

# Correlations between light and heavy flavors near the chiral crossover

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(Received 23 December 2014; published 16 April 2015)

Thermal fluctuations and correlations between the light and heavy-light mesons are explored within a chiral effective theory implementing heavy quark symmetry. We show that various heavy-light flavor correlations indicate a remnant of the chiral criticality in a narrow range of temperature in which the chiral susceptibility exhibits a peak structure. The onset of the chiral crossover, in the heavy-light flavor correlations, is therefore independent from the light flavors. This indicates that the fluctuations carried by strange charmed mesons can also be used to identify the chiral crossover, which is dominated by the nonstrange light quark dynamics.

DOI: 10.1103/PhysRevD.91.074021

PACS numbers: 14.40.Lb, 12.39.Fe, 12.39.Hg

## I. INTRODUCTION

Modifications in magnitude of fluctuations for different observables are usually considered to be an excellent probe of a phase transition or its remnant. In heavy-ion collision, fluctuations related to conserved charges carried by light and strange quarks play an important role in identifying the QCD chiral crossover or deconfinement properties [1,2]. Recently, however, the lattice QCD simulations have revealed that the charmed mesons are deconfined together with light-flavor mesons in the temperature range in which the chiral crossover is partly restored [3]. This result indicates that the light-flavor dynamics interferes nontrivially with the heavy flavors.

In the field theoretical approach, the physics of heavy-light hadrons is constrained by heavy quark symmetry that emerges in the heavy quark mass limit [4,5]. The pseudo-scalar and vector charmed mesons form the lowest spin multiplets  $H$  and their low-energy dynamics is dominated by interactions with Nambu–Goldstone bosons associated with spontaneous chiral symmetry breaking [6–9]. The chiral partner of  $H$  is embodied as the second-lowest spin multiplets  $G$  [10,11]. The mass splitting between  $G$  and  $H$  is proportional to the chiral order parameter, and its thermal/dense evolution characterizes partial restoration of the chiral symmetry in a medium [12–16].

Recently, a self-consistent effective theory implementing the chiral and heavy quark symmetry has been formulated at finite temperature [15]. In the present paper, we will use this effective theory to study the fluctuations in various flavor sectors at finite temperature and vanishing chemical potential. Our special attention will be paid to the properties of heavy-light mixed correlations to be influenced by the underlying heavy quark symmetry in the presence of the chiral crossover. We will show that the onset of the chiral crossover is well identified in the heavy-light flavor correlations and that it is independent of the light flavors.

## II. EFFECTIVE LAGRANGIAN

We utilize the Lagrangian, which includes the mesons with the light and heavy flavors and their couplings.

To quantify the light-flavor dynamics, we introduce the standard linear sigma model with three flavors. The main building block is the chiral field  $\Sigma = T^a \Sigma^a = T^a(\sigma^a + i\pi^a)$ , expressed as a  $3 \times 3$  complex matrix in terms of the scalar  $\sigma^a$  and the pseudoscalar  $\pi^a$  states. The Lagrangian is given by

$$\mathcal{L}_L = \bar{q}(i\partial - gT^a(\sigma^a + i\gamma_5\pi^a))q + \text{tr}[\partial_\mu \Sigma^\dagger \cdot \partial^\mu \Sigma] - V_L(\Sigma), \quad (2.1)$$

with

$$V_L = m^2 \text{tr}[\Sigma^\dagger \Sigma] + \lambda_1 (\text{tr}[\Sigma^\dagger \Sigma])^2 + \lambda_2 \text{tr}[(\Sigma^\dagger \Sigma)^2] - c(\det \Sigma + \det \Sigma^\dagger) - \text{tr}[h(\Sigma + \Sigma^\dagger)]. \quad (2.2)$$

The  $U(1)_A$  breaking effects in Eq. (2.2) are accommodated in the determinant terms, whereas the last term, proportional to  $h = T^a h^a$ , breaks the chiral symmetry explicitly.

Heavy-light meson fields, with negative and positive parity, are introduced as [10,11]

$$H = \frac{1 + \not{x}}{2} [P_\mu^* \gamma^\mu + iP\gamma_5], \quad (2.3)$$

$$G = \frac{1 + \not{x}}{2} [-iD_\mu^* \gamma^\mu \gamma_5 + D], \quad (2.4)$$

and chiral eigenstates are given via

$$\mathcal{H}_{L,R} = \frac{1}{\sqrt{2}} (G \pm iH\gamma_5). \quad (2.5)$$

The relevant operators are transformed under the chiral and heavy quark symmetries as

$$\mathcal{H}_{L,R} \rightarrow S\mathcal{H}_{L,R}g_{L,R}^\dagger, \quad (2.6)$$

$$\Sigma \rightarrow g_L \Sigma g_R^\dagger, \quad (2.7)$$

with the group elements  $g_{L,R} \in SU(3)_{L,R}$  and  $S \in SU(2)_{Q=c}$ .

To formulate thermodynamics, we employ the mean-field approximation. We also assume that there is the SU(2) isospin symmetry in the up- and down-quark sectors. This leads to  $\sigma_0$  and  $\sigma_8$  as nonvanishing condensates, which contain both strange and nonstrange components. The pure nonstrange and strange parts are obtained through the following rearrangement:

$$\begin{pmatrix} \sigma_q \\ \sigma_s \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} \sqrt{2} & 1 \\ 1 & -\sqrt{2} \end{pmatrix} \begin{pmatrix} \sigma_0 \\ \sigma_8 \end{pmatrix}. \quad (2.8)$$

In this base, the effective quark masses read

$$M_q = \frac{g}{2}\sigma_q, \quad M_s = \frac{g}{\sqrt{2}}\sigma_s. \quad (2.9)$$

The chiral fields,  $\sigma_q$  and  $\sigma_s$ , are related with the weak decay constants of pions and kaons, respectively, via the partially conserved axial current hypothesis:

$$\langle \sigma_q \rangle = f_\pi, \quad \langle \sigma_s \rangle = \frac{1}{\sqrt{2}}(2f_K - f_\pi). \quad (2.10)$$

The scalar heavy-light meson states are embedded in the multiplets,

$$D = (D_q, D_q, D_s), \quad (2.11)$$

where, due to the isospin symmetry,  $D_u = D_d = D_q$ . The potential of the heavy-light sector is expressed, in terms of the meson mean fields, as [15]

$$\begin{aligned} V_{\text{HL}} = & 2 \left( m_0 + \frac{1}{4} g_\pi^q \sigma_q \right) D_q^2 + \left( m_0 + \frac{1}{2\sqrt{2}} g_\pi^s \sigma_s \right) D_s^2 \\ & + k_0 (2D_q^2 + D_s^2)^2 + 2k_q \sigma_q D_q^4 + \sqrt{2} k_s \sigma_s D_s^4. \end{aligned} \quad (2.12)$$

The complete thermodynamic potential,

$$\Omega = V_L + V_{\text{HL}} + \Omega_q, \quad (2.13)$$

contains the contributions of light and heavy-light mesons, expressed through Eqs. (2.2) and (2.12), respectively, as well as the quarks contribution

TABLE I. Set of parameters in the light sector with  $m_\sigma = 400$  MeV [17].

$c$ (GeV)	$m$ (GeV)	$\lambda_1$	$\lambda_2$	$h_q$ (GeV <sup>3</sup> )	$h_s$ (GeV <sup>3</sup> )	$g$
4.81	0.495	-5.90	46.48	(0.121) <sup>3</sup>	(0.336) <sup>3</sup>	6.5

TABLE II. Set of parameters in the heavy-light sector [15].

$m_0$ (GeV)	$g_\pi^q$	$g_\pi^s$	$k_0$ (1/GeV <sup>2</sup> )	$k_q$ (1/GeV <sup>3</sup> )	$k_s$ (1/GeV <sup>3</sup> )
1.04	3.78	2.61	-(1/0.74) <sup>2</sup>	-(1/0.44) <sup>3</sup>	-(1/0.53) <sup>3</sup>

$$\Omega_q = 6T \sum_{f=u,d,s} \int \frac{d^3 p}{(2\pi)^3} [\ln(1 - n_f) + \ln(1 - \bar{n}_f)], \quad (2.14)$$

with the Fermi-Dirac distribution functions,  $n_f, \bar{n}_f = 1/(1 + e^{(E_f \mp \mu_f)/T})$ , and the quasiquark energies,  $E_f = \sqrt{p^2 + M_f^2}$ .

In the following, we consider thermodynamics at  $\mu_B = 0$  and account for the charm and strangeness conservation. The model parameters are fixed at  $T = 0$  and are summarized in Tables I and II.

A naive implementation of heavy-light mean fields causes a rather strong mixing to the sigma fields, and it weakens the magnitude of the explicit chiral symmetry breaking at finite  $T$ . However, as discussed in Ref. [15], this defect can be avoided when a certain temperature dependence in the Lagrangian parameters is present. Such intrinsic effects emerge, formally, via integrals of underlying degrees of freedom which interact with a heat bath. Alternatively, they can be extracted from the chiral condensates obtained in lattice QCD, resulting in quenching couplings  $g_\pi^s(T)$  and  $k_s(T)$  [15].<sup>1</sup> In calculating thermodynamic quantities, nontrivial modifications emerge from the temperature-dependent couplings of which the gradients at each temperature are under control. This enables us to perform the calculations in a thermodynamically consistent way.

In the light-flavor sector, we set the sigma meson mass to  $m_\sigma = 400$  MeV so that the chiral crossover temperature comes out to be  $T_{\text{pc}} = 154$  MeV, as obtained in lattice QCD [18].

<sup>1</sup>The thermodynamic potential yields four coupled gap equations for the mean fields  $\sigma_q, \sigma_s, D_q$ , and  $D_s$ . Instead of solving them with a given set of the coupling constants, one uses the chiral condensates extracted from the lattice results,  $\sigma_q$  and  $\sigma_s$ , in the range of  $0.5 < T/T_{\text{pc}} < 1.5$  as input and solves the gap equations for the two mean fields,  $D_q$  and  $D_s$ , and for the two couplings,  $g_\pi^s$  and  $k_s$ .

### III. CORRELATIONS BETWEEN DIFFERENT FLAVORS

#### A. Chiral Susceptibilities

Chiral phase transition is characterized by various fluctuations to which the order parameter couples. The chiral susceptibility  $\hat{\chi}_{\text{ch}}$  is defined through the 2-by-2 matrix [19]

$$\hat{C} = \begin{pmatrix} C_{qq} & C_{qs} \\ C_{sq} & C_{ss} \end{pmatrix}, \quad C_{ij} = \frac{\partial^2 \Omega}{\partial \sigma_i \partial \sigma_j} \quad (3.1)$$

as

$$\hat{\chi}_{\text{ch}} = \hat{C}^{-1} = \begin{pmatrix} \chi_{qq} & \chi_{qs} \\ \chi_{sq} & \chi_{ss} \end{pmatrix}. \quad (3.2)$$

At the chiral symmetry restoration temperature, the chiral susceptibility yields a peak, which is predominantly linked to the inverse of the effective sigma-meson mass squared  $M_\sigma^2$ , which measures the curvature of the potential. Consequently, in the nonstrange sector, the chiral susceptibility is well approximated by

$$\chi_{qq} \sim \frac{\partial^2 \Omega}{\partial \sigma_q^2} \sim M_\sigma^{-2}. \quad (3.3)$$

In Fig. 1, we show the flavor-diagonal  $\chi_{qq}$  and  $\chi_{ss}$  as well as the off-diagonal  $\chi_{qs}$  chiral susceptibilities. All these susceptibilities are dominated by an abrupt change of the  $\sigma_q$ , leading to a peak around  $T_{\text{pc}}$ . At  $T_{\text{pc}}$ , the strange condensate  $\sigma_s$  remains substantially larger than  $\sigma_q$  and melts gradually toward higher temperature. This yields a rather broad bump in  $\chi_{ss}$  around  $T \sim 1.35T_{\text{pc}}$ . The flavor-mixed susceptibility  $\chi_{qs}$  is affected strongly by the nonstrange dynamics, and consequently any remnant of the modification of  $\sigma_s$  is hardly visible there. Such properties of  $\sigma_s$  can be seen also in a pure  $SU(2+1)$  flavor chiral model.<sup>2</sup>

The response of the net quark number  $n_f$  to the chemical potential is quantified by the quark number susceptibilities,

$$\chi_f = \frac{\partial n_f}{\partial \mu_f}. \quad (3.4)$$

<sup>2</sup>In a conventional treatment with constant couplings, a rate change of the chiral order parameter  $\sigma_q$  with temperature is sensitive to the input sigma mass,  $m_\sigma$ . With increasing  $m_\sigma$ , the pseudocritical temperature  $T_{\text{pc}}$  increases, and the crossover becomes smoother. This suppresses the peaks in  $\hat{\chi}_{\text{ch}}$  around  $T_{\text{pc}}$ . Consequently, for a large  $m_\sigma \sim 1$  GeV, the  $\chi_{ss}$  does not yield a pronounced peak at  $T_{\text{pc}}$  but rather a bump at higher temperature.

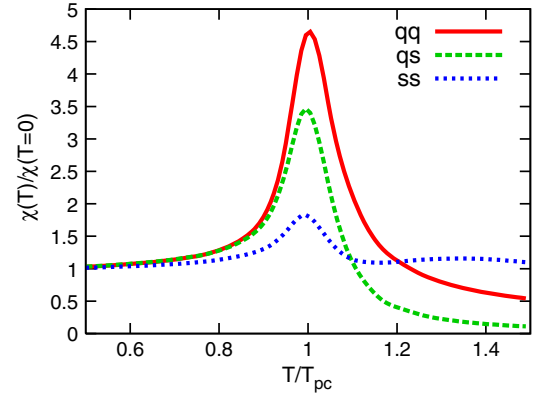


FIG. 1 (color online). Chiral susceptibilities of  $\sigma_q$  and  $\sigma_s$ .

In the mean-field approximation, the  $\chi_q$  and  $\chi_s$  can be expressed as a product of the chiral condensates and the corresponding susceptibilities as [19]

$$\chi_q \sim \chi_q^{(0)} + \sigma_q^2 \cdot \chi_{qq}, \quad \text{and} \quad \chi_s \sim \chi_s^{(0)} + \sigma_s^2 \cdot \chi_{ss}, \quad (3.5)$$

where  $\chi_{q,s}^{(0)}$  correspond to the noninteracting part.

In the present calculation, restricted to vanishing chemical potentials, the peaks of the chiral susceptibilities  $\chi_{qq}$  and  $\chi_{ss}$  are suppressed by the multiplied condensates  $\sigma_{q,s}$ , which monotonically decrease with temperature. Nevertheless, they exhibit sensitivity to the onset of the chiral crossover. At  $\mu_f = 0$ , the chiral criticality in the quark number fluctuations starts to show up in the sixth-order cumulant [20]. At finite chemical potentials, the  $\chi_q$  yields more significance to the criticality, carried by the term  $\partial \sigma_q / \partial \mu_q$  in Eq. (3.4).<sup>3</sup>

#### B. Correlations between light and heavy flavors

To study correlations among the light and heavy-light mesons, we introduce a matrix in the base of the four mean fields:

$$\hat{C} = \begin{pmatrix} C_{qq} & C_{qs} & C_{qD_q} & C_{qD_s} \\ C_{sq} & C_{ss} & C_{sD_q} & C_{sD_s} \\ C_{D_q q} & C_{D_q s} & C_{D_q D_q} & C_{D_q D_s} \\ C_{D_s q} & C_{D_s s} & C_{D_s D_q} & C_{D_s D_s} \end{pmatrix}, \quad (3.6)$$

$$C_{ij} = \frac{\partial^2 \Omega}{\partial \sigma_i \partial \sigma_j}, \quad C_{iD_j} = \sqrt{2M_{D_j}} \frac{\partial^2 \Omega}{\partial \sigma_i \partial D_j},$$

$$C_{D_i D_j} = 2\sqrt{M_{D_i} M_{D_j}} \frac{\partial^2 \Omega}{\partial D_i \partial D_j}.$$

A set of susceptibilities  $\hat{\chi}$  is then defined as

<sup>3</sup>At zero chemical potential, the coefficient of this term vanishes.

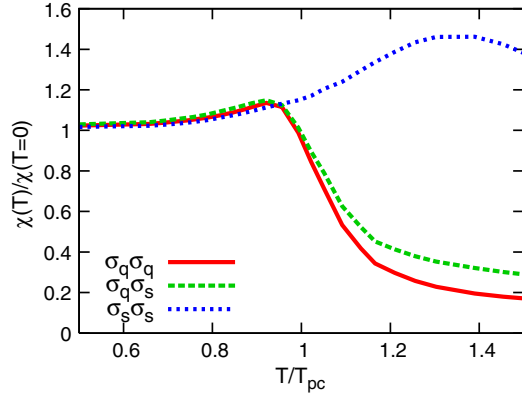


FIG. 2 (color online). Flavor correlations among the light mesons.

$$\hat{\chi} = \hat{C}^{-1}. \quad (3.7)$$

They are associated with the chiral susceptibilities defined in Eq. (3.2). The explicit relations are summarized in the Appendix.

The light-flavor correlations, obtained from Eq. (3.7), are shown in Fig. 2. In striking contrast to the chiral susceptibilities shown in Fig. 1, they are to a large extent suppressed, due to the heavy-light mean fields that act as an extra external source. Nevertheless, the light-flavor correlations change their properties around  $T_{pc}$ . The  $qq$  and  $qs$  components drop just above  $T_{pc}$ , whereas the  $ss$  component shows a broad bump above  $T_{pc}$ , similarly as that seen in  $\hat{\chi}_{ch}$  in Fig. 1.

Correlations, obtained from Eqs. (3.6) and (3.7), between the light and heavy-light mesons, as well as those between the heavy-light mesons, are shown in Figs. 3 and 4, respectively.

There is a clear influence of remnants of the chiral criticality on correlations with heavy flavor. This is the case not only for the susceptibilities involving the non-strange flavors but also those with the strange content, which exhibit a certain qualitative change almost in the same temperature range near  $T_{pc}$ . This characteristic feature of the quantities involving a charm quark is governed by the heavy quark symmetry, which guarantees, in the leading order of the heavy-quark inverse-mass expansion, that the physics is independent of light flavors. The same tendency has also been found in the effective charmed-meson masses [15]. Consequently, the  $\chi_{D_i D_j}$  follows the same pattern of a sensitivity to the chiral crossover as the chiral susceptibilities (see Fig. 1). We note that  $\chi_{\sigma_s D_q}$  in Fig. 3 changes its sign at  $T/T_{pc} \sim 1.3$ , and this does not lead to any instability in the system since the generalized susceptibility is not precisely the curvature of the entire potential; see the Appendix.

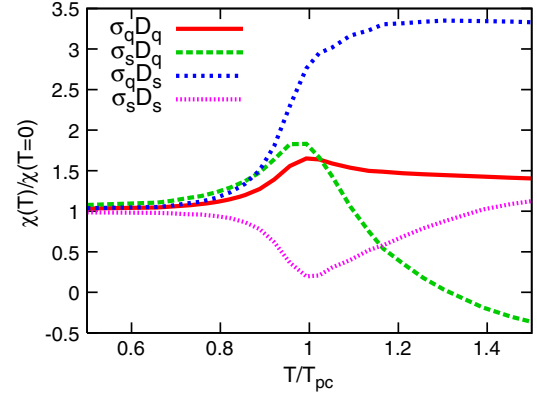


FIG. 3 (color online). Flavor correlations between the light and heavy-light mesons.

Certain thermodynamic quantities, to which the chiral condensates couple, are known to be governed by the underlying  $O(4)$  universality around  $T_{pc}$ , even though the chiral symmetry is not exact but violated only softly. Just like the chiral symmetry, the heavy quark symmetry is also reliable around  $T_{pc}$  since the charm quark mass  $m_c$  is much larger than  $T_{pc}$ , and thus the  $1/m_c$  expansion should hold. The extended flavor correlations given in Figs. 3 and 4 apparently show that the heavy-flavor symmetry constrains chiral thermodynamics. Further fluctuations beyond the mean-field approximation will modify the flavor correlations on a quantitative level. This has been shown on the level of different fluctuations, without heavy flavors, in terms of effective chiral models using the functional renormalization method that accounts for quantum and thermal fluctuations of the chiral fields [21]. The main modifications due to the quantum fluctuations result in smoothing susceptibilities across the chiral crossover; however, the generic structure is preserved. Thus, the main feature of the heavy-light correlations is also considered to be unaffected.

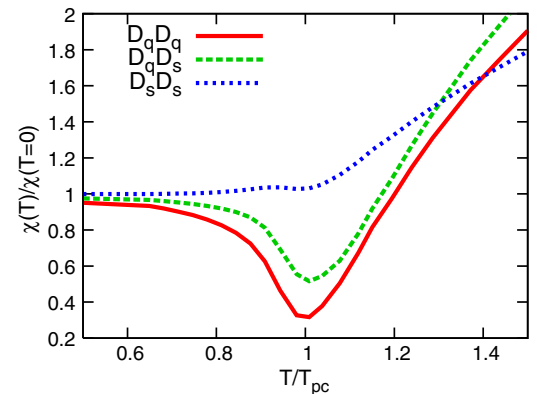


FIG. 4 (color online). Flavor correlations among the heavy-light mesons.

#### IV. CONCLUSIONS

We have studied different correlations between the light and heavy-light flavored mesons at finite temperature within a chiral effective theory implementing heavy quark symmetry. Particular attention was attributed to properties of these correlations near the chiral crossover temperature,  $T_{pc}$ .

We have shown that the chiral properties of the correlations among the light mesons are flavor dependent. In the  $qq$  and  $qs$  sector, the qualitative change of correlations coincides with the chiral crossover. This is, however, not the case in the  $ss$  sector in which the modification of the fluctuations is shifted to a slightly higher temperature above the chiral crossover, due to the large explicit chiral symmetry breaking for the strange quark.

A striking contrast is found in the correlations involving the heavy-light meson mean fields. Those fluctuations exhibit certain qualitative changes around  $T_{pc}$ , and this feature is independent of the light flavors. Theoretically, it is anchored to the heavy quark symmetry, which guarantees that the charm quark does not distinguish the nonstrange from the strange flavor in the limit of  $m_c \rightarrow \infty$ .

In the heavy-light system, the heavy quark dynamics is tied to the light-flavor physics, and the thermodynamics is strongly dragged by the chiral crossover dominated by the nonstrange flavors. Consequently, the fluctuations carried by the strange states can also be used to measure the onset of the chiral symmetry restoration. The situation is essentially different from the pure light-flavored system in which the observables with strangeness, e.g., effective masses of the kaon and its chiral partner, are rather insensitive to the onset of the chiral crossover.

The lattice QCD studies for the  $D_s$  states at finite temperature in Ref. [3] strongly suggest that the charmed mesons start to melt at a temperature close to  $T_{pc} \sim 154$  MeV. Thus, the hadronic picture should not be naively extrapolated to higher temperatures. However, because of the crossover nature, such dissociation may take place gradually in some range of temperature. In particular, the correlations might still be mediated by collective modes like mesonic bound states up to temperatures above the chiral crossover. In fact, a study in the  $N_f = 2$  Nambu–Jona-Lasinio model, which can handle a bound-state nature of the light mesons and their dissociation, shows the presence of residual correlations of mesonlike states even above the chiral crossover [22]. This also suggests similar soft modes in the heavy-light sector. Therefore, the fluctuations extrapolated slightly above  $T_{pc}$  may not be totally unrealistic.

An important application is to explore the flavor fluctuations at finite density and to quantify the chiral

modifications of heavy-light hadrons. There exist a number of works on the charmed mesons in nuclear matter, in the context of QCD sum rules [14,23–25] and effective theories [16,26–32]. In utilizing our effective theory, a central task is to introduce a reliable density dependence of the interaction parameters, which requires a more microscopic prescription [33].

#### ACKNOWLEDGMENTS

This work has been partly supported by the Hessian LOEWE initiative through the Helmholtz International Center for FAIR (HIC for FAIR), and by the Polish Science Foundation (NCN) under Maestro Grant No. DEC-2013/10/A/ST2/00106.

#### APPENDIX: FLUCTUATIONS OF THE LIGHT AND HEAVY-LIGHT MESONS

In the following, we outline how the chiral susceptibilities are embedded in the extended flavor susceptibilities (3.7). We introduce the following  $(2 \times 2)$  block matrices for  $\hat{C}$ :

$$\hat{C} = \begin{pmatrix} \hat{C}_L & \hat{C}_{HL} \\ \hat{C}_{HL} & \hat{C}_D \end{pmatrix}, \quad (\text{A1})$$

with

$$\hat{C}_L = \hat{C}, \quad \hat{C}_{HL} = \begin{pmatrix} C_{qD_q} & C_{qD_s} \\ C_{sD_q} & C_{sD_s} \end{pmatrix},$$

$$\hat{C}_D = \begin{pmatrix} C_{D_q D_q} & C_{D_q D_s} \\ C_{D_s D_q} & C_{D_s D_s} \end{pmatrix}. \quad (\text{A2})$$

The heavy-light mixed susceptibilities (3.7) can also be composed of  $(2 \times 2)$  matrices as

$$\hat{\chi} = \hat{C}^{-1} = \begin{pmatrix} \hat{\chi}_{\sigma\sigma} & \hat{\chi}_{\sigma D} \\ \hat{\chi}_{D\sigma} & \hat{\chi}_{DD} \end{pmatrix}. \quad (\text{A3})$$

Each matrix is expressed in terms of the chiral susceptibility  $\hat{\chi}_{ch}$  and the matrix  $\hat{C}_{HL}$  that represents the curvature of the effective potential in the  $\sigma$ - $D$  direction. One finds that the following relations hold:

$$\begin{aligned} \hat{\chi}_{\sigma\sigma} &= \hat{\chi}_{ch} + \hat{\chi}_{ch} \hat{C}_{HL} \hat{\chi}_D \hat{C}_{HL} \hat{\chi}_{ch}, \\ \hat{\chi}_{\sigma D} &= -\hat{\chi}_{ch} \hat{C}_{HL} \hat{\chi}_D, \\ \hat{\chi}_{D\sigma} &= -\hat{\chi}_D \hat{C}_{HL} \hat{\chi}_{ch}, \\ \hat{\chi}_{DD} &= \hat{C}_D - \hat{C}_{HL} \hat{\chi}_{ch} \hat{C}_{HL} \equiv \hat{\chi}_D. \end{aligned} \quad (\text{A4})$$

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