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Can spin-charge-family theory explain baryon number nonconservation?

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The spin-charge-family theory [1–13], in which spinors, besides the Dirac spin, also carry the second kind of Clifford object—no charges—is a type of Kaluza-Klein theory [14]. The Dirac spinors of one Weyl representation in $d = (13 + 1)$ manifest [1,3,4,10,13,15] in $d = (3 + 1)$ at low energies all of the properties of quarks and leptons assumed by the standard model. The second kind of spin explains the origin of families. Spinors interact with the vielbeins and the two kinds of spin connection fields, the gauge fields of the two kinds of Clifford objects, which, besides the gravity and known gauge vector fields, manifest in $d = (3 + 1)$ also several scalar gauge fields. Scalars with the space index $s \in (7, 8)$ carry the weak charge and the hypercharge ($\mp \frac{1}{2}, \pm \frac{1}{2}$, respectively), thereby explaining the origin of the Higgs and Yukawa couplings. It is demonstrated in this paper that the scalar fields with the space index $t \in (9, 10, \ldots, 14)$ carry the triplet color charges, causing transitions of antileptons and antiquarks into quarks and back, thus enabling the appearance and the decay of baryons. These scalar fields show themselves in the presence of the right-handed neutrino condensate, which breaks the *CP* symmetry, the answer to the question about matter-antimatter asymmetry.

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I. INTRODUCTION

The spin-charge-family theory [1–13] is offering, as a type of Kaluza-Klein-like theory, the explanation for the charges of quarks and leptons (right-handed neutrinos are, in this theory, the regular members of a family) and antiquarks and antileptons [15,16], and for the existence of the corresponding gauge vector fields. The theory explains, by also using, besides the Dirac kind of Clifford algebra object, the second kind of Clifford algebra object (there are only two kinds [3,5–7,17–20] associated with the left and the right multiplication of any Clifford object), the origin of families of quarks and leptons and, correspondingly, the origin of the scalar gauge fields causing the electroweak break. These scalar fields are responsible, after gaining nonzero vacuum expectation values, for the masses and mixing matrices of quarks and leptons [9–11] and for the masses of the weak vector gauge fields. They manifest, carrying the weak charge and the hypercharge equal to $\pm \frac{1}{2}, \mp \frac{1}{2}$, respectively [13], as the Higgs field and the Yukawa couplings of the standard model.

The spin-charge-family theory predicts two decoupled groups of four families [3,4,9–11]: The fourth family of the lower group is expected to be observed at the LHC [10,11], while the lowest of the upper four families constitutes dark matter [12].

This theory also predicts the existence of scalar fields which carry triplet color charges. All of the scalar fields carry the fractional quantum numbers with respect to the scalar index $s \geq 5$, either the ones of the groups $SU(2)$ or the ones of the group $SU(3)$, while they are, with respect to the groups not connected with the space index, in the adjoint representations. Neither these scalar fields nor the scalars causing the electroweak break are the supersymmetric scalar partners of the quarks and leptons since they do not carry all of the charges of a family member.

These scalar fields with the triplet color charges cause transitions of antileptons into quarks and antiquarks into quarks and back, offering, in the presence of the condensate of the two right-handed neutrinos with the family quantum numbers belonging to the upper four families which break the *CP* symmetry, the explanation for the matter-antimatter asymmetry. This is the topic of the present paper.

Let me point out that the spin-charge-family theory overlaps in many points with other unifying theories [21–26] since all of the unifying groups can be seen as the subgroups of a large enough orthogonal group, with the family groups included. However, there are also many differences. While the theories built on chosen groups must, for their choice, propose the Lagrangian densities designed for these groups and representations (which also means that there must be a theory behind these effective Lagrangian densities), the spin-charge-family theory starts with a very simple action, from which all of the properties of spinors and the gauge vector and scalar fields follow, provided that symmetry breaking occurs. And all of the scalar and vector gauge fields, either directly or indirectly, manifest in the low energy regime.

Consequently this theory differs from other unifying theories in the degrees of freedom of spinors and scalar and vector gauge fields which show up in different levels of symmetry breaking, in the unification scheme, in the family degrees of freedom and, correspondingly, also in the evolution of our Universe.

It will be demonstrated in this paper that one condensate of two right-handed neutrinos makes all of the scalar gauge fields and all of the vector gauge fields massive on the scale of the appearance of the condensate, except those vector gauge fields which appear in the standard model action as massless fields before the electroweak break. The scalar gauge fields, which cause the electroweak break while gaining nonzero vacuum expectation values and changing their masses, then explain the masses of quarks and leptons and of the weak bosons.

It is an extremely encouraging fact for this theory that one scalar condensate and nonzero vacuum expectation values of some scalar fields [those with the space index $s = (7, 8)$ carrying the weak and the hypercharge equal to, by the standard model, required charges for the Higgs scalar] can make a simple starting action in $d = (13 + 1)$ to manifest in $d = (3 + 1)$ in the low energy regime the observed phenomena of fermions and bosons, explaining the assumptions of the standard model. The theory can also possibly answer open questions like ones regarding the appearance of family members, families, dark matter, and matter-antimatter asymmetry.

The paper leaves, however, many questions connected with symmetry breaking open. Although the scale of symmetry breaking can be roughly estimated, a careful study of the properties of fermions and bosons in the expanding Universe is needed to provide a trustworthy prediction. It remains to be determined under which conditions in the expanding Universe the starting fields (fermions with the two kinds of spins and the corresponding vielbeins and the two kinds of spin connection fields) manifest after spontaneous symmetry breaking in the observed phenomena. This is a very demanding study, a simple first step which was taken in Refs. [12,27]. The present paper is a step towards understanding the matter-antimatter asymmetry within the spin-charge-family theory.

Section I A presents the action and the assumptions of the spin-charge-family theory, with comments added.

In Secs. II, III, IV, and V, the properties of the scalar and vector gauge fields and of the condensate are discussed. In the appendixes, the discrete symmetries of the spin-charge-family theory and the technique used for representing spinors, with the one Weyl representation of $SO(13, 1)$ and the families in $SO(7, 1)$ included, are briefly presented. The final discussions are presented in Sec. VII.

A. Action of spin-charge-family theory and assumptions

In this subsection all of the assumptions of the spin-charge-family theory are presented and commented upon. This subsection follows, to some extent, a similar subsection of Ref. [13].

(i) The space-time is $d = (13 + 1)$ dimensional. Besides the internal degrees of freedom determined by the Dirac γ^a operators, spinors also carry the second kind of Clifford algebra operator [4–7], called $\tilde{\gamma}^a$'s.

(ii) In the simple action [1,3], fermions ψ carry in $d = (13 + 1)$ only two kinds of spins and no charges and interact correspondingly with only two kinds of spin connection gauge fields, $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$, and the vielbeins, $f^{\alpha}{}_{a}$:

$$
\begin{aligned}
S &= \int d^d x E \mathcal{L}_f + \int d^d x E (\alpha R + \tilde{\alpha} \tilde{R}),\\
\mathcal{L}_f &= \frac{1}{2} (\bar{\psi} \gamma^a p_{0a} \psi) + \text{H.c.},\\
p_{0a} &= f^{\alpha}{}_{a} p_{0\alpha} + \frac{1}{2E} \{p_\alpha, E f^{\alpha}{}_{a}\}_-,\\
p_{0\alpha} &= p_\alpha - \frac{1}{2} S^{ab} \omega_{ab\alpha} - \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{ab\alpha},\\
R &= \frac{1}{2} \{ f^{\alpha[a} f^{\beta b]} (\omega_{ab\alpha,\beta} - \omega_{ca\alpha} \omega^{c}{}_{b\beta}) \} + \text{H.c.},\\
\tilde{R} &= \frac{1}{2} f^{\alpha[a} f^{\beta b]} (\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha} \tilde{\omega}^{c}{}_{b\beta}) + \text{H.c.} \qquad (1)
\end{aligned}
$$

Here [28] $f^{\alpha[a} f^{\beta b]} = f^{\alpha a} f^{\beta b} - f^{\alpha b} f^{\beta a}$. S^{ab} and $\tilde{S}^{ab}$ are generators [Eqs. (5), and (B3)] of the groups $SO(13, 1)$ and $\widetilde{SO}(13, 1)$, respectively, expressible by γ^a and $\tilde{\gamma}^a$.

(iii) The manifold $M^{(13+1)}$ breaks first into $M^{(7+1)}$ times $M^{(6)}$ [which manifests as $SU(3) \times U(1)$], affecting both internal degrees of freedom, $SO(13 + 1)$ and $\widetilde{SO}(13 + 1)$. After this break there are $2^{((7+1)/2-1)}$ massless families, with all of the rest of the families getting heavy masses [29].

Both internal degrees of freedom, the ordinary $SO(13 + 1)$ one (where γ^a determine the spins and charges of the spinors) and the $\widetilde{SO}(13 + 1)$ (where $\tilde{\gamma}^a$ determine the family quantum numbers), break simultaneously with the manifolds.

(iv) There are additional symmetry breaks: The manifold $M^{(7+1)}$ breaks further into $M^{(3+1)} \times M^{(4)}$.

(v) There is a scalar condensate of two right-handed neutrinos with the family quantum numbers of the upper four families, bringing masses of the scale above the unification scale to all of the vector and scalar gauge fields which interact with the condensate.

(vi) There are nonzero vacuum expectation values of the scalar fields with the scalar indices (7,8) which cause the electroweak break and bring masses to the fermions and weak gauge bosons, conserving the electromagnetic and color charge.

Comments on the assumptions:

(i) There are, as written above, two—and only two—kinds of Clifford algebra objects. The Dirac one [Eq. (B1)], γ^a, will be used to describe spins of the spinors (fermions) in $d = (13 + 1)$, manifesting in $d = (3 + 1)$ the spin and all of the fermion charges; the second one [Eq. (B1)], $\tilde{\gamma}^a$, will describe families of the spinors. The representations of γ^a's and $\tilde{\gamma}^a$'s are orthogonal to one another [30]. There are, correspondingly, two groups determining the internal degrees of freedom of spinors in $d = (13+1)$: the Lorentz group $SO(13,1)$ and the group $\widetilde{SO}(13,1)$.

One Weyl representation of $SO(13,1)$ contains, if analyzed [1,3,4,13] with respect to the standard model groups, all of the family members assumed by the standard model, with the right-handed neutrinos included (the family members are presented in Table III). It contains the left-handed weak $[SU(2)_I]$ charged and $SU(2)_{II}$ chargeless color triplet quarks and colorless leptons (neutrinos and electrons) and the right-handed weak chargeless and $SU(2)_{II}$ charged quarks and leptons, as well as the right-handed weak charged and $SU(2)_{II}$ chargeless color antitriplet antiquarks and (anti)colorless antileptons and the left-handed weak chargeless and $SU(2)_{II}$ charged antiquarks and antileptons. The reader can easily check the properties of the representations of the spinors (Table III), presented in the technique way (Appendix B), if using Eqs. (5), (8), (9), (10), and (13).

Each family member carries the family quantum numbers, originating in $\tilde{\gamma}^a$ degrees of freedom. Correspondingly, $\tilde{S}^{ab}$ change the family quantum numbers, leaving the family member quantum number unchanged.

(ii) This starting action enables us to represent the standard model as an effective low energy manifestation of the spin-charge-family theory, which explains all of the standard model assumptions, with the families included. There are gauge vector fields, massless before the electroweak break: gravity, color $SU(3)$ octet vector gauge fields, weak $SU(2)$ [to be called $SU(2)_I$] triplet vector gauge field and hyper-$U(1)$ [to be called $U(1)_I$] singlet vector gauge fields. All are superpositions of $f^{\alpha}{}_{c}\, \omega_{ab\alpha}$. There are eight (rather than the observed three) families of quarks and leptons that are massless before the electroweak break. These eight families are indeed two decoupled groups of four families, each of them in the fundamental representations with respect to $\widetilde{SU}(2)\times\widetilde{SU}(2)$ groups—the subgroups of $\widetilde{SO}(3,1)\times\widetilde{SO}(4)\in\widetilde{SO}(7,1)$.

The scalar gauge fields determining the mass matrices of quarks and leptons carry the scalar index $s\in(7,8)$ and, correspondingly, the weak and the hyper charge of the Higgs scalar (Sec. IV). Those among them which are superpositions of $f^{\sigma}{}_{s}\, \tilde{\omega}_{ab\sigma}$ carry, besides the weak and hypercharges, two kinds of family quantum numbers in the adjoint representations, representing two orthogonal groups. Each of the two groups contains two triplets with respect to $\widetilde{SU}(2)_{\widetilde{SO}(3,1)}\times\widetilde{SU}(2)_{\widetilde{SO}(4)}$ [generators of these subgroups are presented in Eqs. (11) and (12)]. The three singlet scalar fields with the space index $s=(7,8)$ and carrying the quantum numbers (Q,Q',Y') are the superpositions of $f^{\sigma}{}_{s}\, \omega_{ab\sigma}$. They again carry the weak and the hyper charge of the Higgs scalar.

One group of two triplets together with the three singlets determines, after gaining nonzero vacuum expectation values at the electroweak break, the Higgs scalar and the Yukawa couplings of the standard model. The second group of two triplets, the three singlets, and the condensate determine, at the electroweak break, the masses of the upper four families, the stable family of which is the candidate for forming dark matter.

The starting action also contains an additional $SU(2)_{II}$ [from $SO(4)$] vector gauge field and the scalar fields with the space index $s\in(5,6)$ and $t\in(9,10,11,12)$, as well as the auxiliary vector gauge fields expressible with vielbeins [Eqs. (C2) and (C1)] in Appendix C]. They all remain either auxiliary (if there are no spinor sources manifesting their quantum numbers) or become massive after the appearance of the condensate.

(iii), (iv) The assumed first break from $M^{(13+1)}$ into $M^{(7+1)}$ times $M^{(6)}$ [manifesting itself in the symmetry $SU(3)\times U(1)_{II}$] explains why the left-handed members of a family carry the weak charge while the right-handed members do not: In the spinor representation of $SO(7,1)$ there are left-handed weak charged quarks and leptons with the hypercharges $\frac{1}{6}$, $-\frac{1}{2}$, respectively, and the right-handed weak chargeless quarks with a hypercharge of either $\frac{2}{3}$ or $-\frac{1}{3}$, while the right-handed weak chargeless leptons carry a hypercharge equal to either 0 or -1. A further break from $M^{(7+1)}$ into $M^{(3+1)}\times M^{(4)}$ manifests the symmetry $SO(3,1)\times SU(2)_I\times SU(2)_{II}\times U(1)_{II}\times SU(3)$, explaining the observed properties of the family members: There are the colored quarks, left-handed weak charged and $SU(2)_{II}$ chargeless and right-handed weak chargeless and $SU(2)_{II}$ charged, and there are the colorless leptons, again left-handed weak charged and $SU(2)_{II}$ chargeless and right-handed weak chargeless and $SU(2)_{II}$ charged. Quarks carry the spinor charge $\frac{1}{6}$, leptons carry the spinor charge $-\frac{1}{2}$. There are the observed vector gauge fields with the corresponding charges in adjoint representation and there are vector gauge fields which gain mass through the interaction with the condensate and are unobservable at low energies. There are the scalar fields manifesting so far as the Higgs scalar and Yukawa couplings and additional scalar fields, which through interaction with the condensate become massive.

Since the left-handed members distinguish from the right-handed partners in the weak and hypercharges, the family members of all of the families stay massless and mass protected up to the electroweak break [31]. Antiparticles are accessible from particles by $\mathbb{C}_{\mathcal{N}}\cdot\mathcal{P}_{\mathcal{N}}$, as explained in Refs. [15,16] and also briefly in Appendix A. This discrete symmetry operator does not contain $\tilde{\gamma}^a$ degrees of freedom. To each family member there corresponds the antimember with the same family quantum number.

(v) There is a condensate of two right-handed neutrinos with the quantum numbers of the upper four families (Table II), appearing in the energy region above the unification scale ($\geq 10^{16}$ GeV), which makes all of the scalar gauge fields [those with the space index (5, 6, 7, 8), as well as those with the space index $(9,\ldots,14)$] and the vector gauge fields manifesting nonzero quantum numbers τ^4, τ^{23}, Q, Y, $\tilde{\tau}^4$, $\tilde{\tau}^{23}$, $\tilde{Q}$, $\tilde{Y}$, $\tilde{N}^3_R$ [Eqs. (8), (9), (10), (11), (12), and (13)] massive.

(vi) At the electroweak break the scalar fields with the space index $s=(7,8)$—twice the three triplets, the superposition of $\tilde{\omega}_{abs}$, Eq. (15), and the three singlets, the superposition of $\omega_{ts's}$, Eq. (14), carrying the charges (Q,Q',Y'); all of these scalars have weak and hypercharges equal to $\mp\frac{1}{2}$, $\pm\frac{1}{2}$, respectively—get nonzero vacuum expectation values, also changing their own masses and breaking the weak and hypercharge symmetries. These scalars determine the mass matrices of the twice four families, as well as the masses of the weak bosons.

The fourth family belonging to the observed three will (sooner or later) be observed at the LHC. Its properties are under consideration [10,11], while the stable one of the upper four families is the candidate for dark matter constituent.

The above assumptions enable the starting action [Eq. (1)] to manifest effectively in $d=(3+1)$ in the low energy regime fermion and the boson fields assumed by the standard model. The starting action also offers the explanation for the dark matter content and for the matter-antimatter asymmetry in the Universe.

To see that the action in Eq. (1) really manifests in $d=(3+1)$ by the standard model required degrees of freedom of fermions and bosons [1–13], let us formally rewrite the Lagrangian density for a Weyl spinor of Eq. (1), which also includes families, as follows:

$$\begin{aligned}
\mathcal{L}_f &= \bar{\psi}\gamma^m\left(p_m-\sum_{Ai} g^A\tau^{Ai}A^{Ai}_m\right)\psi \\
&\quad+\left\{\sum_{s=7,8}\bar{\psi}\,\gamma^s p_{0s}\psi\right\}+\left\{\sum_{t=5,6,9,\ldots,14}\bar{\psi}\,\gamma^t p_{0t}\psi\right\}, \\
p_{0s} &= p_s-\frac{1}{2}S^{s's''}\omega_{s's''s}-\frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abs}, \\
p_{0t} &= p_t-\frac{1}{2}S^{t't''}\omega_{t't''t}-\frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abt},
\end{aligned} \tag{2}$$

where $m\in(0,1,2,3)$, $s\in 7,8,(s',s'')\in(5,6,7,8)$, (a,b) (appearing in $\tilde{S}^{ab}$) run within $\in(0,1,2,3)$ and $\in(5,6,7,8)$, $t\in(5,6,9,\ldots,13,14)$, $(t',t'')\in(5,6,7,8)$, and $\in(9,10,\ldots,14)$. ψ represents all family members of all of the families. The generators of the charge groups τ^{Ai} [expressed in Eqs. (3), (9), and (10) in terms of S^{ab}] fulfill the commutation relations

$$\begin{aligned}
\tau^{Ai} &= \sum_{a,b} c^{Ai}{}_{ab}S^{ab}, \\
\{\tau^{Ai},\tau^{Bj}\}_- &= i\delta^{AB}f^{Aijk}\tau^{Ak}.
\end{aligned} \tag{3}$$

The spin generators are defined in Eq. (8). These group generators determine all of the internal degrees of freedom of one family members as seen from the point of view of $d=(3+1)$: The spin is determined by the group $SO(3,1)$, the color charge is determined by the group $SU(3)$ [originating in $SO(6)$] and with the generators $\vec{\tau}^3$, the spinor charge is determined by $U(1)_{II}$ [originating in $SO(6)$] and with the generator τ^4, the weak charge is determined by the group $SU(2)_I$ [originating in $SO(4)$] and with the generators $\vec{\tau}^1$, and the second $SU(2)_{II}$ charge [$SU(2)_{II}$ originating in $SO(4)$] has the generators $\vec{\tau}^2$. The group $SU(2)_{II}$ breaks [3] in the presence of the condensate into $U(1)_I$. The generators τ^{23} define, together with τ^4, the hyper charge Y $(=\tau^{23}+\tau^4)$.

The condensate of the two right-handed neutrinos with the family quantum numbers of the upper four families bring masses (of the unifying scale $\geq 10^{16}$ GeV) to all of the scalar fields and those vector gauge fields which are not observed at currently measurable energies, since the observed vectors do not couple to the condensate.

The scalar fields with the scalar index $s=(7,8)$ bring masses, when gaining nonzero vacuum expectation values at the electroweak phase transition, to twice four families and to the weak bosons. I shall comment on all of these fields in what follows.

The first line of Eq. (2) describes [1,3] before the electroweak break the dynamics of eight families of massless fermions in interaction with the massless color $\vec{A}^3_m$, the weak $\vec{A}^1_m$, and the hyper-A^Y_m $(=\sin\vartheta_2 A^{23}_m+\cos\vartheta_2 A^4_m)$ gauge fields, all of which are the superpositions of ω_{abm} [32].

The first term of the second line of the same equation [Eq. (2)] determines the mass term, which, after the electroweak break, brings masses to all of the family members of the eight families and to the weak bosons. The scalar fields responsible—after gaining nonzero vacuum expectation values—for the masses of the family members and of the weak bosons are, namely, included in the second line of Eq. (2) as $-\frac{1}{2}S^{s's''}\omega_{s's''s}-\frac{1}{2}\tilde{S}^{\tilde{a}\tilde{b}}\tilde{\omega}_{\tilde{a}\tilde{b}s}$, $s\in(7,8)$, $(s',s'')\in(5,6,7,8)$, $(\tilde{a},\tilde{b})\in(\tilde{0},\tilde{1},\ldots,\tilde{8})$ [33].

The properties of those scalar fields with the scalar index $s=(7,8)$ are discussed in Sec. IV, where a proof is presented in which they all carry the weak charge and the hypercharge as the standard model Higgs scalar, while they are either triplets with respect to the family quantum numbers or singlets with respect to the charges Q,Q', and Y'. While the two triplets $(\vec{\tilde{A}}^{1}_s,\vec{\tilde{A}}^{\tilde{N}_L}_s)$ interact with the lower four families, $(\vec{\tilde{A}}^{2}_s,\vec{\tilde{A}}^{\tilde{N}_R}_s)$ interact with the upper four families. These twice two triplets are superpositions of

$\frac{1}{2}\tilde{S}^{\tilde{a}\tilde{b}}\tilde{\omega}_{\tilde{a}\tilde{b}s}$, $s = \in (7,8)$, Eq. (15). The three singlets (A^{Q}_{s}, $A^{Q'}_{s}$, and $A^{Y'}_{m}$) are superpositions of $\omega_{s's''s}$, Eq. (14). They interact with the family members of all the families, seeing charges of the family members.

The second term in the second line of Eq. (2) represents fermions in interaction with the rest of the scalar fields. Scalar fields become massive after interacting with the condensate. Those which do not gain nonzero vacuum expectation values keep the heavy masses of the order of the scale of the condensate up to low energies.

The massive scalars with the space index $t \in (5,6)$ transform (Table III) u_R quarks into d_L quarks and ν_R leptons into e_L leptons and back, as well as $\bar{u}_R$ antiquarks into $\bar{d}_L$ antiquarks and back and $\bar{\nu}_R$ antileptons into $\bar{e}_L$ antileptons and back.

Those scalar fields with the space index $t = (9,10,\ldots,14)$ transform antileptons into quarks and antiquarks into quarks and back. They are offering, in the presence of the scalar condensate breaking the CP symmetry, the explanation for the observed matter-antimatter asymmetry, as we shall show in Sec. II.

Let us write down the second term in the second line of Eq. (2), the part of the fermion action which, in the presence of the condensate, offers an explanation for the observed matter-antimatter asymmetry:

$$\mathcal{L}_{f'} = \psi^{\dagger}\gamma^{0}\gamma^{t}\Big\{\sum_{t=(9,10,\ldots 14)}\Big[p_t - \Big(\frac{1}{2}S^{s's''}\omega_{s's''t} + \frac{1}{2}S^{t't''}\omega_{t't''t} + \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abt}\Big)\Big]\Big\}\psi, \tag{4}$$

where $(s',s'') \in (5,6,7,8)$, $(t,t',t'') \in (9,10,\ldots,14)$ and $(a,b) \in (0,1,2,3)$ and $\in (5,6,7,8)$, in agreement with the assumed breaks in Sec. I. Again the operators $\tilde{S}^{ab}$ determine family quantum numbers and S^{ab} determine family member quantum numbers. Correspondingly the superposition of the scalar fields $\tilde{\omega}_{abt}$ and the superposition of the scalar fields ω_{abt} carry the quantum numbers determined by either the superposition of $\tilde{S}^{ab}$ or by the superposition of S^{ab} in the adjoint representations, while they all carry the color charge, determined by the space index $t \in (9,10,\ldots,14)$, in the triplet representation of the $SU(3)$ group, as we shall see. Similarly the scalars with the space index $s \in (7,8)$ carry the weak and the hypercharge in the doublet representations [34].

The condensate of two right-handed neutrinos with the family quantum numbers of the upper four families carries (Table II) $\tau^4 = 1$, $\tau^{23} = -1$, $\tau^{13} = 0$, $Y = 0$, $Q = 0$, and the family quantum numbers of the upper four families and gives masses to scalar and vector gauge fields with the nonzero corresponding quantum numbers. The only vector gauge fields which stay massless up to the electroweak break are the hypercharge field (A^{Y}_{m}), the weak charge field ($\vec{A}^{1}_{m}$), and the color charge field ($\vec{A}^{3}_{m}$) (besides the gravity).

1. Standard model subgroups of $SO(13+1)$ *and* $\widetilde{SO}(13+1)$ *groups and corresponding gauge fields*

This section follows Refs. [3,13]. To calculate the quantum numbers of one Weyl representation presented in Table III in terms of the generators of the standard model charge groups τ^{Ai} ($=\sum_{a,b}c^{Ai}{}_{ab}S^{ab}$), one must look for the coefficients $c^{Ai}{}_{ab}$ [Eq. (3)]. Similarly the spin and the family degrees of freedom also have to be expressed as a superposition of S^{ab} or $\tilde{S}^{ab}$.

The same coefficients $c^{Ai}{}_{ab}$ determine operators which apply either on spinors or on vectors. The difference among the three kinds of operators—the vector and the two kinds of spinor operators—lies in the difference among S^{ab}, $\tilde{S}^{ab}$, and $\mathcal{S}^{ab}$.

While S^{ab} for spins of spinors is equal to

$$S^{ab} = \frac{i}{4}(\gamma^a\gamma^b - \gamma^b\gamma^a), \qquad \{\gamma^a,\gamma^b\}_+ = 2\eta^{ab} \tag{5}$$

and $\tilde{S}^{ab}$ for families of spinors is equal to

$$\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a), \qquad \{\tilde{\gamma}^a,\tilde{\gamma}^b\}_+ = 2\eta^{ab},$$
$$\{\gamma^a,\tilde{\gamma}^b\}_+ = 0, \tag{6}$$

one must take on, when $\mathcal{S}^{ab}$ apply to the spin connections ω_{bde} ($= f^{\alpha}{}_{e}\omega_{bd\alpha}$) and $\tilde{\omega}_{\tilde{b}\tilde{d}e}$ ($= f^{\alpha}{}_{e}\tilde{\omega}_{\tilde{b}\tilde{d}\alpha}$), either the space index e or the indices $(b,d,\tilde{b},\tilde{d})$, the operator

$$(\mathcal{S}^{ab})^{c}{}_{e}A^{d\ldots e\ldots g} = i(\eta^{ac}\delta^{b}_{e} - \eta^{bc}\delta^{a}_{e})A^{d\ldots e\ldots g}. \tag{7}$$

This means that the space index (e) of ω_{bde} transforms according to the requirement of Eq. (7), and so do (b,d) and $(\tilde{b},\tilde{d})$. Here I used again the notation $(\tilde{b},\tilde{d})$ to point out that S^{ab} and $\tilde{S}^{ab}$ ($= \tilde{S}^{\tilde{a}\tilde{b}}$) are the generators of two independent groups [13].

One finds [1–8] for the generators of the spin and the charge groups, which are the subgroups of $SO(13,1)$, the expressions

$$\vec{N}_{\pm}(=\vec{N}_{(L,R)}) := \frac{1}{2}(S^{23} \pm iS^{01}, S^{31} \pm iS^{02}, S^{12} \pm iS^{03}), \tag{8}$$

where the generators $\vec{N}_{\pm}$ determine representations of the two $SU(2)$ invariant subgroups of $SO(3,1)$, the generators $\vec{\tau}^1$ and $\vec{\tau}^2$,

$$\vec{\tau}^{1} := \frac{1}{2}(S^{58} - S^{67}, S^{57} + S^{68}, S^{56} - S^{78}),$$
$$\vec{\tau}^{2} := \frac{1}{2}(S^{58} + S^{67}, S^{57} - S^{68}, S^{56} + S^{78}), \tag{9}$$

determine representations of the $SU(2)_I \times SU(2)_{II}$ invariant subgroups of the group $SO(4)$, which is, further, the subgroup of $SO(7,1)$ [$SO(4), SO(3,1)$ are subgroups of $SO(7,1)$] and the generators $\vec{\tau}^3$, τ^4, and $\tilde{\tau}^4$.

$$\vec{\tau}^3 := \frac{1}{2}\Big\{S^{9\,12} - S^{10\,11}, S^{9\,11} + S^{10\,12}, S^{9\,10} - S^{11\,12}, \times S^{9\,14} - S^{10\,13}, S^{9\,13} + S^{10\,14}, S^{11\,14} - S^{12\,13}, \times S^{11\,13} + S^{12\,14}, \frac{1}{\sqrt{3}}(S^{9\,10} + S^{11\,12} - 2S^{13\,14})\Big\},$$
$$\tau^4 := -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14}),$$
$$\tilde{\tau}^4 := -\frac{1}{3}(\tilde{S}^{9\,10} + \tilde{S}^{11\,12} + \tilde{S}^{13\,14}) \tag{10}$$

determine representations of $SU(3) \times U(1)$, originating in $SO(6)$, and of $\tilde{\tau}^4$, originating in $\widetilde{SO}(6)$.

One finds, correspondingly, the generators of the subgroups of $\widetilde{SO}(7,1)$,

$$\vec{\tilde{N}}_{L,R} := \frac{1}{2}(\tilde{S}^{23} \pm i\tilde{S}^{01}, \tilde{S}^{31} \pm i\tilde{S}^{02}, \tilde{S}^{12} \pm i\tilde{S}^{03}), \tag{11}$$

which determine representations of the two $\widetilde{SU}(2)$ invariant subgroups of $\widetilde{SO}(3,1)$, while

$$\vec{\tilde{\tau}}^1 := \frac{1}{2}(\tilde{S}^{58} - \tilde{S}^{67}, \tilde{S}^{57} + \tilde{S}^{68}, \tilde{S}^{56} - \tilde{S}^{78}),$$
$$\vec{\tilde{\tau}}^2 := \frac{1}{2}(\tilde{S}^{58} + \tilde{S}^{67}, \tilde{S}^{57} - \tilde{S}^{68}, \tilde{S}^{56} + \tilde{S}^{78}) \tag{12}$$

determine representations of $\widetilde{SU}(2)_I \times \widetilde{SU}(2)_{II}$ of $\widetilde{SO}(4)$. Both $\widetilde{SO}(3,1)$ and $\widetilde{SO}(4)$ are subgroups of $\widetilde{SO}(7,1)$.

One further finds [3]

$$Y = \tau^4 + \tau^{23}, \quad Y' = -\tau^4 \tan^2\vartheta_2 + \tau^{23}, \quad Q = \tau^{13} + Y,$$
$$Q' = -Y\tan^2\vartheta_1 + \tau^{13}, \quad \tilde{Y} = \tilde{\tau}^4 + \tilde{\tau}^{23}, \quad \tilde{Y}' = -\tilde{\tau}^4\tan^2\tilde{\vartheta}_2 + \tilde{\tau}^{23},$$
$$\tilde{Q} = \tilde{Y} + \tilde{\tau}^{13}, \quad \tilde{Q}' = -\tilde{Y}\tan^2\tilde{\vartheta}_1 + \tilde{\tau}^{13}. \tag{13}$$

The scalar fields from the first term in the second line of Eq. (2) (let me remind you that they are responsible [1–3] after gaining in the electroweak break nonzero vacuum expectation values for the masses of the family members and of the weak bosons) are expressible in terms of the ω_{abc} fields and $\tilde{\omega}_{abc}$ fields presented in Eqs. (14) and (15). One can find the below expressions by taking into account Eqs. (9), (10), (11), (12), and (13).

$$-\frac{1}{2}S^{s's''}\omega_{s's''s} = -(g^{23}\tau^{23}A^{23}_s + g^{13}\tau^{13}A^{13}_s + g^4\tau^4 A^4_s),$$
$$g^{13}\tau^{13}A^{13}_s + g^{23}\tau^{23}A^{23}_s + g^4\tau^4A^4_s = g^Q Q A^Q_s + g^{Q'}Q'A^{Q'}_s + g^{Y'}Y'A^{Y'}_s,$$
$$A^4_s = -(\omega_{9\,10s} + \omega_{11\,12s} + \omega_{13\,14s}),$$
$$A^{13}_s = (\omega_{56s} - \omega_{78s}), \qquad A^{23}_s = (\omega_{56s} + \omega_{78s}),$$
$$A^Q_s = \sin\vartheta_1 A^{13}_s + \cos\vartheta_1 A^Y_s, \qquad A^{Q'}_s = \cos\vartheta_1 A^{13}_s - \sin\vartheta_1 A^Y_s,$$
$$A^Y_s = \sin\vartheta_2 A^{23}_s + \cos\vartheta_2 A^4_s,$$
$$A^{Y'}_s = \cos\vartheta_2 A^{23}_s - \sin\vartheta_2 A^4_s,$$
$$(s \in (7,8)). \tag{14}$$

In Eq. (14) the coupling constants are explicitly written to see the analogy with the gauge fields in the standard model:

$$-\frac{1}{2}\tilde{S}^{\tilde{a}\tilde{b}}\tilde{\omega}_{\tilde{a}\tilde{b}s} = -(\vec{\tilde{\tau}}^{\tilde{1}}\vec{\tilde{A}}^{\tilde{1}}_s + \vec{\tilde{N}}_{\tilde{L}}\vec{\tilde{A}}^{\tilde{N}_{\tilde{L}}}_s + \vec{\tilde{\tau}}^{\tilde{2}}\vec{\tilde{A}}^{\tilde{2}}_s + \vec{\tilde{N}}_{\tilde{R}}\vec{\tilde{A}}^{\tilde{N}_{\tilde{R}}}_s),$$
$$\vec{\tilde{A}}^{\tilde{1}}_s = (\tilde{\omega}_{\tilde{5}\tilde{8}s} - \tilde{\omega}_{\tilde{6}\tilde{7}s}, \tilde{\omega}_{\tilde{5}\tilde{7}s} + \tilde{\omega}_{\tilde{6}\tilde{8}s}, \tilde{\omega}_{\tilde{5}\tilde{6}s} - \tilde{\omega}_{\tilde{7}\tilde{8}s}),$$
$$\vec{\tilde{A}}^{\tilde{N}_{\tilde{L}}}_s = (\tilde{\omega}_{\tilde{2}\tilde{3}s} + i\tilde{\omega}_{\tilde{0}\tilde{1}s}, \tilde{\omega}_{\tilde{3}\tilde{1}s} + i\tilde{\omega}_{\tilde{0}\tilde{2}s}, \tilde{\omega}_{\tilde{1}\tilde{2}s} + i\tilde{\omega}_{\tilde{0}\tilde{3}s}),$$
$$\vec{\tilde{A}}^{\tilde{2}}_s = (\tilde{\omega}_{\tilde{5}\tilde{8}s} + \tilde{\omega}_{\tilde{6}\tilde{7}s}, \tilde{\omega}_{\tilde{5}\tilde{7}s} - \tilde{\omega}_{\tilde{6}\tilde{8}s}, \tilde{\omega}_{\tilde{5}\tilde{6}s} + \tilde{\omega}_{\tilde{7}\tilde{8}s}),$$
$$\vec{\tilde{A}}^{\tilde{N}_{\tilde{R}}}_s = (\tilde{\omega}_{\tilde{2}\tilde{3}s} - i\tilde{\omega}_{\tilde{0}\tilde{1}s}, \tilde{\omega}_{\tilde{3}\tilde{1}s} - i\tilde{\omega}_{\tilde{0}\tilde{2}s}, \tilde{\omega}_{\tilde{1}\tilde{2}s} - i\tilde{\omega}_{\tilde{0}\tilde{3}s}),$$
$$(s \in (7,8)). \tag{15}$$

Scalar fields from Eq. (15) couple to fermions due to the family quantum numbers, while those from Eq. (14) distinguish between family members.

The vector gauge fields $\vec{A}^3_m$ (the color octet), $\vec{A}^1_m$ [the weak $SU(2)_I$ triplet], $\vec{A}^2_m$ [the $SU(2)_{II}$ triplet] and $\vec{A}^4_m$ [the $U(1)_{II}$ singlet originating in $SO(6)$] can be expressed in terms of the spin connection fields and the vielbeins by taking into account Eqs. (9) and (10). Equivalently one finds the vector gauge fields in the tilde sector.

II. PROPERTIES OF SCALAR AND VECTOR GAUGE FIELDS, CONTRIBUTING TO TRANSITIONS OF ANTILEPTONS INTO QUARKS

In this—the main part of the paper—we study the properties, quantum numbers, and discrete symmetries of those scalar and vector gauge fields appearing in the action [Eqs. (1), (2), and (4)] of the spin-charge-family theory [1–9,12] which cause transitions of antileptons into quarks and back, and antiquarks into quarks and back.

These scalar gauge fields carry the triplet or antitriplet color charge (see Table I) and the fractional hyper- and electromagnetic charge.

The Lagrangian densities from Eqs. (1), (2), and (4) manifest $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$ invariance (Appendix A). All of the

TABLE I. Quantum numbers of the scalar gauge fields carrying the space index $t = (9, 10, \ldots, 14)$, appearing in Eq. (20), are presented. To the color charge of all these scalar fields the space degrees of freedom contribute one of the triplet values. These scalars are, with respect to the two $SU(2)$ charges ($\vec{\tau}^1$ and $\vec{\tau}^2$) and the two $\widetilde{SU}(2)$ charges ($\vec{\tilde{\tau}}^1$ and $\vec{\tilde{\tau}}^2$), triplets (that is, in the adjoint representations of the corresponding groups), and they all carry twice the spinor number (τ^4) of the quarks. The quantum numbers of the two vector gauge fields, the color and the $U(1)_{II}$ ones, are added.

Field	Prop.	τ^4	τ^{13}	τ^{23}	(τ^{33}, τ^{38})	Y	Q	$\tilde{\tau}^4$	$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$
$A^{1\boxed{\pm}}_{9\,10\,(\text{\textcircled{$\pm$}})}$	scalar	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\boxed{\pm}1$	0	$(\text{\textcircled{$\pm$}}\frac{1}{2}, \text{\textcircled{$\pm$}}\frac{1}{2\sqrt{3}})$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\text{\textcircled{$\pm$}}\frac{1}{3} + \boxed{\mp}1$	0	0	0	0	0
$A^{13}_{9\,10\,(\text{\textcircled{$\pm$}})}$	scalar	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	$(\text{\textcircled{$\pm$}}\frac{1}{2}, \text{\textcircled{$\pm$}}\frac{1}{2\sqrt{3}})$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	0	0	0
$A^{1\boxed{\pm}}_{11\,12\,(\text{\textcircled{$\pm$}})}$	scalar	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\boxed{\mp}1$	0	$(\text{\textcircled{$\pm$}}\frac{1}{2}, \text{\textcircled{$\pm$}}\frac{1}{2\sqrt{3}})$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\text{\textcircled{$\pm$}}\frac{1}{3} + \boxed{\mp}1$	0	0	0	0	0
$A^{13}_{11\,12\,(\text{\textcircled{$\pm$}})}$	scalar	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	$(\text{\textcircled{$\pm$}}\frac{1}{2}, \text{\textcircled{$\pm$}}\frac{1}{2\sqrt{3}})$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	0	0	0
$A^{1\boxed{\pm}}_{13\,14\,(\text{\textcircled{$\pm$}})}$	scalar	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\boxed{\mp}1$	0	$(0, \text{\textcircled{$\pm$}}\frac{1}{\sqrt{3}})$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\text{\textcircled{$\pm$}}\frac{1}{3} + \boxed{\mp}1$	0	0	0	0	0
$A^{13}_{13\,14\,(\text{\textcircled{$\pm$}})}$	scalar	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	$(0, \text{\textcircled{$\pm$}}\frac{1}{\sqrt{3}})$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	0	0	0
$A^{2\boxed{\pm}}_{9\,10\,(\text{\textcircled{$\pm$}})}$	scalar	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	$\boxed{\pm}1$	$(\text{\textcircled{$\pm$}}\frac{1}{2}, \text{\textcircled{$\pm$}}\frac{1}{2\sqrt{3}})$	$\text{\textcircled{$\pm$}}\frac{1}{3} + \boxed{\mp}1$	$\text{\textcircled{$\pm$}}\frac{1}{3} + \boxed{\mp}1$	0	0	0	0	0
$A^{23}_{9\,10\,(\text{\textcircled{$\pm$}})}$	scalar	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	$(\text{\textcircled{$\pm$}}\frac{1}{2}, \text{\textcircled{$\pm$}}\frac{1}{2\sqrt{3}})$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	0	0	0
…												
$\tilde{A}^{1\boxed{\pm}}_{9\,10\,(\text{\textcircled{$\pm$}})}$	scalar	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	$(\text{\textcircled{$\pm$}}\frac{1}{2}, \text{\textcircled{$\pm$}}\frac{1}{2\sqrt{3}})$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	$\boxed{\pm}1$	0	0	0
$\tilde{A}^{13}_{9\,10\,(\text{\textcircled{$\pm$}})}$	scalar	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	$(\text{\textcircled{$\pm$}}\frac{1}{2}, \text{\textcircled{$\pm$}}\frac{1}{2\sqrt{3}})$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	0	0	0
…												
$\tilde{A}^{2\boxed{\pm}}_{9\,10\,(\text{\textcircled{$\pm$}})}$	scalar	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	$(\text{\textcircled{$\pm$}}\frac{1}{2}, \text{\textcircled{$\pm$}}\frac{1}{2\sqrt{3}})$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	$\boxed{\pm}1$	0	0
$\tilde{A}^{23}_{9\,10\,(\text{\textcircled{$\pm$}})}$	scalar	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	$(\text{\textcircled{$\pm$}}\frac{1}{2}, \text{\textcircled{$\pm$}}\frac{1}{2\sqrt{3}})$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	0	0	0
…												
$\tilde{A}^{N_L\boxed{\pm}}_{9\,10\,(\text{\textcircled{$\pm$}})}$	scalar	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	$(\text{\textcircled{$\pm$}}\frac{1}{2}, \text{\textcircled{$\pm$}}\frac{1}{2\sqrt{3}})$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	0	$\boxed{\pm}1$	0
$\tilde{A}^{N_L 3}_{9\,10\,(\text{\textcircled{$\pm$}})}$	scalar	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	$(\text{\textcircled{$\pm$}}\frac{1}{2}, \text{\textcircled{$\pm$}}\frac{1}{2\sqrt{3}})$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	0	0	0
…												
$\tilde{A}^{N_R\boxed{\pm}}_{9\,10\,(\text{\textcircled{$\pm$}})}$	scalar	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	$(\text{\textcircled{$\pm$}}\frac{1}{2}, \text{\textcircled{$\pm$}}\frac{1}{2\sqrt{3}})$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	0	0	$\boxed{\pm}1$
$\tilde{A}^{N_R 3}_{9\,10\,(\text{\textcircled{$\pm$}})}$	scalar	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	$(\text{\textcircled{$\pm$}}\frac{1}{2}, \text{\textcircled{$\pm$}}\frac{1}{2\sqrt{3}})$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	0	0	0
…												
$A^{3i}_{9\,10\,(\text{\textcircled{$\pm$}})}$	scalar	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	$(\boxed{\pm}1 + \text{\textcircled{$\pm$}}\frac{1}{2}, \text{\textcircled{$\pm$}}\frac{1}{2\sqrt{3}})$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	0	0	0
…												
$A^{4}_{9\,10\,(\text{\textcircled{$\pm$}})}$	scalar	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	$(\text{\textcircled{$\pm$}}\frac{1}{2}, \text{\textcircled{$\pm$}}\frac{1}{2\sqrt{3}})$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	$\text{\textcircled{$\pm$}}\frac{1}{3}$	0	0	0	0	0
…												
$\vec{A}^3_m$	vector	0	0	0	octet	0	0	0	0	0	0	0
A^4_m	vector	0	0	0	0	0	0	0	0	0	0	0

vector and the spinor gauge fields are before the appearance of the condensate massless and the reactions creating particles from antiparticles and back go in both directions equivalently. Correspondingly there is no matter-antimatter asymmetry.

The spin-charge-family theory breaks the matter-antimatter symmetry through the appearance of the condensate (Sec. III) and also by nonzero vacuum expectation values of the scalar fields causing the electroweak phase transition (Sec. IV). I shall show that there is the condensate of two right-handed neutrinos which breaks this symmetry, giving masses to all of the scalar gauge fields and to all of those vector gauge fields which would contradict the observations.

Let us start by analyzing the Lagrangian density presented in Eq. (4) before the appearance of the condensate. The term $\gamma^t \frac{1}{2} S^{s's''} \omega_{s's''t}$ in Eq. (4) can be rewritten, if taking into account Eq. (B8), as follows:

$$\begin{aligned}
\gamma^t \frac{1}{2} S^{s's''} \omega_{s's''t} &= \sum_{+,-} \sum_{(tt')} \overset{tt'}{(\oplus)} \frac{1}{2} S^{s's''} \omega_{s''s''\overset{tt'}{(\oplus)}}, \\
\omega_{s''s''\overset{tt'}{(\oplus)}} &:= \omega_{s''s''\overset{tt'}{(\pm)}} = (\omega_{s's''t} \mp i\omega_{s's''t'}), \\
\overset{tt'}{(\oplus)} &\coloneqq \overset{tt'}{(\pm)} = \frac{1}{2}(\gamma^t \pm \gamma^{t'}), \\
(tt') &\in ((910), (1112), (1314)).
\end{aligned} \tag{16}$$

I introduced the notations $\overset{tt'}{(\oplus)}$ and $\omega_{s''s''\overset{tt'}{(\oplus)}}$ to distinguish between different superpositions of states in the equations below.

Using Eqs. (9), (10), and (14), the expression $\overset{tt'}{(\oplus)} \frac{1}{2} S^{s's''} \omega_{s''s''\overset{tt'}{(\oplus)}}$ can be further rewritten as follows:

$$\begin{aligned}
\overset{tt'}{(\oplus)} \frac{1}{2} S^{s's''} \omega_{s''s''\overset{tt'}{(\oplus)}} &= \overset{tt'}{(\oplus)} \{\tau^{2+} A^{2+}_{\overset{tt'}{(\oplus)}} + \tau^{2-} A^{2-}_{\overset{tt'}{(\oplus)}} + \tau^{23} A^{23}_{\overset{tt'}{(\oplus)}} + \tau^{1+} A^{1+}_{\overset{tt'}{(\oplus)}} + \tau^{1-} A^{1-}_{\overset{tt'}{(\oplus)}} + \tau^{13} A^{13}_{\overset{tt'}{(\oplus)}}\}, \\
A^{2\boxed{\pm}}_{\overset{tt'}{(\oplus)}} &= (\omega_{58\overset{tt'}{(\oplus)}} + \omega_{67\overset{tt'}{(\oplus)}}) \boxed{\mp} i (\omega_{57\overset{tt'}{(\oplus)}} - \omega_{68\overset{tt'}{(\oplus)}}), \qquad A^{23}_{\overset{tt'}{(\oplus)}} = (\omega_{56\overset{tt'}{(\oplus)}} + \omega_{78\overset{tt'}{(\oplus)}}), \\
A^{1\boxed{\pm}}_{\overset{tt'}{(\oplus)}} &= (\omega_{58\overset{tt'}{(\oplus)}} - \omega_{67\overset{tt'}{(\oplus)}}) \boxed{\mp} i (\omega_{57\overset{tt'}{(\oplus)}} + \omega_{68\overset{tt'}{(\oplus)}}), \qquad A^{13}_{\overset{tt'}{(\oplus)}} = (\omega_{56\overset{tt'}{(\oplus)}} - \omega_{78\overset{tt'}{(\oplus)}}).
\end{aligned} \tag{17}$$

Equivalently one expresses the term $\gamma^t \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{abt}$ in Eq. (4), by using Eqs. (11), (12), and (15), as

$$\begin{aligned}
\gamma^t \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{abt} &= \overset{tt'}{(\oplus)} \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{ab\overset{tt'}{(\oplus)}} = \overset{tt'}{(\oplus)} \{\tilde{\tau}^{2+} \tilde{A}^{2+}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{2-} \tilde{A}^{2-}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{23} \tilde{A}^{23}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{1+} \tilde{A}^{1+}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{1-} \tilde{A}^{1-}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{13} \tilde{A}^{13}_{\overset{tt'}{(\oplus)}} \\
&\quad + \tilde{N}^{+}_{R} \tilde{A}^{N_R+}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{-}_{R} \tilde{A}^{N_R-}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{3}_{R} \tilde{A}^{N_R 3}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{+}_{L} \tilde{A}^{N_L+}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{-}_{L} \tilde{A}^{N_L-}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{3}_{L} \tilde{A}^{N_L 3}_{\overset{tt'}{(\oplus)}}\}, \\
\tilde{A}^{N_R\boxed{\pm}}_{\overset{tt'}{(\oplus)}} &= (\tilde{\omega}_{23\overset{tt'}{(\oplus)}} - i\tilde{\omega}_{01\overset{tt'}{(\oplus)}}) \boxed{\mp} i (\tilde{\omega}_{31\overset{tt'}{(\oplus)}} - i\tilde{\omega}_{02\overset{tt'}{(\oplus)}}), \qquad \tilde{A}^{N_R 3}_{\overset{tt'}{(\oplus)}} = (\tilde{\omega}_{12\overset{tt'}{(\oplus)}} - i\tilde{\omega}_{03\overset{tt'}{(\oplus)}}), \\
\tilde{A}^{N_L\boxed{\pm}}_{\overset{tt'}{(\oplus)}} &= (\tilde{\omega}_{23\overset{tt'}{(\oplus)}} + i\tilde{\omega}_{01\overset{tt'}{(\oplus)}}) \boxed{\mp} i (\tilde{\omega}_{31\overset{tt'}{(\oplus)}} + i\tilde{\omega}_{02\overset{tt'}{(\oplus)}}), \qquad \tilde{A}^{N_R 3}_{\overset{tt'}{(\oplus)}} = (\tilde{\omega}_{12\overset{tt'}{(\oplus)}} + i\tilde{\omega}_{03\overset{tt'}{(\oplus)}}).
\end{aligned} \tag{18}$$

The expressions for $\tilde{A}^{2\boxed{\pm}}_{\overset{tt'}{(\oplus)}}$, $\tilde{A}^{23}_{\overset{tt'}{(\oplus)}}$, $\tilde{A}^{1\boxed{\pm}}_{\overset{tt'}{(\oplus)}}$, and $\tilde{A}^{13}_{\overset{tt'}{(\oplus)}}$ can easily be obtained from Eq. (17) by replacing, in the expressions for $A^{2\boxed{\pm}}_{\overset{tt'}{(\oplus)}}$, $A^{23}_{\overset{tt'}{(\oplus)}}$, $A^{1\boxed{\pm}}_{\overset{tt'}{(\oplus)}}$ and $A^{13}_{\overset{tt'}{(\oplus)}}$, respectively, $\omega_{s's''\overset{tt'}{(\oplus)}}$ by $\tilde{\omega}_{s's''\overset{tt'}{(\oplus)}}$.

There is the additional term in Eq. (4): $\gamma^t \frac{1}{2} S^{t't''} \omega_{t't''t}$. This term can be rewritten with respect to the generators $S^{t't''}$ as one color octet scalar field and one $U(1)_{II}$ singlet scalar field [Eq. (10)]

$$\begin{aligned}
\gamma^t \frac{1}{2} S^{t''t'''} \omega_{t''t'''t} &= \sum_{+,-} \sum_{(tt')} \overset{tt'}{(\oplus)} \{\vec{\tau}^3 \cdot \vec{A}^3_{\overset{tt'}{(\oplus)}} + \tau^4 \cdot A^4_{\overset{tt'}{(\oplus)}}\}, \\
(tt') &\in ((9\ 10), 11\ 12), 13\ 14)).
\end{aligned} \tag{19}$$

Taking all of the above equations [(16), (17), (18), and (19)] into account, Eq. (4) can be rewritten, if we leave out $p_{\overset{tt'}{(\oplus)}}$, since in the low energy limit the momentum does not play any role, as follows:

$$
\begin{aligned}
\mathcal{L}_{f''} = \psi^{\dagger}\gamma^{0}(-)\Big\{\sum_{+,-}\sum_{(tt')}(\stackrel{tt'}{\oplus})\cdot[&\tau^{2+}A^{2+}_{\underset{(\oplus)}{tt'}} + \tau^{2-}A^{2-}_{\underset{(\oplus)}{tt'}} + \tau^{23}A^{23}_{\underset{(\oplus)}{tt'}} + \tau^{1+}A^{1+}_{\underset{(\oplus)}{tt'}} + \tau^{1-}A^{1-}_{\underset{(\oplus)}{tt'}} + \tau^{13}A^{13}_{\underset{(\oplus)}{tt'}}\\
&+ \tilde{\tau}^{2+}\tilde{A}^{2+}_{\underset{(\oplus)}{tt'}} + \tilde{\tau}^{2-}\tilde{A}^{2-}_{\underset{(\oplus)}{tt'}} + \tilde{\tau}^{23}\tilde{A}^{23}_{\underset{(\oplus)}{tt'}} + \tilde{\tau}^{1+}\tilde{A}^{1+}_{\underset{(\oplus)}{tt'}} + \tilde{\tau}^{1-}\tilde{A}^{1-}_{\underset{(\oplus)}{tt'}} + \tilde{\tau}^{13}\tilde{A}^{13}_{\underset{(\oplus)}{tt'}} + \tilde{N}^{+}_{R}\tilde{A}^{N_R+}_{\underset{(\oplus)}{tt'}} + \tilde{N}^{-}_{R}\tilde{A}^{N_R-}_{\underset{(\oplus)}{tt'}} + \tilde{N}^{3}_{R}\tilde{A}^{N_R3}_{\underset{(\oplus)}{tt'}}\\
&+ \tilde{N}^{+}_{L}\tilde{A}^{N_L+}_{\underset{(\oplus)}{tt'}} + \tilde{N}^{-}_{L}\tilde{A}^{N_L-}_{\underset{(\oplus)}{tt'}} + \tilde{N}^{3}_{L}\tilde{A}^{N_L3}_{\underset{(\oplus)}{tt'}} + \tau^{3i}A^{3i}_{\underset{(\oplus)}{tt'}} + \tau^{4}A^{4}_{\underset{(\oplus)}{tt'}}\ ]\Big\}\psi, \qquad (20)
\end{aligned}
$$

where (t,t') run in pairs over $[(9,10),\ldots(13,14)]$ and the summation must go over the $+$ and the $-$ of $\underset{(\oplus)}{tt'}$.

Let us now calculate the quantum numbers of the scalar and vector gauge fields appearing in Eq. (20) by taking into account the fact that the spin of the gauge fields is determined according to Eq. (7) $[(S^{ab})^{c}{}_{e}A^{d\ldots e\ldots g} = i(\eta^{ac}\delta^{b}_{d} - \eta^{bc}\delta^{a}_{d})A^{d\ldots e\ldots g}$, for each index $(\in (d\ldots g))$ of a bosonic field $A^{d\ldots g}$ separately]. We must also take into account the relation among S^{ab} and the charges (the relations are, of course, the same for bosons and fermions) [Eqs. (8), (9), (10), (11), and (12)].

In Table I properties of the scalar gauge fields appearing in Eq. (20) are presented.

The scalar fields with the scalar index $s = (9, 10, \ldots, 14)$, presented in Table I, carry one of the triplet color charges and the spinor charge equal to twice the quark spinor charge, or the antitriplet color charges and the antispinor charge. They carry in addition the quantum numbers of the adjoint representations originating in S^{ab} or in $\tilde{S}^{ab}$. Although carrying the color charge in one of the triplet or antitriplet states, these fields cannot be interpreted as superpartners of the quarks, as required by, let us say, the $N = 1$ supersymmetry. The hypercharges and the electromagnetic charges are, namely, not those required by the supersymmetric partners to the family members.

Let us have a look at what the scalar fields appearing in Eq. (20) and Table I do when applying the left-handed members of the Weyl representation presented in Table III, containing quarks and leptons and antiquarks and antileptons [4,15,35]. Let us choose the 57th line of Table III, which represents in the spinor technique the left-handed positron, $\bar{e}^{+}_{L}$. If we make, let us say, the choice of the term $(\gamma^{0}\stackrel{910}{(+)}\tau^{2\boxminus})\ A^{2\boxminus}_{\underset{(\oplus)}{910}}$ (the scalar field $A^{2\boxminus}_{\underset{(\oplus)}{910}}$ is presented in the seventh line in Table I and in the first line of Eq. (20)], the family quantum numbers will not be affected and thus can be any number. The state carries the spinor (lepton) number $\tau^{4} = \frac{1}{2}$, the weak charge $\tau^{13} = 0$, the second $SU(2)_{II}$ charge $\tau^{23} = \frac{1}{2}$, and the color charge $(\tau^{33}, \tau^{38}) = (0, 0)$. Correspondingly, its hypercharge $[Y(= \tau^{4} + \tau^{23})]$ is 1 and the electromagnetic charge $Q(= Y + \tau^{13})$ is 1.

So, what does the term $\gamma^{0}\stackrel{9\,10}{(+)}\tau^{2\boxminus}\ A^{2\boxminus}_{\underset{(\oplus)}{9\,10}}$ make of this spinor $\bar{e}^{+}_{L}$? Making use of Eqs. (B10), (B12), and (B20) of Appendix B one easily finds that the operator $\gamma^{0}\stackrel{9\,10}{(+)}$ τ^{2-} transforms the left-handed positron into $\stackrel{03}{(+i)}\,\stackrel{12}{(+)}\,|\stackrel{56}{[-]}\,\stackrel{78}{[-]}\,||\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{(-)}\,\stackrel{13\,14}{(-)}$, which is d^{c1}_{R}, presented on line 3 of Table III. Namely, γ^{0} transforms $\stackrel{03}{[-i]}$ into $\stackrel{03}{(+i)}$, $\stackrel{9\,10}{(+)}$ transforms $\stackrel{9\,10}{[-]}$ into $\stackrel{9\,10}{(+)}$, and τ^{2-} $[= -\stackrel{56}{(-)}\,\stackrel{78}{(-)}]$ transforms $\stackrel{56}{(+)}\,\stackrel{78}{(+)}$ into $\stackrel{56}{[-]}\,\stackrel{78}{[-]}$. The state d^{c1}_{R} carries the spinor (quark) number $\tau^{4} = \frac{1}{6}$, the weak charge $\tau^{13} = 0$, the second $SU(2)_{II}$ charge $\tau^{23} = -\frac{1}{2}$, and the color charge $(\tau^{33}, \tau^{38}) = (\frac{1}{2}, \frac{1}{2\sqrt{3}})$. Correspondingly its hypercharge is $(Y = \tau^{4} + \tau^{23} =)\ -\frac{1}{3}$ and the electromagnetic charge $(Q = Y + \tau^{13} =)\ -\frac{1}{3}$. The scalar field $A^{2\boxminus}_{\underset{(\oplus)}{9\,10}}$ carries just the needed quantum numbers, as we can see in the seventh line of Table I.

If the antiquark $\bar{u}^{\bar{c}2}_{L}$, from line 43 (it is not presented, but one can very easily construct it) in Table III, with the spinor

TABLE II. The condensate of the two right-handed neutrinos ν_{R}, with the VIIIth family quantum number, coupled to spin zero and belonging to a triplet with respect to the generators τ^{2i}, is presented, together with its two partners. The condensate carries $\tau^{13} = 0$, $\tau^{23} = 1$, $\tau^{4} = -1$, and $Q = 0 = Y$. The triplet carries $\tilde{\tau}^{4} = -1$, $\tilde{\tau}^{23} = 1$, and $\tilde{N}^{3}_{R} = 1$, $\tilde{N}^{3}_{L} = 0$, $\tilde{Y} = 0$, $\tilde{Q} = 0$. The family quantum numbers are presented in Table IV.

State	S^{03}	S^{12}	τ^{13}	τ^{23}	τ^{4}	Y	Q	$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{\tau}^{4}$	$\tilde{Y}$	$\tilde{Q}$	$\tilde{N}^{3}_{L}$	$\tilde{N}^{3}_{R}$
$(\lvert\nu^{\mathbf{VIII}}_{\mathbf{1R}}\rangle_{1}\lvert\nu^{\mathbf{VIII}}_{\mathbf{2R}}\rangle_{2})$	0	0	0	1	−1	0	0	0	1	−1	0	0	0	1
$(\lvert\nu^{VIII}_{1R}\rangle_{1}\lvert e^{VIII}_{2R}\rangle_{2})$	0	0	0	0	−1	−1	−1	0	1	−1	0	0	0	1
$(\lvert e^{VIII}_{1R}\rangle_{1}\lvert e^{VIII}_{2R}\rangle_{2})$	0	0	0	−1	−1	−2	−2	0	1	−1	0	0	0	1

TABLE III. The left-handed $[\Gamma^{(13,1)} = -1]$ multiplet of spinors—the members of the $SO(13,1)$ group, manifesting the subgroup $SO(7,1)$—of the color charged quarks and antiquarks and the colorless leptons and antileptons is presented on a massless basis using the technique presented in Appendix B. It contains the left-handed $[\Gamma^{(3,1)} = -1]$ weak charged ($\tau^{13} = \pm\frac{1}{2}$) and $SU(2)_{II}$ chargeless ($\tau^{23} = 0$) quarks and the right-handed weak chargeless and $SU(2)_{II}$ charged ($\tau^{23} = \pm\frac{1}{2}$) quarks of three colors [$c^i = (\tau^{33}, \tau^{38})$] with the spinor charge ($\tau^4 = \frac{1}{6}$) and the colorless left-handed weak charged leptons and the right-handed weak chargeless leptons with the spinor charge ($\tau^4 = -\frac{1}{2}$). S^{12} defines the ordinary spin $\pm\frac{1}{2}$. The vacuum state $|\mathrm{vac}\rangle_{\mathrm{fam}}$, on which the nilpotents and projectors operate, is not shown. The reader can find this Weyl representation also in Refs. [3,35]. Left-handed antiquarks and antileptons are weak chargeless and carry opposite charges.

i		$\vert{}^a\psi_i\rangle$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	τ^{33}	τ^{38}	τ^4	Y	Q
		Octet, $\Gamma^{(1,7)} = 1, \Gamma^{(6)} = -1$ of quarks and leptons										
1	u_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
2	u_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
3	d_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
4	d_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
5	d_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
6	d_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
7	u_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
8	u_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
9	u_R^{c2}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
10	u_R^{c2}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
…												
17	u_R^{c3}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
18	u_R^{c3}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
…												
25	ν_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
26	ν_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
27	e_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	-1	-1
28	e_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	-1	-1
29	e_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
30	e_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
31	ν_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
32	ν_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
33	$\bar{d}_L^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
34	$\bar{d}_L^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
35	$\bar{u}_L^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\vert\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
36	$\bar{u}_L^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\vert\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
37	$\bar{d}_R^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\vert\vert\vert\stackrel{910}{[-]}\stackrel{1112}{[+]}\stackrel{1314}{[+]}$	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
38	$\bar{d}_R^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\vert\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
39	$\bar{u}_R^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\vert\vert\vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$

(Table continued)

TABLE III. (*Continued*)

i		$\vert^a\psi_i\rangle$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	τ^{33}	τ^{38}	τ^4	Y	Q
40	$\bar{u}_R^{\bar{c}1}$	$\overset{03}{[-i]}\overset{12}{[-]}\vert\overset{56}{[-]}\overset{78}{(+)}\vert\vert\overset{9\,10}{[-]}\overset{11\,12}{[+]}\overset{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
41	$\bar{d}_L^{\bar{c}2}$	$\overset{03}{[-i]}\overset{12}{(+)}\vert\overset{56}{(+)}\overset{78}{(+)}\vert\vert\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
…												
49	$\bar{d}_L^{\bar{c}3}$	$\overset{03}{[-i]}\overset{12}{(+)}\vert\overset{56}{(+)}\overset{78}{(+)}\vert\vert\overset{9\,10}{(+)}\overset{11\,12}{[+]}\overset{13\,14}{(-)}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
…												
57	$\bar{e}_L$	$\overset{03}{[-i]}\overset{12}{(+)}\vert\overset{56}{(+)}\overset{78}{(+)}\vert\vert\overset{9\,10}{[-]}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	1
58	$\bar{e}_L$	$\overset{03}{(+i)}\overset{12}{[-]}\vert\overset{56}{(+)}\overset{78}{(+)}\vert\vert\overset{9\,10}{[-]}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	1
59	$\bar{\nu}_L$	$\overset{03}{[-i]}\overset{12}{(+)}\vert\overset{56}{[-]}\overset{78}{[-]}\vert\vert\overset{9\,10}{[-]}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	-1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
60	$\bar{\nu}_L$	$\overset{03}{(+i)}\overset{12}{[-]}\vert\overset{56}{[-]}\overset{78}{[-]}\vert\vert\overset{9\,10}{[-]}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
61	$\bar{\nu}_R$	$\overset{03}{(+i)}\overset{12}{(+)}\vert\overset{56}{[-]}\overset{78}{(+)}\vert\vert\overset{9\,10}{[-]}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
62	$\bar{\nu}_R$	$\overset{03}{[-i]}\overset{12}{[-]}\vert\overset{56}{[-]}\overset{78}{(+)}\vert\vert\overset{9\,10}{[-]}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
63	$\bar{e}_R$	$\overset{03}{(+i)}\overset{12}{(+)}\vert\overset{56}{(+)}\overset{78}{[-]}\vert\vert\overset{9\,10}{[-]}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1
64	$\bar{e}_R$	$\overset{03}{[-i]}\overset{12}{[-]}\vert\overset{56}{(+)}\overset{78}{[-]}\vert\vert\overset{9\,10}{[-]}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1

charge $\tau^4 = -\frac{1}{6}$, the weak charge $\tau^{13} = 0$, the second $SU(2)_{II}$ charge $\tau^{23} = -\frac{1}{2}$, the color charge $(\tau^{33}, \tau^{38}) = (\frac{1}{2}, -\frac{1}{2\sqrt{3}})$, the hypercharge $Y(= \tau^4 + \tau^{23} =)$ $-\frac{2}{3}$ and the electromagnetic charge $Q(= Y + \tau^{13} =)$ $-\frac{2}{3}$ submits the $A^{2\boxminus}_{\overset{9\,10}{(\oplus)}}$ scalar field, it transforms into u_R^{c3} from line 17 of Table III, carrying the quantum numbers $\tau^4 = \frac{1}{6}$, $\tau^{13} = 0$, $\tau^{23} = \frac{1}{2}$, $(\tau^{33}, \tau^{38}) = (0, -\frac{1}{\sqrt{3}})$, $Y = \frac{2}{3}$, and $Q = \frac{2}{3}$. These two quarks, d_R^{c1} and u_R^{c3}, can bind together with u_R^{c2} from the ninth line of the same table (at low enough energy, after the electroweak transition, and if they belong in a superposition with the left-handed partners to the first family) into the color chargeless baryon—a proton. This transition is presented in Fig. 1.

The opposite transition at low energies would make the proton decay.

Let us look at one more example. The 63rd line of Table III represents, in the spinor technique, the right-handed positron, $\bar{e}_R^+$. Since we shall not again look at a transition in which scalar fields with the nonzero family quantum numbers are involved, the family quantum number of this positron is not important. The state carries the spinor (lepton) number $\tau^4 = \frac{1}{2}$, the weak charge $\tau^{13} = \frac{1}{2}$, the second $SU(2)_{II}$ charge $\tau^{23} = 0$, and the color charge $(\tau^{33}, \tau^{38}) = (0, 0)$. Correspondingly, its hypercharge $(Y = \tau^4 + \tau^{23})$ is $\frac{1}{2}$ and the electromagnetic charge $Q = Y + \tau^{13}$ is 1.

What does, let us say, the term $\gamma^0 \overset{9\,10}{(+)} \tau^{1\boxminus} A^{1\boxminus}_{\overset{9\,10}{(\oplus)}}$ (the scalar field $A^{1\boxminus}_{\overset{9\,10}{(\oplus)}}$ is presented in the first line of Table I) make on $\bar{e}_R^+$? Making use of Eqs. (B10), (B12), and (B20) of Appendix B, one easily finds that the right-handed positron transforms under the application of $\gamma^0\ \tau^{1-}\ \overset{9\,10}{(+)}$ into $\overset{03}{[-i]}\overset{12}{(+)}|\overset{56}{[-]}\overset{78}{(+)}||\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$, which is d_L^{c1}, presented in line 5 of Table III. Namely, γ^0 transforms $\overset{03}{(+i)}$ into $\overset{03}{[-i]}$, $\overset{9\,10}{(+)}$ transforms $\overset{9\,10}{[-]}$ into $\overset{9\,10}{(+)}$, and $\tau^{1\boxminus}$ $[= \overset{56}{(-)}\ \overset{78}{(+)}]$ transforms $\overset{56}{(+)}\ \overset{56}{[-]}$ into $\overset{56}{[-]}\ \overset{56}{(+)}$. The state d_L^{c1} carries the spinor (quark) number $\tau^4 = \frac{1}{6}$, the weak charge $\tau^{13} = -\frac{1}{2}$, the second $SU(2)_{II}$ charge $\tau^{23} = 0$, and the color charge $(\tau^{33}, \tau^{38}) = (\frac{1}{2}, \frac{1}{2\sqrt{3}})$. Correspondingly its hypercharge is $(Y = \tau^4 + \tau^{23} =)$ $\frac{1}{6}$ and the electromagnetic charge $(Q = Y + \tau^{13} =)$ $-\frac{1}{3}$. The scalar field $A^{1\boxminus}_{\overset{9\,10}{(\oplus)}}$ carries all of the needed quantum numbers, as one can see in Fig. 1.

If the antiquark $\bar{u}_R^{\bar{c}2}$, from line 47 in Table III (the reader can easily find the expression $\overset{03}{(+i)}\overset{12}{(+)}|\overset{56}{[-]}\overset{78}{(+)}||\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{[+]}$), with the spinor charge $\tau^4 = -\frac{1}{6}$, the weak charge $\tau^{13} = -\frac{1}{2}$, the second $SU(2)_{II}$ charge $\tau^{23} = 0$, the color charge $(\tau^{33}, \tau^{38}) = (\frac{1}{2}, -\frac{1}{2\sqrt{3}})$, the hypercharge $(Y = \tau^4 + \tau^{23} =)$ $-\frac{1}{6}$, and the electromagnetic charge $(Q = Y + \tau^{13} =)$ $-\frac{2}{3}$, submits the $A^{1\boxminus}_{\overset{9\,10}{(\oplus)}}$ scalar field, it transforms into u_L^{c3} from line 23 of Table III ($\overset{03}{[-i]}\overset{12}{(+)}|\overset{56}{(+)}\overset{78}{[-]}||\overset{9\,10}{[-]}\overset{11\,12}{(-)}\overset{13\,14}{[+]}$), carrying the quantum numbers $\tau^4 = \frac{1}{6}$, $\tau^{13} = \frac{1}{2}$, $\tau^{23} = 0$, $(\tau^{33}, \tau^{38}) = (0, -\frac{1}{\sqrt{3}})$,

$Y = \frac{1}{6}$, and $Q = \frac{2}{3}$. These two quarks, d_L^{c1} and u_L^{c3}, can bind together (at low enough energy, when making the electroweak transition after the superposition with the right-handed partners) with u_L^{c2} from the 15th line of the same table into the color chargeless baryon—a proton. This transition is presented in Fig. 2.

The opposite transition would make the proton decay.

Similar transitions also go with other scalars from Eq. (20) and Table I. The $\underset{(+)}{\vec{\tilde{A}}{}^{1}_{t't''}}$, $\underset{(+)}{\vec{\tilde{A}}{}^{2}_{t't''}}$, $\underset{(+)}{\vec{\tilde{A}}{}^{N_L}_{t't''}}$, and $\underset{(+)}{\vec{\tilde{A}}{}^{N_L}_{t't''}}$ fields cause transitions among the family members, changing a particular member into an antimember of another color and another family. The term $\gamma^0 \overset{910}{(+)} \tilde{N}_R^- \, A^{\tilde{N}_R -}_{\underset{(\oplus)}{910}}$ transforms $\bar{e}_R^+$ into u_L^{c1}, changing the family quantum numbers.

The action from Eqs. (1), (2), and (4) manifests $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$ invariance. All of the vector and spinor gauge fields are massless.

Since none of the scalar fields from Table I have been observed, nor any vector gauge fields like $\vec{A}^2_m$, A^4_m or other scalar or vector fields, we shall discuss this topic in Sec. V, a mechanism must exist which makes the nonobserved scalar and vector gauge fields massive enough.

Scalar fields from Table I carry the color and the electromagnetic charge. Therefore their nonzero vacuum expectation values would not be in agreement with the observed phenomena. One notices, however, that all of the scalar gauge fields from Table I and several other scalar and vector gauge fields (see Sec. V) couple to the condensate with the nonzero quantum numbers τ^4 and τ^{23} and nonzero family quantum numbers.

It is not difficult to recognize that the desired condensate must have spin zero, $Y = \tau^4 + \tau^{23} = 0$, $Q = Y + \tau^{13} = 0$, and $\vec{\tau}^1 = 0$ in order that, in the low energy limit, the spin-charge-family theory would manifest effectively as the standard model.

I make a choice of the two right-handed neutrinos of the VIIIth family coupled into a scalar, with $\tau^4 = -1$, $\tau^{23} = 1$, and, correspondingly, $Y = 0$, $Q = 0$, and $\vec{\tau}^1 = 0$, and with family quantum numbers [Eqs. (12) and (11)] $\tilde{\tau}^4 = -1$, $\tilde{\tau}^{23} = 1$, $\tilde{N}_R^3 = 1$, and, correspondingly, with $\tilde{Y} = \tilde{\tau}^4 + \tilde{\tau}^{23} = 0$, $\tilde{Q} = \tilde{Y} + \tilde{\tau}^{13} = 0$, and $\vec{\tilde{\tau}}^1 = 0$. The condensate carries the family quantum numbers of the upper four families.

The condensate made out of spinors couples to spinors differently than to antispinors (the anticondensate would, namely, carry $\tau^4 = 1$ and $\tau^{23} = -1$) breaking, correspondingly, the $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$ symmetry: The reactions creating particles from antiparticles are no longer symmetric to those creating antiparticles from particles.

Such a condensate leaves the hyperfield A^Y_m $(= \sin\vartheta_2 A^{23}_m + \cos\vartheta_2 A^4_m)$ [for the choice that $\sin\vartheta_2 = \cos\vartheta_2$ and $g^4 = g^2$, there is no justification for such a choice, $A^Y_m = \frac{1}{\sqrt{2}}(A^{23}_m + A^4_m)$ follows] massless, while it

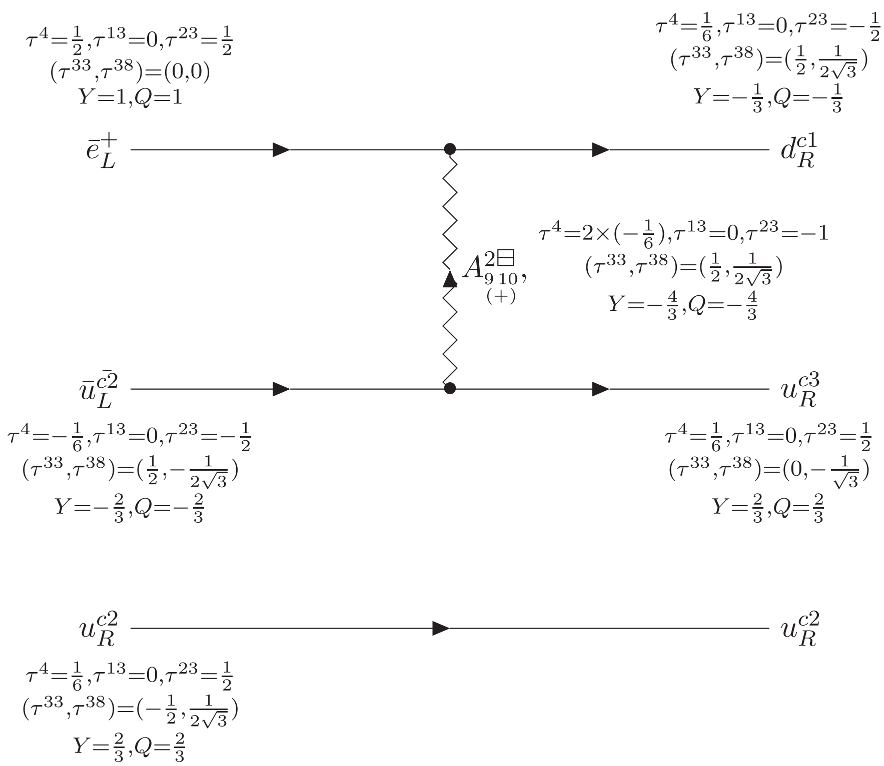

FIG. 1. The birth of a right-handed proton out of a positron $\bar{e}_L^+$, an antiquark $\bar{u}_L^{\bar{c}2}$, and a quark (spectator) u_R^{c2}. The family quantum number can be any number.

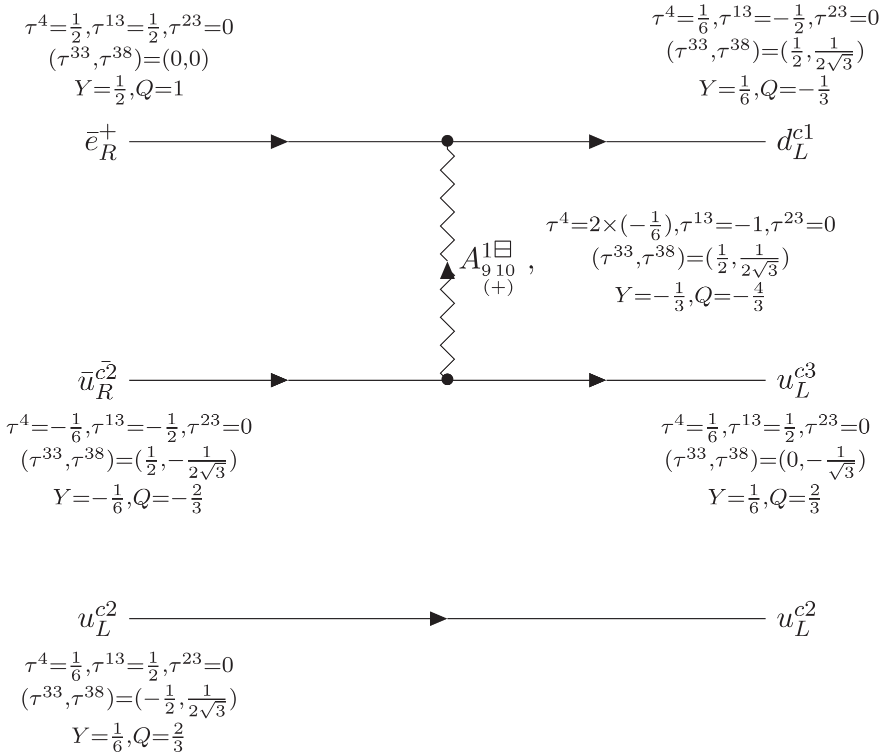

FIG. 2. The birth of a left-handed proton out of a positron $\bar{e}^+_R$, an antiquark $\bar{u}^{\bar{c}2}_R$, and a quark (spectator) u^{c2}_L. The family quantum number can be any number.

gives masses to $A^{2\pm}_m$ and $A^{Y'}_m$ [$= \frac{1}{\sqrt{2}}(A^4_m - A^{23}_m)$ for $\sin\vartheta_2 = \cos\vartheta_2$] and it also gives masses to all of the scalar gauge fields from Table I since they all couple to the condensate through τ^4.

The weak vector gauge fields, $\vec{A}^1_m$, the hypercharge vector gauge fields, A^Y_m, and the color vector gauge fields, $\vec{A}^1_m$, stay massless.

The scalar fields with the scalar space index $s=(7,8)$ (there are three singlets which couple to all eight families, two triplets which couple only to the upper four families, and another two triplets which couple only to the lower four families)—carrying the weak and the hypercharges of the Higgs scalar—wait for gaining nonzero vacuum expectation values to change their masses while causing the electroweak break.

The condensate does what is needed so that in the low energy regime the spin-charge-family theory manifests as an effective theory which agrees with the standard model to such an extent that it is in agreement with the observed phenomena, explaining the standard model assumptions and predicting new fermion and boson fields.

It also may hopefully explain the observed matter-antimatter asymmetry if the conditions in the expanding Universe would be appropriate (Sec. VI). The work needed to check these conditions in the expanding Universe within the spin-charge-family theory is very demanding. Although we do have some experience with following the history of the expanding Universe [12], this study needs much more effort, not only in calculations, but also in understanding the mechanism of the condensate appearance and relations among the velocity of the expansion, the temperature, and the dimension of space-time in the period of the appearance of the condensate. This study has not yet really begun.

III. PROPERTIES OF THE CONDENSATE

In Table II the properties of the condensate of the two right-handed neutrinos ($|\nu^{VIII}_R\rangle_1|\nu^{VIII}_R\rangle_2$), one with spin up and another with spin down (Table III, lines 25 and 26), carrying the family quantum numbers of the VIIIth family (Table IV), are presented. The condensate carries the quantum numbers of $SU(2)_{II}$, $\tau^{23}=1$ [Eq. (9)], of $U(1)_{II}$ originating in $SO(6)$, $\tau^4=-1$ [Eq. (10)], and, correspondingly, $Y=0$, $Q=0$, and the family quantum numbers (Table IV) $\tilde{\tau}^4=-1$ [Eq. (10)], $\tilde{\tau}^{23}=1$ [Eq. (12)], and $\tilde{N}^3_R=1$ [Eq. (11)]. Each of the two neutrinos could belong to a different family of the upper four families. In this case the family quantum numbers of the condensate change.

The condensate is presented in the first line of Table II as a member of a triplet of the group $SU(2)_{II}$ with the generators τ^{2i}. Correspondingly the condensate couples to all of the vector gauge fields which carry nonzero τ^{2i}, τ^4, $\tilde{\tau}^{2i}$, $\tilde{N}^i_R$, and $\tilde{\tau}^4$. The fields $A^Y_m, \vec{A}^3_m$, and $\vec{A}^1_m$ stay massless.

TABLE IV. Eight families of the right-handed u_R^{c1} (III) quark with spin $\frac{1}{2}$, the color charge [$\tau^{33} = 1/2$, $\tau^{38} = 1/(2\sqrt{3})$], and the colorless right-handed neutrino ν_R of spin $\frac{1}{2}$ (III) are presented in the left and right columns, respectively. They belong to two groups of four families: One (I) is a doublet with respect to [$\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^{(1)}$] and a singlet with respect to [$\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^{(2)}$], the other (II) is a singlet with respect to [$\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^{(1)}$] and a doublet with respect to [$\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^{(2)}$]. All of the families follow from the starting one by the application of the operators [$\tilde{N}^{\pm}_{R,L}$, $\tilde{\tau}^{(2,1)\pm}$], Eq. (B20). The generators ($N^{\pm}_{R,L}$, $\tau^{(2,1)\pm}$) [Eq. (B20)] transform u_{1R} to all of the members of one family of the same color. The same generators equivalently transform the right-handed neutrino ν_{1R} to all of the colorless members of the same family.

					$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{N}_L^3$	$\tilde{N}_R^3$	$\tilde{\tau}^4$
I	u_{R1}^{c1}	$\overset{03}{(+i)}\overset{12}{[+]}\vert\overset{56}{[+]}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\overset{11\,12}{[-]}\overset{13\,14}{[-]}$	ν_{R2}	$\overset{03}{(+i)}\overset{12}{[+]}\vert\overset{56}{[+]}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
I	u_{R2}^{c1}	$\overset{03}{[+i]}\overset{12}{(+)}\vert\overset{56}{[+]}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\overset{11\,12}{[-]}\overset{13\,14}{[-]}$	ν_{R2}	$\overset{03}{[+i]}\overset{12}{(+)}\vert\overset{56}{[+]}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
I	u_{R3}^{c1}	$\overset{03}{(+i)}\overset{12}{[+]}\vert\overset{56}{(+)}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\overset{11\,12}{[-]}\overset{13\,14}{[-]}$	ν_{R3}	$\overset{03}{(+i)}\overset{12}{[+]}\vert\overset{56}{(+)}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
I	u_{R4}^{c1}	$\overset{03}{[+i]}\overset{12}{(+)}\vert\overset{56}{(+)}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\overset{11\,12}{[-]}\overset{13\,14}{[-]}$	ν_{R4}	$\overset{03}{[+i]}\overset{12}{(+)}\vert\overset{56}{(+)}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)}$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
II	u_{R5}^{c1}	$\overset{03}{[+i]}\overset{12}{[+]}\vert\overset{56}{[+]}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\overset{11\,12}{[-]}\overset{13\,14}{[-]}$	ν_{R5}	$\overset{03}{[+i]}\overset{12}{[+]}\vert\overset{56}{[+]}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
II	u_{R6}^{c1}	$\overset{03}{(+i)}\overset{12}{(+)}\vert\overset{56}{[+]}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\overset{11\,12}{[-]}\overset{13\,14}{[-]}$	ν_{R6}	$\overset{03}{(+i)}\overset{12}{(+)}\vert\overset{56}{[+]}\overset{78}{[+]}\Vert\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$
II	u_{R7}^{c1}	$\overset{03}{[+i]}\overset{12}{[+]}\vert\overset{56}{(+)}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\overset{11\,12}{[-]}\overset{13\,14}{[-]}$	ν_{R7}	$\overset{03}{[+i]}\overset{12}{[+]}\vert\overset{56}{(+)}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
II	u_{R8}^{c1}	$\overset{03}{(+i)}\overset{12}{(+)}\vert\overset{56}{(+)}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\overset{11\,12}{[-]}\overset{13\,14}{[-]}$	ν_{R8}	$\overset{03}{(+i)}\overset{12}{(+)}\vert\overset{56}{(+)}\overset{78}{(+)}\Vert\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$

The condensate also couples to all of the scalar gauge fields with the scalar indices $s \in (5, 6, 7, 8, 9, \ldots, 14)$ since they all carry either nonzero τ^4 or nonzero τ^{23}.

The coupling of the scalar gauge fields to the condensate is proportional to

$$\begin{aligned}
&(\langle\nu_{1R}^{VIII}|_1\langle\nu_{2R}^{VIII}|)(\gamma^0(\overset{tt'}{\bigoplus})\tau^{Ai}A^{Ai}_{\underset{(\oplus)}{tt'}})^{\dagger}\\
&\times(\gamma^0(\overset{tt'}{\bigoplus})\tau^{Ai}A^{Ai}_{\underset{(\oplus)}{tt'}})(|\nu_{1R}^{VIII}\rangle_1|\nu_{2R}^{VIII}\rangle)\\
&\propto (A^{Ai}_{\underset{(\oplus)}{tt'}})^{\dagger}(A^{Ai}_{\underset{(\oplus)}{tt'}}),\\
&(tt') \in [(56), (78), (9\,10), \ldots, (13\,14)]. \qquad (21)
\end{aligned}$$

The condensate does break the $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$ symmetry. (The anticondensate would, namely, carry $\tau^{23} = -1$ and $\tau^4 = 1$).

The condensate gives masses to all of the scalars from Table I because they couple to the condensate either due to τ^4 or due to the τ^4 and τ^{23} quantum numbers. It also gives masses to all of the scalar fields with $s \in (5, 6, 7, 8)$ since they couple to the condensate due to the nonzero τ^{23}. The scalar fields with the quantum numbers of the upper four families couple in addition through their family quantum numbers.

The condensate also couples to all of the vector gauge fields except to the gauge color octet field $\vec{A}^3_m$, the hypercharge vector fields A^Y_m, and the weak charge vector triplet fields $\vec{A}^1_m$ since they carry zero τ^{23}, τ^4, and Y quantum numbers.

The spin connection fields, of either tilde ($\tilde{S}^{ab}$) or nontilde (S^{ab}) origin, which do not couple to the spinor condensate, are auxiliary fields, expressible with vielbein fields (Appendix C).

Below, the scalar and vector gauge fields, which get their masses through the interaction with the condensate, are presented:

$$\begin{aligned}
&A^{2\boxed{\pm}}_{\underset{(\oplus)}{tt'}},\quad A^{23}_{\underset{(\oplus)}{tt'}},\quad A^{1\boxed{\pm}}_{\underset{(\oplus)}{tt'}},\quad A^{13}_{\underset{(\oplus)}{tt'}},\quad \vec{A}^{3}_{\underset{(\oplus)}{tt'}},\\
&\tilde{A}^{2\boxed{\pm}}_{\underset{(\oplus)}{tt'}},\quad \tilde{A}^{23}_{\underset{(\oplus)}{tt'}},\quad \tilde{A}^{1\boxed{\pm}}_{\underset{(\oplus)}{tt'}},\quad \tilde{A}^{13}_{\underset{(\oplus)}{tt'}},\\
&\tilde{A}^{N_L\boxed{\pm}}_{\underset{(\oplus)}{tt'}},\quad \tilde{A}^{N_L 3}_{\underset{(\oplus)}{tt'}},\quad \tilde{A}^{N_R\boxed{\pm}}_{\underset{(\oplus)}{tt'}},\quad \tilde{A}^{N_R 3}_{\underset{(\oplus)}{tt'}},\\
&(tt') \in [(9\,10), (11\,12), (13\,14)],\\
&A^{2\boxed{\pm}}_{\underset{(\oplus)}{ss'}},\quad A^{Y'}_{\underset{(\oplus)}{ss'}} = \frac{1}{\sqrt{2}}(A^{23}_{\underset{(\oplus)}{ss'}} - A^{4}_{\underset{(\oplus)}{ss'}}),\\
&(ss') \in [(56), (78)],\\
&A^{2\boxed{\pm}}_{m},\quad A^{Y'}_{m} = \frac{1}{\sqrt{2}}(A^{23}_{m} - A^{4}_{m}),\\
&\vec{\tilde{A}}^{2}_{m},\quad \tilde{A}^{4}_{m},\quad \vec{\tilde{A}}^{N_R}_{m},\\
&m \in (0, 1, 2, 3). \qquad (22)
\end{aligned}$$

An expression for $A^{Y'}_{m,s}$ $\vartheta_2 = \frac{\pi}{4}$ is chosen just for simplicity, with no justification so far.

It remains an open question as to what has made the right-handed neutrinos form such a condensate in the history of the Universe.

Since $A^{Ai}_s, s \in (5,6)$ couple to the condensate and get masses, while (by assumption) they do not get nonzero vacuum expectation values during the electroweak break [which changes the masses of the scalar fields $A^{Ai}_s, s \in (7,8)$] the restriction in the sum in Eq. (2) is justified.

The scalar fields, causing the birth of baryons, have the triplet color charges. They resemble the supersymmetric partners of the quarks, but since they do not carry all of the quantum numbers of the quarks, they are not.

IV. PROPERTIES OF SCALAR FIELDS WHICH DETERMINE MASS MATRICES OF FERMIONS

This section is a short overview of Ref. [13].

There are two kinds of scalar gauge fields which gain at the electroweak break nonzero vacuum expectation values and determine, correspondingly, masses of the families of quarks and leptons and masses of gauge weak bosons: the kind originating in $\tilde{\omega}_{\tilde{a}\tilde{b}s}$ and the kind originating in $\omega_{s's''s}$. Both kinds have the space index $s=(7,8)$ and carry, correspondingly, the weak and the hypercharge as the Higgs scalar. These scalar fields are presented in the Lagrangian density for fermions [Eq. (2)] on the second line. The tilde kind influences the family quantum numbers of fermions, the Dirac kind influences the family member quantum numbers.

The two triplets $(\vec{\tilde{A}}^1_s, \vec{\tilde{A}}^{N_L}_s)$ influence the lower four families (the lowest three families are already observed), while $(\vec{\tilde{A}}^2_s, \vec{\tilde{A}}^{N_R}_s)$ influence the upper four families, the stable of which constitute dark matter. Recognizing that $\vec{\tilde{\tau}}^1\vec{\tilde{A}}^1_s + \vec{\tilde{N}}_L\vec{\tilde{A}}^{N_L}_s + \vec{\tilde{\tau}}^2\vec{\tilde{A}}^2_s + \vec{\tilde{N}}_R\vec{\tilde{A}}^{N_R}_s = \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abs},\quad s=(7,8)$, one easily finds, taking into account Eqs. (11) and (12), the expressions $\vec{\tilde{A}}^1_s = (\tilde{\omega}_{58s}-\tilde{\omega}_{67s}, \tilde{\omega}_{57s}+\tilde{\omega}_{68s}, \tilde{\omega}_{56s}-\tilde{\omega}_{78s})$, $\vec{\tilde{A}}^{N_L}_s = (\tilde{\omega}_{23s}+i\tilde{\omega}_{01s}, \tilde{\omega}_{31s}+i\tilde{\omega}_{02s}, \tilde{\omega}_{12s}+i\tilde{\omega}_{03s})$, $\vec{\tilde{A}}^2_s = (\tilde{\omega}_{58s}+\tilde{\omega}_{67s}, \tilde{\omega}_{57s}-\tilde{\omega}_{68s}, \tilde{\omega}_{56s}+\tilde{\omega}_{78s})$, $\vec{\tilde{A}}^{N_R}_s = (\tilde{\omega}_{23s}-i\tilde{\omega}_{01s}, \tilde{\omega}_{31s}-i\tilde{\omega}_{02s}, \tilde{\omega}_{12s}-i\tilde{\omega}_{03s})$, $s=(7,8)$, presented already in Eq. (15). Similarly one finds, taking into account Eqs. (8), (9), (10), and (13), the expressions for A^Q_s, $A^{Q'}_s$, and $A^{Y'}_s$, presented in Eq. (14).

The scalar fields A^Q_s, $A^{Q'}_s$, and $A^{Y'}_s$ distinguish among the family members, coupling to them through the family members quantum numbers Q $[Q=(\tau^{13}+Y)$, Y $(=\tau^{23}+\tau^4)]$, $Q'(=-Y\tan^2\vartheta_1+\tau^{13})$ and $Y' = (\tau^{23}-\tan\vartheta_2\tau^4)$, $\tau^4 = -\frac{1}{3}(S^{9\,10}+S^{11\,12}+S^{13\,14})$.

The scalars, originating in $\tilde{\omega}_{abs}$ and distinguishing among families, couple to the family quantum numbers through $\vec{\tilde{\tau}}^1$ and $\vec{\tilde{N}}_L$, or through $\vec{\tilde{\tau}}^2$ and $\vec{\tilde{N}}_R$. These scalars are all in the adjoint representations of the corresponding subgroups of the $\widetilde{SO}(7,1)$ group.

Let us now prove that all of the scalar fields with the space [scalar with respect to $d=(3+1)$] index $s=(7,8)$ carry the weak and the hypercharge (τ^{13}, Y) equal to either $(-\frac{1}{2},\frac{1}{2})$ or to $(\frac{1}{2},-\frac{1}{2})$. Let us first simplify the notation, using a common name A^{Ai}_s for all of the scalar fields with the scalar index $s=(7,8)$,

$$A^{Ai}_s = (A^Q_s, A^{Q'}_s, A^{Y'}_s, \tilde{A}^4_s, \vec{\tilde{A}}^2_s, \vec{\tilde{A}}^1_s, \vec{\tilde{A}}^{N_R}_s, \vec{\tilde{A}}^{N_L}_s), \tag{23}$$

and let us rewrite the term $\sum_{s=7,8}\bar{\psi}\gamma^s p_{0s}\psi$ in Eq. (2) as follows:

$$\begin{aligned}
\sum_{s=7,8}\bar{\psi}\gamma^s p_{0s}\psi, &= \bar{\psi}\{\overset{78}{(+)}p_{0+} + \overset{78}{(-)}p_{0-}\}\psi, \\
p_{0\pm} &= (p_{07} \mp i p_{08}), \\
(p_{07} \mp i p_{08}) &= (p_7 \mp i p_8) - \tau^{Ai}(A^{Ai}_7 \mp i A^{Ai}_8) \\
\overset{78}{(\pm)} &= \frac{1}{2}(\gamma^7 \pm i\gamma^8).
\end{aligned} \tag{24}$$

Let us now apply the operators Y, Q, Eq. (13), and $\tau^{13} = \frac{1}{2}(S^{56}-S^{78})$, Eq. (9), on the fields $A^{Ai}_{\underset{(\pm)}{78}} = (A^{Ai}_7 \mp i A^{Ai}_8)$. One finds

$$\begin{aligned}
\tau^{13}(A^{Ai}_7 \mp i A^{Ai}_8) &= \pm\frac{1}{2}(A^{Ai}_7 \mp i A^{Ai}_8), \\
Y(A^{Ai}_7 \mp i A^{Ai}_8) &= \mp\frac{1}{2}(A^{Ai}_7 \mp i A^{Ai}_8), \\
Q(A^{Ai}_7 \mp i A^{Ai}_8) &= 0,
\end{aligned} \tag{25}$$

This is, with respect to the weak, the hyper-, and the electromagnetic charge, just what the standard model assumes for the Higgs scalars. The proof is complete.

One can also check, using Eq. (B10), that $\gamma^0\overset{78}{(-)}$ transforms the u^{c1}_R from the first line of Table III into u^{c1}_L from the seventh line of the same table, or ν_R from the 25th line into the ν_L from the 31st line of the same table.

The scalars $A^{Ai}_{\underset{(-)}{78}}$ obviously bring the weak charge $\frac{1}{2}$ and the hypercharge $-\frac{1}{2}$ to the right-handed family members (u_R, ν_R), and the scalars $A^{Ai}_{\underset{(+)}{78}}$ bring the weak charge $-\frac{1}{2}$ and the hypercharge $\frac{1}{2}$ to $(d_R,\ e_R)$.

Let us now prove that the scalar fields of Eq. (23) are either triplets with respect to the family quantum numbers $[\vec{\tilde{N}}_R, \vec{\tilde{N}}_L, \vec{\tilde{\tau}}^2, \vec{\tilde{\tau}}^1$; Eqs. (11) and (12)] or singlets as the gauge fields of $Q=\tau^{13}+Y, Q'=\tau^{13}-Y\tan^2\vartheta_1$, and $Y'=\tau^{23}-\tan^2\vartheta_2\tau^4$. One can prove this by applying $\vec{\tilde{\tau}}^2$, $\vec{\tilde{\tau}}^1$, $\vec{\tilde{N}}_R$, $\vec{\tilde{N}}_L$, and Q, Q', Y' on the states belonging to representations of these operators. Let us calculate, as an example, $\tilde{N}^3_L$ and Q on $\tilde{A}^{N_L\boxplus}_{\underset{(\pm)}{78}}$, $\tilde{A}^{N_L3}_{\underset{(\pm)}{78}}$, and $A^Q_{\underset{(\pm)}{78}}$, taking into account Eqs. (11), (10), (9), and (7):

$$\begin{aligned}
&\tilde{N}^3_L \tilde{A}^{N_L \boxplus}_{\substack{78\\(\pm)}} = \boxplus \tilde{A}^{N_L \boxplus}_{\substack{78\\(\pm)}}, \qquad \tilde{N}^3_L \tilde{A}^{N_L 3}_{\substack{78\\(\pm)}} = 0,\\
&Q A^{Q}_{\substack{78\\(\pm)}} = 0,\\
&\tilde{A}^{N_L \boxplus}_{\substack{78\\(\pm)}} = \{(\tilde{\omega}_{\substack{78\\23(\pm)}} + i\tilde{\omega}_{\substack{78\\01(\pm)}}) \boxminus i(\tilde{\omega}_{\substack{78\\31(\pm)}} + i\tilde{\omega}_{\substack{78\\02(\pm)}})\},\\
&\tilde{A}^{N_L 3}_{\substack{78\\(\pm)}} = (\tilde{\omega}_{\substack{78\\12(\pm)}} + i\tilde{\omega}_{\substack{78\\03(\pm)}})\\
&A^{Q}_{\substack{78\\(\pm)}} = \sin\vartheta_1 A^{13}_{\substack{78\\(\pm)}} + \cos\vartheta_1 (-)(\omega_{\substack{78\\9\,10(\pm)}}\\
&\qquad + \omega_{\substack{78\\11\,12(\pm)}} + \omega_{\substack{78\\13\,14(\pm)}}),
\end{aligned} \tag{26}$$

with $Q = S^{56} + \tau^4 = S^{56} - \frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})$, and with τ^4 defined in Eq. (10).

Nonzero vacuum expectation values of the scalar fields [Eq. (23)], which carry the scalar index $s = (7,8)$, and, correspondingly, the weak and the hypercharges as calculated in Eq. (25), break the mass protection mechanism of quarks and leptons of the lower and upper four families. In the loop corrections besides $\tilde{A}^{Ai}_s$ and the scalar fields which are the gauge fields of Q, Q', Y' also the vector gauge fields contribute to all of the matrix elements of mass matrices of any family members.

The gauge fields of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ contribute only to the masses of the upper four families, while the gauge fields of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ contribute only to the masses of the lower four families. The triplet scalar fields with the scalar index $s = (7,8)$ and the family charges $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ transform any family member belonging to the group of the upper four families into the same family member belonging to another family of the same group of four families, changing the right-handed member into the left-handed partner, while those triplets with the family charges $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ transform any family member of a particular handedness and belonging to the lower four families into its partner of the opposite handedness, belonging to another family of the lower four families.

The scalars $A^{Q}_{\substack{78\\(\pm)}}$ [Eq. (26)], $A^{Q'}_{\substack{78\\(\pm)}}$ $(= \cos\vartheta_1 A^{13}_{\substack{78\\(\pm)}} - \sin\vartheta_1 A^{Y}_{\substack{78\\(\pm)}})$, and $A^{Y'}_{\substack{78\\(\pm)}}$ [Eq. (14)] contribute to all eight families, distinguishing among the family members and not among the families.

The mass matrix of any family member, belonging to any of the two groups of the four families, manifests—due to the $\widetilde{SU}(2)_{(R,L)} \times \widetilde{SU}(2)_{(II,I)}$ [either (R, II) or (L, I)] structure of the scalar fields, which are the gauge fields of the $\vec{\tilde{N}}_{R,L}$ and $\vec{\tilde{\tau}}^{2,1}$—the symmetry presented in Eq. (27)

$$\mathcal{M}^\alpha = \begin{pmatrix} -a_1 - a & e & d & b \\ e & -a_2 - a & b & d \\ d & b & a_2 - a & e \\ b & d & e & a_1 - a \end{pmatrix}^\alpha. \tag{27}$$

In Ref. [11] the mass matrices for the quarks, which are in agreement with the experimental data, are presented and predictions are made. It is demonstrated in this reference that the improved measurements of the quark mixing matrix are in better agreement with the predicted symmetry of the mass matrices by the spin-charge-family theory than the previous ones.

V. CONDENSATE AND NONZERO VACUUM EXPECTATION VALUES OF SCALAR FIELDS MAKE SPINORS AND MOST SCALAR AND VECTOR GAUGE FIELDS MASSIVE

Let us make a short overview of the properties of the scalar and vector gauge fields:

(i.) after two right-handed neutrinos (coupled to spin zero and with the family quantum numbers, Table IV, of the upper four families) make a condensate (Table II) at the scale $\geq 10^{16}$ GeV and

(ii.) after the electroweak break, when the scalar fields with the space index $s = (7,8)$ get nonzero vacuum expectation values.

All of the scalar gauge fields $A^{Ai}_t, t \in (5,6,7,8,9,\ldots,14)$ [Eqs. (2), (20), and (22), Table I] interact with the condensate through the quantum numbers τ^4 and τ^{23}—those with the family quantum numbers of the upper four families also interact through the family quantum numbers $\vec{\tilde{\tau}}^2$ or $\vec{\tilde{N}}_R$—getting masses of the order of the condensate scale [Eq. (22)].

At the electroweak break, the scalar fields $A^{Ai}_s, s \in (7,8)$ from Eq. (23) get nonzero vacuum expectation values, changing, correspondingly, their own masses and determining masses of quarks and leptons, as well as of the weak vector gauge fields.

The vector gauge fields $A^{2\boxplus}_m, A^{Y'}_m, \vec{\tilde{A}}^{2\boxplus}_m, \tilde{A}^{Y'}_m$, and $\vec{\tilde{A}}^{N_R}_m$ [Eq. (22)] get masses due to the interaction with the condensate through τ^{23} and τ^4 (the first two) or due to the family quantum numbers of the upper four families (the last three, respectively).

The vector gauge fields $\vec{A}^3_m, \vec{A}^1_m$, and A^Y_m stay massless up to the electroweak break when the scalar gauge fields, which are weak doublets with the hypercharge making their electromagnetic charge Q equal to zero, give masses to the weak bosons $[A^{1\boxplus}_m = \frac{1}{\sqrt{2}}(A^{11}_m \mp i A^{12}_m)$ and $A^{Q'}_m = \cos\vartheta_1 A^{13}_m - \sin\vartheta_1 A^Y_m]$, while the electromagnetic vector field $(A^Q_m = \sin\vartheta_1 A^{13}_m + \cos\vartheta_1 A^Y_m)$ and the color vector gauge field stay massless.

At the electroweak break, when the nonzero vacuum expectation values of the scalar fields break the weak and the hypercharge global symmetry, all eight families of quarks and leptons also get masses. Up to the electroweak break the families were mass protected since the right-handed partners were distinguished from the left-handed ones in the weak and hypercharges, which were the conserved quantum numbers that disabled them to make the superposition manifesting masses.

VI. SAKHAROV CONDITIONS AS SEEN IN VIEW OF THE SPIN-CHARGE-FAMILY THEORY

The condensate of the right-handed neutrinos, as well as the nonzero vacuum expectation values of the scalar fields $A^{Ai}_{\substack{78\\(\pm)}}$—if leading to the complex matrix elements of the mixing matrices—cause the $\mathbb{C}_{\mathcal{N}}\,\mathcal{P}_{\mathcal{N}}$ violation terms, which generate the matter-antimatter asymmetry.

It is a question whether both generators of the matter-antimatter asymmetry—the condensate and the complex phases of the mixing matrices of quarks and leptons (this last one alone cannot with one complex phase and also very probably not with the three complex phases of the lower four families)—can at all explain the observed matter-antimatter asymmetry of the ordinary matter, that is, the matter consisting mostly of the first family of quarks and leptons.

The lowest of the upper four families determine the dark matter. For dark matter any relation among matter and antimatter is, so far, experimentally allowed.

Both origins of the matter-antimatter asymmetry—the condensate and the nonzero vacuum expectation values of the scalar fields carrying the weak and the hypercharge—(are assumed to) appear spontaneously.

Sakharov [36] states that, for the matter-antimatter asymmetry, three conditions must be fulfilled:

(a.) ($\mathcal{C}_{\mathcal{N}}$ and) $\mathbb{C}_{\mathcal{N}}\,\mathcal{P}_{\mathcal{N}}$ must not be conserved.

(b.) Baryon number nonconserving processes must take place.

(c.) Thermal nonequilibrium must be present not to equilibrate the number of baryons and antibaryons.

Sakharov uses for (c.) the requirement that $\mathcal{CPT}$ must be conserved and that $\{\mathcal{CPT}, H\}_{-} = 0$. In a thermal equilibrium the average number of baryons $\langle n_B\rangle = \mathrm{Tr}(e^{-\beta H} n_B) = \mathrm{Tr}(e^{-\beta H}\mathcal{CPT} n_B (\mathcal{CPT})^{-1}) = \langle \bar{n}_B\rangle$. Therefore $\langle n_B\rangle - \langle \bar{n}_B\rangle = 0$ at the thermal equilibrium and there is no excess of baryons with respect to antibaryons. In the expanding Universe, however, the temperature is changing with time. It is necessary that the discrete symmetry $\mathbb{C}_{\mathcal{N}}\,\mathcal{P}_{\mathcal{N}}$ is broken to break the symmetry between matter and antimatter if the Universe starts with no matter-antimatter asymmetry.

The spin-charge-family theory starting action [Eq. (1)] is invariant under $\mathbb{C}_{\mathcal{N}}\,\mathcal{P}_{\mathcal{N}}$ symmetry. The scalar fields [Eq. (20)] of this theory cause transitions in which a quark is born out of a positron (Figs. 1 and 2) and a quark is born out of antiquark, and back. These reactions go in both directions with the same probability until the spontaneous break of $\mathbb{C}_{\mathcal{N}}\,\mathcal{P}_{\mathcal{N}}$ symmetry is caused by the appearance of the condensate of the two right-handed neutrinos (Table II).

But after the appearance of the condensate [and in addition to the appearance of the nonzero vacuum expectation values of the scalar fields with the space index $s \in (7, 8)$], family members see the vacuum differently than the antimembers. And this *might* explain the matter-antimatter asymmetry, provided that the conditions of the expanding Universe at the appearance of the condensate (and later at the electroweak break) make the matter-antimatter asymmetry strong enough so that it is not later washed out. Massive scalar fields with the color charge also predict the proton decay.

It is, of course, a question as to whether both phenomena can at all explain the observed matter-antimatter asymmetry. I agree completely with the referee of this paper that before answering the question whether or not the spin-charge-family theory explains this observed phenomena, one must do a lot of additional work to find out the following. (i.) Which is the order of the phase transition which leads to the appearance of the condensate? (ii.) How strong is the thermal nonequilibrium which leads to the matter-antimatter asymmetry during the phase transition? (iii.) How rapid is the appearance of the matter-antimatter asymmetry in comparison with the expansion of the Universe? (iv.) Does the later history of the expanding Universe enable the produced asymmetry to survive until today?

Although we do have some experience with solving the Boltzmann equations for fermions and antifermions [12] to follow the history of dark matter within the spin-charge-family theory, the study of the history of the Universe from the very high temperature to the baryon production within the same theory in order to see the matter-antimatter asymmetry in the present time is a much more demanding task. These questions are under consideration, but still at a preliminary point since a lot of things must be understood before we can start on the calculations.

What I can conclude is only that the spin-charge-family theory does offer the opportunity for an explanation of the observed matter-antimatter asymmetry.

VII. CONCLUSIONS

The spin-charge-family [1–13,15] theory is a kind of Kaluza-Klein theory in $d = (13 + 1)$ but with the families introduced by the second kind of gamma operators—the $\tilde{\gamma}^a$ operators in addition to the Dirac γ^a. The theory assumes a simple starting action [Eq. (1)] in $d = (13 + 1)$. This simple action manifests in the low energy regime, after the breaks of symmetries (Sec. I A), all of the degrees of freedom assumed in the standard model, offering an explanation for all of the properties of the quarks and leptons (right-handed neutrinos are, in this theory, the regular members of each family) and antiquarks and antileptons. The theory explains the existence of the observed gauge vector fields. It explains the origin of the scalar fields (the Higgs scalars and the Yukawa couplings) responsible for the quark and lepton masses and the masses of the weak bosons [13].

The theory also offers an explanation for the matter-antimatter asymmetry and for the appearance of dark matter.

The spin-charge-family theory predicts two decoupled groups of four families [3,4,9,12]: The fourth of the lower group of four families will be measured at the LHC [10] and the lowest of the upper four families constitutes the dark matter [12] and was already seen. It also predicts that there might be more scalar fields observable at the LHC. The upper four families manifest, due to their high masses, a new nuclear force among their baryons.

All of these degrees of freedom are contained in the simple starting action. The scalar fields with weak and hypercharges equal to $(\mp\frac{1}{2}, \pm\frac{1}{2})$, respectively (Sec. IV), have the space index $s = (7, 8)$, while they also carry in addition to the weak and the hypercharges either the family quantum numbers originating in $\tilde{S}^{ab}$ [they form two groups of twice $SU(2)$ triplets] or the family member quantum numbers originating in S^{ab} [they form three singlets with the quantum numbers (Q, Q', Y')]. These scalar fields cause the transitions of the right-handed family members into the left-handed partners and back. Those with the family quantum numbers cause at the same time transitions among families within each of the two family groups of the four families. They all gain in the electroweak break nonzero vacuum expectation values, giving masses to both groups of the four families of quarks and leptons and to weak bosons (also changing their own masses).

There are also, in this theory, scalar fields with the space index $s = (5, 6)$. They carry, with respect to this degree of freedom, the weak charge equal to the hypercharge ($\mp\frac{1}{2}$, $\mp\frac{1}{2}$, respectively). They also carry additional quantum numbers [Eq. (22)] like all of the other scalar fields: Either the family quantum numbers originating in $\tilde{S}^{ab}$ or the family member quantum numbers originating in S^{ab}.

There are also the scalar fields with the scalar index $s = (9, 10, \ldots, 14)$. These scalars carry the triplet color charge with respect to the space index and the additional quantum numbers (Table I), originating either in the family quantum numbers $\tilde{S}^{ab}$ or in the family member quantum numbers S^{ab}.

There are no additional scalar gauge fields in this theory.

There are the vector gauge fields with respect to $d = (3 + 1)$: A^{Ai}_m, with Ai staying for the groups $SU(3)$ and $U(1)$ [both originating in $SO(6)$ of $SO(13, 1)$], for the groups $SU(2)_{II}$ and $SU(2)_I$ [both originating in $SO(4)$ of $SO(7, 1)$] and for the groups $SU(2) \times SU(2)$ [$\in SO(3, 1)$] in both sectors, the S^{ab} and $\tilde{S}^{ab}$ ones.

The condensate of the two right-handed neutrinos with the family charges of the upper four families (Table II) gives masses to all of the scalar and vector gauge fields, except for the color octet vector, the hypersinglet vector, and the weak triplet vector gauge fields, to which the condensate does not couple. Those vector gauge fields of either S^{ab} or $\tilde{S}^{ab}$ origin, which do not couple to the condensate, are expressible with the corresponding vielbeins (Appendixes (C1) and (C2); they are auxiliary fields. The condensate breaks the $\mathbb{C}_{\mathcal{N}}\,\mathcal{P}_{\mathcal{N}}$ symmetry (Sec. III and Appendix A).

There are no additional vector gauge fields in this theory.

Nonzero vacuum expectation values of the scalar gauge fields with the space index $s = (7, 8)$ and the quantum numbers as explained in the fourth paragraph of this section change their own masses in the electroweak break, while all of the other scalars or vectors either stay massless (the color octet, the electromagnetic field) or keep the masses of the scale of the condensate. The only vector fields that are massless before the electroweak break which become massive at the electroweak break are the heavy bosons.

It is extremely encouraging that the simple starting action of the spin-charge-family offers, at low energies, explanations for so many observed phenomena. However, it is also true that the starting assumptions (Sec. I A) wait to be derived from the initial and boundary conditions of the expanding Universe.

This paper is a step towards understanding the matter-antimatter asymmetry within the spin-charge-family theory and also predicts the proton decay. The theory obviously offers the possibility that the scalar gauge fields with the space index $s = (9, 10, \ldots, 14)$ explain, after the appearance of the condensate, the matter-antimatter asymmetry. To prove, however, that this indeed happens requires additional study to follow the Universe through the phase transitions which break the $\mathbb{C}_{\mathcal{N}}\,\mathcal{P}_{\mathcal{N}}$ symmetry at the level of the condensate and further through the electroweak phase transition up to now, to determine how much of the matter-antimatter asymmetry is left. The experience when following the history of the expanding Universe to see whether the spin-charge-family theory can explain the dark matter content [12] is of some help. However, answering the question regarding to which extent this theory can explain the observed matter-antimatter asymmetry will require a lot of additional understanding and work.

Let me conclude with the recognition, pointed out already in the Introduction, that the spin-charge-family theory overlaps in many points with other unifying theories [21–26] since all of the unifying groups can be recognized as subgroups of the large enough orthogonal groups, with family groups included. There are also many differences, though: The spin-charge-family theory starts with a very simple action, from which all of the properties of spinors and the gauge vector and scalar fields follow, provided that the breaks of symmetries occur in the desired way.

Consequently it differs from other unifying theories in the degrees of freedom of spinors and scalar and vector gauge fields which show up on different levels of the break of symmetries, in the unification scheme, in the family degrees of freedom, and, correspondingly, also in the evolution of our Universe.

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APPENDIX A: DISCRETE SYMMETRY OPERATORS

I present here the discrete symmetry operators in the second quantized picture, for the description of which the Dirac sea is used. I will follow Ref. [15]. The discrete symmetry operators of this reference are designed for Kaluza-Klein-like theories, in which the total angular momentum in higher than $(3+1)$ dimensions manifests as charges in $d=(3+1)$. The dimension of space-time is even, as it is in the case of the spin-charge-family theory:

$$\begin{aligned}\mathcal{C}_{\mathcal{N}} &= \prod_{\Im\gamma^m, m=0}^{3} \gamma^m \Gamma^{(3+1)} K I_{x^6,x^8,\dots,x^d},\\ \mathcal{T}_{\mathcal{N}} &= \prod_{\Re\gamma^m, m=1}^{3} \gamma^m \Gamma^{(3+1)} K I_{x^0} I_{x^5,x^7,\dots,x^{d-1}},\\ \mathcal{P}_{\mathcal{N}} &= \gamma^0 \Gamma^{(3+1)} \Gamma^{(d)} I_{\vec{x}_3}. \end{aligned} \tag{A1}$$

The operator of handedness in even d-dimensional spaces is defined as

$$\Gamma^{(d)} := (i)^{d/2} \prod_a (\sqrt{\eta^{aa}} \gamma^a), \tag{A2}$$

with products of γ^a in ascending order. We choose γ^0, γ^1 real, γ^2 imaginary, γ^3 real, γ^5 imaginary, γ^6 real, alternating imaginary, and real up to γ^d real. Operators I operate as follows: $I_{x^0} x^0 = -x^0$; $I_x x^a = -x^a$; $I_{x^0} x^a = (-x^0, \vec{x})$; $I_{\vec{x}} \vec{x} = -\vec{x}$; $I_{\vec{x}_3} x^a = (x^0, -x^1, -x^2, -x^3, x^5, x^6, \dots, x^d)$; $I_{x^5,x^7,\dots,x^{d-1}}(x^0, x^1, x^2, x^3, x^5, x^6, x^7, x^8, \dots, x^{d-1}, x^d) = (x^0, x^1, x^2, x^3, -x^5, x^6, -x^7, \dots, -x^{d-1}, x^d)$; $I_{x^6,x^8,\dots,x^d}(x^0, x^1, x^2, x^3, x^5, x^6, x^7, x^8, \dots, x^{d-1}, x^d) = (x^0, x^1, x^2, x^3, x^5, -x^6, x^7, -x^8, \dots, x^{d-1}, -x^d)$, $d = 2n$.

$\mathcal{C}_{\mathcal{N}}$ transforms the state, put on the top of the Dirac sea, into the corresponding negative energy state in the Dirac sea.

The operator, which is called $\mathbb{C}_{\mathcal{N}}$ [1,15,16], is needed, which transforms the starting single particle state on top of the Dirac sea into the negative energy state and then empties this negative energy state. This hole in the Dirac sea is the antiparticle state put on top of the Dirac sea. Both a particle and its antiparticle state (both put on top of the Dirac sea) must solve the Weyl equations of motion.

This $\mathbb{C}_{\mathcal{N}}$ is defined as a product of the operator emptying [1,16] (making transformations into a completely different Fock space),

$$\text{emptying} = \prod_{\Re\gamma^a} \gamma^a K = (-)^{\frac{d}{2}+1} \prod_{\Im\gamma^a} \gamma^a \Gamma^{(d)} K, \tag{A3}$$

and $\mathcal{C}_{\mathcal{N}}$:

$$\begin{aligned}\mathbb{C}_{\mathcal{N}} &= \prod_{\Re\gamma^a, a=0}^{d} \gamma^a K \prod_{\Im\gamma^m, m=0}^{3} \gamma^m \Gamma^{(3+1)} K I_{x^6,x^8,\dots,x^d}\\ &= \prod_{\Re\gamma^s, s=5}^{d} \gamma^s I_{x^6,x^8,\dots,x^d}. \end{aligned} \tag{A4}$$

We shall indeed need only the product of operators $\mathbb{C}_{\mathcal{N}} \mathcal{P}_{\mathcal{N}}$, $\mathcal{T}_{\mathcal{N}}$ and $\mathbb{C}_{\mathcal{N}} \mathcal{P}_{\mathcal{N}} \mathcal{T}_{\mathcal{N}}$ since both $\mathbb{C}_{\mathcal{N}}$ and $\mathcal{P}_{\mathcal{N}}$ have an odd number of γ^a operators in even-dimensional spaces with $d = 2(2n+1)$, transforming, accordingly, states from the representation of one handedness in $d = 2(2n+1)$ into the Weyl of another handedness:

$$\begin{aligned}\mathbb{C}_{\mathcal{N}} \mathcal{P}_{\mathcal{N}} &= \gamma^0 \prod_{\Im\gamma^s, s=5}^{d} \gamma^s I_{\vec{x}_3} I_{x^6,x^8,\dots,x^d},\\ \mathbb{C}_{\mathcal{N}} \mathcal{P}_{\mathcal{N}} \mathcal{T}_{\mathcal{N}} &= \prod_{\Im\gamma^a, a=0}^{d} \gamma^a K I_x. \end{aligned} \tag{A5}$$

APPENDIX B: SHORT PRESENTATION OF TECHNIQUE

I make in this appendix a short review of the technique [18,20] initiated and developed [5–8] when proposing the spin-charge-family theory [1–12] assuming that all of the internal degrees of freedom of spinors, with family quantum number included, are describable in the space of d-anticommuting (Grassmann) coordinates [6] if the dimension of ordinary space is d. There are two kinds of operators in the Grassmann space fulfilling the Clifford algebra which anticommute with one another. The technique was further developed in the present shape together with Nielsen [18,20] by identifying one kind of Clifford object with γ^s's and another with $\tilde{\gamma}^a$'s.

The objects γ^a and $\tilde{\gamma}^a$ have the properties

$$\begin{aligned} &\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab}, \qquad \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab},\\ &\{\gamma^a, \tilde{\gamma}^b\}_+ = 0, \end{aligned} \tag{B1}$$

If B is a Clifford algebra object, let us say a polynomial of γ^a, $B = a_0 + a_a \gamma^a + a_{ab} \gamma^a \gamma^b + \dots + a_{a_1 a_2 \dots a_d} \gamma^{a_1} \gamma^{a_2} \cdots \gamma^{a_d}$, then one finds

$$(\tilde{\gamma}^a B := i(-)^{n_B} B\gamma^a)|\psi_0\rangle,$$
$$B = a_0 + a_{a_0}\gamma^{a_0} + a_{a_1 a_2}\gamma^{a_1}\gamma^{a_2} + \cdots + a_{a_1\ldots a_d}\gamma^{a_1}\cdots\gamma^{a_d}, \tag{B2}$$

where $|\psi_0\rangle$ is a vacuum state, defined in Eq. (B16), and $(-)^{n_B}$ is equal to 1 for the term in the polynomial which has an even number of γ^b's, and to -1 for the term with an odd number of γ^b's.

In this last stage we constructed a spinor basis as products of nilpotents and projectors formed as odd and even objects of γ^a's, respectively, and chosen to be eigenstates of a Cartan subalgebra of the Lorentz groups defined by γ^a's and $\tilde{\gamma}^a$'s.

The technique can be used to construct a spinor basis for any dimension d and any signature in an easy and transparent way. Equipped with the graphic presentation of basic states, the technique offers an elegant way to see all of the quantum numbers of states with respect to the two Lorentz groups, as well as transformation properties of the states under any Clifford algebra object.

The Clifford algebra objects S^{ab} and $\tilde{S}^{ab}$ close the algebra of the Lorentz group

$$\begin{aligned}
S^{ab} &:= (i/4)(\gamma^a\gamma^b - \gamma^b\gamma^a),\\
\tilde{S}^{ab} &:= (i/4)(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a),\\
\{S^{ab}, \tilde{S}^{cd}\}_- &= 0,\\
\{S^{ab}, S^{cd}\}_- &= i(\eta^{ad}S^{bc} + \eta^{bc}S^{ad} - \eta^{ac}S^{bd} - \eta^{bd}S^{ac}),\\
\{\tilde{S}^{ab}, \tilde{S}^{cd}\}_- &= i(\eta^{ad}\tilde{S}^{bc} + \eta^{bc}\tilde{S}^{ad} - \eta^{ac}\tilde{S}^{bd} - \eta^{bd}\tilde{S}^{ac}).
\end{aligned} \tag{B3}$$

We assume the Hermiticity property for γ^a's and $\tilde{\gamma}^a$'s:

$$\gamma^{a\dagger} = \eta^{aa}\gamma^a, \qquad \tilde{\gamma}^{a\dagger} = \eta^{aa}\tilde{\gamma}^a, \tag{B4}$$

in order that γ^a and $\tilde{\gamma}^a$ are compatible with (B1) and formally unitary, i.e., $\gamma^{a\dagger}\gamma^a = I$ and $\tilde{\gamma}^{a\dagger}\tilde{\gamma}^a = I$.

One finds from Eq. (B4) that $(S^{ab})^\dagger = \eta^{aa}\eta^{bb}S^{ab}$.

Recognizing from Eq. (B3) that the two Clifford algebra objects S^{ab}, S^{cd} with all indices different commute and, equivalently, for $\tilde{S}^{ab}, \tilde{S}^{cd}$, we select the Cartan subalgebra of the algebra of the two groups, which form equivalent representations with respect to one another:

$$\begin{aligned}
&S^{03}, S^{12}, S^{56}, \ldots, S^{d-1\,d}, && \text{if } d = 2n \geq 4,\\
&S^{03}, S^{12}, \ldots, S^{d-2\,d-1}, && \text{if } d = (2n+1) > 4,\\
&\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \ldots, \tilde{S}^{d-1\,d}, && \text{if } d = 2n \geq 4,\\
&\tilde{S}^{03}, \tilde{S}^{12}, \ldots, \tilde{S}^{d-2\,d-1}, && \text{if } d = (2n+1) > 4.
\end{aligned} \tag{B5}$$

The choice for the Cartan subalgebra in $d < 4$ is straightforward. It is useful to define one of the Casimirs of the Lorentz group, the handedness Γ ($\{\Gamma, S^{ab}\}_- = 0$) in any d,

$$\begin{aligned}
\Gamma^{(d)} &:= (i)^{d/2}\prod_a(\sqrt{\eta^{aa}}\gamma^a), \quad \text{if } d = 2n,\\
\Gamma^{(d)} &:= (i)^{(d-1)/2}\prod_a(\sqrt{\eta^{aa}}\gamma^a), \quad \text{if } d = 2n+1.
\end{aligned} \tag{B6}$$

One proceeds equivalently for $\tilde{\Gamma}^{(d)}$, substituting $\tilde{\gamma}^a$'s for γ^a's. We understand the product of γ^a's in the ascending order with respect to the index a: $\gamma^0\gamma^1\ldots\gamma^d$. It follows from Eq. (B4) for any choice of the signature η^{aa} that $\Gamma^\dagger = \Gamma, \Gamma^2 = I$. We also find that for d even the handedness anticommutes with the Clifford algebra objects γ^a ($\{\gamma^a, \Gamma\}_+ = 0$), while for d odd it commutes with γ^a ($\{\gamma^a, \Gamma\}_- = 0$).

To make the technique simple we introduce the graphic presentation as follows:

$$\begin{aligned}
&\overset{ab}{(k)} := \frac{1}{2}\left(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b\right), \qquad \overset{ab}{[k]} := \frac{1}{2}\left(1 + \frac{i}{k}\gamma^a\gamma^b\right),\\
&\overset{+}{\circ} := \frac{1}{2}(1+\Gamma), \qquad \overset{-}{\bullet} := \frac{1}{2}(1-\Gamma),
\end{aligned} \tag{B7}$$

where $k^2 = \eta^{aa}\eta^{bb}$. It follows then that

$$\begin{aligned}
&\gamma^a = \overset{ab}{(k)} + \overset{ab}{(-k)}, \qquad \gamma^b = ik\eta^{aa}(\overset{ab}{(k)} - \overset{ab}{(-k)}),\\
&S^{ab} = \frac{k}{2}(\overset{ab}{[k]} - \overset{ab}{[-k]}).
\end{aligned} \tag{B8}$$

One can easily check, by taking into account the Clifford algebra relation [Eq. (B1)] and the definitions of S^{ab} and $\tilde{S}^{ab}$ [Eq. (B3)], that if one multiplies by S^{ab} or $\tilde{S}^{ab}$ from the left-hand side the Clifford algebra objects $\overset{ab}{(k)}$ and $\overset{ab}{[k]}$, it follows that

$$\begin{aligned}
&S^{ab}\overset{ab}{(k)} = \frac{1}{2}k\overset{ab}{(k)}, \qquad S^{ab}\overset{ab}{[k]} = \frac{1}{2}k\overset{ab}{[k]},\\
&\tilde{S}^{ab}\overset{ab}{(k)} = \frac{1}{2}k\overset{ab}{(k)}, \qquad \tilde{S}^{ab}\overset{ab}{[k]} = -\frac{1}{2}k\overset{ab}{[k]},
\end{aligned} \tag{B9}$$

which means that we get the same objects back multiplied by the constant $\frac{1}{2}k$ in the case of S^{ab}, while $\tilde{S}^{ab}$ multiply $\overset{ab}{(k)}$ by k and $\overset{ab}{[k]}$ by $(-k)$ rather than (k). This also means that when $\overset{ab}{(k)}$ and $\overset{ab}{[k]}$ act from the left-hand side on a vacuum state $|\psi_0\rangle$, the obtained states are the eigenvectors of S^{ab}. We further recognize that γ^a transform $\overset{ab}{(k)}$ into $\overset{ab}{[-k]}$, never to $\overset{ab}{[k]}$, while $\tilde{\gamma}^a$ transform $\overset{ab}{(k)}$ into $\overset{ab}{[k]}$, never to $\overset{ab}{[-k]}$:

$$
\begin{aligned}
&\gamma^a \stackrel{ab}{(k)} = \eta^{aa} \stackrel{ab}{[-k]}, \qquad \gamma^b \stackrel{ab}{(k)} = -ik \stackrel{ab}{[-k]}, \qquad \gamma^a \stackrel{ab}{[k]} = \stackrel{ab}{(-k)}, \qquad \gamma^b \stackrel{ab}{[k]} = -ik\eta^{aa} \stackrel{ab}{(-k)},\\
&\tilde{\gamma}^a \stackrel{ab}{(k)} = -i\eta^{aa} \stackrel{ab}{[k]}, \qquad \tilde{\gamma}^b \stackrel{ab}{(k)} = -k \stackrel{ab}{[k]}, \qquad \tilde{\gamma}^a \stackrel{ab}{[k]} = i \stackrel{ab}{(k)}, \qquad \tilde{\gamma}^b \stackrel{ab}{[k]} = -k\eta^{aa} \stackrel{ab}{(k)}. \qquad \text{(B10)}
\end{aligned}
$$

From Eq. (B10) it follows that

$$
\begin{aligned}
&S^{ac} \stackrel{ab}{(k)} \stackrel{cd}{(k)} = -\frac{i}{2}\eta^{aa}\eta^{cc} \stackrel{ab}{[-k]} \stackrel{cd}{[-k]}, \qquad \tilde{S}^{ac} \stackrel{ab}{(k)} \stackrel{cd}{(k)} = \frac{i}{2}\eta^{aa}\eta^{cc} \stackrel{ab}{[k]} \stackrel{cd}{[k]},\\
&S^{ac} \stackrel{ab}{[k]} \stackrel{cd}{[k]} = \frac{i}{2} \stackrel{ab}{(-k)} \stackrel{cd}{(-k)}, \qquad \tilde{S}^{ac} \stackrel{ab}{[k]} \stackrel{cd}{[k]} = -\frac{i}{2} \stackrel{ab}{(k)} \stackrel{cd}{(k)},\\
&S^{ac} \stackrel{ab}{(k)} \stackrel{cd}{[k]} = -\frac{i}{2}\eta^{aa} \stackrel{ab}{[-k]} \stackrel{cd}{(-k)}, \qquad \tilde{S}^{ac} \stackrel{ab}{(k)} \stackrel{cd}{[k]} = -\frac{i}{2}\eta^{aa} \stackrel{ab}{[k]} \stackrel{cd}{(k)},\\
&S^{ac} \stackrel{ab}{[k]} \stackrel{cd}{(k)} = \frac{i}{2}\eta^{cc} \stackrel{ab}{(-k)} \stackrel{cd}{[-k]}, \qquad \tilde{S}^{ac} \stackrel{ab}{[k]} \stackrel{cd}{(k)} = \frac{i}{2}\eta^{cc} \stackrel{ab}{(k)} \stackrel{cd}{[k]}. \qquad \text{(B11)}
\end{aligned}
$$

From Eq. (B11) we conclude that $\tilde{S}^{ab}$ generate the equivalent representations with respect to S^{ab} and the opposite.

Let us deduce some useful relations:

$$
\begin{aligned}
&\stackrel{ab}{(k)} \stackrel{ab}{(k)} = 0, \qquad \stackrel{ab}{(k)} \stackrel{ab}{(-k)} = \eta^{aa} \stackrel{ab}{[k]}, \qquad \stackrel{ab}{(-k)} \stackrel{ab}{(k)} = \eta^{aa} \stackrel{ab}{[-k]}, \qquad \stackrel{ab}{(-k)} \stackrel{ab}{(-k)} = 0,\\
&\stackrel{ab}{[k]} \stackrel{ab}{[k]} = \stackrel{ab}{[k]}, \qquad \stackrel{ab}{[k]} \stackrel{ab}{[-k]} = 0, \qquad \stackrel{ab}{[-k]} \stackrel{ab}{[k]} = 0, \qquad \stackrel{ab}{[-k]} \stackrel{ab}{[-k]} = \stackrel{ab}{[-k]},\\
&\stackrel{ab}{(k)} \stackrel{ab}{[k]} = 0, \qquad \stackrel{ab}{[k]} \stackrel{ab}{(k)} = \stackrel{ab}{(k)}, \qquad \stackrel{ab}{(-k)} \stackrel{ab}{[k]} = \stackrel{ab}{(-k)}, \qquad \stackrel{ab}{(-k)} \stackrel{ab}{[-k]} = 0,\\
&\stackrel{ab}{(k)} \stackrel{ab}{[-k]} = \stackrel{ab}{(k)}, \qquad \stackrel{ab}{[k]} \stackrel{ab}{(-k)} = 0, \qquad \stackrel{ab}{[-k]} \stackrel{ab}{(k)} = 0, \qquad \stackrel{ab}{[-k]} \stackrel{ab}{(-k)} = \stackrel{ab}{(-k)}. \qquad \text{(B12)}
\end{aligned}
$$

We recognize in the first equation of the first line and the first and second equations of the second line the demonstration of the nilpotent and the projector character of the Clifford algebra objects $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$, respectively. Defining

$$
\stackrel{\widetilde{ab}}{(\pm i)} = \frac{1}{2}(\tilde{\gamma}^a \mp \tilde{\gamma}^b), \qquad \stackrel{\widetilde{ab}}{(\pm 1)} = \frac{1}{2}(\tilde{\gamma}^a \pm i\tilde{\gamma}^b), \qquad \text{(B13)}
$$

one recognizes that

$$
\begin{aligned}
&\stackrel{\widetilde{ab}}{(k)} \stackrel{ab}{(k)} = 0, \qquad \stackrel{\widetilde{ab}}{(-k)} \stackrel{ab}{(k)} = -i\eta^{aa} \stackrel{ab}{[k]},\\
&\stackrel{\widetilde{ab}}{(k)} \stackrel{ab}{[k]} = i \stackrel{ab}{(k)}, \qquad \stackrel{\widetilde{ab}}{(k)} \stackrel{ab}{[-k]} = 0. \qquad \text{(B14)}
\end{aligned}
$$

Recognizing that

$$
\stackrel{ab}{(k)}^{\dagger} = \eta^{aa} \stackrel{ab}{(-k)}, \qquad \stackrel{ab}{[k]}^{\dagger} = \stackrel{ab}{[k]}, \qquad \text{(B15)}
$$

we define a vacuum state $|\psi_0\rangle$ so that one finds

$$
\langle \stackrel{ab}{(k)}^{\dagger} \stackrel{ab}{(k)} \rangle = 1, \qquad \langle \stackrel{ab}{[k]}^{\dagger} \stackrel{ab}{[k]} \rangle = 1. \qquad \text{(B16)}
$$

Taking into account the above equations it is easy to find a Weyl spinor irreducible representation for d-dimensional space, with d even or odd.

For d even we simply make a starting state as a product of $d/2$, let us say, only nilpotents $\stackrel{ab}{(k)}$, one for each S^{ab} of the Cartan subalgebra elements [Eq. (B5)], applying it on an (unimportant) vacuum state. For d odd the basic states are products of the $(d-1)/2$ nilpotents and a factor $(1 \pm \Gamma)$. Then the generators S^{ab}, which do not belong to the Cartan subalgebra, being applied on the starting state from the left, generate all of the members of one Weyl spinor.

$$
\begin{aligned}
&\stackrel{0d}{(k_{0d})} \stackrel{12}{(k_{12})} \stackrel{35}{(k_{35})} \cdots \stackrel{d-1\,d-2}{(k_{d-1\,d-2})} \psi_0\\
&\stackrel{0d}{[-k_{0d}]} \stackrel{12}{[-k_{12}]} \stackrel{35}{(k_{35})} \cdots \stackrel{d-1\,d-2}{(k_{d-1\,d-2})} \psi_0\\
&\stackrel{0d}{[-k_{0d}]} \stackrel{12}{(k_{12})} \stackrel{35}{[-k_{35}]} \cdots \stackrel{d-1\,d-2}{(k_{d-1\,d-2})} \psi_0\\
&\qquad\vdots\\
&\stackrel{0d}{[-k_{0d}]} \stackrel{12}{(k_{12})} \stackrel{35}{(k_{35})} \cdots \stackrel{d-1\,d-2}{[-k_{d-1\,d-2}]} \psi_0\\
&\stackrel{od}{(k_{0d})} \stackrel{12}{[-k_{12}]} \stackrel{35}{[-k_{35}]} \cdots \stackrel{d-1\,d-2}{(k_{d-1\,d-2})} \psi_0\\
&\qquad\vdots \qquad \text{(B17)}
\end{aligned}
$$

All of the states have the handedness Γ since $\{\Gamma, S^{ab}\} = 0$. States belonging to one multiplet with respect to the group $SO(q, d-q)$, that is, to one irreducible representation of spinors (one Weyl spinor), can have any phase. We made a choice of the simplest one, taking all phases equal to one.

The above graphic representation demonstrates that for d even all of the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of nilpotents $\stackrel{ab}{(k_{ab})}$, by transforming all possible pairs of $\stackrel{ab}{(k_{ab})}\stackrel{mn}{(k_{mn})}$ into $\stackrel{ab}{[-k_{ab}]}\stackrel{mn}{[-k_{mn}]}$. There are $S^{am}, S^{an}, S^{bm}, S^{bn}$, which all do this. The procedure gives $2^{(d/2-1)}$ states. A Clifford algebra object γ^a being applied from the left-hand side transforms a Weyl spinor of one handedness into a Weyl spinor of the opposite handedness. Both Weyl spinors form a Dirac spinor.

For d odd a Weyl spinor has, besides a product of $(d-1)/2$ nilpotents or projectors, also either the factor $\stackrel{+}{\circ} := \frac{1}{2}(1+\Gamma)$ or the factor $\stackrel{-}{\bullet} := \frac{1}{2}(1-\Gamma)$. As in the case of d even, all of the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of $(1+\Gamma)$ and $(d-1)/2$ nilpotents $\stackrel{ab}{(k_{ab})}$, by transforming all possible pairs of $\stackrel{ab}{(k_{ab})}\stackrel{mn}{(k_{mn})}$ into $\stackrel{ab}{[-k_{ab}]}\stackrel{mn}{[-k_{mn}]}$. However, γ^a's, being applied from the left-hand side, do not change the handedness of the Weyl spinor since $\{\Gamma, \gamma^a\}_- = 0$ for d odd. A Dirac and a Weyl spinor are for d odd identical and a family has, accordingly, $2^{(d-1)/2}$ members of basic states of a definite handedness.

We shall speak about left-handedness when $\Gamma = -1$ and about right-handedness when $\Gamma = 1$ for either d even or odd.

While S^{ab} which do not belong to the Cartan subalgebra [Eq. (B5)] generate all of the states of one representation, $\tilde{S}^{ab}$ which do not belong to the Cartan subalgebra [Eq. (B5)] generate the states of $2^{d/2-1}$ equivalent representations.

Making a choice of the Cartan subalgebra set [Eq. (B5)] of the algebra S^{ab} and $\tilde{S}^{ab}$ a left-handed ($\Gamma^{(13,1)} = -1$) eigenstate of all of the members of the Cartan subalgebra, representing a weak chargeless u_R quark with spin-up, hypercharge (2/3), and color $(1/2, 1/(2\sqrt{3}))$, for example, can be written as

$$\stackrel{03}{(+i)}\stackrel{12}{(+)}|\stackrel{56}{(+)}\stackrel{78}{(+)}||\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}|\psi\rangle$$
$$= \frac{1}{2^7}(\gamma^0 - \gamma^3)(\gamma^1 + i\gamma^2)|(\gamma^5 + i\gamma^6)(\gamma^7 + i\gamma^8)||$$
$$\times (\gamma^9 + i\gamma^{10})(\gamma^{11} - i\gamma^{12})(\gamma^{13} - i\gamma^{14})|\psi\rangle. \qquad \text{(B18)}$$

This state is an eigenstate of all S^{ab} and $\tilde{S}^{ab}$ which are members of the Cartan subalgebra [Eq. (B5)].

The operators $\tilde{S}^{ab}$, which do not belong to the Cartan subalgebra [Eq. (B5)], generate families from the starting u_R quark, transforming the u_R quark from Eq. (B18) to the u_R of another family, keeping all of the properties with respect to S^{ab} unchanged. In particular, $\tilde{S}^{01}$ applied on a right-handed u_R quark, weak chargeless, with spin-up, hypercharge (2/3), and the color charge $[1/2, 1/(2\sqrt{3})]$ from Eq. (B18) generates a state which is again a right-handed u_R quark, weak chargeless, with spin-up, hypercharge (2/3), and the color charge $[1/2, 1/(2\sqrt{3})]$:

$$\tilde{S}^{01}\stackrel{03}{(+i)}\stackrel{12}{(+)}|\stackrel{56}{(+)}\stackrel{78}{(+)}||\stackrel{910}{(+)}\stackrel{1112}{(-)}\stackrel{1314}{(-)}$$
$$= -\frac{i}{2}\stackrel{03}{[+i]}\stackrel{12}{[+]}|\stackrel{56}{(+)}\stackrel{78}{(+)}||\stackrel{910}{(+)}\stackrel{1112}{(-)}\stackrel{1314}{(-)}. \qquad \text{(B19)}$$

Some useful relations [3] are presented below:

$$N^{\pm}_{+} = N^1_+ \pm iN^2_+ = -\stackrel{03}{(\mp i)}\stackrel{12}{(\pm)},$$
$$N^{\pm}_{-} = N^1_- \pm iN^2_- = \stackrel{03}{(\pm i)}\stackrel{12}{(\pm)},$$
$$\tilde{N}^{\pm}_{+} = -\stackrel{03}{\tilde{(\mp i)}}\stackrel{12}{\tilde{(\pm)}}, \qquad \tilde{N}^{\pm}_{-} = \stackrel{03}{\tilde{(\pm i)}}\stackrel{12}{\tilde{(\pm)}},$$
$$\tau^{1\pm} = (\mp)\stackrel{56}{(\pm)}\stackrel{78}{(\mp)}, \qquad \tau^{2\mp} = (\mp)\stackrel{56}{(\mp)}\stackrel{78}{(\mp)},$$
$$\tilde{\tau}^{1\pm} = (\mp)\stackrel{56}{\tilde{(\pm)}}\stackrel{78}{\tilde{(\mp)}}, \qquad \tilde{\tau}^{2\mp} = (\mp)\stackrel{56}{\tilde{(\mp)}}\stackrel{78}{\tilde{(\mp)}}. \qquad \text{(B20)}$$

I present at the end one Weyl representation of $SO(13+1)$ and the family quantum numbers of the two groups of four families.

One Weyl representation of $SO(13+1)$ contains left-handed weak charged and the second $SU(2)$ chargeless colored quarks and colorless leptons and right-handed weak chargeless, and the second $SU(2)$ charged quarks and leptons (electrons and neutrinos). It carries also the family quantum numbers, not mentioned in this table. The table is taken from Ref. [3,4].

The eight families of the first member of the eight-plet of quarks from Table III, for example, that is of the right handed u_{1R} quark, are presented in the left column of Table IV [3]. In the right column of the same table the equivalent eight-plet of the right-handed neutrinos ν_{1R} are presented. All of the other members of any of the eight families of quarks or leptons follow from any member of a particular family by the application of the operators $N^{\pm}_{R,L}$ and $\tau^{(2,1)\pm}$ on this particular member.

The eight-plets separate into two group of four families. One group contains doublets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$; these families are singlets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$. Another group of families contains doublets with respect to

$\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$; these families are singlets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$.

The scalar fields which are the gauge scalars of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ couple only to the four families which are doublets with respect to these two groups. The scalar fields which are the gauge scalars of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ couple only to the four families which are doublets with respect to these last two groups.

APPENDIX C: EXPRESSIONS FOR THE SPIN CONNECTION FIELDS IN TERMS OF VIELBEINS AND THE SPINOR SOURCES

The expressions for the spin connection of both kinds, $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$, in terms of the vielbeins and the spinor sources of both kinds are presented, obtained by a variation of action (1). The expression for the spin connection $\omega_{ab\alpha}$ is taken from Ref. [37]:

$$\begin{aligned}\omega_{ab\alpha} = &-\frac{1}{2E}\left\{e_{e\alpha}e_{b\gamma}\partial_\beta(Ef^{\gamma[e}f^{\beta}{}_{a]}) + e_{e\alpha}e_{a\gamma}\partial_\beta(Ef^{\gamma}{}_{[b}f^{\beta e]}) - e_{e\alpha}e^{e}{}_{\gamma}\partial_\beta(Ef^{\gamma}{}_{[a}f^{\beta}{}_{b]})\right\}\\ &-\frac{e_{e\alpha}}{4}\left\{\bar{\Psi}\left(\gamma_e S_{ab} + \frac{3i}{2}(\delta^e_b\gamma_a - \delta^e_a\gamma_b)\right)\Psi\right\}\\ &-\frac{1}{d-2}\left\{e_{a\alpha}\left[\frac{1}{E}e^{d}{}_{\gamma}\partial_\beta(Ef^{\gamma}{}_{[d}f^{\beta}{}_{b]}) + \frac{1}{2}\bar{\Psi}\gamma^d S_{db}\Psi\right] - e_{b\alpha}\left[\frac{1}{E}e^{d}{}_{\gamma}\partial_\beta(Ef^{\gamma}{}_{[d}f^{\beta}{}_{a]}) + \frac{1}{2}\bar{\Psi}\gamma^d S_{da}\Psi\right]\right\}.\end{aligned} \tag{C1}$$

One notices that if there are no spinor sources, carrying the spinor quantum numbers S^{ab}, then $\omega_{ab\alpha}$ is completely determined by the vielbeins.

Equivalently, one obtains expressions for the spin connection fields carrying family quantum numbers:

$$\begin{aligned}\tilde{\omega}_{ab\alpha} = &-\frac{1}{2E}\left\{e_{e\alpha}e_{b\gamma}\partial_\beta(Ef^{\gamma[e}f^{\beta}{}_{a]}) + e_{e\alpha}e_{a\gamma}\partial_\beta(Ef^{\gamma}{}_{[b}f^{\beta e]}) - e_{e\alpha}e^{e}{}_{\gamma}\partial_\beta(Ef^{\gamma}{}_{[a}f^{\beta}{}_{b]})\right\}\\ &-\frac{e_{e\alpha}}{4}\left\{\bar{\Psi}\left(\gamma_e \tilde{S}_{ab} + \frac{3i}{2}(\delta^e_b\gamma_a - \delta^e_a\gamma_b)\right)\Psi\right\}\\ &-\frac{1}{d-2}\left\{e_{a\alpha}\left[\frac{1}{E}e^{d}{}_{\gamma}\partial_\beta(Ef^{\gamma}{}_{[d}f^{\beta}{}_{b]}) + \frac{1}{2}\bar{\Psi}\gamma^d \tilde{S}_{db}\Psi\right] - e_{b\alpha}\left[\frac{1}{E}e^{d}{}_{\gamma}\partial_\beta(Ef^{\gamma}{}_{[d}f^{\beta}{}_{a]}) + \frac{1}{2}\bar{\Psi}\gamma^d \tilde{S}_{da}\Psi\right]\right\}.\end{aligned} \tag{C2}$$

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[28] ${f^\alpha}_a$ are inverted vielbeins to ${e^a}_\alpha$ with the properties ${e^a}_\alpha {f^\alpha}_b = {\delta^a}_b, {e^a}_\alpha {f^\beta}_a = \delta^\beta_\alpha$, $E = \det({e^a}_\alpha)$. Latin indices $a, b, \ldots, m, n \ldots, s, t, \ldots$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, \ldots, \mu, \nu, \ldots \sigma, \tau, \ldots$ denote an Einstein index (a curved index). Letters from the beginning of both of the alphabets indicate a general index ($a, b, c, \ldots$ and $\alpha, \beta, \gamma, \ldots$), from the middle of both of the alphabets the observed dimensions 0,1,2,3 ($m, n \ldots$ and $\mu, \nu, \ldots$), and indices from the bottom of the alphabets indicate the compactified dimensions ($s, t \ldots$ and $\sigma, \tau, \ldots$). We assume that the signature $\eta^{ab} = diag\{1, -1, -1, \ldots, -1\}$.

[29] A toy model [15,27,38] was studied in $d = (5 + 1)$ with the same action as in Eq. (1). For a particular choice of vielbeins and for a class of spin connection fields the manifold M^{5+1} breaks into $M^{(3+1)}$ times an almost S^2, while $2^{((3+1)/2-1)}$ families stay massless and mass protected. An equivalent assumption, although one not yet proven, is also made in the case in which $M^{(13+1)}$ first breaks into $M^{(7+1)} \times M^{(6)}$. The study is in progress.

[30] One can learn from Eq. (B10) of Appendix B that S^{ab} transforms one state of the representation into another state of the same representation, while $\tilde{S}^{ab}$ transforms the state into a state belonging to another representation.

[31] As long as the left-handed family members and their right-handed partners carry different conserved charges, they cannot behave as massive particles—they are mass protected. It is the appearance of nonzero vacuum expectation values of the scalar fields, carrying the weak and the hypercharge, which causes nonconservation of these two charges, that enables the superposition of the left- and right-handed family members, breaking the mass protection.

[32] These superpositions can easily be found by using Eqs. (10) and (9). They are explicitly written in Ref. [3]. The interaction with the condensate makes the fields $A^{Y'}_m$, Eq. (13), A^{21}_m, and A^{22}_m very massive (at the scale of the condensate).

[33] Indicating that S^{ab} and $\tilde{S}^{ab}$ belong to two different kinds of Clifford algebra objects are the indices (a, b) in $\tilde{S}^{ab}$, in this paragraph written as $(\tilde{a}, \tilde{b})$. Normally only (a, b) will be used for S^{ab} and $\tilde{S}^{ab}$.

[34] Although there are three scalar fields, each with the color charge of one of the triplet representations, and there are also two scalar fields, each with the weak charge of one of the doublet representations, neither a triplet nor a doublet have all of the other quantum numbers needed to be recognized as a supersymmetric partner of any family member.

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