Newtonian self-gravitation in the neutral meson system

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We derive the effect of the Schrödinger-Newton equation, which can be considered as a nonrelativistic limit of classical gravity, for a composite quantum system in the regime of high energies. Such meson-antimeson systems exhibit very unique properties, e.g., distinct masses due to strong and electroweak interactions. This raises an immediate question: what does one mean by mass in gravity for a state that is a superposition of mass eigenstates due to strong and electroweak interactions? We find conceptually different physical scenarios due to lacking of a clear physical guiding principle to explain which mass is the relevant one and due to the fact that it is not clear how the flavor wave function relates to the spatial wave function. There seems to be no principal contradiction. However, a nonlinear extension of the Schrödinger equation in this manner strongly depends on the relation between the flavor wave function and spatial wave function and its particular shape. In opposition to the continuous spontaneous localization collapse models we find a change in the oscillating behavior and not in the damping of the flavor oscillation.

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I. INTRODUCTION

The search for a theory that consistently combines quantum theory and gravitation is certainly one of the bigger challenges of contemporary theoretical physics. While most physicists believe that—whatever the correct quantum theory of gravity is—in the low-energy limit gravity can be described by a perturbative quantum field theory, in full analogy to the low-energy limit of quantum electrodynamics, there is no experimental evidence, to date, that rules out a theory in which gravity remains unquantized, even at the fundamental level. This idea has been raised by many before [1–4], and a behavior of gravity at the quantum level that is different from what one would expect by a naive perturbative quantization of the gravitational field is also discussed as a possible solution of the quantum measurement problem [5–12]. In the context of nonrelativistic quantum mechanics, the problem is basically condensed to the question of how quantum matter sources the gravitational field.

One hypothesis that has been brought into the debate [4,13–15] is that the gravitational interaction for nonrelativistic quantum matter is described by a nonlinear extension of the Schrödinger equation, the Schrödinger-Newton equation

$$\mathrm{i}\hbar\partial_t\psi(t,\mathbf{r}) = \left(-\frac{\hbar^2}{2m}\nabla^2 - Gm^2\int \mathrm{d}^3\mathbf{r}'\,\frac{|\psi(t,\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|}\right)\psi(t,\mathbf{r}), \quad (1)$$

originally proposed as a model for the localization of macroscopic quantum objects [5,10]. The intuition behind such an equation is that the absolute value squared of the wave function corresponds to a mass density sourcing a Newtonian gravitational potential [16]. The equation can also be shown to follow naturally as the nonrelativistic limit of a semiclassical theory of gravity, i.e., a theory in which the gravitational field stays classical even at the fundamental level and quantum matter is coupled by the semiclassical Einstein equations

$$R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}\langle\Psi|\hat{T}_{\mu\nu}|\Psi\rangle, \quad (2)$$

where $\hat{T}_{\mu\nu}$ is the energy-momentum operator and the expectation value is taken in some quantum state [15]. One particularly charming aspect of the Schrödinger-Newton equation is that it most likely can be experimentally tested in the foreseeable future, e.g., with large molecules [18,19] or with crystalline nanospheres [20,21].

In this paper we want to consider neutral meson-antimeson systems that are typically produced at accelerator facilities. In particular we focus for the sake of simplicity on the neutral K-meson system, also dubbed kaons; however, all considerations hold for all nonrelativistic meson systems. These massive systems, that can also be produced even in entangled pairs, have been shown to be a unique laboratory for precision measurements of particle properties and fundamental principles in particle physics (e.g., discrete symmetries) as well as for testing fundamental principles in quantum physics such as superposition and entanglement (for an overview see, e.g., Ref. [22]). For example, a violation of Bell's inequality that is only due to the breaking of a discrete symmetry resulting in a tiny difference between matter and antimatter properties has been discovered [23]. Or due to the existence of two distinct measurement procedures, a special feature of kaons, the very working of a quantum eraser [24,25] or Heisenberg's principle [26] can in a novel way be demonstrated. Proposals for how to test decoherence effects have been developed [27,28] and put to experimental tests [29–31]. Models testing for Lorentz-symmetry violations or assuming intrinsic violations of the *CPT* symmetry induced by quantum gravity [32] have been put to test for K-mesons [29,33]. Recently, also the prediction of collapse models was computed [34,35].

Despite the fact that these meson systems are elementary particle systems, for which one would expect that they must be theoretically treated with the tools of relativistic quantum field theories, the formalism of nonrelativistic quantum mechanics turns out to provide good predictions for almost all interesting effects in these systems.

At first sight neutral kaons, composite systems of a quark and an antiquark, seem not to be good candidates to test for gravitational effects since the mass is very low, approximately half of a proton mass; however, the unique properties of these meson-antimeson systems—as witnessed by the above literature—make it an interesting case to see whether conceptual contradictions can be derived. Let us here quote the famous Feynman lectures [36], where Feynman writes after introducing K-mesons:

> *"If there is any place where we have a chance to test the main principles of quantum mechanics in the purest way—does the superposition of amplitudes work or doesn't it?—this is it."*

The superposition of these two different mass eigenstates due to weak interaction exhibiting oscillations of the eigenstates of the strong interaction has been proven by now at many accelerator facilities. Moreover, since 1964 the unexpected breaking of the *CP* symmetry (*C*… charge conjugation, *P*… parity), a tiny difference between matter and antimatter properties, was discovered.

Taking the point of view that the Schrödinger-Newton equation correctly describes the coupling of quantum matter to gravity—keeping in mind that as a mere hypothesis it could be experimentally falsified at any time—one may immediately ask:

> *Do both eigenstates of the mass Hamiltonian couple independently to the gravitational field? Or is only the rest mass of the neutral K-meson the one relevant for any gravitational effect?*

In this contribution we analyze in detail which options to include a Schrödinger-Newton interaction in the neutral K-meson system are conceptually possible—if any—and which effects they may have on the flavor oscillations. We will briefly review the properties of the neutral kaon system in the second section. There we raise the important question of how the spatial wave function should be treated in the case of neutral mesons. We then discuss different ways to implement the features of the neutral kaon into the Schrödinger-Newton equation in the third section, accounting for the dependence on the right description of the spatial wave function. In the fourth section we compare these results to the previously obtained results for the continuous spontaneous localization (CSL) collapse model. Finally, we discuss the results and draw our conclusions.

II. THE NEUTRAL KAON SYSTEM

Via strong interactions one has to distinguish between two different eigenstates labeled by the strangeness number S, the kaon state $|K^0\rangle$ ($S=1$) and the antikaon $|\bar{K}^0\rangle$ ($S=-1$). Neutral kaons decay via the weak interaction leading to the following non-Hermitian Hamiltonian

$$H = \begin{pmatrix} \langle K^0|H^{(|\Delta S|=0)}|K^0\rangle & \langle \bar{K}^0|H^{(|\Delta S|=2)}|K^0\rangle \\ \langle K^0|H^{(|\Delta S|=2)}|\bar{K}^0\rangle & \langle \bar{K}^0|H^{(|\Delta S|=0)}|\bar{K}^0\rangle \end{pmatrix} = M - \frac{\mathrm{i}}{2}\mathbf{\Gamma} \tag{3}$$

where both the mass matrix M and decay matrix Γ are chosen to be Hermitian. $H^{(|\Delta S|=0)}$ describes the processes which conserve the strangeness number S and $H^{(|\Delta S|=2)}$ describes those which differ by two. The states diagonalizing this Hamiltonian are denoted as mass eigenstates, namely the short- and long-lived states $|K_S\rangle$ and $|K_L\rangle$. If we assume *CPT* conservation (T… time reversal), the two diagonal elements of M have to be equal and have to correspond to the rest mass m_K. Analogously, the two diagonal elements of Γ have to be equal and to correspond to the total decay width Γ of K^0, $\bar{K}^0$.

The complex eigenvalues of the Hamiltonian H are derived to

$$\lambda_{S/L} = m_{S/L} - \frac{\mathrm{i}}{2}\Gamma_{S/L} = m_K - \frac{\mathrm{i}}{2}\Gamma \mp \sqrt{\left(M_{12} - \frac{\mathrm{i}}{2}\mathbf{\Gamma}_{12}\right)\left(M_{12}^* - \frac{\mathrm{i}}{2}\mathbf{\Gamma}_{12}^*\right)} \tag{4}$$

and consequently the mass difference $\Delta m \coloneqq m_L - m_S$ and the decay width difference $\Delta\Gamma \coloneqq \Gamma_L - \Gamma_S$ are given by

$$\Delta m = 2\mathrm{Re}\left\{\sqrt{\left(M_{12} - \frac{\mathrm{i}}{2}\mathbf{\Gamma}_{12}\right)\left(M_{12}^* - \frac{\mathrm{i}}{2}\mathbf{\Gamma}_{12}^*\right)}\right\}$$
$$\Delta\Gamma = -4\mathrm{Im}\left\{\sqrt{\left(M_{12} - \frac{\mathrm{i}}{2}\mathbf{\Gamma}_{12}\right)\left(M_{12}^* - \frac{\mathrm{i}}{2}\mathbf{\Gamma}_{12}^*\right)}\right\}. \tag{5}$$

The mass difference Δm and the two decay widths Γ_S, Γ_L have been measured for all neutral meson systems [37];

however, only for neutral K-mesons the two decay widths differ greatly.

Thus, the time evolution of the mass-Hamilton eigenstates is given by [38]

$$
\begin{aligned}
|K_S(t)\rangle &= \mathrm{e}^{-\mathrm{i}m_S t}\mathrm{e}^{-\frac{\Gamma_S}{2}t}|K_S(t=0)\rangle \\
|K_L(t)\rangle &= \mathrm{e}^{-\mathrm{i}m_L t}\mathrm{e}^{-\frac{\Gamma_L}{2}t}|K_L(t=0)\rangle,
\end{aligned} \tag{6}
$$

preserving their identity in time. The mass eigenstates states are connected via the following basis transformation

$$
\begin{aligned}
|K_S\rangle &= \frac{(1+\varepsilon)|K^0\rangle - (1-\varepsilon)|\bar{K}^0\rangle}{\sqrt{2(1+|\varepsilon|^2)}} \\
|K_L\rangle &= \frac{(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\bar{K}^0\rangle}{\sqrt{2(1+|\varepsilon|^2)}}
\end{aligned} \tag{7}
$$

where ε is the CP violating parameter that equals in a conventional phase choice to

$$
\varepsilon = \frac{(M_{12} - \frac{\mathrm{i}}{2}\mathbf{\Gamma}_{12}) - (M_{12}^* - \frac{\mathrm{i}}{2}\mathbf{\Gamma}_{12}^*)}{(M_{12} - \frac{\mathrm{i}}{2}\mathbf{\Gamma}_{12}) + (M_{12}^* - \frac{\mathrm{i}}{2}\mathbf{\Gamma}_{12}^*)}. \tag{8}
$$

To understand the difference between the dynamical parameters Δm, $\Delta\Gamma$ and the CP violating parameter ε let us introduce two complex numbers X, Y by

$$
X^2 = \frac{M_{12}^* - \frac{\mathrm{i}}{2}\mathbf{\Gamma}_{12}^*}{M_{12} - \frac{\mathrm{i}}{2}\mathbf{\Gamma}_{12}}, \qquad \frac{Y}{X} = M_{12} - \frac{\mathrm{i}}{2}\mathbf{\Gamma}_{12}. \tag{9}
$$

With that we find (up to nonphysical sign changes)

$$
\begin{aligned}
\Delta m &= 2\mathrm{Re}\{Y\} \\
\Delta\Gamma &= -4\mathrm{Im}\{Y\} \\
\varepsilon &= \frac{1-X^2}{1+X^2}.
\end{aligned} \tag{10}
$$

Obviously the values Δm, $\Delta\Gamma$ are independent of ε in the sense that the value of X does not influence the value of these dynamical parameters; however, the time evolution does depend on all three parameters as we show explicitly in the following.

The probabilities of finding a K^0 or a $\bar{K}^0$ after a certain time t if a state $|K^0\rangle$ was produced at time $t=0$ is consequently given by

$$
\begin{aligned}
P(K^0 t; |K^0|) &= |\langle K^0|K^0(t)\rangle|^2 \\
&= \frac{1}{4}(\mathrm{e}^{-\Gamma_S t} + \mathrm{e}^{-\Gamma_L t} + 2\cos(\Delta m t)\cdot \mathrm{e}^{-\Gamma t})
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
P(\bar{K}^0 t; |K^0|) &= |\langle \bar{K}^0|K^0(t)\rangle|^2 \\
&= \frac{|1-\varepsilon|^2}{4|1+\varepsilon|^2}(\mathrm{e}^{-\Gamma_S t} + \mathrm{e}^{-\Gamma_L t} \\
&\quad - 2\cos(\Delta m t)\cdot \mathrm{e}^{-\Gamma t}).
\end{aligned} \tag{12}
$$

Taking the difference we derive the time-dependent asymmetry

$$
\frac{P(K^0 t; |K^0|) - P(\bar{K}^0 t; |K^0|)}{P(K^0 t; |K^0|) + P(\bar{K}^0 t; |K^0|)} = \frac{\frac{2\mathrm{Re}\{\varepsilon\}}{1+|\varepsilon|^2} + \frac{\cos(\Delta m t)}{\cosh(\frac{\Delta\Gamma}{2}t)}}{1 + \frac{2\mathrm{Re}\{\varepsilon\}}{1+|\varepsilon|^2}\frac{\cos(\Delta m t)}{\cosh(\frac{\Delta\Gamma}{2}t)}}, \tag{13}
$$

where for short times the oscillation is visible whereas in the long time limit the CP violation can be measured. In summary one observes a damped oscillation due to Δm and the decay constants where the mass m_K does not enter and for the huge time regime the difference of both probabilities reveals the tiny CP violation, namely $\frac{2\mathrm{Re}\{\varepsilon\}}{1+|\varepsilon|^2} \approx 10^{-3}$.

Let us remark on the nonrelativistic treatment of K-mesons as composite systems. Ordinarily, nonrelativistic systems have excitation energies that are small compared to the component masses. For meson systems, however, these energies are comparable to the quark masses of these models. The complexity of QCD forces one to resort to approximate models, so called bag models (see, e.g., the review article [39]). These bag models divide space into two regions, the interior of the bag in which the quarks have very small (current) masses and feel only weak forces and the exterior in which the quarks are not allowed to propagate having a different (lower) vacuum energy. In the above presented phenomenology, mesons are treated as a single entity, in very good agreement to all current experiments. Relativistic effects—such as the speed of the mesons—do not alter the physics in the flavor space; thus, we do not consider any particular relativistic effects in the following treatment.

Certainly, we can be interested in the kinematics of the neutral meson system; then the relevant Hamiltonian would be

$$
H_{\mathrm{kin}} = m_{\mathrm{inert}}c^2 - \frac{\hbar^2}{2m_{\mathrm{inert}}}\nabla^2. \tag{14}
$$

The inertial mass m_{inert} could be considered as that of the composite K-meson, i.e., m_K, or one may assume that each mass-energy eigenstate $K_{S/L}$ exhibits a different kinetic/spatial wave function due to its different decaying property, namely $m_{S/L}$. Note that $m_{S/L}$ alone—contrary to the difference Δm—has also contributions of m_K; see Eq. (4). Differently stated, considering the spatial wave function of mesons:

Do we have to consider only one unique wave function or do we have to handle it as a two-state system?

We will differentiate between those two scenarios in the following.

Before we proceed let us comment on the validity of a nonrelativistic treatment of the K-meson via the Schrödinger equation. The first point to mention is that we are interested in space-dependent probabilities measuring the strangeness content, i.e., probabilities that a neutral kaon, having propagated a certain macroscopic distance, decays semileptonically or is forced by a matter block to reveal over the subsequent reaction the strangeness property. Flavor oscillation, i.e., the probabilities of observing a particle or antiparticle state at a certain position/time, is the phenomenon one is interested in. This distance is usually converted into a proper time (via $\tau \approx \frac{x}{v}$). Here the relativistic effects matter but of course in a trivial way. Given the Hamiltonian above, the Schrödinger equation is the appropriate nonrelativistic limit of either the Klein-Gordon equation or Dirac equation keeping the rest mass energy. There have been numerous approaches to how the spatial wave function of a strangeness state—that has to be a coherent state composed of a short-lived and long-lived state—has to be treated in an unambiguous way [40–43] (and for the neutrino oscillations, see, e.g., [44]). The main disagreement is whether the energies and/or momenta of the two different mass eigenstates remain unchanged. However, the final probabilities, i.e., Eq. (11), are—within good approximations—always the same, that are also those well tested in various experiments. The authors of [40] apply a different view by defining the probabilities as position measurements with averages over times. In summary, the key observation is that experiments are only sensitive to the mass difference and not sum of mass or energy; thus, experiments are not restricted to favor one approach over the other one and only conceptual arguments apply. Modifications of the standard quantum approach such as the here discussed Schrödinger-Newton equation or collapse models [34,35], however, require an explicit modeling of the relation between the spatial wave function and the flavor space and, therefore, offer a unique laboratory to enlighten the subtle interplay arising from particle-antiparticle mixing.

III. SCHRÖDINGER-NEWTON EQUATION FOR THE NEUTRAL KAON

The Schrödinger-Newton equation is based on the assumption that the wave function sources a gravitational field as if the total mass of the particle would be smeared with the spatial probability density $|\psi|^2$. If the Schrödinger-Newton equation is applied to the kaon system this raises two questions:

(i) Which is the right mass that acts as the source of the gravitational field?

(ii) How can one describe the spatial wave function of the neutral kaon?

And particularly, does this lead to any inconsistencies or unexpected effects that could render the Schrödinger-Newton equation an ill-defined model for describing these systems, or on the contrary lead to observable effects?

Due to Newton the mass entering into the kinetic term of the Hamiltonian is the inertial mass. In the famous experiment with neutrons the authors of Ref. [45] demonstrated that nonrelativistic quantum matter can couple to the gravitational field in the following way:

$$i\hbar\partial_t\psi(t,\mathbf{r}) = \left(-\frac{\hbar^2}{2m_{\text{inert}}}\nabla^2 + m_{\text{grav}}^{\text{passive}}\Phi_{\text{grav}}\right)\psi(t,\mathbf{r}). \tag{15}$$

Here Φ_{grav} is the gravitational potential, and $m_{\text{grav}}^{\text{passive}}$ is the passive gravitational mass, i.e., the coupling of matter to the gravitational field. If the weak equivalence principle holds, these two masses must be equal: $m_{\text{grav}}^{\text{passive}} = m_{\text{inert}}$. Let us remark here that so far in all situations that have been put to test by experiments, Φ_{grav} belongs to an *external* gravitational field.

In the framework of the Schrödinger-Newton equation, however, Φ_{grav} yields the gravitational self-interaction. With the assumptions underlying the Schrödinger-Newton equation the gravitational potential satisfies the Poisson equation

$$\nabla^2\Phi_{\text{grav}} = 4\pi G m_{\text{grav}}^{\text{active}}|\psi(t,\mathbf{r})|^2. \tag{16}$$

The mass entering here as the source of the gravitational field is referred to as the active gravitational mass. Implying conservation of momentum, one can show that $m_{\text{grav}}^{\text{active}} = m_{\text{grav}}^{\text{passive}}$. For classical gravity, this follows simply from Newton's third law, while in the quantum case it can be shown by considering the conserved momentum of a two-particle state, described by the two-particle Schrödinger-Newton equation given in [5,14].

Interestingly, Eq. (16) can be shown to follow from the semiclassical Einstein equations (2) as derived in Refs. [13,15]. The mass density on the right-hand side is then given by the expectation value of the nonrelativistic limit of the energy-momentum operator,

$$m_{\text{grav}}^{\text{active}}|\psi|^2 = \frac{1}{c^2}\langle\psi|\hat{T}_{00}|\psi\rangle. \tag{17}$$

Following this logic, from a quantum field theoretical point of view the mass $m_{\text{grav}}^{\text{active}}$ would be the one appearing in the mass term of the field Lagrangian (after renormalization). But the kaon is a composite system and its mass is mainly binding energy of quarks.

Usually, one assumes that all three masses, inertial as well as active and passive gravitational mass, correspond to

$m_K = (497.614 \pm 0.024)\ \mathrm{MeV}c^{-2}$, the measurable invariant mass of the neutral kaon. Whereas the flavor eigenstates $|K^0\rangle, |\bar{K}^0\rangle$ have equal mass (assuming CPT symmetry) with the value m_K, the long-lived and short-lived eigenstates $|K_S\rangle$, $|K_L\rangle$ of the strong and weak interaction Hamiltonian manifest a small mass difference $\Delta m = (3.483 \pm 0.006)\ \mathrm{MeV}c^{-2}$ that is at another energy scale, as discussed in the previous section.

So, does this mass difference Δm show up in the Schrödinger-Newton equation? If so, for which of the three masses does it show up, and what would be the consequences?

As indicated in the previous section, similar conceptual questions arise concerning the spatial wave function $\psi(\mathbf{r})$ of the nonrelativistic neutral kaon. Again, one could simply consider a Schrödinger equation with one unique spatial wave function that evolves with the invariant mass, m_K, in the kinetic term. But since the states $|K_S\rangle$ and $|K_L\rangle$ diagonalize the Hamiltonian, there should in principle be different wave functions $\psi_S(\mathbf{r})$ and $\psi_L(\mathbf{r})$ evolving with the masses m_S and m_L, respectively. Because of the difference in the free spreading of the wave function due to the mass difference one would then expect additional flavor oscillations in space.

Therefore, we will in the following distinguish two scenarios and discuss their implications.

Scenario 1: Unique spatial wave function.—Let us first assume that we can treat $|K_S\rangle$ and $|K_L\rangle$ as if they would mix only in flavor space, while they are described by one unique spatial wave function. The total wave function is then

$$\psi_{\mathrm{flavor}} \otimes \psi_{\mathrm{space}} = (\alpha(t)\psi_{\mathrm{S,flavor}} + \beta(t)\psi_{\mathrm{L,flavor}}) \otimes \psi_{\mathrm{space}}, \quad (18)$$

where ψ_{space} denotes the spatial wave function and ψ_{flavor} the flavor part. $\psi_{\mathrm{S,flavor}}$ and $\psi_{\mathrm{L,flavor}}$ denote a short-lived and long-lived kaon, respectively. ψ_{space} satisfies the Schrödinger-Newton equation for the mass $m^{\mathrm{inert}} = m_{\mathrm{grav}}^{\mathrm{passive}} = m_K$. The time-dependent coefficients α and β represent the decomposition of the current kaon state in the basis $\{|K_S\rangle, |K_L\rangle\}$. They are time dependent due to the flavor oscillations, i.e.,

$$\alpha(t) = \alpha_0 \mathrm{e}^{-\mathrm{i}\Delta m t}, \qquad \beta(t) = \beta_0 \mathrm{e}^{-\mathrm{i}\Delta m t}. \quad (19)$$

If the active gravitational mass does not depend on the flavor part, the equation can be separated and the spatial wave function will simply satisfy the Schrödinger-Newton equation for the kaon mass m_K, independent of its composition of $|K_S\rangle$ and $|K_L\rangle$.

If, on the other hand, $m_{\mathrm{grav}}^{\mathrm{active}}$ does depend on the flavor part, the most naive ansatz would be $m_{\mathrm{grav}}^{\mathrm{active}} = |\alpha(t)|^2 m_S + |\beta(t)|^2 m_L$. The active gravitational mass is then time dependent, and slightly different from the inertial and passive gravitational mass m_K, and only the time-averaged momentum is conserved. One then obtains the Schrödinger-Newton equation

$$\mathrm{i}\hbar\partial_t\psi_{\mathrm{space}}(t,\mathbf{r}) = \left(H_{\mathrm{kin}}(m_K) - G m_K m_S|\alpha(t)|^2 \int \mathrm{d}^3\mathbf{r}' \frac{|\psi_{\mathrm{space}}(t,\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|} - G m_K m_L |\beta(t)|^2 \int \mathrm{d}^3\mathbf{r}' \frac{|\psi_{\mathrm{space}}(t,\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|}\right)\psi_{\mathrm{space}}(t,\mathbf{r}). \quad (20)$$

Since the mass difference $\Delta m = m_L - m_S$ is small, the effect is only a tiny modification of the already unmeasurable gravitational self-interaction.

Scenario 2: Different spatial wave functions.—Now let us consider the case assuming that $|K_S\rangle$ and $|K_L\rangle$ have different wave functions also in space. Therefore, the total wave function is given by

$$\psi_{\mathrm{flavor}} \otimes \psi_{\mathrm{space}} = \alpha(t)\psi_{\mathrm{S,flavor}} \otimes \psi_{\mathrm{S,space}} + \beta(t)\psi_{\mathrm{L,flavor}} \otimes \psi_{\mathrm{L,space}}. \quad (21)$$

Each of the spatial wave functions will contribute to the total gravitational potential, and both wave functions will see this same gravitational potential. Therefore, we get the following two Schrödinger-Newton equations:

$$\mathrm{i}\hbar\partial_t\psi_S(t,\mathbf{r}) = \left(H_{\mathrm{kin}}(m_S) - G m_S^2 |\alpha(t)|^2 \int \mathrm{d}^3\mathbf{r}' \frac{|\psi_S(t,\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|} - G m_S m_L |\beta(t)|^2 \int \mathrm{d}^3\mathbf{r}' \frac{|\psi_L(t,\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|}\right)\psi_S(t,\mathbf{r}) \quad (22a)$$

$$\mathrm{i}\hbar\partial_t\psi_L(t,\mathbf{r}) = \left(H_{\mathrm{kin}}(m_L) - G m_S m_L |\alpha(t)|^2 \int \mathrm{d}^3\mathbf{r}' \frac{|\psi_S(t,\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|} - G m_L^2 |\beta(t)|^2 \int \mathrm{d}^3\mathbf{r}' \frac{|\psi_L(t,\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|}\right)\psi_L(t,\mathbf{r}), \quad (22b)$$

where we write $\psi_{S,L}$ for the spatial wave functions of the short- and long-lived contributions, respectively.

As discussed in the previous section, both scenarios are compatible with what has been experimentally tested so far, although there seems to be a slight preference for this latter scenario in the literature. Note that this second scenario, where K_S and K_L evolve with different spatial wave functions, is also the only one that is compatible with a derivation of the Schrödinger-Newton equation from a doublet-state formalism; cf. the Appendix.

A. Resulting wave-function dynamics

In experimental situations, the kaon is usually not well localized. Therefore, the wave function is usually assumed to be a plane wave, in very good agreement with the experiment. However, to determine the effect of the Schrödinger-Newton equation, the localization of the wave function must be taken into account. We will therefore model it by a spherically symmetric Gaussian:

$$\psi^f(t,r;m,a) = (\pi a^2)^{-3/4}\left(1+\frac{\mathrm{i}\hbar t}{ma^2}\right)^{-3/2}\exp\left(-\frac{r^2}{2a^2(1+\frac{\mathrm{i}\hbar t}{ma^2})}\right). \tag{23}$$

This is the solution of the free Schrödinger equation, where the width, a, will be assumed to be large. In general, the Schrödinger-Newton dynamics disturb the Gaussian shape of the wave function. Since the gravitational interaction is very weak due to the large value of a and the small mass m, we will approximate the wave function appearing as the mass density in the gravitational potential by the free solution (23). The approximated gravitational potential is then $\Phi_{\mathrm{grav}} = -Gm_{\mathrm{grav}}^{\mathrm{active}} f(t,r;m,a)$ with

$$f(t,r;m,a) = \int \mathrm{d}^3\mathbf{r}' \frac{|\psi^f(t,\mathbf{r}';m,a)|^2}{|\mathbf{r}-\mathbf{r}'|} = \frac{1}{r}\mathrm{erf}\left[\frac{r}{a}\left(1+\frac{\hbar^2t^2}{m^2a^4}\right)^{-1/2}\right]. \tag{24}$$

The function f could now be expanded in terms of the mass, yielding

$$f(t,r;m,a) = \frac{2am}{\sqrt{\pi}\hbar t} + \mathcal{O}(m^3), \tag{25}$$

or in terms of time, yielding

$$f(t,r;m,a) = \frac{\mathrm{erf}(r/a)}{r} - \frac{\hbar^2}{\sqrt{\pi}m^2a^5}\exp\left(-\frac{r^2}{a^2}\right)t^2 + \mathcal{O}(t^4). \tag{26}$$

Here, however, we choose an expansion around $a=\infty$, which is justified in all usual experimental situations—which is why one usually assumes plane-wave solutions. This approximation yields

$$f(t,r;m,a) = \frac{2}{\sqrt{\pi}a}\left(1-\frac{r^2}{3a^2}+\frac{r^4}{10a^4}\right) - \frac{\hbar^2t^2}{\sqrt{\pi}m^2a^5} + \mathcal{O}(a^{-7}), \tag{27}$$

which is time independent up to order a^{-5}.

Scenario 1: Unique spatial wave function.—In the case of Eq. (20) we have

$$\mathrm{i}\hbar\partial_t\psi = H_{\mathrm{kin}}(m_K)\psi - Gm_K(|\alpha|^2m_S + |\beta|^2m_L)f(t,r;m_K,a)\psi. \tag{28}$$

If we then write $m_S = m$, $m_L = m+\Delta m$ and use the expansion (27) we get

$$\mathrm{i}\hbar\partial_t\psi = H_{\mathrm{kin}}(m_K)\psi - \frac{2Gm_K}{\sqrt{\pi}a}(m_K + |\beta(t)|^2\Delta m)\psi. \tag{29}$$

This is time dependent only through the coefficient β. We also used (4) to replace m by m_K in the approximation.

Scenario 2: Different spatial wave function.—If we assume two different wave functions, as in Eqs. 22, we can write them as

$$\mathrm{i}\hbar\partial_t\psi_S = H_{\mathrm{kin}}(m_S)\psi_S - Gm_S^2|\alpha(t)|^2 f(t,r;m_S,a)\psi_S - Gm_Sm_L|\beta(t)|^2 f(t,r;m_L,a)\psi_S \tag{30a}$$

$$\mathrm{i}\hbar\partial_t\psi_L = H_{\mathrm{kin}}(m_L)\psi_S - Gm_Sm_L|\alpha(t)|^2 f(t,r;m_S,a)\psi_L - Gm_L^2|\beta(t)|^2 f(t,r;m_L,a)\psi_L. \tag{30b}$$

If we then again write $m_S = m$, $m_L = m+\Delta m$ and use the expansion (27) we get

$$\mathrm{i}\hbar\partial_t\psi_S = H_{\mathrm{kin}}(m)\psi_S - \frac{2Gm^2}{\sqrt{\pi}a}\left[1+\left(\frac{1}{m}+\frac{\hbar^2t^2}{m^3a^4}\right)|\beta(t)|^2\Delta m\right]\psi_S \tag{31a}$$

$$\mathrm{i}\hbar\partial_t\psi_L = H_{\mathrm{kin}}(m_L)\psi_L - \frac{2Gm_L^2}{\sqrt{\pi}a}\left[1+\left(\frac{1}{m}+\frac{\hbar^2t^2}{m^3a^4}\right)|\beta(t)|^2\Delta m\right]\psi_L. \tag{31b}$$

B. Gravity-induced energy shift

Above we obtained the nonlinear Schrödinger equation which describes the dynamics of the kaon system in the presence of gravity, and therefore an approximation of the space-dependent Hamiltonian. The Hamiltonian that governs the flavor oscillations is modified by the energy shift due to the gravitational interaction. In order to calculate this energy shift, we must consider the expectation value of the Hamiltonian. This expectation value is proportional to

$$\langle\psi^f|f(t,r;m,a)|\psi^f\rangle = \sqrt{\frac{2}{\pi a^2}}\left(1+\frac{\hbar^2 t^2}{m^2 a^4}\right)^{-1/2}, \quad (32)$$

where we approximated the wave function by the solution of the free Schrödinger equation, as previously explained. For the mass $m+\Delta m$ we can expand this and obtain up to first order in Δm:

$$\langle\psi^f|f(t,r;m+\Delta m,a)|\psi^f\rangle = \langle\psi^f|f(t,r;m,a)|\psi^f\rangle + \frac{\hbar^2 t^2}{m^3 a^4}\left(1+\frac{\hbar^2 t^2}{m^2 a^4}\right)^{-3/2}\Delta m. \quad (33)$$

Scenario 1: Unique spatial wave function.—From Eq. (28) we get the energy shift

$$\Delta E = -Gm_K(|\alpha|^2 m_S + |\beta|^2 m_L) \times \langle\psi^f|f(t,r;m_K,a)|\psi^f\rangle = -\sqrt{\frac{2}{\pi}}\frac{Gm_K}{a}(m+|\beta|^2\Delta m) + \mathcal{O}(a^{-6}). \quad (34)$$

Both states $|K_S\rangle$ and $|K_L\rangle$ obtain a constant energy shift, but these only yield a constant phase shift. The contribution proportional to Δm, however, adds to the flavor oscillations (11). It acts like a shift of the mass difference:

$$\Delta m \to (1-\Delta_{SN})\Delta m \quad (35)$$

with

$$\Delta_{SN} = \sqrt{\frac{2}{\pi}}\frac{Gm_K}{c^2 a}. \quad (36)$$

Scenario 2: Different spatial wave function.—From Eqs. (30) one obtains

$$\Delta E_S = -\sqrt{\frac{2}{\pi}}\frac{Gm}{a}[m+|\beta|^2\Delta m] + \mathcal{O}(a^{-4}) \quad (37a)$$

$$\Delta E_L = -\sqrt{\frac{2}{\pi}}\frac{Gm}{a}[m+(1+|\beta|^2)\Delta m] + \mathcal{O}(a^{-4}), \quad (37b)$$

where higher order terms in $1/a$ have been omitted. The shift in Δm therefore is twice the one before:

$$\Delta m \to (1-2\cdot\Delta_{SN})\Delta m, \quad (38)$$

where we assume that $m\approx m_K$. Inserting the kaon mass, one finds that a large effect is only expected if the wave function becomes close to or narrower than about 10^{-54} m, far below the Planck length. This result does not change for other meson types.

Hence, we conclude that, although the effect is unobservably small for the kaon, the resulting effect depends on the treatment of the spatial wave function, either by a unique wave function or by different wave functions for the different mass eigenstates. Let us also remark here that the effect does depend on the particular shape of the spatial wave function about which not much is known from the experiments. Thus, dedicated experiments to study the spatial wave function of mesons would help to understand the right treatment for possible nonlinear modifications of the Schrödinger equation.

In addition to the energy shift, the Schrödinger-Newton equation of course also leads to the usual localization of the wave function as it has been described in [18], which is a very small effect due to the weakness of the gravitational self-interaction in the situation at hand (large wave function, small masses).

IV. COMPARISON TO THE CSL COLLAPSE MODEL

Collapse models [17] predict the spontaneous collapse of the wave function, in order to avoid the emergence of macroscopic superpositions. In their mass-dependent formulation, they claim that the collapse of any system's wave function depends on its mass. Recently, the most popular collapse model, the mass-proportional CSL model was applied to the meson-antimeson systems [35]. Here, the crucial point was again to connect the spatial with the flavor wave function part. The authors chose the sum of the kinetic contribution of the short- and long-lived components. After a cumbersome computation solving the stochastic nonlinear differential equation they found the following probability

$$P(K^0 t;|K^0|) = \frac{1}{4}\Bigg(\mathrm{e}^{-\Gamma_S t} + \mathrm{e}^{-\Gamma_L t} + 2\cos(\Delta m t)\mathrm{e}^{-\Gamma t}\underbrace{\mathrm{e}^{-\frac{\gamma\Delta m^2}{16\pi^{3/2} r_C^3 m_0^2}t}}_{\text{effect due to CSL model}}\Bigg) \quad (39)$$

where the collapse rate γ and the coherence length r_C are parameters of the collapse model and m_0 is a reference mass. Thus, in opposition to the solution of the Schrödinger-Newton equation, the mass-dependent collapse effect leads to a damping proportional to Δm^2 and has, consequently, to be compared to decoherence effects.

V. SUMMARY AND CONCLUSIONS

The aim of this contribution was to investigate how Newtonian self-gravitation can be included in the standard framework to handle flavor oscillations of neutral meson systems. Provided that such a hypothetical nonlinear modification of the quantum dynamics due to gravity would be correct, it is not straightforwardly physically intuitive which mass is the relevant one for the different terms in the Hamiltonian. Moreover, for any nonlinear extension of the Schrödinger evolution, as in the case of the Schrödinger-Newton equation or spatial collapse models, one has to assume a certain relation between the spatial and flavor wave functions. We have considered two possible scenarios, a separable and entangled ansatz, and derived the effect of the Schrödinger-Newton equation under certain assumptions. Although the effect turns out to be unobservably small, we find that there is a conceptually different result for the two scenarios, namely a shift in energy which is twice the one for the scenario of a unique wave function of the two mass-energy eigenstates. The correct treatment of the spatial wave function in the quantum mechanical description of nonrelativistic elementary particle systems therefore is a crucial question, which deserves further consideration, independently of the question if the kind of self-gravitation discussed in this paper actually exists.

Let us remark that the non-Hermitian part of the Hamiltonian was not considered or affected by the Schrödinger-Newton equation. This is consistent with the results in Ref. [46] where the authors showed that the non-Hermitian part can be removed by changing the Schrödinger equation into a Lindblad equation by extending the Hilbert space. Also the *CP* violation in mixing was not affected by self-gravitation since it does not enter in the eigenvalues of the considered Hamiltonian. It is not clear what happens with direct *CP* violating effects, i.e., where the violation occurs on the amplitude level.

Let us also mention that neutrinos exhibit a similar oscillation feature in flavor space; hence, the same conceptual problems apply to this case, although the relevant masses are even smaller.

Our current understanding of the nonrelativistic limit of particle physics, as well as the foundations of the Schrödinger-Newton equation (provided it is correct), is not sufficient to derive the Schrödinger-Newton effects unambiguously. In particular, there is no definite answer for which are the relevant masses and how to treat the spatial wave function. However, none of the possibilities we discussed leads to any conceptional contradictions between the Schrödinger-Newton equation and the treatment of kaons as a nonrelativistic quantum system. In all considered cases the effects within our assumptions were found to be irrelevant in practical situations; therefore, no contradictions to already performed or envisaged experiments in elementary particle physics seem to exist.

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APPENDIX: DERIVATION OF THE SCHRÖDINGER-NEWTON EQUATION FROM A DOUBLET-STATE FORMALISM

In [15] the Schrödinger-Newton equation has been derived from semiclassical gravity for a scalar field. In the nonrelativistic limit, one ends up with the gravitational interaction Hamiltonian

$$\hat{H}_{\text{int}} = -G \int \mathrm{d}^3 r \mathrm{d}^3 r' \frac{\langle \Psi | \hat{\varrho}(\mathbf{r}') | \Psi \rangle}{|\mathbf{r} - \mathbf{r}'|} \hat{\varrho}(\mathbf{r}), \tag{A1}$$

where $\hat{\varrho} = m\hat{\psi}^\dagger \hat{\psi}$ is the mass-density operator for only one kind of particles. This, together with the kinetic energy operator

$$\hat{T} = \int \mathrm{d}^3 r \hat{\psi}^\dagger(\mathbf{r}) \left(-\frac{\hbar^2}{2m} \nabla^2 \right) \hat{\psi}(\mathbf{r}), \tag{A2}$$

yields the Schrödinger-Newton equation in second quantized formalism,

$$\mathrm{i}\hbar \partial_t |\Psi\rangle = (\hat{T} + \hat{H}_{\text{int}}) |\Psi\rangle. \tag{A3}$$

The standard, first quantized Schrödinger-Newton equation follows for the one-particle state

$$|\Psi\rangle = \int \mathrm{d}^3 r \Psi(t, \mathbf{r}) \hat{\psi}^\dagger(\mathbf{r}) |0\rangle, \tag{A4}$$

where $\Psi(t, \mathbf{r})$ is the spatial wave function.

Let us extend this to a doublet field, as in the case of the neutral kaon. Hence, we consider the one-particle states

$$|\Psi\rangle = \int \mathrm{d}^3 r \begin{pmatrix} \Psi_1(t, \mathbf{r}) \hat{\psi}_1{}^\dagger(\mathbf{r}) \\ \Psi_2(t, \mathbf{r}) \hat{\psi}_2{}^\dagger(\mathbf{r}) \end{pmatrix} |0\rangle, \tag{A5}$$

where the indices 1 and 2 denote a not yet specified orthonormal basis in the space of the field operators. Note that this in principle covers the case where both fields evolve differently in space, as well as the case where both have the same unique distribution in space. The latter case is obtained by demanding $\Psi_1 \equiv \Psi_2$. The normalization condition $\langle \Psi | \Psi \rangle = 1$ requires $|\Psi_1|^2 + |\Psi_2|^2 = 1$.

We will not specify the form of the kinetic energy and mass-density operators, yet, but make the general ansatz

$$\hat{T} = \int \mathrm{d}^3 r \begin{pmatrix} \hat{\psi}_1(\mathbf{r}) \\ \hat{\psi}_2(\mathbf{r}) \end{pmatrix}^\dagger \begin{pmatrix} \hat{T}_{11} & \hat{T}_{12} \\ \hat{T}_{21} & \hat{T}_{22} \end{pmatrix} \begin{pmatrix} \hat{\psi}_1(\mathbf{r}) \\ \hat{\psi}_2(\mathbf{r}) \end{pmatrix} \tag{A6}$$

$$\hat{\rho}(\mathbf{r}) = \begin{pmatrix} \hat{\psi}_1(\mathbf{r}) \\ \hat{\psi}_2(\mathbf{r}) \end{pmatrix}^\dagger \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} \hat{\psi}_1(\mathbf{r}) \\ \hat{\psi}_2(\mathbf{r}) \end{pmatrix}, \tag{A7}$$

where $\hat{T}_{ij}$ are operators acting on the spatial wave function. Applied to the state (A5) these yield

$$\hat{T}|\Psi\rangle = \int \mathrm{d}^3 r \begin{pmatrix} \hat{T}_{11}\Psi_1(t,\mathbf{r})\hat{\psi}_1^\dagger + \hat{T}_{21}\Psi_1(t,\mathbf{r})\hat{\psi}_2^\dagger \\ \hat{T}_{12}\Psi_2(t,\mathbf{r})\hat{\psi}_1^\dagger + \hat{T}_{22}\Psi_2(t,\mathbf{r})\hat{\psi}_2^\dagger \end{pmatrix} |0\rangle \tag{A8}$$

$$\hat{\rho}(\mathbf{r})|\Psi\rangle = \begin{pmatrix} M_{11}\Psi_1(t,\mathbf{r})\hat{\psi}_1^\dagger + M_{21}\Psi_1(t,\mathbf{r})\hat{\psi}_2^\dagger \\ M_{12}\Psi_2(t,\mathbf{r})\hat{\psi}_1^\dagger + M_{22}\Psi_2(t,\mathbf{r})\hat{\psi}_2^\dagger \end{pmatrix} |0\rangle, \tag{A9}$$

and the expectation value of the mass-density operator is

$$\langle\Psi|\hat{\rho}(\mathbf{r})|\Psi\rangle_t = M_{11}|\Psi_1(t,\mathbf{r})|^2 + M_{22}|\Psi_2(t,\mathbf{r})|^2. \tag{A10}$$

By inserting these expressions into the second quantized Schrödinger-Newton equation (A3), and multiplying with $\langle 0|\hat{\psi}_{1,2}$ from the left, one obtains the following coupled system of equations:

$$\begin{aligned} \mathrm{i}\hbar\partial_t\Psi_1(t,\mathbf{r}) = {} & \hat{T}_{11}\Psi_1(t,\mathbf{r}) \\ & - GM_{11}\int \mathrm{d}^3 r' \left(M_{11}\frac{|\Psi_1(t,\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|} + M_{22}\frac{|\Psi_2(t,\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|}\right)\Psi_1(t,\mathbf{r}) \end{aligned} \tag{A11a}$$

$$\begin{aligned} \mathrm{i}\hbar\partial_t\Psi_2(t,\mathbf{r}) = {} & \hat{T}_{22}\Psi_2(t,\mathbf{r}) \\ & - GM_{22}\int \mathrm{d}^3 r' \left(M_{11}\frac{|\Psi_1(t,\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|} + M_{22}\frac{|\Psi_2(t,\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|}\right)\Psi_2(t,\mathbf{r}) \end{aligned} \tag{A11b}$$

$$\begin{aligned} 0 = {} & \hat{T}_{21}\Psi_1(t,\mathbf{r}) \\ & - GM_{21}\int \mathrm{d}^3 r' \left(M_{11}\frac{|\Psi_1(t,\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|} + M_{22}\frac{|\Psi_2(t,\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|}\right)\Psi_1(t,\mathbf{r}) \end{aligned} \tag{A11c}$$

$$\begin{aligned} 0 = {} & \hat{T}_{12}\Psi_2(t,\mathbf{r}) \\ & - GM_{12}\int \mathrm{d}^3 r' \left(M_{11}\frac{|\Psi_1(t,\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|} + M_{22}\frac{|\Psi_2(t,\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|}\right)\Psi_2(t,\mathbf{r}). \end{aligned} \tag{A11d}$$

In the case of free evolution, without gravity, the last two equations require the off-diagonal terms of the kinetic energy operator to vanish. The first two equations are completely decoupled in this case. Therefore, at this point, no restriction on the relation of the two wave functions can be made. In particular, it is possible that $\Psi_1 \equiv \Psi_2$, which then implies $\hat{T}_{11} = \hat{T}_{22}$.

However, if the Schrödinger-Newton term is considered, the off-diagonal elements of the kinetic energy operator vanish if and only if the mass matrix is diagonal. The self-gravitational interaction also leads to a coupling of Eqs. (A11a) and (A11b). Having $\Psi_1 \equiv \Psi_2$ then not only requires $\hat{T}_{11} = \hat{T}_{22}$ but also $M_{11} = M_{22}$.

Hence, if there is a mass difference of the states, as for the neutral kaon system, a doublet-state formalism is only compatible with the Schrödinger-Newton equation if both states evolve in with their respective masses, having different wave functions $\Psi_1 \neq \Psi_2$.

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