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Prediction of leptonic *CP* phase in A_4 symmetric model

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We consider minimal modifications to the tribimaximal (TBM) mixing matrix which accommodate nonzero mixing angle θ_{13} and *CP* violation. We derive four possible forms for the minimal modifications to TBM mixing in a model with A_4 flavor symmetry by incorporating symmetry breaking terms appropriately. We show how possible values of the Dirac-type *CP* phase δ_D can be predicted with regards to two neutrino mixing angles in the standard parametrization of the neutrino mixing matrix. Carrying out a numerical analysis based on the recent updated experimental results for neutrino mixing angles, we predict the values of the *CP* phase for all possible cases. We also confront our predictions for the *CP* phase with the updated fit.

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I. INTRODUCTION

Establishing leptonic *CP* violation (LCPV) is one of the most challenging tasks for future neutrino experiments [1]. The relatively large value of the reactor mixing angle measured with high precision in neutrino epxeriments [2] has opened up a wide range of possibilities to explore *CP* violation in the lepton sector. The LCPV can be induced by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix [3], which contains, in addition to the three angles, a Dirac-type *CP* violating phase in general as it exists in the quark sector, and two extra phases if neutrinos are Majorana particles. Although we do not yet have compelling evidence for the LCPV, the current global fit to the available neutrino data indicates nontrivial values of the Dirac-type *CP* phase [4,5]. In this situation, we must attempt to predict the possible size of the LCPV detectable through neutrino oscillations. From the point of view of *calculability*, much attention has been paid to the prediction of the Dirac-type LCPV phase with regards to some of the observables [6]. Recently, it has been shown [7] that the Dirac-type leptonic *CP* phase can be particularly predictable in terms of the neutrino mixing angles in the standard parametrization of the PMNS mixing matrix [8].

Before the measurements of the reactor mixing angle, the fit to the neutrino data was consistent with the so-called tribimaximal (TBM) neutrino mixing matrix, U_0^{TBM}, which is theoretically a well motivated flavor mixing pattern [9]. However, it should be modified to accommodate a nonzero reactor mixing angle as well as *CP* violation. Although the current neutrino data rule out the exact TBM mixing pattern, the exact TBM mixing can be regarded as a leading order approximation. Among the various possible modifications to U_{TBM}, as discussed in [7], the minimal modification is useful to predict the Dirac-type *CP* phase. The minimal modification is to multiply U_0^{TBM} by a rotation matrix in the (i,j) plane with an angle θ and a *CP* phase ξ, $U_{ij}(\theta,\xi)$, whose form is given as either $U_{ij}^\dagger(\theta,\xi)U_0^{\mathrm{TBM}}$ or $U_0^{\mathrm{TBM}}U_{ij}(\theta,\xi)$ [10]. Among them, $U_{23}^\dagger(\theta,\xi)U_0^{\mathrm{TBM}}$ and $U_0^{\mathrm{TBM}}U_{12}(\theta,\xi)$ are ruled out because they lead to a zero reactor mixing angle. So, all possible forms of minimal modification to the TBM mixing matrix are as follows:

$$V=\begin{cases} U_0^{\mathrm{TBM}}U_{23}(\theta,\xi)(\mathrm{case\,A}),\\ U_0^{\mathrm{TBM}}U_{13}(\theta,\xi)(\mathrm{case\,B}),\\ U_{12}^\dagger(\theta,\xi)U_0^{\mathrm{TBM}}(\mathrm{case\,C}),\\ U_{13}^\dagger(\theta,\xi)U_0^{\mathrm{TBM}}(\mathrm{case\,D}).\end{cases} \tag{1}$$

While the study in [7] did not account for the origin of such a modification to U_0^{TBM}, in this paper, we first study how such a minimally modified TBM mixing pattern can be achieved in a neutrino model with A_4 flavor symmetry by incorporating A_4 symmetry breaking terms appropriately. Then, following [7], we investigate how the Dirac-type *CP* phase can be predicted based on the updated fit results for neutrino mixing angles [5]. As shown later, in a comparison with the results obtained in [7], the Dirac-type *CP* phase prediction based on the updated fit results has different implications, particularly at 1σ C.L.

II. MINIMAL MODIFICATIONS TO TRIBIMAXIMAL MIXING IN AN A_4 SYMMETRIC MODEL

In [11], an A_4 symmetric model for neutrino masses and mixing has been proposed to accommodate the nonzero mixing angle θ_{13} on top of the TBM mixing. Based on the A_4 symmetric model, we study how the forms given in Eq. (1) can be derived by incorporating the appropriate A_4 symmetry breaking terms.

A. Case A

As proposed in [11], A_4 flavor symmetry allows the charged-lepton mass matrix to be diagonalized by the Cabibbo-Wolfenstein matrix [12],

$$U_{\rm CW} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \tag{2}$$

where $\omega = e^{2\pi i/3}$, with three independent eigenvalues, m_e, m_μ, m_τ. This can be realized by the lepton assignments $L_i = (\nu_i, l_i) \sim \underline{3}$, $l_1^c \sim \underline{1}$, $l_2^c \sim \underline{1}'$, $l_3^c \sim \underline{1}''$, with three Higgs doublets $\Phi_i = (\phi_i^0, \phi_i^-) \sim \underline{3}$. Introducing three heavy A_4 Higgs singlets and three triplets,

$$\eta_1 \sim \underline{1}, \qquad \eta_2 \sim \underline{1}', \qquad \eta_3 \sim \underline{1}'', \qquad \eta_{i(=4,5,6)} \sim \underline{3}, \tag{3}$$

where $\eta_i = (\eta_i^{++}, \eta_i^{+}, \eta_i^0)$, one can obtain the neutrino mass matrix in the A_4 basis [11]

$$M_\nu = \begin{pmatrix} a+b+c & f & e \\ f & a+\omega b+\omega^2 c & d \\ e & d & a+\omega^2 b+\omega c \end{pmatrix}, \tag{4}$$

where a comes from $\langle\eta_1^0\rangle$, b from $\langle\eta_2^0\rangle$, c from $\langle\eta_3^0\rangle$, d from $\langle\eta_4^0\rangle$, e from $\langle\eta_5^0\rangle$, and f from $\langle\eta_6^0\rangle$. To achieve the TBM mixing pattern of the neutrino mixing matrix, A_4 flavor symmetry should be broken into Z_2 in such a way that $b = c$ and $e = f = 0$. Then, the neutrino mass matrix in the flavor basis where the charged-lepton mass matrix is diagonal is given by

$$M_\nu^{(e,\mu,\tau)} = U_{\rm CW}^\dagger M_\nu U_{\rm CW}^* = \begin{pmatrix} a+(2d/3) & b-(d/3) & b-(d/3) \\ b-(d/3) & b+(2d/3) & a-(d/3) \\ b-(d/3) & a-(d/3) & b+(2d/3) \end{pmatrix}, \tag{5}$$

which is diagonalized by the TBM mixing matrix $U_0^{\rm TBM}$. To achieve nonzero mixing angle θ_{13} so as to accommodate neutrino data from reactor experiments, we take $b = c$ and $e = -f \equiv -\epsilon \neq 0$ in Eq. (4), and the neutrino mass matrix in the A_4 basis is then given by

$$M_\nu = \begin{pmatrix} a+2b & -\epsilon & \epsilon \\ -\epsilon & a-b & d \\ \epsilon & d & a-b \end{pmatrix}. \tag{6}$$

In the flavor basis, the neutrino mass matrix can be rewritten as

$$M_\nu^{(e,\mu,\tau)} = \begin{pmatrix} a+(2d/3) & b-(d/3) & b-(d/3) \\ b-(d/3) & b+(2d/3) & a-(d/3) \\ b-(d/3) & a-(d/3) & b+(2d/3) \end{pmatrix} + \frac{i}{\sqrt{3}}\begin{pmatrix} 0 & -\epsilon & \epsilon \\ -\epsilon & -2\epsilon & 0 \\ \epsilon & 0 & 2\epsilon \end{pmatrix}. \tag{7}$$

Rotating the mass matrix given in Eq. (7) by the TBM mixing matrix, we get

$$\begin{pmatrix} a-b+d & 0 & 0 \\ 0 & a+2b & X \\ 0 & X & b-a+d \end{pmatrix}, \tag{8}$$

where $X = \sqrt{2}i\epsilon$ and nonzero entries are complex in general. It can be easily shown that the mass matrix given by Eq. (7) can be diagonalized by

$$V' = U_0^{\rm TBM}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta e^{-i\xi} \\ 0 & \sin\theta e^{i\xi} & \cos\theta \end{pmatrix} \cdot P_\beta, \tag{9}$$

where $P_\beta = {\rm Diag}[e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3}]$.

Now, let us check the testability of the cases in this neutrino model by taking into account the sum rules among the light neutrino masses [13]. In the leading order, the mass eigenvalues are given by $m_1^0 = a-b+d$, $m_2^0 = a+2b$, $m_3^0 = b-a+d$, and thus we get the mass sum rules

$$\begin{aligned} m_3^0 &= m_2^0 + m_1^0, \quad \text{for } a = 0, \\ m_1^0 &= 2m_2^0 + m_3^0, \quad \text{for } b = 0. \end{aligned} \tag{10}$$

Including the perturbation given by the second matrix in Eq. (7), we get the following sum rule,

$$\hat{m}_2 + \hat{m}_3 = \hat{m}_2^0 + \hat{m}_3^0, \tag{11}$$

where $\hat{m}_2 \equiv m_2 e^{-i\xi}$, $\hat{m}_3 \equiv m_3 e^{i\xi}$, $\hat{m}_2^0 \equiv m_2^0 e^{-i\xi}$, $\hat{m}_3^0 \equiv m_3^0 e^{i\xi}$, with $m_{i(=1,2,3)}$ representing the mass eigenvalues obtained by diagonalizing the mass matrix equation (8). Plugging Eq. (10) into Eq. (11), we can get the following sum rules for $\xi = 0$, π, 2π,

$$\begin{aligned} m_1 + m_2 - m_3 &= 2\delta m_2, \quad \text{for } a = 0, \\ 2m_2 + m_3 - m_1 &= \delta m_2, \quad \text{for } b = 0, \end{aligned} \tag{12}$$

where $\delta m_2 \equiv m_2 - m_2^0$ and we have used $m_1 = m_1^0$. The sum rules for $\xi = \pi/2, (3\pi/2)$ are

$$m_1 + m_2 = m_3, \quad \text{for } a = 0,$$
$$2m_2 + m_3 - m_1 = 3\delta m_2, \quad \text{for } b = 0. \tag{13}$$

B. Case B

To realize case B, we add the breaking terms δM_ν to M_ν in the A_4 basis, which is given by

$$\delta M_\nu = \begin{pmatrix} g+h & 0 & 0 \\ 0 & \omega g + \omega^2 h & 0 \\ 0 & 0 & \omega^2 g + \omega h \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & -A \end{pmatrix}, \tag{14}$$

where $g = -h$ and $A = \sqrt{3} i g$.

Then, the matrix given in Eq. (14) becomes in the flavor basis as follows:

$$\begin{pmatrix} 0 & g & -g \\ g & -g & 0 \\ -g & 0 & g \end{pmatrix}. \tag{15}$$

Then, the mass matrix $M_\nu + \delta M_\nu$ can be diagonalized by

$$V = U_0^{\mathrm{TBM}} \begin{pmatrix} \cos\theta & 0 & -\sin\theta e^{-i\xi} \\ 0 & 1 & 0 \\ \sin\theta e^{i\xi} & 0 & \cos\theta \end{pmatrix} \cdot P_\beta. \tag{16}$$

For case B, the sum rules at the leading order are the same as in Eq. (10). Including the perturbation given by the second matrix in Eq. (15), we get the following sum rule:

$$\hat{m}_1 + \hat{m}_3 = \hat{m}_1^0 + \hat{m}_3^0, \tag{17}$$

where $\hat{m}_1 \equiv m_1 e^{-i\xi}$, $\hat{m}_3 \equiv m_3 e^{i\xi}$, $\hat{m}_1^0 \equiv m_1^0 e^{-i\xi}$, $\hat{m}_3^0 \equiv m_3^0 e^{i\xi}$, with $m_{i(=1,2,3)}$ representing the mass eigenvalues obtained by diagonalizing the mass matrix $M_\nu + \delta M_\nu$. Plugging Eq. (10) into Eq. (17), we can get the following sum rules for $\xi = 0, \pi, 2\pi$,

$$m_3 - m_2 - m_1 = 2\delta m_3, \quad \text{for } a = 0,$$
$$m_3 + 2m_2 - m_1 = 2\delta m_3, \quad \text{for } b = 0, \tag{18}$$

where $\delta m_3 \equiv m_3 - m_3^0$ and we have used $m_2 = m_2^0$. The sum rules for $\xi = \pi/2, (3\pi/2)$ are

$$m_1 - m_3 + m_2 = 0, \quad \text{for } a = 0,$$
$$m_1 - 2m_2 - m_3 = 0, \quad \text{for } b = 0. \tag{19}$$

C. Case C

Case C can be realized by adding the A_4 breaking term δM_l to the charged-lepton mass matrix M_l :

$$\delta M_l = \begin{pmatrix} g_1 v_1 & g_2 v_1 & 0 \\ g_1 \omega v_2 & g_2 v_2 & 0 \\ g_1 \omega^2 v_3 & g_3 v_3 & 0 \end{pmatrix}. \tag{20}$$

Taking $v_1 = v_2 = v_3$ and $g_1 = g_2 = g$, the matrix given by (20) becomes

$$\delta M_l = \begin{pmatrix} gv & gv & 0 \\ g\omega v & gv & 0 \\ g\omega^2 v & gv & 0 \end{pmatrix} = U_{\mathrm{CW}} \begin{pmatrix} 0 & g & 0 \\ g & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \sqrt{3} v. \tag{21}$$

Because of the addition of δM_l, the PMNS mixing matrix should be changed to $U_{12}^\dagger(\theta, \xi) U_0^{\mathrm{TBM}} P_\beta$.

For case C, the sum rules are given by Eq. (10).

D. Case D

Similarily, case D can be achieved by adding the following matrix δM_l to the charged-lepton mass matrix M_l:

$$\delta M_l = \begin{pmatrix} g_1 v_1 & 0 & g_2 v_1 \\ g_1 \omega^2 v_2 & 0 & g_2 v_2 \\ g_1 \omega v_3 & 0 & g_3 v_3 \end{pmatrix}. \tag{22}$$

Taking $v_1 = v_2 = v_3$ and $g_1 = g_2 = g$, the matrix given in (22) becomes

$$\delta M_l = \begin{pmatrix} gv & 0 & gv \\ g\omega^2 v & 0 & gv \\ g\omega v & 0 & gv \end{pmatrix} = U_{\mathrm{CW}} \begin{pmatrix} 0 & 0 & g \\ 0 & 0 & 0 \\ g & 0 & 0 \end{pmatrix} \sqrt{3} v. \tag{23}$$

The addition of δM_l causes the PMNS mixing matrix to be changed to $U_{13}^\dagger(\theta, \xi) U_0^{\mathrm{TBM}} P_\beta$.

For case D, the sum rules are given by Eq. (10).

III. PREDICTIONS FOR THE DIRAC-TYPE *CP* PHASE

Now, let us review how to predict the Dirac-type *CP* phase in the PMNS mixing matrix with regards to the neutrino mixing angles presented in [7]. Multiplying the V

given in Eq. (1) by the phase matrices P_α and P_β that can arise from the charged-lepton sector and the neutrino sector, respectively, we can equate it with the standard parametrization of the PMN mixing matrix as follows:

$$P_\alpha \cdot V \cdot P_\beta = U^{\rm ST} = U_0^{\rm PMNS} \cdot P_\phi = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_D} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_D} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_D} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_D} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_D} & c_{23}c_{13} \end{pmatrix} P_\phi. \tag{24}$$

The equivalence between both parametrizations dictates the following relations,

$$V_{ij}e^{i(\alpha_i+\beta_j)} = U_{ij}^{\rm ST} = (U_0^{\rm PMNS})_{ij}e^{i\phi_j}. \tag{25}$$

A. Cases A and B

Applying $|V_{13}| = |U_{13}^{\rm ST}|$ and $|V_{11}/V_{12}| = |U_{11}^{\rm ST}/U_{12}^{\rm ST}|$, we obtain the relations

$$\sin^2\theta = 3s_{13}^2,$$
$$\cos^2\theta = \begin{cases} 2\tan^2\theta_{12} & (\text{case A}), \\ \frac{1}{2}\cot^2\theta_{12} & (\text{case B}), \end{cases} \tag{26}$$

which lead to this relation between the solar and reactor mixing angles,

$$s_{12}^2 = \begin{cases} 1 - \frac{2}{3(1-s_{13}^2)} & (\text{case A}), \\ \frac{1}{3(1-s_{13}^2)} & (\text{case B}). \end{cases} \tag{27}$$

Those relations indicate that the nonzero values of s_{13}^2 lead to $s_{12}^2 < 1/3$ for case A and $s_{12}^2 > 1/3$ for case B. From $|V_{23}/V_{33}| = |U_{23}^{\rm ST}/U_{33}^{\rm ST}|$, we also get the relations

$$|\cos\eta| = \begin{cases} \frac{c_{13}^2(s_{23}^2 - c_{23}^2)}{2s_{13}\sqrt{2-6s_{13}^2}} & (\text{case A}), \\ \frac{c_{13}^2(c_{23}^2 - s_{23}^2)}{s_{13}\sqrt{2-3s_{13}^2}} & (\text{case B}). \end{cases} \tag{28}$$

Now, we demonstrate how to derive δ_D in terms of the neutrino mixing angles in the standard parametrization. From the components of the neutrino mixing matrix for case A, we see that

$$\frac{V_{23}+V_{33}}{V_{22}+V_{32}} = \frac{V_{13}}{V_{12}}. \tag{29}$$

From the relation (25), we get the relations

$$\frac{U_{13}^{\rm ST}}{U_{12}^{\rm ST}} = \frac{U_{23}^{\rm ST} + U_{33}^{\rm ST}e^{-i(\alpha_3-\alpha_2)}}{U_{22}^{\rm ST} + U_{32}^{\rm ST}e^{-i(\alpha_3-\alpha_2)}}, \tag{30}$$

$$\frac{U_{3i}^{\rm ST}}{U_{2i}^{\rm ST}} = \frac{V_{3i}}{V_{2i}}e^{i(\alpha_3-\alpha_2)}. \tag{31}$$

Since $V_{21} = V_{31}$,

$$e^{i(\alpha_3-\alpha_2)} = \frac{U_{31}^{\rm ST}}{U_{21}^{\rm ST}}. \tag{32}$$

Plugging Eq. (32) into Eq. (30), we finally obtain the relation

$$\frac{U_{13}^{\rm ST}}{U_{12}^{\rm ST}} = \frac{U_{23}^{\rm ST}U_{31}^{\rm ST} + U_{33}^{\rm ST}U_{21}^{\rm ST}}{U_{22}^{\rm ST}U_{31}^{\rm ST} + U_{32}^{\rm ST}U_{21}^{\rm ST}}. \tag{33}$$

Notice that the Majorana phases in Eq. (33) are canceled. Presenting $U_{ij}^{\rm ST}$ explicitly in terms of the neutrino mixing angles as well as δ_D, we find the equation for δ_D to be

$$\cos\delta_D = \frac{-1}{2\tan 2\theta_{23}} \times \frac{1-5s_{13}^2}{s_{13}\sqrt{2-6s_{13}^2}}. \tag{34}$$

Notice that the imaginary part in Eq. (33) is automatically canceled.

Similarly, we get the relation for case B,

$$\cos\delta_D = \frac{1}{2\tan 2\theta_{23}} \times \frac{2-4s_{13}^2}{s_{13}\sqrt{2-3s_{13}^2}}. \tag{35}$$

B. Cases C and D

Applying $|V_{13}| = |U_{13}^{\rm ST}|$ and $|V_{23}/V_{33}| = |U_{23}^{\rm ST}/U_{33}^{\rm ST}|$, we obtain the relations

$$\sin^2\theta = 2s_{13}^2, \quad \cos^2\theta = \begin{cases} \tan^2\theta_{23} & (\text{case C}), \\ \cot^2\theta_{23} & (\text{case D}). \end{cases} \tag{36}$$

which lead to the relation between the atmospheric and reactor mixing angles,

$$s_{23}^2 = \begin{cases} 1 - \frac{1}{2(1-s_{13}^2)} & (\text{case C}), \\ \frac{1}{2(1-s_{13}^2)} & (\text{case D}). \end{cases} \tag{37}$$

TABLE I. Three neutrino mixing parameters from the fit to the global data after the NOW 2014 conference [5].

	Normal ordering ($\Delta\chi^2 = 0.97$)		Inverted ordering (best fit)	
	bfp $\pm$ 1σ	3σ range	bfp $\pm$ 1σ	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.270 \to 0.344$	$0.304^{+0.013}_{-0.012}$	$0.270 \to 0.344$
$\sin^2\theta_{23}$	$0.452^{+0.052}_{-0.028}$	$0.382 \to 0.643$	$0.579^{+0.025}_{-0.037}$	$0.389 \to 0.644$
$\sin^2\theta_{13}$	$0.218^{+0.0010}_{-0.0010}$	$0.0186 \to 0.0250$	$0.219^{+0.0011}_{-0.0010}$	$0.0188 \to 0.0251$

Those relations indicate that the nonzero values of s^2_{13} lead to $s^2_{23} < 1/2$ for case C and $s^2_{23} > 1/2$ for case D. From $|V_{11}/V_{12}| = |U^{\rm ST}_{11}/U^{\rm ST}_{12}|$, we also get the relation

$$|\cos\eta| = \frac{3c^2_{13}s^2_{12} - 1}{2s_{13}\sqrt{2 - 4s^2_{13}}}. \quad (38)$$

We note that both cases lead to the same relation for $|\cos\eta|$.

Following the same procedures for obtaining Eqs. (34) and (35), we get the relations

$$\cos\delta_D = \begin{cases} \frac{s^2_{13}-(1-3s^2_{12})(1-3s^2_{13})}{6s_{12}c_{12}s_{13}\sqrt{1-2s^2_{13}}} & (\text{case C}), \\ \frac{(1-3s^2_{12})(1-3s^2_{13})-s^2_{13}}{6s_{12}c_{12}s_{13}\sqrt{1-2s^2_{13}}} & (\text{case D}). \end{cases} \quad (39)$$

Substituting the experimental values for the neutrino mixing angles into Eqs. (34), (35), and (39), we can estimate the values of δ_D for each case.

IV. NUMERICAL RESULTS

For our numerical analysis, we use the current experimental data for three neutrino mixing angles as inputs taken from Ref. [5], which are given in Table 1. As shown in Table 1, prominent differences between normal and inverted ordering of the neutrino masses are the values of $\sin^2\theta_{23}$ at 1σ C.L., and its lower limit at 3σ C.L. Among the relations for δ_D, cases A and B are sensitive to the values θ_{23}. This analysis is, in fact, to update the numerical results for the prediction of δ_D given in [7] by taking the new fit to the data [5]. However, as will be shown later, the results based on 1σ data are completely different from those in [7]. Using experimental results for three neutrino mixing angles, we first check to see if the relations in Eqs. (27) and (37) hold and then estimate the values of δ_D in terms of the neutrino mixing angles for those four cases.

A. Results for 1σ C.L.

Plugging the experimental data for s^2_{13} at 1σ C.L. into Eqs. (27) and (37), we predict the values of the mixing parameters s^2_{12} (A and B) and s^2_{23} (C and D) as follows:

$$s^2_{12} = \begin{cases} 0.318 - 0.319 & (\text{A}), \\ 0.340 - 0.341 & (\text{B}), \end{cases} \quad (40)$$

$$s^2_{23} = \begin{cases} 0.488 - 0.489 & (\text{C}), \\ 0.510(0.511) - 0.511(0.512) & (\text{D}), \end{cases} \quad (41)$$

where the values for cases A, B, and C are the same for both neutrino mass orderings, respectively, but the numbers in parentheses correspond to the inverted ordering, whereas the others correspond to the normal ordering for D. We see that s^2_{12} and s^2_{23} are very narrowly determined for the 1σ region of s^2_{13}. Comparing the experimental values of s^2_{12} and s^2_{23} with the above predictions, we see that only C is consistent with the experimental results at 1σ C.L.

In Fig. 1, we show the prediction for δ_D in terms of the s^2_{12} based on the experimental data at 1σ C.L. The results for both neutrino mass orderings are the same. The upper curve in Fig. 1 indicates $1.32\pi \le \delta_D \le 1.52\pi$, which is consistent with the result of the fit for *CP* phase ($1.3\pi \le \delta_D \le 1.92\pi$) shown in [5].

B. Results for 3σ C.L.

Plugging the experimental data for $\sin^2\theta_{13}$ at 3σ C.L. into Eqs. (27) and (37), we predict the values for the mixing

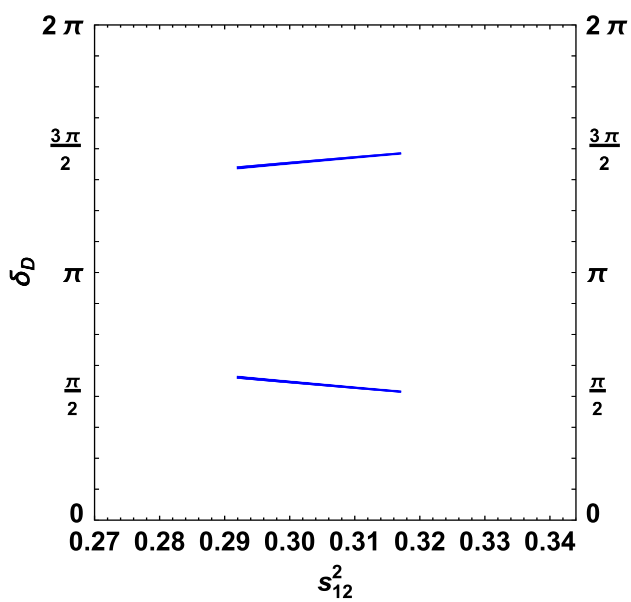

FIG. 1 (color online). Prediction of δ_D in terms of s^2_{12} for C based on the 1σ experimental data.

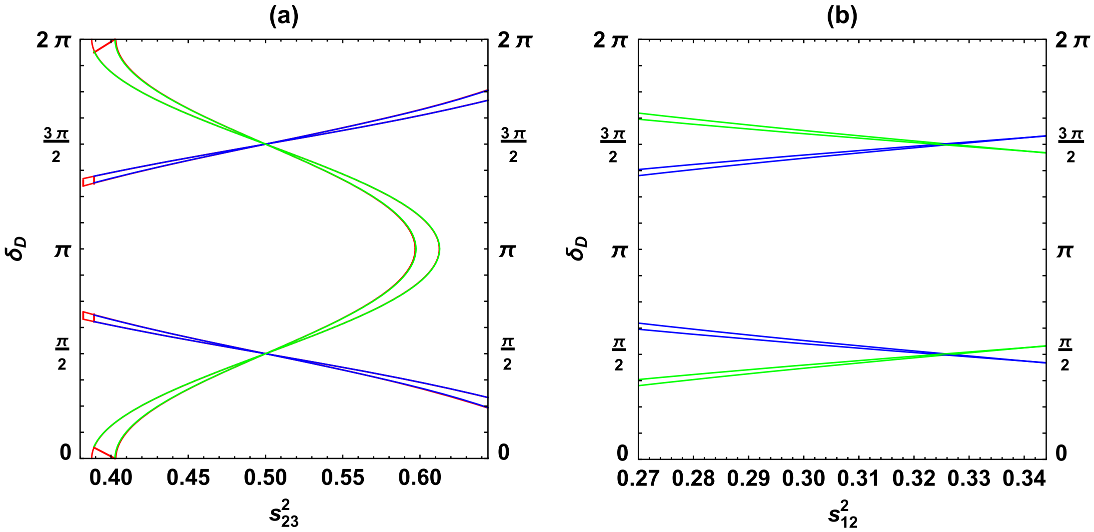

FIG. 2 (color online). Prediction for δ_D in terms of (a) s_{23}^2 for s A and B, and (b) s_{12}^2 for s C and D, based on 3σ experimental data. Regions surrounded by blue (green) lines correspond to s (A, C) and (B, D). These are almost equally predicted for both neutrino mass orderings. The small regions surrounded by the red line corresponding to $s_{23}^2 \leq 0.41$.

parameters $\sin^2\theta_{12}$ (A and B) and $\sin^2\theta_{23}$ (C and D) as follows:

$$s_{12}^2 = \begin{cases} 0.316 - 0.321 & \text{(A)} \\ 0.340 - 0.342 & \text{(B)} \end{cases} \quad (42)$$

$$s_{23}^2 = \begin{cases} 0.487 - 0.491 & \text{(C)} \\ 0.509 - 0.513 & \text{(D)} \end{cases} \quad (43)$$

We see that these results are almost the same for both neutrino mass orderings. Comparing the experimental values of s_{12}^2 and s_{23}^2 with the above predictions, we see that they are all consistent with the experimental results at 3σ C.L. In particular, the prediction of s_{12}^2 for B is preferable to the nearly upper limit of 3σ allowed region.

Figure 2 shows the predictions of δ_D in terms of s_{23}^2 [(a): cases A and B] and s_{12}^2 [(b): cases C and D] based on the corresponding experimental data given at 3σ C.L. Regions surrounded by blue and green lines correspond to cases (A, C) and (B, D), respectively. The width of each band implies the variation of the other mixing angles, s_{12}^2 (cases A and B) and s_{23}^2 (cases C and D). Note that these regions are almost equally predicted for both neutrino mass orderings. However, the small regions surrounded by the red line in Fig. 2(a) are predicted only for the normal mass ordering. We see that the almost maximal $\delta_D \sim \pi/2$, $3\pi/2$ can be achieved by $s_{23}^2 \sim 0.5$ for cases (A, B) and by $s_{12}^2 \sim 0.325$ for cases (C, D). The values around $3\pi/2$ are consistent with the current fit of the Dirac-type *CP* phase [5].

Comparing these results with the corresponding ones presented in [7], we see that the shapes of the curves in each are nearly unchanged, but the widths of each band get much narrower. The allowed regions of s_{12}^2 above 0.344 for cases C and D are excluded in the updated analysis.

V. CONCLUSION

As a summary, we have considered how the nonzero mixing angle θ_{13} and *CP* violation can be accommodated in a model with A_4 flavor symmetry by incorporating symmetry breaking terms appropriately. The four possible forms of the neutrino mixing matrix we considered are minimal modifications to the TBM mixing matrix and are factorized by the TBM mixing form and a unitary mixing matrix with an angle and a *CP* phase corresponding to a rotation in a plane. We have shown that the possible size of the Dirac-type *CP* phase δ_D can be predicted with regards to two neutrino mixing angles in the standard parametrization of the neutrino mixing matrix. This has been achieved by equating one of the minimally modified TBM mixing matrices with the standard parametrization of the PMNS one. Based on the current fit results for the neutrino mixing angles and the *CP* phase, we have seen that the neutrino mixing matrix corresponding to C is consistent with the current fit data at 1σ C.L., whereas others are not so. This result is different from that in [7]. Extending the analysis to 3σ C.L., all cases are consistent with the current fit data. We have presented the numerical results for the predictions of δ_D in terms of either s_{12}^2 or s_{23}^2 for those cases.

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