Explicit versus spontaneous diffeomorphism breaking in gravity

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Gravitational theories with fixed background fields break local Lorentz and diffeomorphism invariance either explicitly or spontaneously. In the case of explicit breaking it is known that conflicts can arise between the dynamics and geometrical constraints, while spontaneous breaking evades this problem. It is for this reason that in the gravity sector of the Standard-Model extension (SME) it is assumed that the background fields (SME coefficients) originate from spontaneous symmetry breaking. However, in other examples, such as Chern-Simons gravity and massive gravity, diffeomorphism invariance is explicitly broken by the background fields, and the potential conflicts between the dynamics and geometry can be avoided in most cases. An analysis of how this occurs is given, and the conditions that are placed on the metric tensor and gravitational structure as a result of the presence of an explicit-breaking background are described. The gravity sector of the SME is then considered for the case of explicit breaking. However, it is found that a useful post-Newtonian limit is only obtained when the symmetry breaking is spontaneous.

I. INTRODUCTION

The idea that local Lorentz symmetry and diffeomorphism symmetry might not hold exactly is frequently cited as a key signature of new physics in quantum gravity and modified gravity theories that go beyond the Standard Model of particle physics and Einstein's general relativity (GR) [1]. The breaking of Lorentz invariance also allows breaking of the discrete spacetime symmetry *CPT*, which consists of the combination of charge conjugation, parity, and time reversal.

Gravitational models that incorporate local Lorentz and diffeomorphism violation at the level of effective field theory typically do so by including fixed background fields that break the spacetime symmetries either explicitly or spontaneously. Examples include vector-tensor theories motivated from string theory, in which local Lorentz and diffeomorphism invariance are spontaneously broken by background vacuum expectation values, as well as Chern-Simons gravity and massive gravity models, where the symmetries are explicitly broken.

Phenomenological investigations of Lorentz and diffeomorphism violation use the theoretical framework known as the Standard-Model extension (SME) [2–4]. In the SME, fixed background fields, referred to as SME coefficients, couple to the matter and gravitational fields. It is the presence of these coefficients that causes the Lorentz and diffeomorphism breaking. Numerous experiments have been performed in recent years searching for violations of Lorentz symmetry, *CPT* and diffeomorphism invariance [5]. These include gravity tests where a post-Newtonian limit of the SME is used as a framework in Riemann spacetime [6,7]. The sensitivities in these experiments are expressed as bounds on the SME coefficients. Extensive data tables for these bounds can be found in Ref. [8].

In the absence of gravity, in the context of special relativity, the SME coefficients can be treated as due to either explicit or spontaneous Lorentz breaking [2]. However, with gravity in a curved spacetime, the SME coefficients are assumed to arise from a process of spontaneous local Lorentz and diffeomorphism breaking. One reason supporting this view is that the original motivation for developing the SME stemmed from the idea that Lorentz symmetry might be spontaneously broken in the context of a quantum theory of gravity such as string theory [9]. At the same time, spontaneous symmetry breaking is an elegant method that has wide application in physics compared to explicit symmetry breaking. However, it was also found by Kostelecký that in the context of gravity explicit Lorentz and diffeomorphism breaking can lead to conflicts between the dynamics and geometrical constraints that must hold, while spontaneous breaking of these symmetries evades these potential conflicts [3]. It is mainly for this reason that the gravity sector of the SME assumes the background coefficients stem from spontaneous symmetry breaking.

In contrast, however, Chern-Simons and massive gravity models have background fields in their Lagrangians, which explicitly break local Lorentz and diffeomorphism invariance. Nonetheless, these types of models are for the most part able to evade the potential conflicts between dynamics and geometrical constraints. This therefore raises the question of what the critical differences are in gravitational theories with fixed background fields when the symmetry breaking is explicit versus spontaneous. A related question is whether the gravity sector of the SME remains consistent when the coefficients are interpreted as explicitly breaking local Lorentz and diffeomorphism invariance.

The main goal of this paper is to address these issues and to examine the differences between the processes of explicit

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and spontaneous diffeomorphism breaking in gravitational theories in Riemann spacetime. This includes looking at the differences in interpretation and physical behavior of the background fields that cause the symmetry breaking. Examples of theories that are considered include Chern-Simons gravity, massive gravity, models with spontaneous Lorentz breaking, and the gravity sector of the SME.

Traditionally, gravitational theories with fixed background fields have been viewed as less compelling than GR. This is because they can contain what are called "absolute objects" or involve "prior geometry" [10,11]. An absolute object cannot have backreactions, although it can affect the other fields in the theory. The term prior geometry usually implies that parts of the metric and background curvature are predetermined. In GR, features such as these do not occur, and there are natural backreactions between the matter and gravitational fields. This results in a direct link between geometry and the energy-momentum density. However, this is not necessarily the case when diffeomorphisms are broken, although the extent of the departure from GR can depend on whether the symmetry breaking is explicit versus spontaneous. Thus, a further related goal of this paper is to compare theories with explicit or spontaneous diffeomorphism breaking with GR and to examine whether they can retain the natural features found in GR or whether they are fundamentally different from GR.

The next section begins with an overview of the different types of symmetry breaking that are relevant. It then gives a general overview of gravitational effective theories with background tensors that break diffeomorphism invariance. The potential conflicts between dynamics and geometrical constraints in the case of explicit diffeomorphism breaking are examined and discussed in Sec. III. This is followed in Sec. IV by looking at specific examples of models with fixed background fields. Section V examines the post-Newtonian limit of the SME and considers the possibility that the symmetry breaking is explicit instead of spontaneous. A summary and conclusions are given in Sec. VI.

II. SYMMETRY BREAKING

In theories with spacetime symmetry breaking, it is important to make two sets of distinctions that characterize the symmetry breaking [2,3]. The first is between what are called particle and observer transformations. The second is between explicit versus spontaneous symmetry breaking.

When testing a theory with fixed background fields, it is not possible to alter the experimental setups in a way that makes active changes in the background tensors. The definition of particle transformations takes this into account. Under particle transformations, the dynamical tensor fields transform while the background fields are left unchanged. The coordinate system used to describe the spacetime manifold is left unchanged as well. On the other hand, under observer transformations, which are passive transformations, all of the tensor fields (including the

background) are left unchanged, while the coordinate system transforms. In the absence of symmetry breaking, these two transformations acting on tensor components are inversely related. However, when a fixed background is present, the particle symmetry need not hold, in which case the action is not invariant under the particle transformations. Nonetheless, a physical theory should continue to be observer invariant even when there is a fixed background field. Thus, the observer symmetry must continue to hold. This requires that all of the terms in the Lagrangian must be scalars under observer transformations.

The particle symmetry breaking can then be characterized as either explicit or spontaneous. If it is explicit, it is due to the appearance of the fixed background tensor directly in the Lagrangian. However, if it is spontaneous, the background tensor does not initially appear in the Lagrangian at a fundamental level, but instead it appears in the vacuum solution for the theory. In this case, there is a dynamical field that acquires a vacuum expectation value, and the full action remains invariant under both particle and observer transformations.

In GR, diffeomorphisms are particle transformations, consisting of mappings $x^\mu \rightarrow x^\mu + \xi^\mu$, where the changes in dynamical tensors are given by the Lie derivative $\mathcal{L}_\xi$ along the direction of the vectors ξ^μ. In contrast, general coordinate transformations are observer transformations to a new coordinate system $x^\mu \rightarrow x^{\mu'}(x)$. By choosing $x^{\mu'}(x)$ as an infinitesimal coordinate transformation to $x^\mu - \xi^\mu$, using an opposite sign for ξ^μ, a set of observer transformations can be found that have the same mathematical form as the particle diffeomorphisms when acting on dynamical tensor components. These are called observer diffeomorphisms.

In many applications in GR, it is mathematically equivalent to use either particle or observer diffeomorphisms, since dynamical tensors transform the same way under either symmetry. For example, the components of the metric field transform as

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + \mathcal{L}_\xi g_{\mu\nu} = g_{\mu\nu} + D_\mu \xi_\nu + D_\nu \xi_\mu, \qquad (1)$$

where D_μ is the covariant derivative.

To examine the breaking of diffeomorphism invariance, consider a classical gravitational action with a Lagrangian density $\mathcal{L}$. Symmetry breaking occurs when a fixed background tensor $\bar{k}_\chi$ appears in the theory, where χ generically denotes the spacetime indices on the background tensor, which can be contravariant, covariant, or mixed. If the breaking is explicit, $\bar{k}_\chi$ appears directly in the Lagrangian. However, with spontaneous breaking $\bar{k}_\chi$ does not initially appear in the Lagrangian at a fundamental level. Instead, it appears as a vacuum solution for the theory. In this case, there is a dynamical field k_χ that acquires a vacuum expectation value denoted as $\bar{k}_\chi = \langle k_\chi \rangle$, and the full action remains diffeomorphism invariant.

In many cases of theories with spontaneous breaking, particularly when perturbation theory is used, or when massive degrees of freedom are integrated out, it is useful to work with an effective field theory for $\mathcal{L}$ in which the field k_χ is truncated to its background value $\bar{k}_\chi$ plus small excitations about the background. For the vacuum solution itself, the excitations can be set to zero. However, to maintain the diffeomorphism invariance of the action the Nambu-Goldstone (NG) modes for the broken symmetry would need to be included along with the vacuum value.

With these considerations in mind, a generic action describing a gravitational effective theory with a background field, matter fields, and either explicit or spontaneous diffeomorphism violation can be written in a low-energy limit as

$$S = S_{\rm EH} + S_{\rm LV} + S_{\rm LI}. \tag{2}$$

In Riemann spacetime, it is assumed that the leading gravitational contribution to the action is the Einstein-Hilbert term of GR. It is given as

$$S_{\rm EH} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} R, \tag{3}$$

where R is the Ricci scalar and $\kappa = 8\pi G$. For simplicity, a cosmological constant term has been omitted. The second term in S contains the Lorentz- and diffeomorphism-violating background $\bar{k}_\chi$, and it has the form

$$S_{\rm LV} = \int d^4x \sqrt{-g} \mathcal{L}_{\rm LV}(g_{\mu\nu}, f^\psi, \bar{k}_\chi). \tag{4}$$

The Lagrangian $\mathcal{L}_{\rm LV}$ also depends on the metric, $g_{\mu\nu}$, and the conventional matter fields denoted as f^ψ. The spacetime label ψ collectively denotes all the component indices of the tensor f^ψ. The third term contains the Lorentz- and diffeomorphism-invariant matter terms:

$$S_{\rm LI} = \int d^4x \sqrt{-g} \mathcal{L}_{\rm LI}(g_{\mu\nu}, f^\psi). \tag{5}$$

It includes kinetic contributions and symmetry-preserving interaction terms for f^ψ.

When written as an effective field theory with a fixed background tensor, it is not immediately obvious whether the diffeomorphism breaking is explicit or spontaneous. However, the interpretation and behavior of the background field depends on which type of symmetry breaking is being implemented. For example, if the symmetry breaking is explicit, then there are no excitations in the background fields $\bar{k}_\chi$. The background is fixed and there are no dynamical field variations of it in the action. On the other hand, if the breaking is spontaneous, then $\bar{k}_\chi$ is the vacuum value of a dynamical field k_χ. The full theory contains additional excitations of k_χ around $\bar{k}_\chi$, including the NG modes. However, the effective theory in terms of $\bar{k}_\chi$ alone is

a truncated version of the full theory. It has vacuum solutions as exact solutions, or it can be viewed as a gauge-fixed theory where the diffeomorphism symmetry is hidden as opposed to being explicitly broken.

Regardless of the type of particle symmetry breaking, each term in the action S is invariant under general coordinate transformations, including the special case of observer diffeomorphisms. For these transformations, all of the fields $g_{\mu\nu}$, f^ψ, and $\bar{k}_\chi$ transform using the Lie derivative. For example,

$$\bar{k}_\chi \xrightarrow{\rm obsv} \bar{k}_\chi + \mathcal{L}_\xi \bar{k}_\chi, \tag{6}$$

under observer diffeomorphisms, where the expression for the Lie derivative depends on the type and number of indices that $\bar{k}_\chi$ has.

In contrast, the term $S_{\rm LV}$ is not invariant under particle diffeomorphisms. These are broken by the background field $\bar{k}_\chi$, which mathematically obeys $\mathcal{L}_\xi \bar{k}_\chi \neq 0$ for the broken diffeomorphisms ξ^μ. Here, the breaking occurs because the field $\bar{k}_\chi$ remains a fixed background under particle diffeomorphisms and does not transform along with the other fields in the theory. Its transformation rule is therefore

$$\bar{k}_\chi \xrightarrow{\rm part} \bar{k}_\chi \tag{7}$$

under particle diffeomorphisms.

In the effective field theory, when the symmetry breaking is spontaneous, and the field k_χ acquires a vacuum value $\bar{k}_\chi = \langle k_\chi \rangle$, then an exact solution must involve a corresponding vacuum solution for the metric, $\bar{g}_{\mu\nu} = \langle g_{\mu\nu} \rangle$. The ordinary background fields are assumed for simplicity to have vanishing vacuum values $\langle f^\psi \rangle = 0$. Although $\bar{k}_\chi$ is a fixed background, when the symmetry breaking is spontaneous it is still a solution of the k_χ equations of motion and therefore obeys

$$\int d^4x \sqrt{-g} \frac{\delta \mathcal{L}_{\rm LV}}{\delta \bar{k}_\chi} \delta \bar{k}_\chi = 0 \ \text{(spontaneous)} \tag{8}$$

when the other fields take their vacuum values $g_{\mu\nu} = \bar{g}_{\mu\nu}$ and $\langle f^\psi \rangle = 0$.

On the other hand, when the symmetry breaking is explicit, then the fixed background $\bar{k}_\chi$ represents a non-dynamical field, which does not have equations of motion and can therefore obey

$$\int d^4x \sqrt{-g} \frac{\delta \mathcal{L}_{\rm LV}}{\delta \bar{k}_\chi} \delta \bar{k}_\chi \neq 0 \ \text{(explicit)}. \tag{9}$$

III. EXPLICIT BREAKING

General features of gravitational theories with local Lorentz and diffeomorphism breaking were investigated in Ref. [3]. For the case of explicit symmetry breaking,

potential inconsistencies were found, which were shown not to occur when the symmetry breaking is spontaneous. The formalism in [3] uses a vierbein description in order to include fermions in a gravitational theory. A vierbein treatment is also useful in that it reveals that when local Lorentz symmetry is broken by a background field so is diffeomorphism invariance [12]. It allows for dynamical torsion in addition to curvature in a geometry that is Riemann-Cartan [13]. The potential incompatibility for the case of explicit breaking found in [3] involves considering the dynamical equations of motion, the conservation laws for energy-momentum and spin density, and geometric identities for the curvature and torsion.

In this section, the potential conflicts stemming from explicit diffeomorphism breaking are examined in further detail for gravitational theories restricted to Riemann spacetime. In this case, and in the absence of fermions, there is no torsion and a vierbein is not needed. In this restricted treatment, the action takes the generic form as given in Eq. (2).

A. Consistency requirements

With explicit symmetry breaking, the tensor $\bar{k}_\chi$ is a fixed background, which is nondynamical and does not transform under particle diffeomorphisms. The action term $S_{\rm LI}$ therefore only involves conventional matter terms and the metric, and its variation with respect to the metric gives a contribution to the energy-momentum tensor, which can be labeled as $T_{\rm LI}^{\mu\nu}$.

The symmetry-breaking term $S_{\rm LV}$ in general can involve derivative functions of the metric and the connection, such as the curvature tensor or its contractions. Variation of this term with respect to the metric involves using integration by parts. Assuming contributions from the boundaries vanish, these variations can be written as

$$\delta S_{\rm LV} = \int d^4x \frac{\delta(\sqrt{-g}\mathcal{L}_{\rm LV})}{\delta g_{\mu\nu}} \delta g_{\mu\nu}$$
$$\equiv \frac{1}{2}\int d^4x \sqrt{-g}\, T_{\rm LV}^{\mu\nu} \delta g_{\mu\nu}, \qquad (10)$$

which defines a quantity denoted as $T_{\rm LV}^{\mu\nu}$.

Variation of the full action gives Einstein's equations:

$$G^{\mu\nu} = \kappa(T_{\rm LI}^{\mu\nu} + T_{\rm LV}^{\mu\nu}), \qquad (11)$$

where $G^{\mu\nu}$ is the Einstein tensor. The remaining equations of motion are obtained by variation of the conventional matter fields f^ψ, which gives

$$\int d^4x \sqrt{-g}\left(\frac{\delta\mathcal{L}_{\rm LI}}{\delta f^\psi} + \frac{\delta\mathcal{L}_{\rm LV}}{\delta f^\psi}\right)\delta f^\psi = 0. \qquad (12)$$

Taking the covariant divergence of Einstein's equations and using the contracted Bianchi identity, $D_\mu G^{\mu\nu} = 0$, gives an on-shell condition that must hold:

$$D_\mu(T_{\rm LI}^{\mu\nu} + T_{\rm LV}^{\mu\nu}) = 0. \qquad (13)$$

Next, consider the variation of S with respect to observer diffeomorphisms. Although these variations are not physically significant on their own, mathematically they can be combined with dynamical field variations to give results that are meaningful. Under observer diffeomorphisms all three sets of fields (including $\bar{k}_\chi$) transform mathematically, where the variations are given by the Lie derivatives $\delta g_{\mu\nu} = \mathcal{L}_\xi g_{\mu\nu}$, $\delta f^\psi = \mathcal{L}_\xi f^\psi$, and $\delta \bar{k}_\chi = \mathcal{L}_\xi \bar{k}_\chi$. Since S is invariant under general coordinate transformations, including observer diffeomorphisms, it follows that $(\delta S)_{\rm observer} = 0$ under these transformations. At the same time, with appropriate boundary conditions, these observer diffeomorphism variations for $g_{\mu\nu}$ and f^ψ are subsets of the dynamical field variations $\delta g_{\mu\nu}$ and δf^ψ. This allows the dynamical equations of motion to be combined with the observer diffeomorphism transformations.

In addition to overall invariance, each individual term in (2) is an observer scalar as well. When the variations $\delta g_{\mu\nu} = \mathcal{L}_\xi g_{\mu\nu}$ are applied to the Einstein-Hilbert term, and integration by parts is used, the result is the contracted Bianchi identity, which vanishes. Performing observer diffeomorphism variations on the term with $\mathcal{L}_{\rm LI}$ gives the condition

$$\int d^4x\left(\frac{\delta(\sqrt{-g}\mathcal{L}_{\rm LI})}{\delta g_{\mu\nu}}\mathcal{L}_\xi g_{\mu\nu} + \sqrt{-g}\frac{\delta\mathcal{L}_{\rm LI}}{\delta f^\psi}\mathcal{L}_\xi f^\psi\right) = 0. \quad (14)$$

Making similar observer variations in the term with $\mathcal{L}_{\rm LV}$ gives

$$\int d^4x\left[\frac{\delta(\sqrt{-g}\mathcal{L}_{\rm LV})}{\delta g_{\mu\nu}}\mathcal{L}_\xi g_{\mu\nu}\right.$$
$$\left.+ \sqrt{-g}\left(\frac{\delta\mathcal{L}_{\rm LV}}{\delta f^\psi}\mathcal{L}_\xi f^\psi + \frac{\delta\mathcal{L}_{\rm LV}}{\delta\bar{k}_\chi}\mathcal{L}_\xi\bar{k}_\chi\right)\right] = 0. \quad (15)$$

Adding (14) and (15), integrating by parts for the terms involving $\mathcal{L}_\xi g_{\mu\nu} = D_\mu\xi_\nu + D_\nu\xi_\mu$, and using the dynamical equations (12) for the matter fields gives the relation

$$\int d^4x\sqrt{-g}\left[D_\mu(T_{\rm LI}^{\mu\nu} + T_{\rm LV}^{\mu\nu})\xi_\nu - \frac{\delta\mathcal{L}_{\rm LV}}{\delta\bar{k}_\chi}\mathcal{L}_\xi\bar{k}_\chi\right] = 0. \quad (16)$$

This equation must hold due to the combination of general covariance, the Bianchi identity, and the dynamical equations of motion.

In contrast, under particle diffeomorphism transformations, when there is explicit breaking the symmetry does not hold, and $(\delta S)_{\rm particle} \neq 0$. However, the only difference mathematically between the particle diffeomorphism transformations and the observer transformations is that the variations involving $\mathcal{L}_\xi\bar{k}_\chi$ are missing in $(\delta S)_{\rm particle}$ because $\bar{k}_\chi$ does not transform under the broken particle diffeomorphisms. Combining this result with (16) allows the condition for diffeomorphism violation to be written as

$$(\delta S)_{\text{particle}} = \int d^4x \sqrt{-g} \frac{\delta \mathcal{L}}{\delta \bar{k}_\chi} \mathcal{L}_\xi \bar{k}_\chi \neq 0. \qquad (17)$$

The potential conflict between the dynamics and the contracted Bianchi identity for the case of explicit diffeomorphism breaking is then readily apparent. In particular, if (13) is applied on shell in (16), the result is

$$\int d^4x \sqrt{-g} \frac{\delta \mathcal{L}}{\delta \bar{k}_\chi} \mathcal{L}_\xi \bar{k}_\chi = 0. \qquad (18)$$

This appears to be in conflict with the condition of explicit breaking in Eq. (17). That is, unless the integral in both (17) and (18) vanishes on shell, the resulting gravity theory with explicit diffeomorphism breaking is inconsistent.

Notice that for the case of spontaneous diffeomorphism breaking, the interpretation of the background fields $\bar{k}_\chi$ as vacuum expectation values of a dynamical field gives the condition in (8). Therefore, the integral in (17) and (18) vanishes on shell and the potential conflict is avoided for spontaneous breaking.

B. Extra degrees of freedom

The source of the potential inconsistency in Riemann spacetime stems from the fact that a theory with explicit diffeomorphism breaking must still be covariant under general coordinate transformations. Since observer diffeomorphisms are special cases of general coordinate transformations and have the same mathematical form as the broken particle diffeomorphisms, this would seem problematic. However, the number of independent degrees of freedom changes as well when diffeomorphisms are explicitly broken, and the structure of the dynamical equations of motion that apply when the theory is on shell is altered.

In GR, in addition to the matter-field degrees of freedom, the metric $g_{\mu\nu}$ has only two physical propagating degrees of freedom. This follows because in the Einstein-Hilbert action, four of the ten metric components can be shown to be auxiliary fields that do not propagate as physical modes. This leaves at most six independent propagating modes for the metric. In addition, with diffeomorphism invariance, four of the metric components are gauge degrees of freedom, which can be eliminated using the four local symmetry transformations given in terms of the vectors ξ^μ. Thus, only two of the metric modes remain as physical degrees of freedom.

However, in theories where diffeomorphism invariance is explicitly broken by the presence of a fixed background, the action is no longer invariant under these transformations. As a result, there are four extra components in the metric compared to theories where diffeomorphism invariance holds. These four extra metric components are the degrees of freedom that would be gauge if diffeomorphism invariance were not broken.

Further insight can be gained by examining the structure of the gravitational equations of motion. First, consider the ten Einstein equations $G^{\mu\nu} = \kappa T^{\mu\nu}$. One way of interpreting these equations is that with the contracted Bianchi identity $D_\mu G^{\mu\nu} = 0$ there are effectively only six components in $G^{\mu\nu}$ that are dynamically independent and act as kinetic terms for the metric $g_{\mu\nu}$. This coincides with the maximal number of physical propagating degrees of freedom in $g_{\mu\nu}$. The remaining four equations, obtained after taking the covariant derivative, reduce to the condition of covariant energy-momentum conservation, $D_\mu T^{\mu\nu} = 0$. However, in a theory with diffeomorphism invariance, these equations are not satisfied using any of the six potentially physical degrees of freedom in $g_{\mu\nu}$. Instead, they are directly linked to the matter-field equations of motion and hold on shell as a result of the matter dynamics. For example, in the absence of matter, $T^{\mu\nu} = 0$, and the equations are trivially satisfied.

On the other hand, in a theory where diffeomorphism symmetry is explicitly violated, there are again ten Einstein equations as shown in (11). Because of the contracted Bianchi identity, it is still possible to make a distinction between six of these equations being associated with the six dynamically independent components in $G^{\mu\nu}$, while the remaining four reduce to the requirement given in (13). However, unlike a theory with diffeomorphism invariance, the background $\bar{k}_\chi$ is not dynamical and it cannot cause the equations in (13) to hold. Furthermore, in the absence of conventional matter, $T^{\mu\nu}_{\text{LI}} = 0$, while the tensor $T^{\mu\nu}_{\text{LV}}$ in (13) does not vanish. Thus, the condition $D_\mu T^{\mu\nu}_{\text{LV}} = 0$ must hold in the absence of conventional matter. The only remaining degrees of freedom that can be used to account for these equations are the four extra would-be-gauge degrees of freedom that appear in the metric when diffeomorphisms are explicitly broken.

C. Role of extra degrees of freedom

It is straightforward to show that the leading-order effect of the four extra metric modes that appear with explicit breaking is that they impose (13) as their equations of motion. To see this, introduce a field redefinition for the metric $g_{\mu\nu}$, dividing it into six degrees of freedom denoted as $\tilde{g}_{\mu\nu}$ and four would-be-gauge degrees of freedom defined in terms of a vector Ξ_μ. This gives

$$g_{\mu\nu} = \tilde{g}_{\mu\nu} + D_\mu \Xi_\nu + D_\nu \Xi_\mu. \qquad (19)$$

In this redefinition, $\tilde{g}_{\mu\nu}$ consists of ten fields obeying four conditions. Effectively, it is a gauge-fixed form of the metric, which is not equivalent to $g_{\mu\nu}$ because of the diffeomorphism breaking. At leading order in Ξ_μ, the covariant derivatives in this expression can be computed using $\tilde{g}_{\mu\nu}$. Using this field redefinition in the action (2) and performing the field variations $\delta\Xi_\mu$ with respect to Ξ_μ gives the equations of motion

$$\delta S = \int d^4x \frac{(\sqrt{-g}\mathcal{L})}{\delta g_{\mu\nu}} (D_\mu \delta \Xi_\nu + D_\nu \delta \Xi_\mu) = 0. \quad (20)$$

Substituting for $\mathcal{L}$ and integrating by parts gives the result

$$\int d^4x \sqrt{-g}(D_\mu G^{\mu\nu} - \kappa D_\mu(T_{\mathrm{LI}}^{\mu\nu} + T_{\mathrm{LV}}^{\mu\nu}))\delta \Xi_\mu = 0. \quad (21)$$

Combining this with the contracted Bianchi identity shows that at leading order (13) holds as a result of the equations of motion for the would-be-gauge components in the metric defined in terms of Ξ_μ.

Note that even in a diffeomorphism-invariant theory a similar argument can be made involving the gauge degrees of freedom. In this case, the result that would follow is that variation of the action with respect to the gauge components in the metric leads to the requirement of covariant energy-momentum conservation. However, in diffeomorphism-invariant theories, when the field redefinition involving (19) is made, there are always compensating field redefinitions that can be made in the matter fields as well, such that the gauge degrees of freedom drop out. In this case, the condition of covariant energy-momentum conservation stems from the dynamics of the matter fields that are not gauge related, and there are no physically distinct solutions that arise from variation with respect to the gauge modes.

Also note that it is not sufficient if Ξ_μ only gives rise to the condition in (13) as an equation of motion. The condition itself needs to be satisfied as well without requiring that the background $\bar{k}_\chi$ must vanish. If it turns out that the full expression for the divergence of $T_{\mathrm{LV}}^{\mu\nu}$ does not itself involve the would-be-gauge modes in the metric, then nontrivial solutions might not exist. For example, when using an ansatz form for the metric, such as in cosmology, this might be a problem. If an assumed ansatz suppresses too many of the would-be-gauge modes, it may not be possible for the condition in (13) to hold for nonzero values of the background $\bar{k}_\chi$. Similarly if $T_{\mathrm{LV}}^{\mu\nu}$ is constructed out of tensors that are invariant under infinitesimal diffeomorphisms, such as the curvature tensor in linearized gravity, then the fields Ξ_μ might not appear except at higher order. In this case, perturbative treatments can become problematic.

In a linearized treatment in a flat background, the extra metric modes also drop out of the linearized Einstein tensor $G^{\mu\nu}$. This suggests that there are no kinetic terms for these modes at leading order unless such terms are generated by the interactions with the fixed background. However, even if kinetic terms do arise for the extra metric modes, they would likely appear with an indefinite sign and could therefore behave as ghost modes. Assuming as well that the ordinary matter sector is covariantly coupled with the metric, there will not be any interactions at leading order between conventional matter particles and the four

additional metric modes. Thus, the most likely scenario appears to be that the extra metric modes are limited to behaving as auxiliary degrees of freedom that impose covariant energy-momentum conservation as their equations of motion but which otherwise lack a physical existence of their own.

It is important to keep in mind, however, that the interaction terms involving the fixed background can also generate mass terms for the metric modes that do not have the form of gauge excitations. In this case, kinetic terms for these modes can arise in the Einstein tensor. As a result, modifications to the propagation of gravitational waves in theories with explicit diffeomorphism breaking should be expected. How this occurs and how many physical massive metric modes can propagate depends on the detailed form of the theory. The question of whether specific choices of models are ghost-free is then of primary importance in this context.

D. Potential terms

In many cases of interest, the symmetry-breaking term $\mathcal{L}_{\mathrm{LV}}$ takes the form of an interaction term involving only the metric (without derivatives) and the background $\bar{k}_\chi$. The matter fields f^ψ do not couple to $\bar{k}_\chi$. In this case, $\mathcal{L}_{\mathrm{LV}}$ is a potential term and can be written as

$$\mathcal{L}_{\mathrm{LV}} = -\mathcal{U}(g_{\mu\nu}, \bar{k}_\chi). \quad (22)$$

An example of this form is massive gravity where the mass terms involve only the contractions of the metric and a fixed background field.

With the conventional matter fields only appearing in $\mathcal{L}_{\mathrm{LI}}$, the tensor $T_{\mathrm{LI}}^{\mu\nu}$ becomes the matter energy-momentum tensor. It is assumed to be conserved and therefore obeys $D_\mu T_{\mathrm{LI}}^{\mu\nu} = 0$ on shell. The symmetry-breaking contribution $T_{\mathrm{LV}}^{\mu\nu}$ in this case can be interpreted as the energy-momentum tensor for the background field $\bar{k}_\chi$. It is conserved only if the consistency requirements can be nontrivially resolved.

With these restrictions, the consistency analysis simplifies. Performing variations consisting of infinitesimal observer diffeomorphisms on $\mathcal{U}$, which is a scalar, the condition in (16) can be shown to reduce to

$$\int d^4x \sqrt{-g}\left[(D_\mu T_{\mathrm{LV}}^{\mu\nu})\xi_\nu + \frac{\delta \mathcal{U}}{\delta \bar{k}_\chi}\mathcal{L}_\xi \bar{k}_\chi\right] = 0. \quad (23)$$

Here, the variations with respect to $\bar{k}_\chi$ need not vanish because $\bar{k}_\chi$ is not dynamical, and $\mathcal{L}_\xi \bar{k}_\chi$ need not vanish either because $\bar{k}_\chi$ explicitly breaks diffeomorphisms. These two conditions would appear to prevent $D_\mu T_{\mathrm{LV}}^{\mu\nu} = 0$ from holding on shell.

However, it can be shown in general that the two terms in the integrand in (23) differ by a total derivative. In that case,

when $D_\mu T_{\mathrm{LV}}^{\mu\nu} = 0$ holds on shell, the remaining integral becomes a surface term. The condition

$$\int d^4x \sqrt{-g} \frac{\delta\mathcal{U}}{\delta\bar{k}_\chi} \mathcal{L}_\xi \bar{k}_\chi = 0 \qquad (24)$$

then holds on shell despite the fact that the integrand does not vanish. To show this, first consider the definition of $T_{\mathrm{LV}}^{\mu\nu}$. Since there are no derivatives involving $g_{\mu\nu}$ in $\mathcal{U}$, standard Euler-Lagrange variations can be used, giving

$$T_{\mathrm{LV}}^{\mu\nu} = -g^{\mu\nu}\mathcal{U} - 2\frac{\delta\mathcal{U}}{\delta g_{\mu\nu}}. \qquad (25)$$

Taking a divergence of this and multiplying by ξ_ν gives

$$(D_\mu T_{\mathrm{LV}}^{\mu\nu})\xi_\nu = -\xi^\mu D_\mu \mathcal{U} + D_\mu\left(2\frac{\delta\mathcal{U}}{\delta g^{\alpha\beta}} g^{\mu\alpha} g^{\beta\nu}\right)\xi_\nu. \qquad (26)$$

Combining this with the expression for the Lie derivative along ξ^μ acting on $\mathcal{U}(g_{\mu\nu}, \bar{k}_\chi)$ gives the off-shell result

$$(D_\mu T_{\mathrm{LV}}^{\mu\nu})\xi_\nu + \frac{\delta\mathcal{U}}{\delta\bar{k}_\chi}\mathcal{L}_\xi \bar{k}_\chi = D_\mu\left(2\frac{\delta\mathcal{U}}{\delta g^{\alpha\beta}} g^{\mu\alpha}\xi^\beta\right). \qquad (27)$$

Thus, on shell when $D_\mu T_{\mathrm{LV}}^{\mu\nu} = 0$, the condition in (24) does indeed hold, since the integrand becomes a total derivative.

This is a general result that holds for a large class of theories with a potential $\mathcal{U}$. However, there can be exceptions. If, for example, the total divergence in (27) vanishes for certain values of $g_{\mu\nu}$, $\bar{k}_\chi$ and ξ^μ, while the second term on the left-hand side does not, then an inconsistency can still arise. In this case, the only resolution would be that the background $\bar{k}_\chi$ must vanish.

Alternatively, if $\mathcal{L}_\xi \bar{k}_\chi = 0$ holds for a subset of transformations with vectors ξ^μ, then the condition in (27) shows that on shell both terms on the left-hand side would vanish. Therefore, the total covariant divergence on the right-hand side would vanish on shell as well. Multiplying by $\sqrt{-g}$ gives the result

$$\partial_\mu\left(2\sqrt{-g}\frac{\delta\mathcal{U}}{\delta g^{\alpha\beta}} g^{\mu\alpha}\xi^\beta\right) = 0. \qquad (28)$$

In a gravitational theory, these equations impose conditions on the metric $g_{\mu\nu}$, which can lead to restrictions on the allowed geometry of the theory. In particular, in approaches using an ansatz form for the metric and background tensor, it may turn out that the condition in (28) is restrictive enough to rule out specific types of solutions.

IV. EXAMPLES

This section looks at specific examples of gravitational theories with fixed background fields in Riemann spacetime. In each case, an examination is made concerning how the potential conflicts between dynamics and geometrical identities are either evaded or not evaded.

A. Spacetime-dependent cosmological constant

A simple illustration of when a fatal conflict arises is provided by the case of a gravitational theory with a prescribed spacetime-dependent cosmological constant $\Lambda(x)$. Adding such a term to the Einstein-Hilbert action with conventional matter gives

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa}(R - 2\Lambda(x)) + \mathcal{L}_{\mathrm{M}}\right]. \qquad (29)$$

Here, the fixed background is a scalar $\bar{k}_\chi = \Lambda(x)$, $\mathcal{L}_{\mathrm{LI}} = \mathcal{L}_{\mathrm{M}}$ for the matter sector, and the symmetry-breaking term is identified as $\mathcal{L}_{\mathrm{LV}} = -\Lambda(x)/\kappa$.

For this theory, with $\Lambda(x) \neq 0$, particle diffeomorphisms are explicitly broken and the change in the action under these transformations in (17) can be verified to be nonzero. First, the variation of $\mathcal{L}_{\mathrm{LV}}$ with respect to $\bar{k}_\chi$ equals $-1/\kappa$ and is nonzero. Second, the Lie derivative acting on $\Lambda(x)$ gives $\mathcal{L}_\xi \Lambda(x) = \xi^\mu \partial_\mu \Lambda(x)$, which also does not vanish. Combining these confirms (17).

However, Einstein's equations in this case are $G^{\mu\nu} = -\Lambda(x)g^{\mu\nu} + \kappa T_{\mathrm{M}}^{\mu\nu}$, where $T_{\mathrm{M}}^{\mu\nu}$ is the energy-momentum for matter and $T_{\mathrm{LV}}^{\mu\nu} = -g^{\mu\nu}\Lambda(x)/\kappa$. Taking the divergence of Einstein's equations, using the contracted Bianchi identity and $D_\mu T_{\mathrm{M}}^{\mu\nu} = 0$, gives the condition $D_\mu T_{\mathrm{LV}}^{\mu\nu} = -(1/\kappa)g^{\mu\nu}\partial_\mu \Lambda(x) = 0$. Acting on the latter equation with $g_{\nu\alpha}$ shows that $\partial_\alpha \Lambda(x) = 0$ must hold. This clearly contradicts the assumption that the theory has explicit breaking and $\Lambda(x) \neq 0$. The theory is therefore inconsistent, with the only acceptable resolution being that Λ must be constant, which restores the diffeomorphism invariance and (17) no longer applies.

Note that in this example there are no solutions involving would-be-gauge modes for the metric. The scalar background $\Lambda(x)$ does not couple to the metric in a way that allows these extra degrees of freedom to step in and impose the condition of energy-momentum conservation. Instead, these equations involve only the background $\Lambda(x)$, which has no dynamics.

B. Chern-Simons gravity

The Chern-Simons term was originally introduced in three-dimensional gauge field theory and gravity models [14]. The possibility of modifying a four-dimensional gravity theory was subsequently considered as well [15]. Its construction involves introducing a prescribed nondynamical scalar $\theta(x)$ and an associated embedding coordinate $v_\mu = \partial_\mu\theta \neq 0$, which explicitly break diffeomorphism invariance [16].

One form of the action for four-dimensional Chern-Simons gravity can be written as [15]

$$S_{CS} = \int d^4x \left(\frac{1}{2\kappa} \left(\sqrt{-g}R + \frac{1}{4}\theta^*RR \right) + \sqrt{-g}\mathcal{L}_M \right). \quad (30)$$

Here, $^*RR = {}^*R^{\kappa}{}_{\lambda}{}^{\mu\nu}R^{\lambda}{}_{\kappa\mu\nu}$ is the gravitational Pontryagin density, with $^*R^{\kappa}{}_{\lambda}{}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}R^{\kappa}{}_{\lambda\alpha\beta}$. In this context, the fixed background $\bar{k}_{\chi}$ becomes the prescribed scalar $\theta(x)$. The energy-momentum $T_M^{\mu\nu}$ stemming from the matter term $\mathcal{L}_M$ is assumed to be conserved, obeying $D_{\mu}T_M^{\mu\nu} = 0$.

The variation of the Chern-Simons action with respect to the metric gives Einstein's equations:

$$G^{\mu\nu} + C^{\mu\nu} = \kappa T_M^{\mu\nu}, \quad (31)$$

where $C^{\mu\nu}$ is the four-dimensional analogue of the Cotton tensor. Here, $C^{\mu\nu} = \kappa T_{LV}^{\mu\nu}$ takes the place of the tensor associated with the symmetry-breaking term. The divergence of $C^{\mu\nu}$ can be computed explicitly, giving the result [15]

$$D_{\mu}C^{\mu\nu} = \frac{1}{8\sqrt{-g}}(D^{\nu}\theta)^*RR. \quad (32)$$

To examine the symmetry breaking, variation of the Chern-Simons action under particle diffeomorphisms can be performed as in (17). The result is

$$(\delta S)_{particle} = \int d^4x \sqrt{-g}\frac{1}{4}{}^*RR\xi^{\mu}D_{\mu}\theta. \quad (33)$$

Explicit diffeomorphism breaking occurs when $D_{\mu}\theta \neq 0$ and the Pontryagin density does not vanish.

The conflict with dynamics and geometry occurs when Einstein's equations are combined with the contracted Bianchi identity. Taking a covariant derivative in (31) gives that $D_{\mu}C^{\mu\nu} = 0$ must hold on shell. This requires that the product $^*RR(D^{\nu}\theta)$ must vanish on shell. Thus, either $D^{\nu}\theta = 0$, restoring diffeomorphism invariance, or the geometry must be restricted so that only spacetimes with a vanishing Pontryagin density, $^*RR = 0$, are allowed.

This behavior, that the consistency of the theory requires the condition $^*RR = 0$, was noted and discussed in Ref. [15]. Here, however, it is used to illustrate how the potential inconsistency between dynamics and geometry due to explicit diffeomorphism breaking can in some cases be evaded by restricting the possible geometry that can occur.

C. Multiplicative background scalars

In addition to the two specific examples described above, theories with a multiplicative background scalar can be examined in general as well.

Consider a symmetry-breaking term of the form $\mathcal{L}_{LV} = \varphi(x)\mathcal{F}$. Here, $\varphi(x)$ is a prescribed nondynamical scalar background and $\mathcal{F}$ is an arbitrary dynamical scalar function constructed from the metric and its derivatives. For example, $\mathcal{F}$ could consist of products of contractions of the curvature tensor. Variation of the metric in a theory with a term of this form defines a tensor $T^{\mu\nu}$ as in (10), where integrations by parts are used. Consistency with Einstein's equations and the contracted Bianchi identity then requires that $D_{\mu}T_{LV}^{\mu\nu} = 0$ must hold on shell.

At the same time, the term with $\mathcal{L}_{LV}$ is a scalar under observer diffeomorphisms, obeying $(\delta S_{LV})_{observer} = 0$. This leads to a condition that can be written as

$$\int d^4x\sqrt{-g}\xi_{\nu}(D_{\mu}T^{\mu\nu} - (D^{\nu}\varphi)\mathcal{F}) = 0, \quad (34)$$

which must hold for arbitrary ξ_{ν}. Here, the integrand in (34) does not equal a total derivative. This is because the Lie derivative $\mathcal{L}_{\xi}\varphi$ only involves factors of ξ^{μ} and there are no derivatives acting on ξ^{μ}. This is different from the expression in (23), which holds for potentials made out of vectors or tensors, where the Lie derivatives give rise to derivatives acting on ξ_{ν}. In the absence of a total derivative term in (34), the expression in parentheses must vanish, and therefore

$$D_{\mu}T^{\mu\nu} = (D^{\nu}\varphi)\mathcal{F}. \quad (35)$$

This means that on shell there are only two possibilities for this type of theory. Either $D^{\nu}\varphi = 0$ and the symmetry is restored or $\mathcal{F} = 0$ and the geometry is restricted.

In the example of a spacetime-dependent cosmological constant, $\varphi(x) = \Lambda(x)$ and $\mathcal{F} = -1/\kappa \neq 0$. Thus, the only option is that $D^{\nu}\Lambda(x) = 0$ must hold. However, in Chern-Simons gravity, $\varphi(x) = \theta(x)$ and $\mathcal{F} = (1/\sqrt{-g})^*RR$. In this case, there is a nontrivial option, which is that geometrically $^*RR = 0$ must hold.

As an additional example, consider a symmetry-breaking term given as $\mathcal{L}_{LV} = (-1/2\kappa)\bar{u}(x)R$, where $\bar{u}(x)$ is a background scalar and R is the curvature scalar. This term has the form of one of the leading-order terms in the gravity sector of the SME, which is discussed in more detail in Sec. V. Here, the possibility that $\bar{u}(x)$ can appear as a fixed background scalar that explicitly breaks diffeomorphisms is considered. Matching to the above expressions gives $\varphi = \bar{u}(x)$ and $\mathcal{F} = (-1/2\kappa)R$, and the result that follows is that $D_{\mu}T^{\mu\nu} = (-1/2\kappa)R(D^{\nu}\bar{u})$. Thus, when the theory is on shell, either $R = 0$ or $D^{\nu}\bar{u}(x) = 0$ must hold. Since the gravity sector of the SME is intended for phenomenological tests in curved spacetimes with matter, the restriction to spacetimes with $R = 0$ is not desirable. Thus, the restriction that $\bar{u}$ must equal a constant would need to be imposed. In that case, however, the factor of $\bar{u}$ can simply be absorbed by a redefinition of the coupling in the Einstein-Hilbert action and it would have no observable effects.

D. Massive gravity

In gravity, it is not possible to form a conventional mass term for the graviton using only the metric since scalar quadratic products of $g_{\mu\nu}$ are equal to constants. To avoid this problem, in their original formulation of a massive gravity theory, Fierz and Pauli used a perturbative approach in Minkowski spacetime, creating mass terms out of metric excitations instead of the metric itself [17]. However, in massive gravity theories in curved spacetime that go beyond the perturbative level, a background tensor is introduced, which can be denoted generically as a symmetric two-tensor $\bar{f}_{\mu\nu}$. The mass terms for the metric are then formed out of products of $g_{\mu\nu}$ contracted with $\bar{f}_{\mu\nu}$.

In general, these types of massive gravity models are known to suffer from the presence of a ghost mode, known as a Boulware-Deser ghost, as well as difficulty in merging with GR in the massless limit [18,19]. Only recently have models been found that seem promising in being able to avoid these issues. They are known as de Rham, Gabadadze, Tolley (dRGT) massive gravity models. The mass term in dRGT gravity is generated from a potential $\mathcal{U}(g_{\mu\nu}, \bar{f}_{\mu\nu})$ that is constructed in a way that eliminates the Boulware-Deser ghost mode to all orders in a nonlinear treatment [20–22]. In the original dRGT models, a Minkowski tensor, $\bar{f}_{\mu\nu} = \eta_{\mu\nu}$, is used as the background field. It has also been shown that ghost-free theories can be obtained using a more general background, $\bar{f}_{\mu\nu}$, which is different from the Minkowski tensor [21].

Since $\bar{f}_{\mu\nu}$ is a fixed background tensor, particle diffeomorphisms are explicitly broken, and variations of the mass term with respect to these transformations give the general off-shell condition

$$\int d^4x \sqrt{-g} \frac{\delta \mathcal{U}}{\delta \bar{f}_{\mu\nu}} \mathcal{L}_\xi \bar{f}_{\mu\nu} \neq 0. \tag{36}$$

At the same time, the theory is generally covariant, and therefore under observer diffeomorphisms, a second off-shell condition is

$$\int d^4x \sqrt{-g} \left[(D_\mu T_{\mathrm{LV}}^{\mu\nu})\xi_\nu + \frac{\delta \mathcal{U}}{\delta \bar{f}_{\mu\nu}} \mathcal{L}_\xi \bar{f}_{\mu\nu} \right] = 0. \tag{37}$$

With $D_\mu T_{\mathrm{LV}}^{\mu\nu} = 0$ holding on shell, a potential inconsistency arises between (36) and (37). However, as shown in the previous section, the two terms in the integrand differ off shell by a total derivative term:

$$(D_\mu T_{\mathrm{LV}}^{\mu\nu})\xi_\nu + \frac{\delta \mathcal{U}}{\delta \bar{f}_{\mu\nu}} \mathcal{L}_\xi \bar{f}_{\mu\nu} = D_\mu \left(2 \frac{\delta \mathcal{U}}{\delta g_{\alpha\beta}} g^{\mu\alpha} \xi^\beta \right). \tag{38}$$

As long as the total derivative does not vanish, the condition in (37) can hold on shell. In this case, the integral

in (36) vanishes on shell as well, while off shell the theory remains diffeomorphism violating.

When exact solutions exist in massive gravity, it is because the would-be-gauge modes are able to impose the on-shell condition $D_\mu T_{\mathrm{LV}}^{\mu\nu} = 0$ as a result of their equations of motion. In general, these modes are able to appear in $T_{\mathrm{LV}}^{\mu\nu}$, since the contractions of $\bar{f}_{\mu\nu}$ with $g_{\mu\nu}$ can involve all ten components of the metric. However, if an ansatz form for the metric is used, as in cosmology when the universe is assumed to be spatially homogeneous and isotropic, then there may not be enough degrees of freedom in the metric to satisfy the consistency requirements. For example, with $\bar{f}_{\mu\nu} = \eta_{\mu\nu}$ and using a spatially flat metric in a homogeneous and isotropic universe, it has been shown that no exact solution for dRGT gravity exists [23]. However, by using other forms for the background and metric, which introduce one or more additional components to work with, exact solutions describing a spatially flat homogeneous and isotropic universe have been obtained [24].

In dRGT massive gravity with $\bar{f}_{\mu\nu} = \eta_{\mu\nu}$, additional considerations arise because a Minkowski background leaves invariant a subset of diffeomorphisms with vectors ξ^μ equal to constants. For these vectors, the Lie derivatives $\mathcal{L}_\xi \eta_{\mu\nu} = 0$. As a result, one of the terms in (38) is removed, and the metric must obey the condition in (28) on shell, which can lead to restrictions on the allowed geometry. For example, in a Robertson-Walker model with a spatially flat homogeneous and isotropic metric that depends on a scale parameter $a(t)$, (28) is only satisfied if $a(t)$ is constant, which is the same result as in [23].

While massive gravity models can satisfy the consistency conditions, the resulting structure of the theory is very different from GR and theories with diffeomorphism invariance. In particular, the backgrounds $\bar{f}_{\mu\nu}$ are fixed nondynamical tensors that are inserted by hand into the theory, and they are unable to undergo backreactions. Covariant energy-momentum conservation is only maintained by the appearance of the extra modes in the metric, which act as a buffer between the fixed background and the other fields in the theory. The natural interplay between geometry and matter that occurs in GR is disrupted since the background $\bar{f}_{\mu\nu}$ must remain fixed.

Note that in massive gravity, it is common to promote the background fields to dynamical fields by introducing four scalar fields ϕ^a, with $a = 0, 1, 2, 3$, known as Stückelberg fields. The background is rewritten as $\bar{f}_{\mu\nu} = D_\mu \phi^a D_\nu \phi^b f_{ab}(\phi)$, where f_{ab} is defined so that when $D_\mu \phi^a = \delta_\mu^a$ the fixed background $\bar{f}_{\mu\nu}$ is reproduced. By having the fields ϕ^a transform as scalars under diffeomorphisms, this restores the diffeomorphism invariance. However, the number of degrees of freedom is unaltered, since adding the four new fields offsets the addition of the four symmetries. Moreover, by choosing a gauge such that $D_\mu \phi^a = \delta_\mu^a$, the equations of motion reduce to the same set

as in the explicit-breaking case. While the use of Stückelberg fields may appear to restore some of the natural features of GR, the physical nature of these additional fields remains somewhat contrived. Similar to the would-be-gauge modes, these fields do not have direct interactions with matter, and they largely act as a kind of camouflage for the fixed background.

There are, however, alternative approaches to massive gravity which do not explicitly break diffeomorphisms and which do not introduce extra fields. These are models that spontaneously break Lorentz and diffeomorphism invariance, where the massive modes for the metric arise as Higgs excitations or through a Higgs mechanism. In this case, the background fields are vacuum expectation values, which arise dynamically, and diffeomorphism invariance still holds in the action due to the presence of NG modes. With spontaneous breaking, the four extra modes in the metric remain purely gauge, and it is the dynamics of the observable matter fields that ensures covariant conservation of energy-momentum. There are also no potential conflicts between the dynamics and geometrical identities, and natural backreactions between the geometry and matter fields occur. Thus, in these approaches, many of the natural features in GR still hold. For further discussions of gravitational Higgs approaches, see, for example, Refs. [9,12,25], and the references therein.

E. Spontaneous diffeomorphism breaking

As a final example, gravitational theories with spontaneous local Lorentz and diffeomorphism breaking can be examined and compared with theories where the breaking is explicit. In theories with spontaneous breaking, there is typically a potential term $\mathcal{U}$ in the Lagrangian that is a function of the metric and an additional tensor field, which induces nonzero vacuum expectation values. Examples of models with spontaneous spacetime symmetry breaking include bumblebee models [3,9,26], cardinal models [27], and models with an antisymmetric two-tensor [28]. The symmetry breaking occurs when nonzero vacuum values are formed for the tensor and metric fields, which minimize the potential $\mathcal{U}$. The vacuum values can be denoted generically as $\bar{k}_\chi$ and $\bar{g}_{\mu\nu}$. For simplicity, and for comparison with explicit-breaking models, it is assumed that the kinetic terms for k_χ vanish in the vacuum solution.

In general, the potentials $\mathcal{U}$ consist of functions of a finite number of independent scalars X_i that are formed out of contractions of the background tensor and the metric. The number of possible values of i depends on the type of tensor. For example, with a vector background, there is only one independent scalar, X, and the potential is a function $\mathcal{U}(X)$. However, if the potential is formed out of the metric and a symmetric two-tensor, the potential $\mathcal{U}$ can be shown to consist of a function of four independent scalars, X_1, X_2, X_3, and X_4. These are given as traces of products made out of the metric and the background tensor.

Denoting the generic form of the potential as $\mathcal{U}(X_i)$, the tensor $T_{\text{LV}}^{\mu\nu}$ can be computed using Euler-Lagrange variations, and the consistency condition $D_\mu T_{\text{LV}}^{\mu\nu} = 0$ can be written as

$$\sum_i \left[\left(\frac{\delta^2 \mathcal{U}(X_i)}{\delta X_i^2} \right) (D_\mu X_i) \frac{\delta X_i}{\delta g_{\mu\nu}} \right.$$
$$\left. + \frac{\delta \mathcal{U}(X_i)}{\delta X_i} \left(\frac{1}{2} g^{\mu\nu} (D_\mu X_i) + D_\mu \left(\frac{\delta X_i}{\delta g_{\mu\nu}} \right) \right) \right] = 0. \quad (39)$$

Using this expression, some of the key differences between the processes of explicit and spontaneous diffeomorphism breaking can be examined.

With explicit breaking, the tensor $\bar{k}_\chi$ is a fixed background that breaks particle diffeomorphism invariance. In this case, the four equations in (39) become the four equations of motion for the would-be-gauge modes in the metric. These equations combine with the remaining independent Einstein equations to determine the full set of metric components. The scalars X_i contain the metric excitations, and in order for massive modes to appear there needs to be quadratic dependence on these terms in $\mathcal{U}$. Thus, the first and second variations of the potential $\mathcal{U}$ with respect to the scalars X_i in general need not vanish for the massive excitations.

In contrast, when the symmetry breaking is spontaneous the vacuum solutions $\bar{k}_\chi$ and $\bar{g}_{\mu\nu}$ minimize the potential, and

$$\frac{\delta \mathcal{U}(X_i)}{\delta X_i} = 0 \quad (40)$$

holds for these solutions. Thus, only the first set of terms remains in (39), which obeys

$$\sum_i \left(\frac{\delta^2 \mathcal{U}(X_i)}{\delta X_i^2} \right) (D_\mu X_i) \frac{\delta X_i}{\delta g_{\mu\nu}} = 0. \quad (41)$$

For the vacuum solution to be an extremum of the potential $\mathcal{U}$, the second variations of $\mathcal{U}$ with respect to X_i should in general be nonzero. The variations of X_i with respect to the metric need not vanish either. Thus, a general vacuum solution that satisfies (41) holds when the four scalars obey $D_\mu X_i = 0$. As a result, the potential $\mathcal{U}(X_i)$ for the vacuum solution can only consist of scalar combinations of $\bar{k}_\chi$ and $\bar{g}_{\mu\nu}$ that are constants and have no explicit spacetime dependence.

In addition to the vacuum solutions, theories with spontaneous symmetry breaking can also have massless NG modes and massive Higgs modes. The NG modes are excitations generated by the broken symmetry that stay in the degenerate vacuum, obeying (40). Since the broken symmetries are diffeomorphisms, the vacuum scalars X_i remain constant for the NG excitations. Thus, the NG modes satisfy $D_\mu X_i = 0$ as well. However, the massive Higgs modes are not generated by diffeomorphisms. They

therefore do not have to obey (40) and the scalars X_i need not remain constant. In this case, the solutions to (39) are nontrivial and depend on the form of $\mathcal{U}$ and the nature of the tensor fields k_χ.

V. POST-NEWTONIAN LIMIT

The SME is used in phenomenological investigations of Lorentz and diffeomorphism violation in gravity and particle physics. The full SME includes both power-counting renormalizable and nonrenormalizable operators [29,30]. Restrictions to subset models can be defined for the gravity sector [3,6,7,30], quantum electrodynamics [31], and both relativistic and nonrelativistic quantum mechanics [32,33].

To examine gravitational experiments that test for corrections to GR, a post-Newtonian limit of the SME has been developed [6]. It has been used in analyses of data obtained from lunar laser ranging [34], atom interferometry [35], short-range gravitational tests [36], analyses of baryon number asymmetry [37], satellite ranging [38], gyroscope precession [39], pulsar timing [40], and perihelion and solar-spin tests [6,38]. These experiments have achieved sensitivities to Lorentz violation down to levels on the order of parts in 10^{10}.

A. Spontaneous breaking

In the gravity sector of the SME, it is assumed that the SME coefficients arise through a process of spontaneous symmetry breaking. The SME coefficients in this case are vacuum expectation values, and the theory must also account for the NG modes and massive Higgs modes associated with the symmetry breaking.

The development of the post-Newtonian limit of the SME is described in detail in Refs. [6,30]. Here, only a brief summary is given. The starting point is the action in the gravity sector of the SME in Riemann spacetime. It consists of three terms written as

$$S = S_{EH} + S_{LV} + S'. \qquad (42)$$

The first is the Einstein-Hilbert action. The second contains the interactions involving the vacuum values and the gravitational fields. It consists of a series of covariant gravitational operators of increasing mass dimension:

$$S_{LV} = \frac{1}{2\kappa} \int \sqrt{-g}\, d^4x (\mathcal{L}_{LV}^{(4)} + \mathcal{L}_{LV}^{(5)} + \mathcal{L}_{LV}^{(6)} + \cdots), \quad (43)$$

where the superscripts denote the mass dimension.

The leading-order terms are of dimension four and have the form

$$\mathcal{L}_{LV}^{(4)} = -uR + s^{\mu\nu}R_{\mu\nu}^{T} + t^{\kappa\lambda\mu\nu}C_{\kappa\lambda\mu\nu}, \qquad (44)$$

where $R_{\mu\nu}^{T}$ is the trace-free Ricci tensor and $C_{\kappa\lambda\mu\nu}$ is the Weyl conformal tensor. The coefficients u, $s^{\mu\nu}$ and $t^{\kappa\lambda\mu\nu}$ are dynamical fields that acquire vacuum values denoted as $\bar{u}$,

$\bar{s}^{\mu\nu}$ and $\bar{t}^{\kappa\lambda\mu\nu}$ in a process of spontaneous local Lorentz and diffeomorphism breaking. The coefficients $s^{\mu\nu}$ and $t^{\kappa\lambda\mu\nu}$ have symmetries that match those of the Ricci tensor and the Riemann curvature tensor, respectively. The coefficients $s^{\mu\nu}$ are traceless and all of the traces of $t^{\kappa\lambda\mu\nu}$ vanish.

In a process of spontaneous symmetry breaking the fields u, $s^{\mu\nu}$, and $t^{\kappa\lambda\mu\nu}$ consist of their background vacuum values as well as small fluctuations denoted using tildes:

$$u = \bar{u} + \tilde{u},$$
$$s^{\mu\nu} = \bar{s}^{\mu\nu} + \tilde{s}^{\mu\nu},$$
$$t^{\kappa\lambda\mu\nu} = \bar{t}^{\kappa\lambda\mu\nu} + \tilde{t}^{\kappa\lambda\mu\nu}. \qquad (45)$$

The fluctuations include the NG excitations and massive modes. The third term in the action, S', describes the dynamics of these excitations as well as the dynamics of the ordinary matter fields. It includes the kinetic terms for the fluctuations $\tilde{u}$, $\tilde{s}^{\mu\nu}$, and $\tilde{t}^{\kappa\lambda\mu\nu}$.

In the post-Newtonian limit, spacetime is assumed to be asymptotically flat, and the metric is expanded perturbatively to first order around a Minkowski metric $\eta_{\mu\nu}$. To obtain results involving only gravity corrections, the excitations in the fields u, $s^{\mu\nu}$, and $t^{\kappa\lambda\mu\nu}$ must be decoupled and eliminated in terms of the metric fluctuations. In general it might seem that this is not possible without giving definite expressions for the action term S'. However, as described in Ref. [6], by making a series of assumptions and by exploiting the diffeomorphism invariance, a general post-Newtonian expansion involving only the vacuum values $\bar{u}$, $\bar{s}^{\mu\nu}$, and $\bar{t}^{\kappa\lambda\mu\nu}$ can be obtained. Central among these assumptions is that the contracted Bianchi identities hold. These combined with conditions stemming from the diffeomorphism invariance of the linearized theory allow the background fluctuations $\tilde{u}$, $\tilde{s}^{\mu\nu}$, and $\tilde{t}^{\kappa\lambda\mu\nu}$ to be decoupled from the vacuum values and metric excitations. The result is a post-Newtonian expansion involving only the metric, the SME coefficients $\bar{u}$, $\bar{s}^{\mu\nu}$, and $\bar{t}^{\kappa\lambda\mu\nu}$, and the relevant parameters describing a given self-gravitating system.

Recently, the post-Newtonian limit including gravitational operators of dimension five and six has been worked out as well [30]. At dimension five, the operators in $\mathcal{L}_{LV}^{(5)}$ consist of terms having the form of covariant derivatives acting on the curvature tensor, $D^\lambda R^{\alpha\beta\gamma\delta}$. However, this type of term is *CPT* odd and would represent pseudovector contributions to the Newtonian gravitational force rather than conventional vector contributions. It therefore does not have any effects on nonrelativistic gravity. The dimension-six operators contribute terms of the form

$$\mathcal{L}_{LV}^{(6)} = \left(\frac{1}{2} (k_1^{(6)})_{\alpha\beta\gamma\delta\kappa\lambda}(D^\kappa D^\lambda R^{\alpha\beta\gamma\delta} + D^\lambda D^\kappa R^{\alpha\beta\gamma\delta}) \right.$$
$$\left. + (k_2^{(6)})_{\alpha\beta\gamma\delta\kappa\lambda\mu\nu}R^{\kappa\lambda\mu\nu}R^{\alpha\beta\gamma\delta} \right), \qquad (46)$$

where the indices on the coefficients $(k_1^{(6)})_{\alpha\beta\gamma\delta\kappa\lambda}$ and $(k_2^{(6)})_{\alpha\beta\gamma\delta\kappa\lambda\mu\nu}$ have symmetries that match the operators that they multiply. The vacuum values for the coefficients are denoted as $(\bar{k}_1^{(6)})_{\alpha\beta\gamma\delta\kappa\lambda}$ and $(\bar{k}_2^{(6)})_{\alpha\beta\gamma\delta\kappa\lambda\mu\nu}$. As described in [30], a procedure for eliminating the fluctuations in the coefficients about their vacuum values has been worked out, and a post-Newtonian limit including contributions from these higher-dimensional terms has been obtained as well.

B. Explicit breaking

This procedure for finding the post-Newtonian limit of the SME clearly depends on the assumption that the local Lorentz and diffeomorphism breaking is spontaneous. The possibility of explicit diffeomorphism breaking is not considered in [6] due to the conflicts that can arise between the dynamics and geometrical identities. However, in light of the fact that there do exist gravitational theories with explicit breaking that evade these conflicts, it is appropriate to examine whether a gravity sector of the SME with explicit breaking can be defined in a consistent way.

To modify the gravity sector of the SME for the case of explicit breaking, the action is assumed to depend on the fixed background values from the start. The leading-order contributions come from the dimension-four terms in $\mathcal{L}_{\mathrm{LV}}^{(4)}$, which in the case of explicit breaking have the form

$$\mathcal{L}_{\mathrm{LV}}^{(4)} = -\bar{u}R + \bar{s}^{\mu\nu}R_{\mu\nu}^T + \bar{t}^{\kappa\lambda\mu\nu}C_{\kappa\lambda\mu\nu}. \qquad (47)$$

Here, $\bar{u}$, $\bar{s}^{\mu\nu}$ and $\bar{t}^{\kappa\lambda\mu\nu}$ are treated as nondynamical background fields that explicitly break diffeomorphisms. Since there are no excitations in the background fields, the action S' reduces to terms for the ordinary matter sector, which can be ignored in the post-Newtonian limit. Thus, the Einstein equations in this case become $G^{\mu\nu} = T_{\mathrm{LV}}^{\mu\nu}$, where $T_{\mathrm{LV}}^{\mu\nu}$ is defined in (10).

Consistency of the theory requires that $D_\mu T_{\mathrm{LV}}^{\mu\nu} = 0$ must hold on shell. In a theory with explicit breaking, this is possible as long as the extra would-be-gauge modes in the metric have solutions consistent with this condition. Writing out this expression in terms of the background fields $\bar{u}$, $\bar{s}^{\mu\nu}$, and $\bar{t}^{\kappa\lambda\mu\nu}$ gives the equation

$$-R(D^\nu\bar{u}) + 2g^{\mu\nu}(D_\alpha R_{\mu\beta})\bar{s}^{\alpha\beta} + 2g^{\mu\nu}R_{\mu\beta}(D_\alpha\bar{s}^{\alpha\beta})$$
$$+ (D^\nu\bar{s}^{\alpha\beta})R_{\alpha\beta} + 4(D_\alpha R^\nu_{\ \lambda\mu\kappa})\bar{t}^{\alpha\lambda\mu\kappa}$$
$$+ 4R^\nu_{\ \lambda\mu\kappa}(D_\alpha\bar{t}^{\alpha\lambda\mu\kappa}) + R_{\alpha\beta\gamma\delta}(D^\nu\bar{t}^{\alpha\beta\gamma\delta}) = 0. \qquad (48)$$

At the nonlinear level, the would-be-gauge modes can appear in this equation, and solutions for these modes can be obtained in principle. The six independent Einstein equations can then be used to solve for the remaining metric modes. Thus, in principle a nonlinear gravity sector of the

SME could be constructed in the case where the symmetry breaking is explicit.

However, in practice, the usefulness of the SME stems from the fact that the post-Newtonian limit provides a framework in which the leading-order corrections to Newtonian gravity due to Lorentz violation can be computed. In this limit, a linearization of the theory is used, where the metric is written as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and the equations of motion are expanded to lowest order in the excitations $h_{\mu\nu}$. After carrying out the linearization, the equations in (48) become

$$-R(\partial^\nu\bar{u}) + 2\eta^{\mu\nu}(\partial_\alpha R_{\mu\beta})\bar{s}^{\alpha\beta} + 2\eta^{\mu\nu}R_{\mu\beta}(\partial_\alpha\bar{s}^{\alpha\beta})$$
$$+ (\partial^\nu\bar{s}^{\alpha\beta})R_{\alpha\beta} + 4(\partial_\alpha R^\nu_{\ \lambda\mu\kappa})\bar{t}^{\alpha\lambda\mu\kappa}$$
$$+ 4R^\nu_{\ \lambda\mu\kappa}(\partial_\alpha\bar{t}^{\alpha\lambda\mu\kappa}) + R_{\alpha\beta\gamma\delta}(\partial^\nu\bar{t}^{\alpha\beta\gamma\delta}) \simeq 0. \qquad (49)$$

With this result, it can be verified directly that the linearized equations are invariant under diffeomorphisms, since the linearized curvature tensor and its contractions are gauge invariant. Thus, any would-be-gauge modes that appear in $h_{\mu\nu}$ in the form $(\partial_\mu\Xi_\nu + \partial_\nu\Xi_\mu)$ completely drop out. This means that the four equations in (49) must be solved by imposing restrictions on the curvature tensor. These restrictions can be obtained treating each of the background fields $\bar{u}$, $\bar{s}^{\mu\nu}$, and $\bar{t}^{\kappa\lambda\mu\nu}$ independently. For the background $\bar{u}$, the result is already given in Sec. IV C. There it was shown that either $\bar{u}$ must be a constant or $R = 0$ must hold. For the tensor backgrounds $\bar{s}^{\mu\nu}$ and $\bar{t}^{\kappa\lambda\mu\nu}$, it is not sufficient that these coefficients are constants. For consistency, the gravitational excitations at the linearized level also need to obey

$$\partial_\alpha R_{\mu\nu} \simeq 0, \qquad \partial_\alpha R_{\kappa\lambda\mu\nu} \simeq 0. \qquad (50)$$

Thus, a spacetime with nonconstant curvature is in general incompatible with the presence of the explicit-breaking background fields at the linearized level. The only consistent solutions either set the background fields to zero or require that the curvature contributions are constant. It is therefore unlikely that a useful post-Newtonian limit of the SME exists when the symmetry breaking is explicit.

Note these conclusions continue to hold when dimension-six terms in $\mathcal{L}_{\mathrm{LV}}^{(6)}$ are included. With explicit breaking, the SME coefficients $(\bar{k}_1^{(6)})_{\alpha\beta\gamma\delta\kappa\lambda}$ and $(\bar{k}_2^{(6)})_{\alpha\beta\gamma\delta\kappa\lambda\mu\nu}$ are treated as fixed backgrounds, and consistency requires that $D_\mu T_{\mathrm{LV}}^{\mu\nu} = 0$ must hold with these terms included. In the linearized limit in a post-Newtonian framework, the terms involving $(\bar{k}_2^{(6)})_{\alpha\beta\gamma\delta\kappa\lambda\mu\nu}$ have no contributions to first order in the metric fluctuations $h_{\mu\nu}$. However, the terms involving $(\bar{k}_1^{(6)})_{\alpha\beta\gamma\delta\kappa\lambda}$ do have contributions linear in $h_{\mu\nu}$. The resulting conditions are again found to be diffeomorphism invariant in the linearized case.

Repeating the same line of reasoning as above, the consistency conditions can be shown to hold provided the background coefficients are constant and the curvature tensor has vanishing derivatives:

$$\partial_\alpha \partial^\mu \partial^\nu R^{\alpha\beta\gamma\delta} \simeq 0,$$
$$\partial_\mu \partial^\mu \partial^\nu R^{\alpha\beta\gamma\delta} \simeq 0. \qquad (51)$$

Thus, the same conclusion holds with explicit breaking stemming from dimension-six operators. A spacetime with nonzero curvature is incompatible with the presence of explicit-breaking backgrounds in the post-Newtonian limit.

It is also possible to consider in a more general fashion the expected behavior of symmetry-breaking terms with dimension greater than six. In Riemann spacetime, these would have a generic form consisting of contractions of a fixed background with products of the curvature tensor and covariant derivatives of the curvature tensor. In the post-Newtonian limit, at leading order in $h_{\mu\nu}$, these contributions would reduce to terms consisting of the background coefficients contracted with higher-order partial derivatives of the linearized curvature tensor. However, since the linearized curvature tensor in a flat background is invariant under diffeomorphisms, there would again be no contributions from the would-be-gauge modes in the metric excitations. As a result, the independence of the background coefficients would require that either the background coefficients must vanish or they can be constant and instead the higher-order derivatives of the linearized curvature tensor would be required to vanish. Thus, the same conclusions as above would be expected to hold with terms greater than dimension six.

Comparing with the other models considered previously, it can be seen that the SME with explicit breaking in a post-Newtonian limit is similar to the case of Chern-Simons gravity in that both theories require that geometrical conditions must be imposed to maintain consistency. For Chern-Simons gravity, the condition is the vanishing of the Pontryagin density, while for the SME in the post-Newtonian limit it is the more severe requirement that derivatives of the linearized curvature tensor must vanish. In contrast, in the case of massive gravity, the symmetry-breaking terms consist of contractions of the metric tensor with the background field. Since these terms are not gauge invariant, the would-be-gauge modes can appear in these models. As a result, there are no restrictions placed on the geometry, unless an ansatz form of the metric is imposed. However, the resulting theory in this case has a structure that is fundamentally different from GR due to the appearance of the extra would-be-gauge modes in the metric.

These conclusions for the post-Newtonian limit of the SME with explicit breaking are in sharp contrast to what happens in the case of spontaneous breaking. When the symmetry breaking is spontaneous, there are NG modes that occur in the theory. These restore the diffeomorphism invariance and there are no conflicts between the Bianchi identities and covariant energy-momentum conservation. As a result, there are no restrictions that are imposed on the curvature in the linearized limit. Thus, for the case of spontaneous diffeomorphism breaking, the post-Newtonian limit of the SME is well defined and can be used in experimental tests.

VI. SUMMARY AND CONCLUSIONS

The idea that local Lorentz and diffeomorphism invariance might not hold exactly in physics that goes beyond Einstein's GR and the Standard Model has been the subject of much theoretical and experimental investigation in recent years. The SME provides the framework used in phenomenological tests searching for Lorentz and diffeomorphism violation. The symmetry-breaking terms in the SME can be probed and measured experimentally, often with very high sensitivity. In the gravity sector of the SME, a post-Newtonian limit can be derived and used to look for departures from Newtonian gravity caused by the spacetime symmetry breaking.

The background fields in the gravity sector of the SME are assumed to arise from a process of spontaneous symmetry breaking. However, other modified theories of gravity include fixed backgrounds that explicitly break local Lorentz symmetry and diffeomorphisms. Examples of these include Chern-Simons gravity and massive gravity.

This paper has looked at the differences between explicit and spontaneous diffeomorphism breaking in gravitational effective field theories that contain a background field. In particular, it has been shown that very different interpretations hold for the background fields when the symmetry breaking is explicit versus spontaneous. In the case of spontaneous breaking, the background fields arise as vacuum expectation values. They are therefore dynamical in nature. There are also NG modes associated with the symmetry breaking. However, when the breaking is explicit, the background fields are nondynamical, the diffeomorphism invariance is destroyed from the outset, and there are no NG modes.

A central feature that distinguishes explicit breaking from spontaneous breaking of diffeomorphisms is that a potential conflict arises between the dynamics and geometrical identities for the case of explicit breaking, while these conflicts are avoided when the breaking is spontaneous. In Riemann spacetime, the conflict arises because even when particle diffeomorphisms are explicitly broken, general coordinate invariance must still hold. As a result, mathematical conditions resulting from observer diffeomorphism transformations lead to a potential conflict between the on-shell Einstein equations and the Bianchi identities. For theories with covariantly coupled conventional matter, the consistency condition becomes that $D_\mu T_{LV}^{\mu\nu} = 0$ must hold even when off-shell diffeomorphism invariance is lost.

At the same time, however, when diffeomorphisms are explicitly broken, four extra independent degrees of freedom appear in the metric. These are the degrees of freedom that would be gauge in a theory with diffeomorphism invariance, such as GR or a theory with spontaneous breaking. With explicit breaking, the would-be-gauge components have independent equations of motion that impose the condition $D_\mu T^{\mu\nu}_{\text{LV}} = 0$, which can allow the potential conflict to be evaded.

Examples of how this occurs or does not occur have been examined. In certain cases, the conflict is avoided when the integrand in the variation of the action due to the broken particle diffeomorphisms equals a total divergence. However, in other cases, the total divergence is absent and the conflict remains. As a result, either geometrical restrictions must be imposed, or the theory is ruled out. For example, in Chern-Simons gravity, the consistency condition requires that the spacetime must have a vanishing Pontryagin density; otherwise, it is inconsistent. Similarly, in massive gravity, there are ansatz solutions for the metric that cannot be reconciled with the consistency conditions for particular choices of the background field.

In the gravity sector of the SME, since it is assumed that the diffeomorphism breaking is spontaneous, there are no potential conflicts. However, since theories with explicit breaking have been found to evade the potential conflicts in a variety of cases, the question arises as to whether the SME coefficients in the gravity sector can be treated as being due to explicit breaking.

To address this question, the gravity sector of the SME was truncated to include only explicit-breaking fixed backgrounds. It was found that in principle a consistent model is possible at the nonlinear level. However, to be useful in phenomenology, a consistent post-Newtonian limit should exist, which can then be used to investigate experimental tests of Lorentz and diffeomorphism violation in the presence of gravity. The result found here is that, in the post-Newtonian limit, consistency only holds if the curvature tensor is a constant. This effectively rules out using the post-Newtonian limit in gravity experiments when the symmetry breaking is explicit. In contrast, however, when the symmetry breaking is spontaneous, the post-Newtonian limit is known to be consistent, and it has been used in numerous experimental tests with gravity.

Lastly, it is important to keep in mind that, even when a theory with explicit breaking is able to evade the potential inconsistency, its resulting structure is fundamentally different from GR, and many of the compelling features that occur in GR are lost. For example, when diffeomorphism invariance holds, there is a natural link between the dynamics of the matter fields and the geometry of spacetime, and both the matter and metric fields have natural backreactions with each other. However, with explicit breaking, these connections are lost, since the fixed background is unable to have backreactions or to exchange energy-momentum density. To compensate for this, the extra degrees of freedom in the form of the would-be-gauge modes in the metric must step in and act as a buffer between the fixed background and the other fields in the theory. This behavior of the metric is thus very different from GR.

In contrast, when diffeomorphisms are spontaneously broken, many of the natural features of GR are retained and no absolute objects or prior geometry enter into the theory. The background in this case is a vacuum solution of the equations of motion. The NG modes combined with the vacuum solution maintain diffeomorphism invariance, and together they have natural backreactions with the other fields in the theory.

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