|}{|A(\bar{K}^0\pi^0)|\tan^2\theta_C} = \frac{|A(K^0\eta_8)|}{|A(\bar{K}^0\eta_8)|\tan^2\theta_C}$$
$$= 1 + 2\mathrm{Re}\epsilon'_0 + \mathcal{O}(\epsilon_0^2, \epsilon_0'^2). \tag{78}$$

Branching ratio measurements of the above two DCS decay modes are not feasible because a final state neutral kaon is identified in a K_S or a K_L state. This involves an interference between CF and DCS decays. A method for measuring this interference has been proposed in Ref. [36]. Defining a rate asymmetry between decays involving K_S^0 and K_L^0,

$$R(D^0, M^0) \equiv \frac{\Gamma(D^0 \to K_S^0 M^0) - \Gamma(D^0 \to K_L^0 M^0)}{\Gamma(D^0 \to K_S^0 M^0) + \Gamma(D^0 \to K_L^0 M^0)},$$

$$(M^0 = \pi, \eta, \eta') \tag{79}$$

one obtains, to leading order in the ratio of DCS and CF amplitudes,

$$R(D^0, M^0) = -\frac{2\mathrm{Re}[A(\bar{K}^0 M^0)A^*(K^0 M^0)]}{|A(\bar{K}^0 M^0)|^2}. \tag{80}$$

Equations (76) predict equal asymmetries for $M^0 = \pi$ and $M^0 = \eta_8$ up to second-order U-spin breaking. The two asymmetries are given by

$$R(D^0, \eta_8) = R(D^0, \pi^0)$$
$$= 2\tan^2\theta_C[1 + 2\mathrm{Re}\epsilon'_0 + \mathcal{O}(\epsilon_0^2, \epsilon_0'^2)]. \tag{81}$$

Comparing Eqs. (78) and (81), we find

$$\frac{|A(K^0 M^0)|}{|A(\bar{K}^0 M^0)|} = \frac{1}{2}R(D^0, M^0)[1 + \mathcal{O}(\epsilon_0^2, \epsilon_0'^2)],$$

$$(M^0 = \pi^0, \eta_8). \tag{82}$$

That is, although the branching fractions for DCS decays $D^0 \to K^0\pi^0$ and $D^0 \to K^0\eta_8$ cannot be measured directly, they may be obtained up to second-order U-spin breaking corrections from corresponding CF branching fractions and $K_S^0 - K_L^0$ asymmetries:

$$\mathcal{B}(D^0 \to K^0 M^0) = \frac{1}{4}[R(D^0, M^0)]^2 \mathcal{B}(D^0$$
$$\to \bar{K}^0 M^0)[1 + \mathcal{O}(\epsilon_0^2, \epsilon_0'^2)],$$

$$(M^0 = \pi, \eta_8). \tag{83}$$

Table IV summarizes current relevant information on branching ratios and amplitudes for D^0 decays into pairs of neutral pseudoscalars. We do not include the $\eta\eta$ mode (and decays involving the η'), as $D^0 \to \eta_8\eta_8$ would include $D^0 \to \eta'\eta'$ which has zero phase space.

An estimate of U-spin breaking is given by a ratio of SCS and CF decay amplitudes which equals one in the symmetry limit [see (67) (68)],

$$\frac{|A(\pi^0\pi^0)|}{|A(\bar{K}^0\pi^0)|\tan\theta_C} - 1 = -0.21 \pm 0.02, \tag{84}$$

Another quantity measuring U-spin breaking is

TABLE IV.   Branching fractions and amplitudes for D^0 decays to pairs of neutral pseudoscalar mesons [17].

Decay mode	Branching fraction	p (GeV/c)	$|A| = \sqrt{\mathcal{B}/p}(\text{GeV}/c)^{-1/2}$
$D^0 \to K_S^0\pi^0$	$(1.19 \pm 0.04) \times 10^{-2}$	0.860	––
$D^0 \to K_L^0\pi^0$	$(1.00 \pm 0.07) \times 10^{-2}$	0.860	––
$D^0 \to \bar{K}^0\pi^0$	$(2.29 \pm 0.07) \times 10^{-2}$ [a]	0.860	0.163 ± 0.002
$D^0 \to \bar{K}^0\eta$	$(0.890 \pm 0.079) \times 10^{-2}$	0.772	0.107 ± 0.005
$D^0 \to \pi^0\pi^0$	$(8.20 \pm 0.35) \times 10^{-4}$	0.923	0.0298 ± 0.0006
$D^0 \to \eta\pi^0$	$(6.8 \pm 0.7) \times 10^{-4}$	0.846	0.0284 ± 0.0014

[a]Branching ratio calculated as twice the average of $\mathcal{B}(D^0 \to K_S^0\pi^0)$ and $\mathcal{B}(D^0 \to K_L^0\pi^0)$.

$$\frac{\sqrt{3}|A(\bar{K}^0\eta)|}{|A(\bar{K}^0\pi^0)|} - 1 = 0.14 \pm 0.05. \qquad (85)$$

In order to determine $\mathrm{Re}\epsilon_0$ from (77), one would have to know also $|A(\bar{K}^0\eta')|$ and the relative strong phase between this amplitude and $A(\bar{K}^0\eta)$, using a favored value for the η-η' mixing angle [37]. The other U-spin breaking parameter, $\mathrm{Re}\epsilon_0'$, is obtained from (81) using a $K_S^0 - K_L^0$ asymmetry measurement by the CLEO collaboration [38], $R(D^0, \pi^0) = 0.108 \pm 0.035$,

$$\mathrm{Re}\epsilon_0' = \frac{R(D^0, \pi^0) - 2\tan^2\theta_C}{4\tan^2\theta_C} = 0.00 \pm 0.16. \qquad (86)$$

Arguments favoring small U-spin breaking in the asymmetries $R(D^0, M^0)$ have been presented in Ref. [39] adopting a diagrammatic flavor SU(3) approach.

The numerical values of $\mathrm{Re}\epsilon_0'$ in (86) and of the above other two measured U-spin breaking quantities imply that second-order U-spin breaking terms in Eqs. (77)–(83) are at most of order several percent. Neglecting these contributions involves an approximation that is quantitatively similar to the one used to obtain (80), where terms which are second order in the ratio of DCS and CF amplitudes have been neglected.

XII. CONCLUSION

We described a new approach to hadronic D^0 decay amplitudes applying a perturbative expansion in U-spin breaking parameters and treating isospin breaking carefully. We have identified a class of two-body and quasi two-body decays involving charged pseudoscalars (*P*) and vector mesons (*V*), for which in each case adequate hierarchies have been shown to occur between U-spin breaking terms of increasing order.

Nonlinear amplitude relations were predicted in each one of three cases, $D^0 \to P^+P^-$, $D^0 \to V^+P^-$ and $D^0 \to P^+V^-$, which hold up to fourth-order U-spin breaking and isospin-breaking terms suppressed also by second-order U-spin breaking. The three predicted relations have been shown to hold experimentally at a very high precision varying between 10^{-3} and 10^{-4}, in agreement with our estimates of high-order terms. More precise tests require a first robust measurement of $\mathcal{B}(D^0 \to K^+\rho^-)$ and improving branching ratio measurements for D^0 decays to $K^{*+}\pi^-, \pi^+K^{*-}, \pi^+\rho^-$ and K^+K^{*-}. So far, no unexpected flavor symmetry breaking down to this very low level has been found, which would indicate physics beyond the standard model. This provides useful constraints on new $|\Delta C| = 1$ operators, potentially originating in new physics at energies much above a TeV [40].

Finally, we also studied decays to two neutral pseudoscalar mesons, deriving much less precise amplitude relations and relations for rate asymmetries between decays involving K_S^0 and K_L^0, which hold up to second-order U-spin breaking terms at a level of several percent.

ACKNOWLEDGMENTS

[1] R. L. Kingsley, S. B. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D **11**, 1919 (1975); M. B. Voloshin, V. I. Zakharov, and L. B. Okun, Pis'ma Zh. Eksp. Teor. Fiz. **21**, 403 (1975) [JETP Lett. **21**, 183 (1975)]; L. L. Wang and F. Wilczek, Phys. Rev. Lett. **43**, 816 (1979); C. Quigg, Z. Phys. C **4**, 55 (1980).

[2] M. J. Savage, Phys. Lett. B **257**, 414 (1991); W. Kwong and S. P. Rosen, Phys. Lett. B **298**, 413 (1993); I. Hinchliffe and T. A. Kaeding, Phys. Rev. D **54**, 914 (1996).

[3] For a recent study including *CP* asymmetries, see D. Pirtskhalava and P. Uttayarat, Phys. Lett. B **712**, 81 (2012).

[4] L. L. Chau and H. Y. Cheng, Phys. Rev. Lett. **56**, 1655 (1986); Phys. Rev. D **36**, 137 (1987); Phys. Lett. B **280**, 281 (1992); J. L. Rosner, Phys. Rev. D **60**, 114026 (1999); M. Gronau and J. L. Rosner, Phys. Lett. B **500**, 247 (2001).

[5] C. W. Chiang, Z. Luo, and J. L. Rosner, Phys. Rev. D **67**, 014001 (2003); B. Bhattacharya and J. L. Rosner, Phys. Rev. D **77**, 114020 (2008); **79**, 034016 (2009); **81**, 099903 (E) (2010); **81**, 014026 (2010); H. Y. Cheng and C. W. Chiang, Phys. Rev. D **86**, 014014 (2012).

[6] F. Buccella, M. Forte, G. Miele, and G. Ricciardi, Z. Phys. C **48**, 47 (1990); F. Buccella, M. Lusignoli, G. Miele, and A. Pugliese, Z. Phys. C **55**, 243 (1992); F. Buccella, M. Lusignoli, G. Miele, A. Pugliese, and P. Santorelli, Phys. Rev. D **51**, 3478 (1995); M. Zhong, Y. L. Wu, and W. Y. Wang, Eur. Phys. J. C **32**, s191 (2004); Y. L. Wu, M. Zhong, and Y. F. Zhou, Eur. Phys. J. C **42**, 391 (2005); D. N. Gao, Phys. Lett. B **645**, 59 (2007).

[7] G. Hiller, M. Jung, and S. Schacht, Phys. Rev. D **87**, 014024 (2013).

[8] Y. Grossman and D. J. Robinson, J. High Energy Phys. 04 (2013) 067.

[9] M. Gronau, Phys. Lett. B **730**, 221 (2014); **735**, 282(E) (2014).

[10] M. Gronau, Phys. Rev. D **90**, 117901 (2014).

[11] In Ref. [9] it was stated without proof that these isospin-breaking terms involve an additional first-order SU(3) breaking.

[12] S. Meshkov, G. A. Snow, and G. B. Yodh, Phys. Rev. Lett. **13**, 212 (1964); H. J. Lipkin, Phys. Rev. **174**, 2151 (1968).

[13] R. L. Kingsley, S. B. Treiman, F. Wilczek, and A. Zee, Ref. [1].

[14] M. Gronau and J. L. Rosner, Ref. [4].

[15] M. Gronau and J. L. Rosner, Phys. Rev. D **86,** 114029 (2012).

[16] A. F. Falk, Y. Grossman, Z. Ligeti, and A. A. Petrov, Phys. Rev. D **65,** 054034 (2002).

[17] K. A. Olive et al. (Particle Data Group Collaboration), Chin. Phys. C **38,** 090001 (2014).

[18] See e. g. J. Brod, A. L. Kagan, and J. Zupan, Phys. Rev. D **86,** 014023 (2012).

[19] M. Golden and B. Grinstein, Phys. Lett. B **222,** 501 (1989).

[20] B. Bhattacharya, M. Gronau, and J. L. Rosner, Phys. Rev. D **85,** 054014 (2012); B. Bhattacharya, M. Gronau, and J. L. Rosner, *Proceedings of the Tenth International Conference on Flavor Physics and CP Violation—FPCP2012, May 21–25, 2012, Hefei, SLAC eConf C120521,* (SLAC, Menlo Park, CA 2012).

[21] M. Suzuki, Phys. Rev. Lett. **43,** 818 (1979).

[22] L. F. Abbott, P. Sikivie, and M. B. Wise, Phys. Rev. D **21,** 768 (1980).

[23] M. J. Savage, Ref. [2].

[24] T. Feldmann, S. Nandi, and A. Soni, J. High Energy Phys. 06 (2012) 007.

[25] J. Brod, Y. Grossman, A. L. Kagan, and J. Zupan, J. High Energy Phys. 10 (2012) 161.

[26] It was noted in Ref. [8] that first-order U-spin breaking cancels in R_3.

[27] It was shown in Ref. [15] that first-order U-spin breaking cancels also in $|A(D^0 \to K^+K^-)|^2 + |A(D^0 \to \pi^+\pi^-)|^2 -$ $2|A(D^0 \to \pi^+K^-)||A(D^0 \to K^+\pi^-)|,$ contributing to $D^0 - \bar{D}^0$ mixing.

[28] M. Antonelli, D. M. Asner, D. A. Bauer, T. G. Becher, M. Beneke, A. J. Bevan, M. Blanke, C. Bloise et al., Phys. Rep. **494,** 197 (2010).

[29] T. A. Aaltonen et al. (CDF Collaboration), Phys. Rev. Lett. **111,** 231802 (2013); R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. **111,** 251801 (2013); B. R. Ko et al. (Belle Collaboration), Phys. Rev. Lett. **112,** 111801 (2014).

[30] D. M. Asner et al. (CLEO Collaboration), Phys. Rev. D **70,** 091101 (2004); B. Aubert et al. (BABAR Collaboration), Phys. Rev. D **78,** 034023 (2008).

[31] X. C. Tian et al. (Belle Collaboration), Phys. Rev. Lett. **95,** 231801 (2005).

[32] B. Aubert et al. (BABAR Collaboration), Phys. Rev. Lett. **97,** 221803 (2006).

[33] B. Aubert et al. (BABAR Collaboration), Phys. Rev. Lett. **103,** 211801 (2009).

[34] A similar prediction assuming $R_3'' = 1$ with no uncertainty was obtained in Ref. [8].

[35] See, e.g., C. Quigg in Ref. [1] and Ref. [15].

[36] I. I. Y. Bigi and H. Yamamoto, Phys. Lett. B **349,** 363 (1995).

[37] See, e.g., F. G. Cao, Phys. Rev. D **85,** 057501 (2012) and references therein.

[38] Q. He et al. (CLEO Collaboration), Phys. Rev. Lett. **100,** 091801 (2008).

[39] J. L. Rosner, Phys. Rev. D **74,** 057502 (2006).

[40] M. Gronau, Phys. Lett. B **738,** 136 (2014).