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Superfield effective potential for the supersymmetric topologically massive gauge theory in four dimensions

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We explicitly calculate the one-loop Kählerian effective potential for the supersymmetric topologically massive gauge theory in four dimensions that involves two gauge superfields, the usual scalar one and the spinor one originally introduced by Siegel, coupled to a chiral scalar matter.

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I. INTRODUCTION

It is well known (see, e.g., [1]) that in a four-dimensional space-time, different supersymmetry multiplets are represented by different superfields, and hence different field theory models exist. The simplest one is a minimal scalar multiplet described by the chiral and antichiral superfields that are well studied being the basic ingredients of the Wess-Zumino model and many other field theories [2,3]. Another well-studied important example is a vector multiplet described by the real scalar superfield that represents itself as a basic ingredient for supergauge theories such as super-QED and super-Yang-Mills theory (for different aspects of supergauge theories, see [1–3] and many other textbooks). These models, although they are most studied within different contexts, do not exhaust the set of physically interesting theories.

In this paper, we consider another supersymmetry multiplet, that is, the tensor multiplet. Originally inspired by the paper [4], this tensor multiplet was elaborated in [5], using a spinor chiral superfield. As it was shown in [5], this multiplet allows the consideration of a new supergauge model and is responsible for a topological mass term when it is coupled to a real gauge scalar superfield. (In the superfield language, it is represented by a spinor chiral superfield that was originally inspired by the paper [4] and has been introduced in [5] where it was shown to correspond to the so-called tensor multiplet and to allow for introducing, first, a new supergauge model, and second, a topological mass term in the case of the coupling of the spinor gauge superfield to the usual real gauge scalar superfield.) Another more interesting feature of this model is that the gauge invariant strength, corresponding to spinor chiral and antichiral superfields, is just a linear superfield, different from the chiral one, that occurs for the real scalar superfield [1]. While in [5] only the free theory has been considered, we study here its coupling to a chiral scalar matter. Classical aspects of this model were discussed in [6]. An alternative coupling for the linear superfield and tensor multiplet has been discussed in [7], where some of its string-related aspects were considered (for applications of this multiplet see also the references therein). Other important issues related to tensor multiplets were considered in [8]; also, beside the standard tensor multiplet discussed in [5], improved tensor multiplets were introduced for both the massless [9] and massive [10] cases.

Using a superfield effective potential methodology [11–14] previously developed, we calculate the one-loop superfield effective potential for a theory involving the coupling of the spinor gauge superfield with the usual scalar gauge superfield and the chiral scalar matter. We emphasize that, up to now, there were no examples of quantum calculations involving chiral spinor superfields.

The structure of the paper is as follows. In Sec. II, we discuss the classical action of the chiral spinor gauge superfield, coupled to the usual scalar gauge superfield and a chiral matter. In Sec. III we calculate the one-loop effective potential in this theory, and Sec. IV contains the summary of our results.

II. SUPERSYMMETRIC TOPOLOGICALLY MASSIVE GAUGE THEORY

Let us start our study with the supersymmetric topologically massive gauge theory that will be used to find the one-loop Kählerian effective potential (KEP). In the pure gauge sector, we have [5]

$$S_G = \frac{1}{2}\int d^6z W^\alpha W_\alpha - \frac{1}{2}\int d^8z G^2 - m\int d^8z VG, \qquad (1)$$

where m is a constant with mass dimension equal to 1, and

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$$W_\alpha = i\bar{D}^2 D_\alpha V, \qquad G = -\frac{1}{2}(D^\alpha \psi_\alpha + \bar{D}^{\dot\alpha}\bar{\psi}_{\dot\alpha}), \quad (2)$$

where ψ_α, $\bar{\psi}_{\dot\alpha}$ are chiral and antichiral spinor superpotentials corresponding to the tensor multiplet [1], and V is a usual real gauge superpotential. Actually, G is a linear superfield satisfying the relation $D^2 G = \bar{D}^2 G = 0$. The superfield strengths W_α, G, and the action (1) are invariant under the Abelian gauge transformations,

$$\delta V = i(\bar{\Lambda} - \Lambda), \qquad \delta\psi_\alpha = i\bar{D}^2 D_\alpha L, \qquad \delta\bar{\psi}_{\dot\alpha} = -iD^2\bar{D}_{\dot\alpha} L, \quad (3)$$

where Λ is a chiral superfield, $\bar{\Lambda}$ is an antichiral one, and $L = \bar{L}$ is a real scalar one [1].

Let us show that the theory (1) describes a massive gauge theory. For this, let us extract the equations of motion by varying the action (1) with respect to the superfields V and ψ_α. Then, we get

$$\frac{\delta S_G}{\delta V} = iD_\alpha W^\alpha - mG = 0, \quad (4)$$

$$\frac{\delta S_G}{\delta \psi_\alpha} = \bar{D}^2 D^\alpha G - imW^\alpha = 0. \quad (5)$$

On the one hand, if we multiply Eq. (4) by $\bar{D}^2 D^\beta$ and use $\bar{D}^2 D^\beta D_\alpha W^\alpha = \Box W^\beta$, we get

$$(\Box - m^2) W^\alpha = 0. \quad (6)$$

On the other hand, if we multiply Eq. (5) by D_α and use $D_\alpha \bar{D}^2 D^\alpha G = \Box G$, we get

$$(\Box - m^2) G = 0. \quad (7)$$

Therefore, from Eqs. (6) and (7), we can conclude that the superfield strengths W_α and G satisfy massive Klein-Gordon equations.

To perform quantum calculations, we must add to (1) a gauge-fixing term. In particular, we will consider the following one [5]:

$$S_{GF} = -\frac{1}{2\alpha}\int d^8 z V\{D^2, \bar{D}^2\} V - \frac{1}{8\beta}\int d^8 z (D^\alpha\psi_\alpha - \bar{D}^{\dot\alpha}\bar{\psi}_{\dot\alpha})^2, \quad (8)$$

where α and β are the gauge-fixing parameters. The ghosts are completely factorized since the theory is Abelian.

Now, let us introduce interaction between the (anti)chiral scalar superfield and the gauge superfields [2]. Under the usual gauge transformation, the chiral and antichiral matter superfields transform as [1]

$$\Phi' = e^{2ig\Lambda}\Phi, \qquad \bar{\Phi}' = \bar{\Phi} e^{-2ig\bar{\Lambda}}. \quad (9)$$

The interaction term that we will consider in this paper, which is invariant under the combined transformations (3) and (9), is given by [15]

$$S_M = \int d^8 z \bar{\Phi} e^{2gV} \Phi e^{4hG}. \quad (10)$$

The coupling constants g and h have mass dimensions 0 and -1, respectively. The reasons for choosing this model are the following: First, this model was considered in [6] in the study of the formation of cosmic strings. Second, our aim consists in calculating the one-loop Kählerian effective potential, which means that we should introduce the coupling of the spinor gauge superfield (actually, of the strength G, to achieve gauge invariance) to the chiral matter. Otherwise, at one-loop order the contribution of the spinor gauge superfield to the effective potential would be trivial. In principle, we could introduce an arbitrary dimensionless function $f(hG)$ instead of e^{4hG}, but the exponential interaction was chosen only for the sake of concreteness. Unfortunately, any coupling of the strength G is nonrenormalizable, but this is the price we pay for the coupling to produce nontrivial one-loop results.

It follows from this expression that the tree-level KEP is

$$K^{(0)} = \Phi\bar{\Phi}. \quad (11)$$

Finally, the supersymmetric topologically massive gauge theory that we will study in this work follows from (1), (8), and (10):

$$S = -\frac{1}{2}\int d^8 z V\left(-D^\alpha\bar{D}^2 D_\alpha + \frac{1}{\alpha}\{D^2, \bar{D}^2\}\right)V - \frac{1}{8}\int d^8 z\left\{\left(1 + \frac{1}{\beta}\right)[\psi_\alpha D^\alpha D^\beta \psi_\beta + \bar{\psi}_{\dot\alpha}\bar{D}^{\dot\alpha}\bar{D}^{\dot\beta}\bar{\psi}_{\dot\beta}] + 2\left(1 - \frac{1}{\beta}\right)\psi_\alpha D^\alpha \bar{D}^{\dot\beta}\bar{\psi}_{\dot\beta}\right\} + \frac{m}{2}\int d^8 z V(D^\alpha\psi_\alpha + \bar{D}^{\dot\alpha}\bar{\psi}_{\dot\alpha}) + \int d^8 z \bar{\Phi} e^{2gV}\Phi e^{-2h(D^\alpha\psi_\alpha + \bar{D}^{\dot\alpha}\bar{\psi}_{\dot\alpha})}, \quad (12)$$

where we explicitly wrote the gauge superpotentials.

The standard method of calculating the effective action is based on the methodology of the loop expansion [16]. To do this, we make a shift $\Phi \to \Phi + \phi$ in the superfield Φ (together with the analogous shift for $\bar{\Phi}$), where now Φ is a background (super)field and ϕ is a quantum one. We assume that the gauge superfields V, ψ_α, and $\bar{\psi}_{\dot\alpha}$ are quantum. To calculate the effective action at the one-loop level, we have to keep only the quadratic terms in the quantum superfields. By using this prescription, we get from (12)

$$S_2[\bar{\Phi}, \Phi; \bar{\phi}, \phi, \psi_\alpha, \bar{\psi}_{\dot\alpha}, V] = S_q + S_{\rm int}, \tag{13}$$

$$S_q = \frac{1}{2}\int d^8z\left[-V\Box\left(\Pi_{1/2} + \frac{1}{\alpha}\Pi_0\right)V - \frac{1}{4}\left[\left(1+\frac{1}{\beta}\right)(\psi_\alpha D^\alpha D^\beta \psi_\beta + \bar{\psi}_{\dot\alpha}\bar{D}^{\dot\alpha}\bar{D}^{\dot\beta}\bar{\psi}_{\dot\beta}) + 2\left(1-\frac{1}{\beta}\right)\psi_\alpha D^\alpha \bar{D}^{\dot\beta}\bar{\psi}_{\dot\beta}\right] + 2\bar{\phi}\phi\right], \tag{14}$$

$$S_{\rm int} = \frac{1}{2}\int d^8z\{(m - 8gh\bar{\Phi}\Phi)V(D^\alpha\psi_\alpha + \bar{D}^{\dot\alpha}\bar{\psi}_{\dot\alpha}) + 2(2g)\bar{\Phi}V\phi + 2(2g)\Phi\bar{\phi}V$$
$$+ (2g)^2\bar{\Phi}\Phi V^2 - 4h\bar{\Phi}(D^\alpha\psi_\alpha + \bar{D}^{\dot\alpha}\bar{\psi}_{\dot\alpha})\phi - 4h\Phi\bar{\phi}(D^\alpha\psi_\alpha + \bar{D}^{\dot\alpha}\bar{\psi}_{\dot\alpha})$$
$$+ (2h)^2\bar{\Phi}\Phi[(D^\alpha\psi_\alpha)D^\beta\psi_\beta + (\bar{D}^{\dot\alpha}\bar{\psi}_{\dot\alpha})\bar{D}^{\dot\beta}\bar{\psi}_{\dot\beta} + 2(D^\alpha\psi_\alpha)\bar{D}^{\dot\alpha}\bar{\psi}_{\dot\alpha}]\}, \tag{15}$$

where the irrelevant terms were omitted, including those involving covariant derivatives of the background (anti) chiral superfields. Moreover, we used the projection operators $\Pi_{1/2} \equiv -\Box^{-1}D^\alpha\bar{D}^2D_\alpha$ and $\Pi_0 \equiv \Box^{-1}\{D^2, \bar{D}^2\}$.

The one-loop approximation does not depend on how we break the Lagrangian into free and interacting parts [17]. However, by convenience, we will extract the propagators from the terms that are independent of the background superfields and the vertices from the ones in which the quantum superfields interact with the background ones.

In the gauges $\alpha = 0$ and $\beta = -1$, we obtain from S_q the propagators

$$\langle V(1)V(2)\rangle = -\frac{1}{p^2}(\Pi_{1/2})_1\delta_{12},$$
$$\langle \psi_\alpha(1)\bar{\psi}_{\dot\alpha}(2)\rangle = \frac{2p_{\alpha\dot\alpha}}{p^4}\delta_{12},$$
$$\langle \phi(1)\bar{\phi}(2)\rangle = \frac{1}{p^2}\delta_{12}. \tag{16}$$

Before we start the calculation of the one-loop supergraphs, we first notice from (15) that there is a factor $D^\alpha\bar{D}^2$ in a vertex at one end of the propagator $\langle\psi_\alpha(1)\bar{\psi}_{\dot\alpha}(2)\rangle$, and there is a factor $\bar{D}^{\dot\alpha}D^2$ in the other vertex at the other end of the same propagator. Here the factors $\bar{D}^2$ and D^2 are present in the vertices due to the chirality (antichirality) of the superfield ψ_α ($\bar{\psi}_{\dot\alpha}$) just as in the usual Wess-Zumino model, because of the properties of the variational derivatives with respect to the chiral superfields (see [1–3]), and the D^α, $\bar{D}^{\dot\alpha}$ arise from the explicit form of the vertices. It is convenient to go from the above used formulation of propagators where the derivatives D^2, $\bar{D}^2$ are associated with the vertices to a formulation where these derivatives are incorporated into the propagators (these two manners to introduce the Feynman supergraphs exist also in the Wess-Zumino model; see, e.g., [1]). In other words, we associate the covariant derivatives with the propagator $\langle\psi_\alpha(1)\bar{\psi}_{\dot\alpha}(2)\rangle$ (instead of to the vertices) and define a new scalar field $\psi = D^\alpha\psi_\alpha$ with the propagator,

$$\langle\psi(1)\bar{\psi}(2)\rangle \equiv D_1^\alpha\bar{D}_1^2\bar{D}_2^{\dot\alpha}D_2^2\langle\psi_\alpha(1)\bar{\psi}_{\dot\alpha}(2)\rangle = 2(\Pi_{1/2})_1\delta_{12}, \tag{17}$$

where we used the fact that $\bar{D}_2^{\dot\alpha}D_2^2\delta_{12} = -D_1^2\bar{D}_1^{\dot\alpha}\delta_{12}$, and the factors D^2, $\bar{D}^2$ emerged due to properties of variational derivatives. We can also apply the same reasoning for the propagator $\langle\phi(1)\bar{\phi}(2)\rangle$ and for the vertices involving the scalar (anti)chiral superfields.

In summary, by transferring all covariant derivatives from the vertices (15) to the propagators (16), we get

$$\langle V(1)V(2)\rangle = -\frac{1}{p^2}(\Pi_{1/2})_1\delta_{12}, \tag{18}$$

$$\langle\psi(1)\bar{\psi}(2)\rangle = \langle\bar{\psi}(1)\psi(2)\rangle = 2(\Pi_{1/2})_1\delta_{12}, \tag{19}$$

$$\langle\phi(1)\bar{\phi}(2)\rangle = -(\Pi_-)_1\delta_{12}, \qquad \langle\bar{\phi}(1)\phi(2)\rangle = -(\Pi_+)_1\delta_{12}, \tag{20}$$

where $\Pi_- \equiv \Box^{-1}\bar{D}^2D^2$ and $\Pi_+ \equiv \Box^{-1}D^2\bar{D}^2$ are projection operators. These propagators will connect the following new vertices:

$$\tilde{S}_{\rm int} = \frac{1}{2}\int d^8z\{2MV(\psi+\bar\psi) + 2(2g)\bar\Phi V\phi + 2(2g)\Phi\bar\phi V + (2g)^2\bar\Phi\Phi V^2 - 4h\bar\Phi(\psi+\bar\psi)\phi - 4h\Phi\bar\phi(\psi+\bar\psi) + (2h)^2\bar\Phi\Phi[\psi^2+\bar\psi^2+2\psi\bar\psi]\}, \quad (21)$$

where $M \equiv \frac{1}{2}(m - 8gh\bar\Phi\Phi)$. Therefore, now the vertices involve only scalar superfields.

In the next section, we will perform the calculations of the one-loop supergraphs using the propagators (18)–(20), written in terms of projection operators, and the vertices (21), written only in terms of scalar superfields, instead of the original propagators (16) and the original vertices (15).

III. ONE-LOOP CALCULATIONS

Now, let us start the calculations of the one-loop supergraphs contributing to the KEP. Since $\Pi_{1/2}\Pi_- = \Pi_-\Pi_{1/2} = \Pi_{1/2}\Pi_+ = \Pi_+\Pi_{1/2} = 0$, it follows from (18)–(20) that there can be no mixed contributions containing both gauge and matter propagators at one-loop order. Therefore, the basic supergraphs contributing to the effective action in the theory under consideration are of three types: first, those with internal lines composed of propagators $\langle\psi(1)\bar\psi(2)\rangle$ only; second, those composed of propagators $\langle V(1)V(2)\rangle$ only; and third, those involving alternating propagators $\langle\psi(1)\bar\psi(2)\rangle$ and $\langle V(1)V(2)\rangle$. In our graphical notation, the dashed line is for $\langle\psi\bar\psi\rangle$ propagator, the wavy line is for $\langle VV\rangle$ propagator, and the double one is for Φ or $\bar\Phi$ background fields.

It is easy to verify that the contribution to the effective action generated by the sum of supergraphs at Fig. 1, with simple propagators (19), and the vertices $2(2h)^2(\Phi\bar\Phi)\psi\bar\psi$ is zero. Indeed, it is equal to

$$\Gamma_0 = \sum_{n=1}^{\infty}\frac{1}{2n}[4(2h^2)\Phi\bar\Phi\langle\psi\bar\psi\rangle]^n, \quad (22)$$

where the coefficient 4 is caused by two different contractions. Using the explicit form of the propagators (19), we get

$$\Gamma_0 = \sum_{n=1}^{\infty}\int d^8z_1\frac{1}{2n}[4(2h^2)\Phi\bar\Phi\Pi_{1/2}]^n\delta^8(z_1-z_2)|_{z_1=z_2}. \quad (23)$$

Then, we take into account that $(\Pi_{1/2})^n = \Pi_{1/2}$, and $\Pi_{1/2}\delta^8(z_1-z_2)|_{z_1=z_2} = -2\frac{1}{\Box}\delta^4(x_1-x_2)|_{x_1=x_2}$. Carrying out the Fourier transform, we have

$$\Gamma_0 = \sum_{n=1}^{\infty}\frac{1}{2n}\int d^8z[4(2h^2)\Phi\bar\Phi]^n\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2}, \quad (24)$$

but within the dimensional regularization framework implemented through the replacement $d^4k \to \mu^{4-2\omega}d^{2\omega}k$, one has $\int\frac{d^{2\omega}k}{(2\pi)^{2\omega}}\frac{1}{k^2} = 0$. Hence, this contribution vanishes.

Now, let us sum over the vertices $(2h)^2\bar\Phi\Phi\psi^2$ and $(2h)^2\bar\Phi\Phi\bar\psi^2$. The corresponding supergraphs again exhibit structures similar to Fig. 1 with only an even number of vertices. However, it is worthwhile to point out that we can insert an arbitrary number of vertices $(2h)^2\bar\Phi\Phi\psi\bar\psi$ into the propagators $\langle\psi(1)\bar\psi(2)\rangle$. Therefore, we should first introduce a "dressed" propagator. In this propagator, the summation over all vertices $(2h)^2\bar\Phi\Phi\psi\bar\psi$ is performed (see Fig. 2). As a result, this dressed propagator is equal to

$$\langle\psi(1)\bar\psi(2)\rangle_D = \langle\psi(1)\bar\psi(2)\rangle + \int d^4\theta_3\langle\psi(1)\bar\psi(3)\rangle[(2h)^2\bar\Phi\Phi]_3\langle\psi(3)\bar\psi(2)\rangle + \int d^4\theta_3 d^4\theta_4\langle\psi(1)\bar\psi(3)\rangle[(2h)^2\bar\Phi\Phi]_3\langle\psi(3)\bar\psi(4)\rangle[(2h)^2\bar\Phi\Phi]_4\langle\psi(4)\bar\psi(2)\rangle + \cdots. \quad (25)$$

By using (19), integrating by parts, and summing the resultant series, we arrive at

$$\langle\psi(1)\bar\psi(2)\rangle_D = \left(\frac{2\Pi_{1/2}}{1-2(2h)^2\bar\Phi\Phi}\right)_1\delta_{12}. \quad (26)$$

Afterwards, we can compute all the contributions by noting that each one-loop supergraph above is formed by n vertices like those given by Fig. 3.

Hence, the contribution of this vertex is given by

$$Q_{13} = \int d^4\theta_2[(2h)^2\bar\Phi\Phi]_1\left[\left(\frac{2\Pi_{1/2}}{1-2(2h)^2\bar\Phi\Phi}\right)_1\delta_{12}\right][(2h)^2\bar\Phi\Phi]_2\left[\left(\frac{2\Pi_{1/2}}{1-2(2h)^2\bar\Phi\Phi}\right)_2\delta_{23}\right] = \left(\frac{2(2h)^2\bar\Phi\Phi}{1-2(2h)^2\bar\Phi\Phi}\Pi_{1/2}\right)_1^2\delta_{13}. \quad (27)$$

It follows from the result above that the contribution of a supergraph formed by n vertices is given by

$$\begin{aligned} I_n &= \int d^4x \frac{1}{2n} \int d^4\theta_1 d^4\theta_3 \cdots d^4\theta_{2n-1} \int \frac{d^4p}{(2\pi)^4} Q_{13} Q_{35} \cdots Q_{2n-3,2n-1} Q_{2n-1,1} \\ &= \int d^4x \frac{1}{2n} \int d^4\theta_1 d^4\theta_3 d^4\theta_5 \cdots d^4\theta_{2n-1} \int \frac{d^4p}{(2\pi)^4} \left[\left(\frac{2(2h)^2\bar\Phi\Phi}{1-2(2h)^2\bar\Phi\Phi} \Pi_{1/2} \right)^2_1 \delta_{13} \right] \\ &\quad \times \left[\left(\frac{2(2h)^2\bar\Phi\Phi}{1-2(2h)^2\bar\Phi\Phi} \Pi_{1/2} \right)^2_3 \delta_{35} \right] \cdots \left[\left(\frac{2(2h)^2\bar\Phi\Phi}{1-2(2h)^2\bar\Phi\Phi} \Pi_{1/2} \right)^2_{2n-1} \delta_{2n-1,1} \right] \\ &= \int d^8z \frac{1}{2n} \int \frac{d^4p}{(2\pi)^4} \left(\frac{2(2h)^2\bar\Phi\Phi}{1-2(2h)^2\bar\Phi\Phi} \right)^{2n} \Pi_{1/2} \delta_{\theta\theta'}|_{\theta=\theta'}. \end{aligned} \tag{28}$$

By using $\Pi_{1/2}\delta_{\theta\theta'}|_{\theta=\theta'} = 2/p^2$, we get the effective action

$$\Gamma_1^{(1)} = \sum_{n=1}^{\infty} I_n = -\int d^8z \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2} \ln\left[1 - \left(\frac{2(2h)^2\bar\Phi\Phi}{1-2(2h)^2\bar\Phi\Phi} \right)^2 \right]. \tag{29}$$

The integral over the momenta vanishes within the dimensional regularization scheme. Therefore,

$$\Gamma_1^{(1)} = 0. \tag{30}$$

We will not calculate explicitly the one-loop supergraphs involving the gauge superfield propagators $\langle V(1)V(2)\rangle$ connecting the vertices $(2g)^2\bar\Phi\Phi V^2$, because the result is already known and described in [14]. Therefore, it is given by

$$\Gamma_2^{(1)} = -\int d^8z \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2} \ln\left[1 + \frac{(2g)^2\bar\Phi\Phi}{p^2} \right]. \tag{31}$$

Finally, let us move on to the last type of one-loop supergraphs, which involve the propagators $\langle\psi(1)\bar\psi(2)\rangle$ and $\langle V(1)V(2)\rangle$ in the internal lines connecting the vertices $MV\psi$ and $MV\bar\psi$ (see Fig. 4). As before, we can insert an arbitrary number of vertices $(2h)^2\bar\Phi\Phi\psi\bar\psi$ into the propagators $\langle\psi(1)\bar\psi(2)\rangle$. Moreover, we can also insert an arbitrary number of pairs of the vertices $(2h)^2\bar\Phi\Phi\psi^2$ and $(2h)^2\bar\Phi\Phi\bar\psi^2$ into $\langle\psi(1)\bar\psi(2)\rangle$. Since $\langle\psi(1)\bar\psi(2)\rangle$ has already been dressed by $(2h)^2\bar\Phi\Phi\psi\bar\psi$ in (25) and (26), it follows that the desired dressed propagator $\langle\psi(1)\bar\psi(2)\rangle_{2D}$ is equal to the summation over all pairs of the vertices $(2h)^2\bar\Phi\Phi\psi^2$ and $(2h)^2\bar\Phi\Phi\bar\psi^2$ into $\langle\psi(1)\bar\psi(2)\rangle_D$ (see Fig. 5). Therefore, we get

$$\begin{aligned} \langle\psi(1)\bar\psi(2)\rangle_{2D} &= \langle\psi(1)\bar\psi(2)\rangle_D + \int d^4\theta_3 d^4\theta_4 \langle\psi(1)\bar\psi(3)\rangle_D [(2h)^2\bar\Phi\Phi]_3 \langle\bar\psi(3)\psi(4)\rangle_D \\ &\quad \times [(2h)^2\bar\Phi\Phi]_4 \langle\psi(4)\bar\psi(2)\rangle_D + \int d^4\theta_3 d^4\theta_4 d^4\theta_5 d^4\theta_6 \langle\psi(1)\bar\psi(3)\rangle_D [(2h)^2\bar\Phi\Phi]_3 \\ &\quad \times \langle\bar\psi(3)\psi(4)\rangle_D [(2h)^2\bar\Phi\Phi]_4 \langle\psi(4)\bar\psi(5)\rangle_D [(2h)^2\bar\Phi\Phi]_5 \langle\bar\psi(5)\psi(6)\rangle_D \\ &\quad \times [(2h)^2\bar\Phi\Phi]_6 \langle\psi(6)\bar\psi(2)\rangle_D + \cdots. \end{aligned} \tag{32}$$

After some algebraic work, we find

$$\langle\psi(1)\bar\psi(2)\rangle_{2D} = (2f(\bar\Phi\Phi)\Pi_{1/2})_1\delta_{12}, \quad \text{where} \quad f(\bar\Phi\Phi) \equiv \frac{1}{1-4(2h)^2\bar\Phi\Phi}. \tag{33}$$

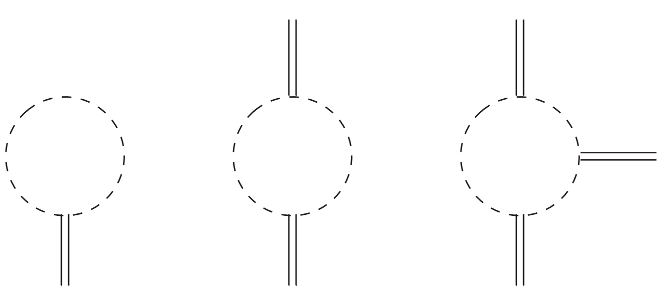

FIG. 1. One-loop supergraphs composed by propagators $\langle\psi(1)\bar\psi(2)\rangle$.

FIG. 2. Dressed propagator $\langle\psi(1)\bar\psi(2)\rangle_D$. The vertices are $2(2h)^2(\Phi\bar\Phi)\psi\bar\psi$.

Additionally, we can also insert an arbitrary number of vertices $(2g)^2\bar{\Phi}\Phi V^2$ into the propagators $\langle V(1)V(2)\rangle$. In this case, the dressed propagator $\langle V(1)V(2)\rangle_D$ is already known in the literature, and it is given by [18]

$$\langle V(1)V(2)\rangle_D = \left(\frac{-\Pi_{1/2}}{p^2+(2g)^2\bar{\Phi}\Phi}\right)_1 \delta_{12}. \quad (34)$$

As before, we can compute all the contributions by noting that each supergraph above (Fig. 4) is formed by n fragments, like those depicted in Fig. 6. This fragment yields the contribution

$$R_{13} = \int d^4\theta_2 (M)_1 \left[\left(\frac{-\Pi_{1/2}}{p^2+(2g)^2\bar{\Phi}\Phi}\right)_1 \delta_{12}\right] \times (M)_2[(2f\Pi_{1/2})_2\delta_{23}] = \left(\frac{-2fM^2\Pi_{1/2}}{p^2+(2g)^2\bar{\Phi}\Phi}\right)_1 \delta_{13}. \quad (35)$$

It follows from the result above that the contribution of a supergraph formed by n subgraphs is given by

$$\begin{aligned} J_n &= \int d^4x \frac{1}{2n}\int d^4\theta_1 d^4\theta_3 \cdots d^4\theta_{2n-1}\int \frac{d^4p}{(2\pi)^4} R_{13}R_{35}\cdots R_{2n-3,2n-1}R_{2n-1,1} \\ &= \int d^4x \frac{1}{2n}\int d^4\theta_1 d^4\theta_3 d^4\theta_5 \cdots d^4\theta_{2n-1}\int \frac{d^4p}{(2\pi)^4}\left[\left(\frac{-2fM^2\Pi_{1/2}}{p^2+(2g)^2\bar{\Phi}\Phi}\right)_1 \delta_{13}\right] \\ &\times \left[\left(\frac{-2fM^2\Pi_{1/2}}{p^2+(2g)^2\bar{\Phi}\Phi}\right)_3 \delta_{35}\right]\cdots\left[\left(\frac{-2fM^2\Pi_{1/2}}{p^2+(2g)^2\bar{\Phi}\Phi}\right)_{2n-1} \delta_{2n-1,1}\right] \\ &= \int d^8z \frac{1}{2n}\int \frac{d^4p}{(2\pi)^4}\left(\frac{-2fM^2}{p^2+(2g)^2\bar{\Phi}\Phi}\right)^n \Pi_{1/2}\delta_{\theta\theta'}|_{\theta=\theta'}. \end{aligned} \quad (36)$$

Again, by using $\Pi_{1/2}\delta_{\theta\theta'}|_{\theta=\theta'} = 2/p^2$, we get the effective action

$$\Gamma_3^{(1)} = \sum_{n=0}^{\infty} J_n = -\int d^8z \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2}\ln\left[1+\frac{2fM^2}{p^2+(2g)^2\bar{\Phi}\Phi}\right]. \quad (37)$$

By summing (30), (31), and (37) we obtain the total one-loop effective action

$$\Gamma^{(1)}[\bar{\Phi},\Phi] = -\int d^8z \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2}\ln[p^2+(2g)^2\bar{\Phi}\Phi+2fM^2]. \quad (38)$$

Substituting the explicit form for M and f, we arrive at the following result for the KEP:

$$K^{(1)}(\bar{\Phi},\Phi) = -\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2}\ln\left[p^2+\frac{1}{2-8(2h)^2\bar{\Phi}\Phi}(m-8gh\bar{\Phi}\Phi)^2+(2g)^2\bar{\Phi}\Phi\right]. \quad (39)$$

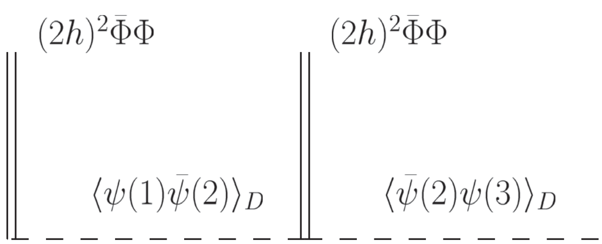

FIG. 3. A typical vertex in one-loop supergraphs involving $(2h)^2\bar{\Phi}\Phi\psi^2$ and $(2h)^2\bar{\Phi}\Phi\bar{\psi}^2$.

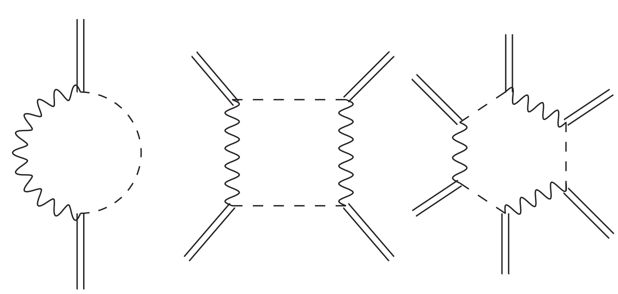

FIG. 4. One-loop supergraphs composed by propagators $\langle\psi(1)\bar{\psi}(2)\rangle$ and $\langle V(1)V(2)\rangle$.

FIG. 5. Dressed propagator $\langle\psi(1)\bar{\psi}(2)\rangle_{2D}$.

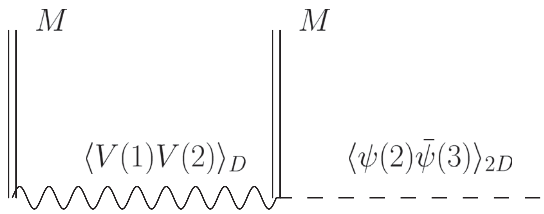

FIG. 6. A typical vertex in one-loop supergraphs involving $MV\psi$ and $MV\bar{\psi}$.

The integral above is well known and can be computed by using the dimensional regularization. Finally, in the limit $\omega \to 2$ we find

$$K^{(1)}(\bar{\Phi},\Phi) = K^{(1)}_{\rm div}(\bar{\Phi},\Phi) + K^{(1)}_{\rm fin}(\bar{\Phi},\Phi), \tag{40}$$

where

$$K^{(1)}_{\rm div}(\bar{\Phi},\Phi) = \frac{1}{16\pi^2(2-\omega)}\left[\frac{1}{2-8(2h)^2\bar{\Phi}\Phi}(m-8gh\bar{\Phi}\Phi)^2 + (2g)^2\bar{\Phi}\Phi\right], \tag{41}$$

$$K^{(1)}_{\rm fin}(\bar{\Phi},\Phi) = -\frac{1}{16\pi^2}\left[\frac{1}{2-8(2h)^2\bar{\Phi}\Phi}(m-8gh\bar{\Phi}\Phi)^2 + (2g)^2\bar{\Phi}\Phi\right] \times \ln\frac{1}{\mu^2}\left[\frac{1}{2-8(2h)^2\bar{\Phi}\Phi}(m-8gh\bar{\Phi}\Phi)^2 + (2g)^2\bar{\Phi}\Phi\right], \tag{42}$$

and μ is an arbitrary scale required on dimensional grounds.

Notice that the one-loop KEP (40)–(42) is divergent. Moreover, we notice that the divergent part (41) is given by an infinite power series in $\bar{\Phi}\Phi$. Therefore, the theory under consideration is nonrenormalizable, and it must be interpreted as an effective field theory below some energy scale chosen on the basis of phenomenological considerations [19].

In particular, let us take $h = 0$ in (40). This choice corresponds to a minimal coupling between the gauge scalar superfield and the matter chiral superfields [see (10)]. Therefore,

$$K^{(1)}_{\rm div}(\bar{\Phi},\Phi) = \frac{(2g)^2\bar{\Phi}\Phi}{16\pi^2(2-\omega)}, \tag{43}$$

$$K^{(1)}_{\rm fin}(\bar{\Phi},\Phi) = -\frac{1}{32\pi^2}[m^2 + 2(2g)^2\bar{\Phi}\Phi] \times \ln\frac{1}{2\mu^2}[m^2 + 2(2g)^2\bar{\Phi}\Phi]. \tag{44}$$

In this case, we notice that the divergent term (43) is proportional to $\bar{\Phi}\Phi$. Therefore, to remove divergences, we can insert a similar one-loop counterterm as the one used in the supersymmetric quantum electrodynamics. Moreover, if we take the massless case in (43)–(44), we recover the one-loop KEP for the usual supersymmetric quantum electrodynamics [14].

IV. SUMMARY

We formulated a new theory involving coupling of three superfields of different natures: a chiral spinor gauge superfield originally introduced in [5] together with the usual real scalar gauge superfield and the chiral scalar matter superfield. For this theory, we developed a superfield procedure for calculating the one-loop effective potential, which we successfully found. The procedure does not essentially differ from the usual supergauge theories [14] with the rather similar structure of the one-loop contribution. The fact that the new theory is nonrenormalizable is not unexpected since many nonpolynomial supersymmetric theories are nonrenormalizable [13,20]. We expect that the importance of the theory is not exhausted by the classical studies in the cosmic string contexto, it can be used as an ingredient of possible phenomenologically interesting supersymmetric gauge theories involving several gauge (super)fields with some of them being massive.

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