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# Standard model fermions and $N = 8$ supergravity

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In a scheme originally proposed by Gell-Mann, and subsequently shown to be realized at the $\mathrm{SU}(3) \times \mathrm{U}(1)$ stationary point of maximal gauged SO(8) supergravity by Warner and one of the present authors, the 48 spin-$\frac{1}{2}$ fermions of the theory remaining after the removal of eight Goldstinos can be identified with the 48 quarks and leptons (including right-chiral neutrinos) of the Standard model, provided one identifies the residual SU(3) with the diagonal subgroup of the color group $\mathrm{SU}(3)_c$ and a family symmetry $\mathrm{SU}(3)_f$. However, there remained a systematic mismatch in the electric charges by a spurion charge of $\pm\frac{1}{6}$. We here identify the "missing" U(1) that rectifies this mismatch, and that takes a surprisingly simple, though unexpected form.

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Maximal gauged $N = 8$ supergravity [1] admits six anti–de Sitter (AdS) vacua (critical points) at which the SO(8) symmetry is broken to a subgroup containing SU(3) [2]. Of these, the one with unbroken $\mathrm{SU}(3) \times \mathrm{U}(1)$ symmetry is in several ways the most interesting [3]. In addition to the residual gauge symmetry, it preserves $N = 2$ supersymmetry, such that its properties can be fully analyzed by means of $N = 2$ AdS supermultiplets [3,4]. Furthermore, the group $\mathrm{SU}(3)_c \times \mathrm{U}(1)_{em}$ is the gauge symmetry that survives to the lowest energies in the Standard model. However, a naive identification of the supergravity SU(3) with the color group $\mathrm{SU}(3)_c$ does not work, as is immediately obvious from the decompositions displayed below [cf. Eq. (7)]. For this reason, Gell-Mann introduced an additional family symmetry $\mathrm{SU}(3)_f$ that acts between the three particle families (generations) and proposed to identify the residual SU(3) of supergravity with the *diagonal subgroup* of color and family [5]. This scheme "almost" works in the sense that, after the removal of eight Goldstinos (as required for a *complete* breaking of supersymmetry) there is complete agreement of the SU(3) assignments, but there remains a systematic mismatch between the U(1) charges: the electric charges of the supergravity fermions are systemically off by $\pm\frac{1}{6}$ from those of the quarks and leptons. Nevertheless, and especially in view of the persistent failure by LHC to detect any new fundamental spin-$\frac{1}{2}$ degrees of freedom (so we "may have already seen it all"), the agreement between the observed number of quarks and leptons, and the number of physical spin-$\frac{1}{2}$ fermions in maximal supergravity remaining after complete breaking of supersymmetry is a tantalizing coincidence [6].

In this article we identify the "missing" U(1) symmetry [designated by $\mathrm{U}(1)_q$] that rectifies the mismatch in the electric charge assignments. As it turns out its action on the original 56 fermions is surprisingly simple, but requires a "deformation" of the residual $\mathrm{SU}(3) \times \mathrm{U}(1)$ symmetry reminiscent of the deformation that appears in nontrivial coproducts. We do not know whether and how such a deformation could be realized dynamically, but the final result [see (14) below] is of such a suggestive simplicity that we may take it as a hint of some nontrivial underlying dynamics that could also lead to new ways of dynamically breaking supersymmetry, possibly in a framework beyond maximal supergravity. Consequently, one main message here is that the "linear" decompositions of group representations commonly employed (often in cascadelike sequences of symmetry breakings) to obtain the particle content of the low energy theory may not suffice to explain the emergence of the Standard model from a unified Planck scale theory.

Let us begin by briefly recalling some basic properties of $N = 8$ supergravity. In its original ungauged version [7] the theory possesses a linearly realized global $\mathrm{E}_{7(7)}$ symmetry and a local chiral SU(8) symmetry, with composite SU(8) gauge fields. Upon choosing a special SU(8) gauge the local SU(8) symmetry collapses to a global (or "rigid") SU(8); in this gauge the noncompact part of $\mathrm{E}_{7(7)}$ is realized nonlinearly. There is no potential for the scalar fields ("moduli"), hence there remains a large vacuum degeneracy. This degeneracy is lifted by gauging the theory. To this aim one promotes an SO(8) subgroup of $\mathrm{E}_{7(7)}$ to a local symmetry, with the 28 spin-1 fields of $N = 8$ supergravity as the Yang-Mills vector bosons [1] [thanks to modern techniques based on the embedding tensor there now exists a large variety of other gaugings, see e.g. [8,9], but the SO(8) gauging remains the only one with a compact simple gauge group]. To maintain full local supersymmetry, the

 

Lagrangian must be modified by Yukawa couplings and a scalar potential, which has been found to display a wealth of stationary points (see Ref. [10], which lists 41 extrema, and Ref. [11] for a more recent survey that also discusses other gaugings). Properties of the $SU(3)\times U(1)$ stationary point are discussed at length in [3], to which we refer for further details. Let us also note that the group theoretical decompositions presented here are independent of the dynamics, and thus to some extent also independent of the specifics of the stationary point. They could thus also apply to some of the $SU(3)\times U(1)$ extrema of the new gaugings recently studied in [11].

In the remainder we focus on the fermionic sector of the theory, which consists of eight gravitinos $\psi^i_\mu$ transforming in the **8**, and a trispinor of spin-$\frac{1}{2}$ fermions $\chi^{ijk}$ transforming in the **56** of SU(8), whence $\chi^{ijk}$ is fully antisymmetric in the SU(8) indices $i,j,k$. We here follow the conventions and notations of [1], so complex conjugation raises (or lowers) indices, such that for instance $\chi^{ijk}=(\chi_{ijk})^*$; at the same time the upper (lower) position of the SU(8) indices indicates positive (negative) chirality. Hence the chiral SU(8) transformations act as

$$\chi^{ijk}\to U^i{}_l U^j{}_m U^k{}_n \chi^{lmn},\quad \chi_{ijk}\to U_i{}^l U_j{}^m U_k{}^n \chi_{lmn} \tag{1}$$

with $U\in SU(8)$, and $U_i{}^j\equiv (U^i{}_j)^*$, whence the unitarity relation $U^\dagger U=\mathbf{1}$ is equivalently expressed by $U^i{}_k U_j{}^k=\delta^i_j$. When a special SU(8) gauge is chosen, the remaining local SO(8) acts by real orthogonal transformations $O^i{}_j$, and thus no longer chirally on the fermions.

The group SO(8) admits a subgroup $U(3)\times U(1)$, via the embedding $SO(6)\times SO(2)\subset SO(8)$. To study the relevant decompositions we introduce boldface indices and their complex conjugates according to [3]

$$\begin{aligned} V^{\mathbf{1}} &\equiv V^1+iV^2, & V^{\bar{\mathbf{1}}} &= V^1-iV^2,\\ V^{\mathbf{2}} &\equiv V^3+iV^4, & V^{\bar{\mathbf{2}}} &= V^3-iV^4,\\ V^{\mathbf{3}} &\equiv V^5+iV^6, & V^{\bar{\mathbf{3}}} &\equiv V^5-iV^6,\\ V^{\mathbf{4}} &\equiv V^7+iV^8, & V^{\bar{\mathbf{4}}} &\equiv V^7-iV^8 \end{aligned}$$

so that the complex conjugate representations are indicated by putting a bar on these indices. The U(3) acts on the first three indices $\mathbf{a},\mathbf{b},\ldots=\mathbf{1},\mathbf{2},\mathbf{3}$. The boldface indices thus furnish a compact way of writing the SU(3) representations; writing them out in terms of the original SU(8) fermions $\chi^{ijk}$ we have, for instance,

$$\begin{aligned}\chi^{\mathbf{12\bar{4}}} &= \chi^{137}+i\chi^{237}+i\chi^{147}-i\chi^{138}\\ &\quad -\chi^{247}+\chi^{238}+\chi^{148}+i\chi^{248}\\ \chi^{\mathbf{1\bar{1}4}} &= -2i\chi^{127}+2\chi^{128}\end{aligned} \tag{2}$$

and so on. The group $U(1)\times U(1)$ is a two parameter Abelian subgroup whose associated Lie algebra is embedded as follows into SO(8):

$$Y(\alpha,\beta)=\begin{pmatrix} 0&\alpha&0&0&0&0&0&0\\ -\alpha&0&0&0&0&0&0&0\\ 0&0&0&\alpha&0&0&0&0\\ 0&0&-\alpha&0&0&0&0&0\\ 0&0&0&0&0&\alpha&0&0\\ 0&0&0&0&-\alpha&0&0&0\\ 0&0&0&0&0&0&0&\beta\\ 0&0&0&0&0&0&-\beta&0 \end{pmatrix}. \tag{3}$$

This matrix commutes with $U(3)\times U(1)\subset SO(8)$ for all $\alpha,\beta$. Consequently, for each choice of $\alpha$ and $\beta$ the above matrix defines an $SU(3)\times U(1)$ subgroup of SO(8) where we denote $U(1)\equiv U(1)_{\alpha,\beta}$ for simplicity.

Given some choice of $\alpha,\beta$, one easily reads off the $SU(3)\times U(1)$ assignments for the gravitinos

$$\begin{aligned}\psi^{\mathbf{a}}_\mu &\in (\mathbf{3},\alpha), & \psi^{\bar{\mathbf{a}}}_\mu &\in (\bar{\mathbf{3}},-\alpha),\\ \psi^{\mathbf{4}}_\mu &\in (\mathbf{1},\beta), & \psi^{\bar{\mathbf{4}}}_\mu &\in (\mathbf{1},-\beta).\end{aligned} \tag{4}$$

The 56 spin-$\frac{1}{2}$ fermions are split into six Goldstinos:

$$\chi^{\mathbf{a4\bar{4}}}\in(\mathbf{3},\alpha),\qquad \chi^{\mathbf{\bar{a}4\bar{4}}}\in(\bar{\mathbf{3}},-\alpha), \tag{5}$$

two "would-be Goldstinos":

$$\chi^{\mathbf{abc}}\in(\mathbf{1},3\alpha),\qquad \chi^{\mathbf{\bar{a}\bar{b}\bar{c}}}\in(\mathbf{1},-3\alpha) \tag{6}$$

and the remaining 48 spin-$\frac{1}{2}$ fermions:

$$\begin{aligned} \chi^{\mathbf{ab4}} &\in (\bar{\mathbf{3}},2\alpha+\beta), & \chi^{\mathbf{ab\bar{4}}} &\in (\bar{\mathbf{3}},2\alpha-\beta)\\ \chi^{\mathbf{\bar{a}\bar{b}4}} &\in (\mathbf{3},-2\alpha+\beta), & \chi^{\mathbf{\bar{a}\bar{b}\bar{4}}} &\in (\mathbf{3},-2\alpha-\beta)\\ \chi^{\mathbf{ab\bar{c}}} &\in (\mathbf{3},\alpha)\oplus(\bar{\mathbf{6}},\alpha), & \chi^{\mathbf{\bar{a}\bar{b}c}} &\in (\bar{\mathbf{3}},-\alpha)\oplus(\mathbf{6},-\alpha)\\ \chi^{\mathbf{a\bar{b}4}} &\in (\mathbf{8},\beta)\oplus(\mathbf{1},\beta), & \chi^{\mathbf{a\bar{b}\bar{4}}} &\in (\mathbf{8},-\beta)\oplus(\mathbf{1},-\beta). \end{aligned} \tag{7}$$

At the $SU(3)\times U(1)$ stationary point [2] the $N=8$ supersymmetry is broken into $N=2$ supersymmetry, with two massless gravitinos $\psi^{\mathbf{4}}_\mu\equiv\psi^7_\mu+i\psi^8_\mu$ and $\psi^{\bar{\mathbf{4}}}_\mu\equiv\psi^7_\mu-i\psi^8_\mu$, while the six Goldstinos (5) are eaten to give six massive gravitinos $\psi^{\mathbf{a}}_\mu$ and $\psi^{\bar{\mathbf{a}}}_\mu$. As shown in [3], all particles fit properly into multiplets of $N=2$ AdS supersymmetry. The mass eigenstates at the stationary point actually mix those fermions lying in the same $SU(3)\times U(1)$ representations (see [3] for explicit formulas and a full analysis of the AdS mass spectrum), but these would anyhow have to regroup along the "deformed" $SU(3)\times U(1)$ to be presented below, if the latter is dynamically excited. Furthermore, in terms of the original chiral SU(8) we still have a residual chiral U(2) which, in terms of the original SU(8) acts on the indices $i, j, \ldots = 7, 8$ and commutes with the SU(3) factor.

To get agreement with the nonsupersymmetric low energy world, the residual $N = 2$ supersymmetry must, of course, also be broken, and this must happen through some as yet unknown dynamical mechanism. In this last step the remaining massless gravitinos $\psi_\mu^{\mathbf{4}}$ and $\psi_\mu^{\bar{\mathbf{4}}}$ would eat the would-be Goldstinos from (6) to become massive, whence we are left with the fermions listed in (7). The challenge is then to match these remaining 48 spin-$\frac{1}{2}$ fermions with those of the Standard model.

Now, as shown in [3], the residual $N = 2$ supersymmetry and the structure of (long and short) multiplets of $N = 2$ AdS supersymmetry [3,4] require

$$\alpha = \frac{1}{6}, \qquad \beta = \frac{1}{2}. \tag{8}$$

Remarkably, this choice is also the one required for the matching with quarks and leptons, modulo a spurion charge $q$ [12]. Namely, if—besides the standard color charge assignments—we assign all fermions to triplets or antitriplets of a new family group $\mathrm{SU}(3)_f$ in the way indicated below, the identification (after removing all eight Goldstinos) with quarks and leptons is [5]

$$\begin{array}{llr}
\chi^{\mathbf{a}\bar{\mathbf{b}}\mathbf{4}}\colon (u, c, t)_L & \mathbf{3}_c \times \bar{\mathbf{3}}_f \to \mathbf{8} \oplus \mathbf{1} & \frac{2}{3} = \frac{1}{2} + q \\
\chi^{\bar{\mathbf{a}}\mathbf{b}\bar{\mathbf{4}}}\colon (\bar{u}, \bar{c}, \bar{t})_L & \bar{\mathbf{3}}_c \times \mathbf{3}_f \to \mathbf{8} \oplus \mathbf{1} & -\frac{2}{3} = -\frac{1}{2} - q \\
\chi^{\bar{\mathbf{a}}\bar{\mathbf{b}}\mathbf{c}}\colon (d, s, b)_L & \mathbf{3}_c \times \mathbf{3}_f \to \mathbf{6} \oplus \bar{\mathbf{3}} & -\frac{1}{3} = -\frac{1}{6} - q \\
\chi^{\mathbf{a}\mathbf{b}\bar{\mathbf{c}}}\colon (\bar{d}, \bar{s}, \bar{b})_L & \bar{\mathbf{3}}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{6}} \oplus \mathbf{3} & \frac{1}{3} = \frac{1}{6} + q \\
\chi^{\mathbf{a}\mathbf{b}\bar{\mathbf{4}}}\colon (\nu_e, \nu_\mu, \nu_\tau)_L & \mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}} & 0 = -\frac{1}{6} + q \\
\chi^{\bar{\mathbf{a}}\bar{\mathbf{b}}\mathbf{4}}\colon (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)_L & \mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3} & 0 = \frac{1}{6} - q \\
\chi^{\bar{\mathbf{a}}\bar{\mathbf{b}}\bar{\mathbf{4}}}\colon (e^-, \mu^-, \tau^-)_L & \mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3} & -1 = -\frac{5}{6} - q \\
\chi^{\mathbf{a}\mathbf{b}\mathbf{4}}\colon (e^+, \mu^+, \tau^+)_L & \mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}} & 1 = \frac{5}{6} + q
\end{array} \tag{9}$$

where we made use of the fact (well known to grand unified theory practitioners) that right-chiral particles can be equivalently described by their left-chiral antiparticles. The most important feature here is that the SU(3) of $N = 8$ supergravity is *not* identified with the QCD color group $\mathrm{SU}(3)_c$, but rather with the diagonal subgroup of color and family symmetry, that is, we identify

$$\mathrm{SU}(3) \equiv [\mathrm{SU}(3)_c \times \mathrm{SU}(3)_f]_{\mathrm{diag}}. \tag{10}$$

Breaking color and family symmetry to the diagonal subgroup may look strange, but a not so dissimilar scheme does appear to work surprisingly well in pure QCD with three flavors, if one assumes that the product of color and flavor SU(3) symmetries is broken to the diagonal SU(3) subgroup by a diquark condensate [13] ("flavor-color locking"). The last column in (9) shows the physical electric charges (on the left) in comparison with the U(1) charges as obtained from the decomposition of the $N = 8$ fermions [on the right, that is (7) with the particular choice (8)]. As we see, the latter differ from the quark and lepton charges systematically by the spurion charge $q$, with negative (positive) sign for family triplets (antitriplets). Accordingly the spurion charge must be taken $q = \frac{1}{6}$ to get agreement with the electric charges of quarks and leptons [5]. Importantly, the electroweak $\mathrm{SU}(2)_w$ would *not* commute with $\mathrm{SU}(3)_f$, as the upper and lower components of the would-be electroweak doublets are assigned to opposite representations of $\mathrm{SU}(3)_f$. More precisely, the upper components of the would-be electroweak doublets [that is, $(u, c, t)_L$ and $(\nu_e, \nu_\mu, \nu_\tau)_L$] are assigned to the $\bar{\mathbf{3}}_f$ of $\mathrm{SU}(3)_f$, while their lower components [that is, $(d, s, b)_L$ and $(e^-, \mu^-, \tau^-)_L$)] are assigned to the $\mathbf{3}_f$ of $\mathrm{SU}(3)_f$. As a consequence, the residual chiral SU(2) $R$ symmetry at the stationary point *cannot* be identified with the electroweak $\mathrm{SU}(2)_w$.

We now look for an implementation of the missing $q$ rotation on the 56 spin-$\frac{1}{2}$ fermions of $N = 8$ supergravity. It is not immediately obvious that this is possible at all, since the extra rotation must transform the family triplets $\mathbf{3}_f$ and antitriplets $\bar{\mathbf{3}}_f$ with *opposite* phases, and it is *a priori* unclear whether and how such a transformation could be realized on the original 56 fermions of $N = 8$ supergravity. Furthermore, enlarging SO(8) to the chiral SU(8) cannot help, as we know that the U(1) that is associated with the electric charges must be vectorlike.

First we write out the correspondence more explicitly:

$$\begin{array}{lll}
\chi^{\boldsymbol{\alpha}\bar{\mathbf{1}}\mathbf{4}} \equiv u^{\boldsymbol{\alpha}}, & \chi^{\boldsymbol{\alpha}\bar{\mathbf{2}}\mathbf{4}} \equiv c^{\boldsymbol{\alpha}}, & \chi^{\boldsymbol{\alpha}\bar{\mathbf{3}}\mathbf{4}} \equiv t^{\boldsymbol{\alpha}} \\
\chi^{\boldsymbol{\alpha}\bar{\mathbf{2}}\bar{\mathbf{3}}} \equiv d^{\boldsymbol{\alpha}}, & \chi^{\boldsymbol{\alpha}\bar{\mathbf{3}}\bar{\mathbf{1}}} \equiv s^{\boldsymbol{\alpha}}, & \chi^{\boldsymbol{\alpha}\bar{\mathbf{1}}\bar{\mathbf{2}}} \equiv b^{\boldsymbol{\alpha}} \\
\chi^{\mathbf{23}\bar{\mathbf{4}}} \equiv \nu_e, & \chi^{\mathbf{31}\bar{\mathbf{4}}} \equiv \nu_\mu, & \chi^{\mathbf{12}\bar{\mathbf{4}}} \equiv \nu_\tau \\
\chi^{\bar{\mathbf{2}}\bar{\mathbf{3}}\bar{\mathbf{4}}} \equiv e^-, & \chi^{\bar{\mathbf{3}}\bar{\mathbf{1}}\bar{\mathbf{4}}} \equiv \mu^-, & \chi^{\bar{\mathbf{1}}\bar{\mathbf{2}}\bar{\mathbf{4}}} \equiv \tau^-
\end{array} \tag{11}$$

where the boldface index $\boldsymbol{\alpha}$ is the $\mathrm{SU}(3)_c$ index [but remember that the diagonal SU(3) rotates *all* indices different from $\mathbf{4}$ and $\bar{\mathbf{4}}$], and where we ignore possible subtleties concerning the proper mass eigenstates, in particular possible mixing with the Goldstino and would-be Goldstino representations in (5) and (6). Idem for the complex conjugate representations which describe the associated antiparticles. Hence, the searched for $\mathrm{U}(1)_q$ rotation must act as follows:

$$\begin{aligned} &\delta u^\alpha = -iu^\alpha, && \delta c^\alpha = -ic^\alpha, && \delta t^\alpha = -it^\alpha \\ &\delta d^\alpha = +id^\alpha, && \delta s^\alpha = +is^\alpha, && \delta b^\alpha = +ib^\alpha \\ &\delta \nu_e = -i\nu_e, && \delta \nu_\mu = -i\nu_\mu, && \delta \nu_\tau = -i\nu_\tau \\ &\delta e^- = +ie^-, && \delta \mu^- = +i\mu^-, && \delta \tau^- = +i\tau^-. \end{aligned} \tag{12}$$

To find out whether and how this transformation can be realized on the original spin-$\frac{1}{2}$ fermions of the theory, we express the latter in terms of the physical fermions, then perform the desired $\mathrm{U}(1)_q$ rotation, and finally transform back to the original basis. Although the intermediate expressions are quite messy, the final result takes a very simple form. To this aim, consider the (vectorlike) SO(8) generator [same as (3) with $\alpha = \beta = 1$]

$$T = Y(1,1) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}. \tag{13}$$

Next, introduce the following $56 \times 56$ matrix acting on the antisymmetrized product of three **8** representations:

$$\mathcal{I} := \frac{1}{2}(T \wedge \mathbf{1} \wedge \mathbf{1} + \mathbf{1} \wedge T \wedge \mathbf{1} + \mathbf{1} \wedge \mathbf{1} \wedge T + T \wedge T \wedge T). \tag{14}$$

Note that this is *not* the direct coproduct that one would expect from (1) with $U = \exp(\omega T)$ acting on each of the three indices, and thus not even an element of SU(8). Indeed, the extra term is reminiscent of the modification ("twist") required to deform a trivial into a nontrivial coproduct. We note that, from $T^2 = -\mathbf{1}$,

$$\mathcal{I}^2 = -\mathbb{1} \tag{15}$$

with the $56 \times 56$ unit matrix $\mathbb{1}$, which shows that (14) can be trivially exponentiated to a $\mathrm{U}(1)_q$ phase rotation. Examples of the action of $\mathcal{I}$ are

$$\begin{aligned} &\chi^{137} \to \frac{1}{2}(+\chi^{237} + \chi^{147} + \chi^{138} + \chi^{248}) \\ &\chi^{247} \to \frac{1}{2}(-\chi^{147} - \chi^{237} + \chi^{248} + \chi^{138}) \\ &\chi^{125} \to \chi^{126}, \qquad \chi^{346} \to -\chi^{345}, \end{aligned} \tag{16}$$

and so on. While the commutation of $T \wedge T \wedge T$ with an arbitrary element of SO(8) or SU(8) in the **56** representation would enlarge either Lie algebra to a bigger one, this is not the case for the residual $\mathrm{SU}(3) \times \mathrm{U}(1)$ because $T$ (representing the imaginary unit) commutes with this subgroup. Hence, it results in a genuine deformation, not an enlargement, of the residual $\mathrm{SU}(3) \times \mathrm{U}(1)$ symmetry at the stationary point.

Our main observation now is that $\mathcal{I}$ does realize (12), namely the transformation

$$\chi^{ijk} \to (\mathcal{I} \circ \chi)^{ijk} \tag{17}$$

yields precisely the phase rotations shown in (12), as is most easily verified by observing that the phase is negative on $\chi$'s with zero or one barred index, and positive on $\chi$'s with two or three barred indices [without the twist term in (14), the fermions would transform with a phase factor $\exp(inq)$, where $n$ counts the number of barred minus unbarred indices]. Therefore, assigning all fermions the charge $q = \frac{1}{6}$ under $\mathrm{U}(1)_q$ and combining the action of $\mathrm{U}(1)_q$ with that of the supergravity U(1), we obtain the correct electric charges for all 48 quarks and leptons. We emphasize that the simple formula (14) appears to work only with the choice (8).

An obvious question at this point concerns the possible implementation, within the present scheme, of chiral gauge interactions corresponding to the full electroweak $\mathrm{SU}(2)_w \times \mathrm{U}(1)_Y$ symmetry [which our new $\mathrm{U}(1)_q$ would be part of]. While we have so far no definite answer to this question, we would like to emphasize the following point. As already shown in [1], the gauging can be done while maintaining the "composite" local SU(8) of [7]. In that formulation the theory has a local $\mathrm{SO}(8) \times \mathrm{SU}(8)$ symmetry [1], which might play a role in explaining the emergence of chirality. Indeed, when embedding the SU(3) subgroup of SO(8) into the (chiral) SU(8), the maximal symmetry that commutes with it is a (chiral) U(2) [3], and this statement remains true with the deformed U(1) identified here. While it is evident already from the discussion in [3] that this U(2) by itself cannot produce the correct electroweak charge assignments, because $\mathrm{SU}(2)_w$ would not commute with $\mathrm{SU}(3)_f$, a twist similar to the one introduced here may be required to make things work. What is clear, however, is that in such a scheme the $W^\pm$ and $Z$ vector bosons would have to be composite, in a partial realization of the conjecture already made in [7], that SU(8) becomes dynamical. We recall that the composite chiral SU(8) symmetry does not suffer from anomalies [14], and the same should be true for any subgroup of SU(8) that becomes dynamical.

The results of this article lend further credence to the remarkable coincidence, already exhibited in [5] and [3], between the fermionic sector of $N = 8$ supergravity and the observed 48 spin-$\frac{1}{2}$ fermions of the Standard model.

Evidently this agreement would be spoilt if any new fundamental spin-$\frac{1}{2}$ degrees of freedom (as predicted by all models of $N = 1$ low energy supersymmetry) were to be found at LHC. While the numerology is thus very suggestive, there remain, of course, the thorny open problems already listed in [3] (huge negative cosmological constant, mass spectrum, etc.), whose resolution would demand some new, and as yet unknown, dynamics which would also have to account for the final breaking of $N = 2$ supersymmetry. So the above coincidence between theory and observation may yet turn out to be a mirage. At any rate, and in view of the complete absence so far of any "new physics" at LHC, it appears worthwhile to search for unconventional alternatives, of the type considered here, to currently popular ideas. In particular, the actual realization of supersymmetry in particle physics may require a more sophisticated implementation of this beautiful concept than in the $N = 1$ models currently thought to be phenomenologically viable.

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