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Leptonic *CP* problem in left-right symmetric model

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We find using the minimal left-right symmetric model that the presence of leptonic *CP* violation can radiatively generate a strong *CP* phase at the one-loop level itself, which can be beyond the current bounds established by the neutron electric dipole moment experiments. If there are no axions or unnatural cancellations, this leads to the testable prediction that leptonic *CP* violation must be negligibly small (Dirac phase $\delta_{CP} = 0$ or π), in wide and interesting regions of parameter space.

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I. INTRODUCTION

One of the most attractive extensions of the standard model is the left-right symmetric model [1] based on $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$ that restores parity as a good symmetry of the Lagrangian. The model requires the introduction of three right-handed neutrinos, which are parity partners of the left-handed neutrinos, and thereby provides a strong reason for neutrino masses and mixings. Interestingly, the QCD vacuum angle $\theta_{\rm QCD}$ is absent in the left-right P symmetric (LR) model as it is parity odd, and the strong *CP* phase $\bar{\theta}$ [that contributes to the neutron's electric dipole moment (EDM)] is calculable in terms of the other parameters of the Lagrangian.

In the standard model, if we set the tree-level strong *CP* parameter $\bar{\theta}$ to zero by hand, it is not produced radiatively till the third loop and is negligibly small ($\bar{\theta} \sim 10^{-16}$) [2]. In the left-right symmetric model, the parity-odd $\bar{\theta}$ would be zero had P remained unbroken. However a single *CP* violating quartic term in the Higgs potential can generate a large $\bar{\theta}$ at the tree level once parity is spontaneously broken. If there are no unnatural cancellations between tree-level and radiative contributions to $\bar{\theta}$, the coupling α_2 of this quartic term must be nearly real (or *CP* conserving), so that its tree-level contribution to $\bar{\theta}$ is within the experimental bound $\lesssim 10^{-10}$ (or zero). We can ask in which loop order it is generated from other phases. Just like in the standard model, it was shown that even in the left-right model $\bar{\theta}$ is not generated up to the third loop [3]. However, this calculation (see the last two paragraphs of Ref [4]) in the left-right model had only looked at *CP* violating radiative corrections from the Cabibbo-Kobayashi-Maskawa (CKM) phase of the quark sector.

In this paper we show that *CP* violation in the leptonic sector can radiatively generate a complex phase in α_2 and thereby the strong *CP* phase $\bar{\theta}$ at the one-loop level itself. Moreover, if the Dirac-type Yukawa terms in the leptonic sector are similar to their quark sector counterparts, for a wide region in parameter space that includes all of type 2 dominance and some interesting regions of type 1 seesaw mechanism with right handed symmetry breaking scale $v_R \lesssim 10^{15}$ GeV, the strong *CP* phase generated from the leptonic phases exceeds the bound $\bar{\theta} \leq 10^{-10}$ set by neutron EDM experiments. Thus, we predict that leptonic *CP* violation must be absent or unobservably small in the left-right symmetric model with the above provisions, and we may thereby be able to test if parity is restored in the laws of nature at some scales well beyond collider reach. Moreover, if all the neutrino Dirac Yukawas are similar to the electron's Yukawa coupling, we find using a type 2 seesaw that $\delta_{CP} \lesssim 1/30$ if $v_R \sim 1\,\mathrm{TeV}$, while for $v_R \sim 10\,\mathrm{TeV}$ an observable neutron EDM is generated.

It is worth noting that there are axionless solutions to the strong *CP* problem that restore *CP* at a relaxation scale above v_R, such that, even after spontaneous or soft *CP* breaking, α_2 is naturally real at the relaxation or cutoff scale, while the CKM phase is generated [5]. These solutions are discussed toward the end of the paper.

That a complex α_2 is generated from phases in the leptonic sector should have shown up in the one-loop renormalization group (RG) equations of the left-right symmetric model which were evaluated in Ref. [6]. However the RG equations obtained in that work do not contain the contribution to the imaginary part of α_2 (denoted by λ_{11} in Ref. [6] and α_{2I} in this work) from the phases in leptonic Yukawa matrices. More recent work such as Ref. [7] also does not find *CP* violating one-loop contributions to $\bar{\theta}$, if α_2 is real or *CP* conserving at tree level. Our result is a significant departure from all previous works which concluded or assumed that α_{2I} (and therefore $\bar{\theta}$), once set to zero at the tree level, does not pick up *CP* violating radiative corrections at the one-loop level in the LR model.

The excessive one-loop corrections imply that for the smallness of $\bar{\theta}$ to be natural in a technical sense, in important regions of parameter space of the nonsupersymmetric LR model, the strong *CP* problem *must* be solved by introducing an axion, or else the leptonic *CP* violating phases must be suppressed, which is testable.

In supersymmetic models, that phases from *soft* trilinear terms involving the sleptons can contribute to $\bar{\theta}$ in one loop was found in Ref. [8]. However, that work had to abandon

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the calculation of $\bar{\theta}$ from leptonic phases in nonsupersymmetric models with two Higgs doublets, since there are contributions to $\bar{\theta}$ from imaginary parts of parameters of quadratically divergent dimension-2 Higgs mass terms, which *have* to be fine-tuned due to the hierarchy problem (see Ref. [8]). However, in the LR model, all dimension-2 Higgs mass parameters are real due to P, and there is no imaginary part to quadratic divergences. Thus, there is no roadblock to calculating the radiative contribution from leptonic phases, and it is well known that $\bar{\theta}$ is finite and calculable in the LR model with P.

II. CONNECTION BETWEEN STRONG AND LEPTONIC *CP* VIOLATION

We consider the minimal left-right symmetric model [1] based on $G_{\rm LR} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$, with scalar triplets Δ_R (1, 1, 3, 2) and Δ_L (1, 3, 1, 2) and bidoublet ϕ (1, 2, 2, 0). Under parity (P), the space-time coordinates $(x,t) \to (-x,t), \phi \to \phi^\dagger$, and subscripts $L \leftrightarrow R$ for all other fields (see for example Ref. [9]). The scalar fields have the form

$$\phi = \begin{pmatrix} \phi_1^o & \phi_2^+ \\ \phi_1^- & \phi_2^o \end{pmatrix}, \qquad \Delta_{L,R} = \begin{pmatrix} \delta_{L,R}^+/\sqrt{2} & \delta_{L,R}^{++} \\ \delta_{L,R}^o & -\delta_{L,R}^+/\sqrt{2} \end{pmatrix}. \tag{1}$$

As is well known, all parameters of the Higgs potential are real due to P, except α_2 in the *CP* violating term [9]

$$V = [\alpha_2 \mathrm{Tr}(\Delta_R^\dagger \Delta_R) + \alpha_2^\star \mathrm{Tr}(\Delta_L^\dagger \Delta_L)] \mathrm{Tr}(\tilde{\phi}^\dagger \phi) + \mathrm{h.c.}, \tag{2}$$

where $\tilde{\phi} = \tau_2 \phi^\star \tau_2$. If α_2 is complex, once δ_R^o picks up a vacuum expectation value (VEV) $\langle \delta_R^o \rangle = v_R/\sqrt{2} \gg \langle \delta_L^o \rangle$, the above term generates a *CP* violating coupling between the two standard model doublets in the bidoublet ϕ. This causes the VEV $\langle \phi_2^o \rangle \equiv k_2 e^{i\alpha}/\sqrt{2}$ of the neutral component of the second standard model doublet to pick up a phase (α), where we have chosen a basis so that $\langle \phi_1^o \rangle \equiv k_1/\sqrt{2}$ and $\langle \delta_R^o \rangle$ are real, and the weak scale $v_{wk}^2 = |k_1|^2 + |k_2|^2$. The up and down quark mass matrices (M_u and M_d) are no longer Hermitian, as they are obtained from Yukawa couplings of the quarks that are Hermitian (due to P) and Higgs bidoublet VEVs that are no longer all real. The strong *CP* phase

$$\bar{\theta} = \arg\ \det(M_u M_d) \sim (\alpha_{2I}/\alpha_3)(m_t/m_b) \tag{3}$$

therefore gets generated from the non-Hermitian mass matrices and has been written in terms of the imaginary part of $\alpha_2 (= \alpha_{2R} + i\alpha_{2I})$ and the top to bottom quark mass ratio. α_3 is the real coupling of the term $\alpha_3 \mathrm{Tr}(\phi^\dagger \phi \Delta_R \Delta_R^\dagger)$ that keeps the potential stable and as noted in Ref. [9] generates the mass $\sqrt{\alpha_3/2} v_R$ for the second standard model doublet. Since experimentally [10] $\bar{\theta} \lesssim 10^{-10}$, it is crucial that α_{2I} is close to zero to a high degree. However, we will show that, if α_2 is chosen to be real at the tree level (with a cutoff scale $> v_R$), a dangerous contribution to α_{2I} is generated at the one-loop level from the leptonic Yukawa terms given as

$$h_{ij}^\ell \bar{L}_{iL} \phi L_{jR} + \tilde{h}_{ij}^\ell \bar{L}_{iL} \tilde{\phi} L_{jR} + i f_{ij} (L_{iR}^T \tau_2 C \Delta_R L_{jR} + L_{iL}^T \tau_2 C \Delta_L L_{jL}) + \mathrm{H.c.}, \tag{4}$$

where for example $\bar{L}_{1R}$ is the first-generation leptonic doublet $(\bar{\nu}\bar{e})_R$ and 3×3 Yukawa matrices h^ℓ and $\tilde{h}^\ell$ are Hermitian due to P, while f is a complex, symmetric 3×3 matrix that generates Majorana terms for neutrinos.

The above Lagrangian radiatively generates a logarithmically divergent contribution to α_{2I} through the box diagrams of Fig. 1, so that

$$\alpha_{2I} \sim \frac{i}{16\pi^2} \mathrm{Tr}(f^\dagger f [h^\ell, \tilde{h}^\ell]) \ln (v_R/M_{\rm Pl})^2, \tag{5}$$

where we note that $i[h^\ell, \tilde{h}^\ell]$ and $f^\dagger f$ are Hermitian matrices and the cutoff has been taken to be at the Planck scale $M_{\rm Pl} > v_R$. Note that there is no suppression in Eq. (5) by a factor such as $(v_{wk}/v_R)^2$ or $(v_R/M_{\rm Pl})^2$, and hence this contribution can be dangerously large, even if the v_R scale is well above the TeV scale.

Equation (5) with Eq. (3) generates $\bar{\theta}$ in the quark sector from leptonic Yukawas and *CP* violation therein and provides the severe constraint on the leptonic sector that is being probed by neutrino experiments,

$$|\mathrm{Tr}(f^\dagger f [h^\ell, \tilde{h}^\ell])| \lesssim 3 \times 10^{-11}, \tag{6}$$

where we have substituted $\bar{\theta} \leq 10^{-10}$, $m_t/m_b \sim 40$, $\alpha_3 \lesssim 1$, and have taken the logarithm to have a generic value ~ 10. Taking the Hermitian conjugate of (6), it can be seen that, if $f^\dagger f$, h^ℓ, and $\tilde{h}^\ell$ have all real matrix elements (conserve CP), then the left-hand side vanishes. On the other hand, as we will now see, if they have complex phases, these can be constrained by the above equation.

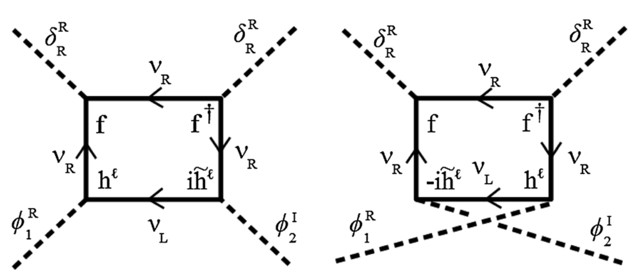

FIG. 1. One-loop contribution to α_{2I} from leptonic Yukawas of Eq. (4), with $\phi_1^o = \phi_1^R + i\phi_1^I, \phi_2^o = \phi_2^R + i\phi_2^I$, and $\delta_R^o = \delta_R^R + i\delta_R^I$ in Eq. (1). This leads to a VEV for ϕ_2^I and generates $\bar{\theta}$.

We will now consider the well-motivated case where the leptonic Dirac Yukawa couplings are similar to their quark counterparts. This for example would be the case if there is an ultraviolet completion with a semiunified theory such as the Pati–Salam model with $SU(4)_c \times SU(2)_L \times SU(2)_R$ or grand unified theory such as $SO(10)$. This implies that the matrix $[h^\ell, \tilde{h}^\ell]$ has some off-diagonal matrix elements that are of the order $\sim V_{ts}(m_b/m_t) \gtrsim 3\times 10^{-4}$, in a basis in which either h^ℓ or $\tilde{h}^\ell$, like its quark counterpart, is diagonal. Even if there is no unification, that the quark and leptonic Dirac type Yukawa matrices could be similar is hinted by the fact that the charged lepton masses are similar to the down sector quark masses.

With the above, Eq. (6) implies that some off-diagonal matrix elements

$$|(f^\dagger f)_{ij}| \lesssim 10^{-7} \tag{7}$$

if there are $O(1)$ CP phases present in $f^\dagger f$ or in $[h^\ell, \tilde{h}^\ell]$.

The Yukawa matrix f leads to Majorana mass terms for neutrinos once δ_R^o picks up a large VEV $\sim v_R/\sqrt{2}$ and δ_L^o picks up an induced VEV $\sim \gamma v_{wk}^2/v_R$ where γ which is symbolically β/ρ is obtained from real quartic couplings β_i and ρ_i of the Higgs potential [9] [which has terms such as $\rho_1^2 \mathrm{Tr}(\Delta_R^\dagger \Delta_R)^2 + R \to L$] and is real at tree level. Since no symmetries can protect ρ_i, we have in general $\rho_i \gtrsim 0.01$ and therefore $|\gamma| \lesssim 100$. The light neutrino mass matrix is given by the well-known seesaw mechanism [11] and has the form

$$M_\nu = \frac{v_{wk}^2}{v_R}\left[\gamma f - h_D\left(\frac{1}{f}\right)h_D^T\right], \tag{8}$$

where $h_D = (k_1 h^\ell + k_2 e^{-i\alpha}\tilde{h}^\ell)/v_{wk}$ is the Dirac-type Yukawa matrix for the neutrinos.

If the first term in the square brackets of Eq. (8) dominates over the second term, we have a type-2 seesaw mechanism. Taking the third-generation Yukawas to be larger than the rest of the Yukawas, for the first term to dominate, we must have $f_{33} > h_{D_{33}}/\sqrt{|\gamma|}$. Since we have assumed that leptonic Dirac type and quark Yukawas are similar, we take $h_{D_{33}} \sim h_t \gtrsim 0.3$. Substituting $|\gamma| \lesssim 100$ we have for the type-2 seesaw mechanism $f_{33} \gtrsim 0.03$. Since $M_\nu \approx f\gamma v_{wk}^2/v_R$ for the type-2 seesaw, and we know by light neutrino experiments that the leptonic mixing angles are large, we have for the off-diagonal matrix elements $f_{3j} \sim f_{33}$ to $f_{33}/10$. Thus, we obtain $|(f^\dagger f)_{3j}| \gtrsim 10^{-4}$.

Comparing the above with Eq. (7), we can see that the leptonic Dirac phase δ_{CP} cannot be of the order 1 and must be less than 10^{-3} from its CP conserving value of 0 or π. Note that in this case of type-2 seesaw, the Majorana CP violating phases are unconstrained since they do not occur in $M_\nu^\dagger M_\nu \propto f^\dagger f$ or in $[h^\ell, \tilde{h}^\ell]$.

Currently experiments are being planned or are underway to measure δ_{CP} with a sensitivity of 5^o (or ~0.1) [12], similar to sensitivity achieved for the CKM phase. The absence of a measurable δ_{CP} (modulo π) for the above well-motivated case is a key prediction of this work.

We now consider the case of the type-1 seesaw where the second term in Eq. (8) dominates. Substituting $h_{D_{33}} \sim 0.3$ we find that for the type-1 seesaw $f_{33}v_R \sim 10^{14}$ GeV, so that $h_{D_{33}}^2 v_{wk}^2/(f_{33}v_R) \sim \sqrt{|\Delta m_{32}^2|} \sim 0.05$ eV, where we have used the mass-squared difference of light neutrinos [12] $|\Delta m_{32}^2| \sim 0.0023$ eV^2 and $v_{wk} \sim 246$ GeV.

If $v_R \sim 10^{18}$ GeV, we must have $f_{33} \sim 10^{-4}$, and we can see that the off-diagonal terms of $f^\dagger f$ will satisfy Eq. (7), and there is no constraint on the leptonic CP phases. On the other hand, if $v_R \sim 10^{15\,\mathrm{to}\,14}$ GeV, then $f_{33} \sim 0.1$ to 1. For the type-1 seesaw, since f is more hierarchical, we take $f_{23} \sim f_{33}/1000$ (f_{23} would be about $f_{33}/100$, but we allow for an additional factor of 10 since the phases in f_{23} could be order $\delta_{CP}/10$). Comparing now with Eq. (7), we once again find that leptonic CP phases cannot be order 1 and must be constrained to be $\lesssim 10^{-(2\,\mathrm{to}\,4)}$, modulo π. This is another key prediction. Note also that for the type-1 seesaw, since both Majorana and Dirac phases can be present in $f^\dagger f$, all the CP phases are constrained.

So far we have looked at cases where quark and leptonic Dirac Yukawas are similar. We now relax this assumption to consider an example where v_R can be at the TeV scale. We use a basis where the charged lepton masses are diagonal. We will assume that all the neutrino Dirac Yukawas, including those of the third generation, are similar to the smallest known Yukawa coupling (that of the electron), so that $h_{D_{3j}} \sim 10^{-6}$ to $10^{-5} \sim h_{3j}^\ell$, and v_R can be ~1 TeV. This determines $f_{33} \sim 1$, since the second term in Eq. (8) should not give a contribution much greater than the observed 0.0023 eV^2 for light neutrino mass-squared differences. Moreover, $[h^\ell, \tilde{h}^\ell]$ can have off-diagonal elements of the order $10^{-6}\times 10^{-2} = 10^{-8}$, since both h_D and charged fermion masses (with Yukawa of $\tau^-, h_\tau \sim 10^{-2} \sim \tilde{h}_{33}^\ell$) must arise from h^ℓ and $\tilde{h}^\ell$. For the type-2 seesaw, $f_{3j} \sim f_{33}/10 \sim 1/10$, and so we find that the left-hand side of Eq. (6) is $\sim 10^{-9}\delta_{CP}$, and therefore δ_{CP} cannot be order 1 and is ~1/30. However, if $v_R \sim 10$ TeV (so that $f_{33} \gtrsim 0.1$), then $\delta_{CP} \sim 1$ may be allowed and can result in an observable neutron EDM ($\bar{\theta} \gtrsim 3\times 10^{-11}\delta_{CP}$). Null results in future neutron EDM experiments probing $\bar{\theta} \sim 10^{-12}$ may further constrain δ_{CP}.

III. STRONG *CP* SOLUTION

We have not assumed anything beyond the minimal left-right symmetric model to obtain the connection between leptonic and strong CP violation. It is clear that, if the leptonic CP phases are $O(1)$, then $\bar{\theta}$ may have to be fine-tuned to cancel excessive one-loop radiative corrections, making it technically unnatural. On the other hand, if leptonic phases and α_{2I} are zero at the tree level, there could be an underlying symmetry reason. This motivates us to look at solutions to the strong *CP* problem.

If we extend the model by adding an axion, then $\bar{\theta}$ dynamically relaxes to zero [13]. However, since *P* sets the QCD vacuum angle to zero, historically it has been hoped that there would be an axionless solution in the LR model. An early attempt was made in Ref. [14] by adding an additional bidoublet and invoking a discrete symmetry. However, after symmetry breaking the CKM phase was not generated, and the problem remained unsolved. Later it was noted that α_2 is automatically absent in supersymmetric left-right (SUSY LR) models and the strong *CP* problem can be thus solved [15]. However, it was shown for the SUSY LR solution that, without any further constraints, the radiatively generated $\bar{\theta} \sim 10^{-8}$ to 10^{-10} [16], which is uncomfortably close to the experimental bound, while a solution in the LR model (without supersymmetry) continued to be elusive [17] at the turn of the century.

Recently, progress was made by adding one heavy vectorlike quark family to the minimal LR model and breaking both *P* and *CP* spontaneously so that α_{2I} naturally vanishes at the *CP* restoration scale [5]. *CP* is spontaneously broken by the VEV of a *P*-even, *CP*-odd real scalar singlet of which the Yukawa couplings mix the usual and vectorlike quarks and generate the CKM phase. Since *P* is not broken by the singlet VEV, the resultant tree-level quark mass matrices are Hermitian, thus solving the strong *CP* problem without requiring supersymmetry. The solution also works without the scalar singlet, if *CP* is broken softly by mass terms involving the vectorlike quarks [5].

The interesting thing is that in the minimal version of the above solution, since a vectorlike lepton family is not introduced, no *CP* violation is generated in the lepton sector. Thus, it predicts that not only α_{2I} but also the Dirac (δ_{CP}) and Majorana leptonic phases vanish (modulo π), as noted in Ref. [18] and detailed in Ref. [19]. It is remarkable that the solution addresses perfectly the issues raised in this work.

However, if a vectorlike lepton family is introduced, then δ_{CP} can be generated. This work shows that it would in turn generate too high a $\bar{\theta}$, in wide and interesting regions of parameter space, and hence the solution that includes a vectorlike lepton family may be disfavored.

IV. NEUTRON EDM IN AXIONLESS LR SOLUTION

If $v_R \ll M$, we just have the minimal LR model below *M* (mass of the vectorlike quark family or equivalently the scale of *CP* breaking). Radiative corrections from heavy quarks introduce a slight non-Hermiticity in the light quark mass matrices and generate a finite and calculable $\bar{\theta}$. This was estimated in Refs. [5,18], and it was found that contribution from terms at the one-loop level are of the form

$$\bar{\theta} \sim \frac{1}{16\pi^2}(\text{Product of Yukawas})\left[\frac{v_R}{M}\right]^2, \qquad (9)$$

where the term in the round brackets is essentially a product of a string of up and down quark Yukawa matrices with the standard model Higgs doublet, and includes one Yukawa inverse. The two-loop contribution has terms of a similar form with a longer Yukawa string in the product and an additional factor of $(4\pi)^{-2}$.

If $M \sim M_{\rm Pl} \sim 10^{18}$ GeV and $v_R \sim 10^{14}$ to 10^{15} GeV, then the above implies $\bar{\theta} \sim 10^{-12}$ to 10^{-10} if the product of Yukawas in the round brackets is $\sim h_t^2 V_{ts} \sim 1/100$. If some unknown Yukawas involving heavy quarks are smaller, it can further reduce $\bar{\theta}$.

An important feature in the above radiative corrections due to the heavy quarks is the suppression factor $(v_R/M)^2$. This is because as $M \to \infty$ the vectorlike quarks decouple. Additionally, if there are Planck scale corrections due to nonrenormalizable terms, they will induce a $\bar{\theta}_{\rm Pl} \sim \lambda v_R^2/(M M_{\rm Pl})$, where λ is a dimensionless parameter and *M* is the soft *CP* breaking scale for the minimal Higgs content without a singlet. Thus, the radiatively generated $\bar{\theta}$ due to vectorlike quarks and Planck scale corrections are both suppressed by $(v_R/M_{\rm Pl})^2$ for $M \sim M_{\rm Pl}$. Even if v_R is small enough that both are undetectable, we still have the testable prediction that leptonic phases vanish (modulo π) in the axionless model.

V. CONCLUSIONS

We have shown that presence of leptonic *CP* violation can radiatively generate an excessive strong *CP* phase at the one-loop level in the minimal left-right symmetric model with *P*, that is beyond the limits already established by the neutron EDM experiments for the following interesting regions of parameter space:

(1) if the leptonic and quark Dirac-type Yukawa couplings are similar, then
   (a) for all regions of the type-2 dominant seesaw.
   (b) for $v_R \lesssim 10^{15}$ GeV with the type-1 seesaw.
(2) if the Dirac Yukawa couplings of the neutrinos, $h_{D_{3j}} \sim 10^{-5}$ to 10^{-6}, so that $v_R \sim 1$ TeV, then for the type-2 seesaw.

In the above significant regions of parameter space, technical naturalness implies that leptonic *CP* phases (particularly δ_{CP}) must vanish or be negligibly small (modulo π), unless there are axions. The result is important as it gives us a way to test if parity is restored in the laws of nature, even if it is at scales $\lesssim 10^{15}$ GeV that are well beyond collider reach, in some well motivated regions of parameter space of the axionless minimal LR model. In the second bullet point of the above, if $v_R \sim 10$ TeV then an observable neutron EDM ($\bar{\theta} \gtrsim 3 \times 10^{-11} \delta_{CP}$) is generated.

In general, we can think of a theory as being afflicted with a leptonic CP problem if leptonic phases generate an excessive strong CP phase radiatively or through RG equation running from higher scales.

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