Quasimatter domination parameters in bouncing cosmologies

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A fine set of parameters is introduced for bouncing cosmologies in order to describe the nearly matter-dominated phase, and which play the same role that the usual slow-roll parameters play in inflationary cosmology. It is shown that, as in the inflation case, the spectral index and the running parameter for scalar perturbations in bouncing cosmologies can be best expressed in terms of these small parameters. Further, they explicitly exhibit the duality that exists between a nearly matter-dominated universe in its contracting phase and the quasi–de Sitter regime in the expanding one. The results obtained also confirm and extend the known evidence that the spectral index for an exactly matter-dominated universe (i.e., a pressureless universe) in the contracting phase is, in fact, the same as the spectral index for an exact de Sitter regime in the expanding phase. Finally, in both the inflationary and the matter bounce scenarios, the theoretical values of the spectral index and of the running parameter are compared with their experimental counterparts, obtained from the most recent Planck data, with the result that the bouncing models discussed here fit accurate astronomical observations well.

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I. INTRODUCTION

Matter bounce scenarios (MBSs) [1] are characterized by the Universe being matter dominated at very early times in the contracting phase and evolving towards a bounce, then entering into an expanding regime, where it matches the behavior of the standard hot Friedmann universe. They constitute a viable alternative to the inflationary paradigm.

It is also well known that matter domination in the contracting phase leads to the same spectral index, n_s, as for the case of the de Sitter regime in the expanding universe, namely, $n_s = 1$ [2]. This value does not agree with the experimental one, $n_s = 0.9603 \pm 0.0073$, which has been obtained from the most recent Planck data [3]. In contrast, this observational value can actually be accounted for in inflationary cosmology, because the Universe does not inflate exactly following a de Sitter regime. Instead, the inflaton field slow roll in its potential drives the Universe to a quasi–de Sitter stage. In such a slow-roll regime, the leading perturbative term of the spectral index depends on two small parameters, so-called slow-roll parameters [4], which are obtained explicitly as functions of the potential and its derivatives. By conveniently fitting these parameters, one is able to match the theoretical value of the spectral index with the corresponding experimental one.

Following the inflationary paradigm, in order to obtain a correct theoretical value of the spectral index in a MBS—when we consider a single scalar field only—we will introduce some dimensionless parameters at very early times in the contracting phase, which we will call quasi-matter domination parameters. When these parameters are less than 1, the universe will be nearly matter dominated in the contracting phase, in exact analogy with the inflationary universe case, where a small value of the slow-roll parameters leads to a universe in the expanding phase, near the de Sitter regime.

The aim of the present work is to construct viable bouncing cosmologies where the matter part of the Lagrangian is composed of a scalar field and, therefore, which have to go beyond general relativity, since the flat Friedmann-Lemaître-Robertson-Walker (FLRW) geometry forbids bounces when one deals with a single field. (Recall that bounces are allowed for FLRW geometries with a positive spatial curvature [5].) Hence, for the flat FLRW geometry, theories such as holonomy-corrected loop quantum cosmology [6], where a big bounce appears owing to the discrete structure of space-time [7], teleparallelism [8], or modified $F(R)$ gravity [9], must be taken into account. When dealing with these theories, in order to obtain a theoretical value of the spectral index that may fit well with current experimental data, a quasimatter-dominated regime in the contracting phase has to be introduced, which is conveniently fixed by the quasimatter domination parameters. Moreover, in slow-roll inflation,

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one also considers the running of the spectral index corresponding to N e-folds before the end of the inflation, which in general is of the order N^{-2}. This value turns out to be very small when one substitutes for N the minimum number of e-folds that are needed to solve the horizon and flatness problems in inflationary cosmology ($N > 50$), as compared with its corresponding observational value coming from the most recent Planck data, -0.0134 ± 0.009 [3]; this shows that these slow-roll models are less favored by observations. In contrast, in MBSs the number of e-folds before the end of the quasimatter domination regime can be relatively small, because the horizon problem does not exist in bouncing cosmologies and the flatness problem is neutral [10]. This gives ground for the viability of such models, thus making it possible that for certain MBSs the theoretical values of the spectral index and the running parameter agree well with Planck observations.

II. QUASIMATTER DOMINATION PARAMETERS

In general relativity, for the flat FLRW geometry, the Friedmann and conservation equations for a single scalar field are

$$H^2 = \frac{1}{3}\left(\frac{\dot{\varphi}^2}{2} + V\right); \qquad \ddot{\varphi} + 3H\dot{\varphi} + V_\varphi = 0. \quad (1)$$

Assuming quasimatter domination at early times in the contracting phase, i.e., $\dot{\varphi}^2 \cong 2V \Rightarrow \ddot{\varphi} \cong V_\varphi$, these equations become

$$\begin{cases} H^2 & = \frac{2}{3}V \\ 3H\dot{\varphi} + 2V_\varphi & = 0 \end{cases} \Leftrightarrow \begin{cases} \mathcal{H}^2 & = \frac{2}{3}a^2 V \\ 3\mathcal{H}\varphi' + 2a^2 V_\varphi & = 0. \end{cases} \quad (2)$$

Now, in complete analogy to the slow-roll regime in inflationary cosmology, we define our quasimatter domination parameters as

$$\bar{\epsilon} = -1 - \frac{2}{3}\frac{\dot{H}}{H^2} = -\frac{2}{3}\left(\frac{1}{2} + \frac{\mathcal{H}'}{\mathcal{H}^2}\right) \cong \frac{1}{3}\left(\frac{V_\varphi}{V}\right)^2 - 1, \quad (3)$$

$$\bar{\delta}^2 = \frac{\dot{\bar{\epsilon}}}{2H(1+\bar{\epsilon})} \cong -\left(\frac{V_\varphi}{V}\right)_\varphi, \quad (4)$$

and

$$\bar{\xi}^3 = -\frac{1}{H}\frac{d\bar{\delta}^2}{dt} \cong -\frac{V_\varphi}{V}\left(\frac{V_\varphi}{V}\right)_{\varphi\varphi}, \quad (5)$$

which characterize this regime through the condition that $|\bar{\epsilon}| \ll 1$.

In view of subsequent calculations, it is important to obtain the evolution of the parameters $\bar{\epsilon}$ and $\bar{\delta}^2$, which is given by

$$\dot{\bar{\epsilon}} \cong 2H\bar{\delta}^2, \qquad \frac{d\bar{\delta}^2}{dt} = -H\bar{\xi}^3. \quad (6)$$

Since a potential of the form $e^{-\sqrt{3}|\varphi|}$ generates exact matter domination, we will reexpress our potential V as $V(\varphi) = e^{\sqrt{3}\varphi}W(\varphi)$ for negative values of the field, thus obtaining

$$\bar{\epsilon} = \frac{2}{\sqrt{3}}\frac{W_\varphi}{W}, \quad \bar{\delta}^2 \cong -\left(\frac{W_\varphi}{W}\right)_\varphi, \quad \bar{\xi}^3 \cong -\sqrt{3}\left(\frac{W_\varphi}{W}\right)_{\varphi\varphi}. \quad (7)$$

This means that, for a very flat potential W, these parameters are very small and nearly constant. In fact, since the expressions in (7) resemble those of the slow-roll parameters, we conclude that we can choose W as the new potential, namely, the same potential as is used in slow-roll inflation. Note also that from (7) one gets the following hierarchy: $|\bar{\xi}^3| \ll |\bar{\delta}^2| \ll |\bar{\epsilon}|$.

As an example, for the potential $W(\varphi) = \lambda\varphi^{2n}$ one has

$$\bar{\epsilon} = \frac{4n}{\sqrt{3}\varphi} + \frac{n^2}{3\varphi^2} \cong \frac{4n}{\sqrt{3}\varphi}, \quad \bar{\delta}^2 = -\frac{2n}{\varphi^2}, \quad \bar{\xi}^3 = \frac{4\sqrt{3}n}{\varphi^3}. \quad (8)$$

One can also introduce, in the same way as in the inflation setup, the number of e-folds before the end of the quasimatter domination period, as follows: $a(N) = e^N a_f$, where a_f is the value of the scale factor at the end of this regime.

With this definition, in the quasimatter approximation the number of e-folds can be calculated as

$$N = -\int_{t_f}^{t_N} H(t)dt \cong \int_{\varphi(N)}^{\varphi_f} \frac{V}{V_\varphi}d\varphi, \quad (9)$$

which, in terms of the potential W, becomes

$$N \cong \int_{\varphi(N)}^{\varphi_f} \frac{1}{\sqrt{3} + \frac{W_\varphi}{W}}d\varphi. \quad (10)$$

For the particular case of the potential $W(\varphi) = \lambda\varphi^{2n}$, for instance, one has

$$N \cong -\frac{1}{\sqrt{3}}(\varphi(N) - \varphi_f) - \frac{2n}{3}\ln\left|\frac{\varphi_f + \frac{2n}{\sqrt{3}}}{\varphi(N) + \frac{2n}{\sqrt{3}}}\right|. \quad (11)$$

Choosing the value of φ_f when $\bar{\epsilon} = -1$, one finally obtains

$$N \cong -\frac{1}{\sqrt{3}}\left(\varphi(N) + \frac{4n}{\sqrt{3}}\right) - \frac{2n}{3}\ln\left|\frac{4n}{\sqrt{3}\varphi(N) + 2n}\right|. \quad (12)$$

A. The spectral index in bouncing cosmologies

It is well known that when one considers only a scalar field, general relativity dealing with the flat FLRW geometry forbids bounces from the contracting to the expanding phase; this is best seen by looking to the Raychaudury equation, $\dot{H} = -\frac{1}{2}\dot{\varphi}^2 < 0$: as the Hubble parameter always decreases, it is absolutely impossible to pass from negative to positive values. For this reason, when the matter part of the Lagrangian is given in terms of a single scalar field, one is led to use cosmologies beyond the realm of general relativity, e.g., loop quantum cosmology, teleparallel $F(T)$ gravity, or $F(R)$ gravities.

Common to all these cases is the Mukhanov-Sasaki [11] equation for scalar perturbations in Fourier space, which can be expressed as

$$v_k'' + \left(k^2 - \frac{z''}{z}\right)v_k = 0, \quad (13)$$

where, for very low energy densities and curvatures, $z = a\frac{\dot{\varphi}}{H} = a\frac{\varphi'}{\mathcal{H}}$. The explicit expressions for z in the cases of $F(T)$ and $F(R)$ gravities have been obtained in [12,13], respectively.

To derive the expression $\frac{z''}{z}$ in the contracting phase, during the quasimatter domination happening at very low energy densities and curvatures, we first calculate

$$\frac{z'}{\mathcal{H}z} = 1 + \bar{\delta}^2. \quad (14)$$

Now, using the same method as in [14] (pp. 54–55), and the second formula of (6), we obtain

$$\frac{z''}{z} = \mathcal{H}(\bar{\delta}^2)' + \mathcal{H}'\frac{z'}{\mathcal{H}z} + \mathcal{H}^2\left(\frac{z'}{\mathcal{H}z}\right)^2$$
$$\cong -\mathcal{H}^2\bar{\xi}^3 + \mathcal{H}'(1+\bar{\delta}^2) + \mathcal{H}^2(1+2\bar{\delta}^2). \quad (15)$$

Finally, solving Eq. (3) for $\bar{\epsilon}$ constant (because $\frac{d\bar{\epsilon}}{dN} = \frac{\dot{\bar{\epsilon}}}{H} \cong 2\bar{\delta}^2 \ll \bar{\epsilon}$), i.e., taking $\mathcal{H} = \frac{2}{\eta(1+3\bar{\epsilon})} \cong \frac{2}{\eta}(1-3\bar{\epsilon})$ and replacing this expression in (16), we get, up to first order,

$$\frac{z''}{z} \cong \frac{2}{\eta^2}(1-9\bar{\epsilon}). \quad (16)$$

It is clear from this result that the Mukhanov-Sasaki equation (13) during the quasimatter domination epoch can be approximated by

$$v_k'' + \left(k^2 - \frac{1}{\eta^2}\left(\nu^2 - \frac{1}{4}\right)\right)v_k = 0 \quad \text{where } \nu \cong \frac{3}{2} - 6\bar{\epsilon}. \quad (17)$$

Then, in order to obtain the adiabatic Bunch-Davies vacuum, one has to choose as a solution of (17)

$$v_k = \frac{\sqrt{\pi|\eta|}}{2}e^{i(1+2\nu)\frac{\pi}{4}}H_\nu^{(1)}(k|\eta|). \quad (18)$$

For modes well outside the Hubble radius $k|\eta| \ll 1$, Eq. (17) becomes

$$v_k'' - \frac{1}{\eta^2}\left(\nu^2 - \frac{1}{4}\right)v_k = 0, \quad (19)$$

the solution of which is given by

$$v_k = C_1(k)|\eta|^{\frac{1}{2}+\nu} + C_2(k)|\eta|^{\frac{1}{2}-\nu} \cong C_2(k)|\eta|^{\frac{1}{2}-\nu}. \quad (20)$$

On the other hand, if one chooses as a scale factor in the quasimatter domination period $a(t) \cong t^{2/3} \Rightarrow a \cong \frac{\eta^2}{9} \Rightarrow z \cong \frac{\eta^2}{3\sqrt{3}}$, the solution (20) can be written as follows:

$$v_k \cong \frac{1}{\sqrt{3}}C_2(k)\left(z(\eta)\int_{-\infty}^{\eta}\frac{d\bar{\eta}}{z^2(\bar{\eta})}\right)|\eta|^{\frac{3}{2}-\nu}. \quad (21)$$

For modes well outside of the Hubble radius the solution (18) should match (21). Using the small-argument approximation in the Hankel function and the expression (20), for these modes we get

$$v_k \cong -i\sqrt{\frac{1}{6}}k^{-3/2}e^{i(1+2\nu)\frac{\pi}{4}}\frac{\Gamma(\nu)}{\Gamma(3/2)}\left(z(\eta)\int_{-\infty}^{\eta}\frac{d\bar{\eta}}{z^2(\bar{\eta})}\right)\left(\frac{k|\eta|}{2}\right)^{\frac{3}{2}-\nu}. \quad (22)$$

Such modes will reenter the Hubble radius at late times in the expanding phase, when the universe is matter dominated. Then, the power spectrum is given by

$$\mathcal{P}_S(k) = \frac{1}{12\pi^2}\left(\frac{\Gamma(\nu)}{\Gamma(3/2)}\right)^2\left(\int_{-\infty}^{\eta}\frac{d\bar{\eta}}{z^2(\bar{\eta})}\right)^2\left(\frac{k}{aH}\right)^{3-2\nu}, \quad (23)$$

where we have used the matter domination condition, i.e., the relation $aH = \frac{2}{\eta}$.

Evaluating this quantity at the reentry time ($aH = k$) and taking into account that this happens at very late times, we obtain the final formula for the power spectrum corresponding to scalar perturbations,

$$\mathcal{P}_S(k) = \frac{1}{12\pi^2}\left(\int_{-\infty}^{+\infty}\frac{d\eta}{z^2(\eta)}\right)^2_{k=aH}, \quad (24)$$

where the approximation $\Gamma(\nu) \cong \Gamma(3/2)$ has been performed.

Note that in slow-roll inflation, the power spectrum could be expressed in terms of the slow-roll parameter $\bar{\epsilon}_{sr}$. This is due to the fact that in inflationary cosmology, for modes well outside of the Hubble radius, the dominant mode is constant being the other one decreasing in the expanding phase. For this reason, one could write the power spectrum in terms of $\bar{\epsilon}_{\rm sr}$, because it only depends on the slow-roll regime. Unfortunately, when one deals with bouncing cosmologies, in the contracting phase for modes well outside of the Hubble radius, the dominant mode is not the constant one, because the other one increases. Then, the spectrum depends on the whole background evolution and not only on the quasimatter domination regime.

In our case, the spectral index for scalar perturbations, n_s, is obtained from (23), giving as a result

$$n_s - 1 \equiv \frac{\ln \mathcal{P}(k)}{\ln k} = 3 - 2\nu = 12\bar{\epsilon}. \tag{25}$$

We can also calculate the running of the spectral tilt,

$$\alpha_s \equiv \left(\frac{dn_s}{d\ln k}\right)_{k=aH} = \frac{n_s'}{(\ln aH)'} = -\frac{2n_s'}{\mathcal{H}} \cong -\frac{24\bar{\epsilon}'}{\mathcal{H}} = -48\bar{\delta}^2, \tag{26}$$

where we have used the formula (3) and the first formula of (6).

In terms of the pressure and energy density, P and ρ, respectively, one has

$$\bar{\epsilon} = \frac{P}{\rho}, \qquad \bar{\delta}^2 = \frac{1}{2H}\frac{d}{dt}\ln\left(1 + \frac{P}{\rho}\right), \tag{27}$$

which leads to the equivalent expression for the spectral index and the running parameter,

$$n_s - 1 = 12\frac{P}{\rho}, \qquad \alpha_s = -\frac{24}{H}\frac{d}{dt}\ln\left(1 + \frac{P}{\rho}\right). \tag{28}$$

In the same way, for tensor perturbations one obtains the following power spectrum:

$$\mathcal{P}_T(k) = \frac{2}{9\pi^2}\left(\int_{-\infty}^{+\infty}\frac{d\eta}{z_T^2(\eta)}\right)^2_{k=aH}, \tag{29}$$

where, for very low energy densities and curvatures, $z_T = a$. The exact expression of z_T in holonomy-corrected loop quantum cosmology was obtained in [15], in teleparallel $F(T)$ gravity in [12], and in modified $F(R)$ gravity in [16].

The ratio of tensor-to-scalar perturbations is given by

$$r = \frac{8}{3}\left(\frac{\int_{-\infty}^{+\infty}\frac{d\eta}{z_T^2(\eta)}}{\int_{-\infty}^{+\infty}\frac{d\eta}{z^2(\eta)}}\right)^2_{k=aH}. \tag{30}$$

Finally, it is instructive to compare these parameters with the slow-roll ones commonly used in inflation,

$$\bar{\epsilon}_{\rm sr} = -\frac{\dot{H}}{H^2} \cong \frac{1}{2}\left(\frac{V_\varphi}{V}\right)^2, \qquad \bar{\eta}_{\rm sr} = 2\bar{\epsilon}_{\rm sr} - \frac{\dot{\bar{\epsilon}}_{\rm sr}}{2H\bar{\epsilon}_{\rm sr}} \cong \frac{V_{\varphi\varphi}}{V}, \tag{31}$$

which are related to the quasimatter domination parameters $\bar{\epsilon}$ and $\bar{\delta}^2$ via the formulas

$$\bar{\epsilon}_{\rm sr} = \frac{3}{2}(\bar{\epsilon} + 1), \qquad \bar{\eta}_{\rm sr} = 3(\bar{\epsilon} + 1) - \frac{9}{4}\bar{\delta}^2. \tag{32}$$

In slow-roll inflation, the spectral index and its running are given by

$$n_s - 1 = 2\bar{\eta}_{\rm sr} - 6\bar{\epsilon}_{\rm sr}, \qquad \alpha_s = 16\bar{\epsilon}_{\rm sr}\bar{\eta}_{\rm sr} - 24\bar{\epsilon}_{\rm sr}^2 - 2\bar{\xi}_{\rm sr}^2, \tag{33}$$

where $\bar{\xi}_{\rm sr}^2 \cong \frac{V_\varphi V_{\varphi\varphi\varphi}}{V^2}$ is a second-order slow-roll parameter .

Moreover, in inflationary cosmology, the scalar/tensor ratio is related to the slow-roll parameter $\bar{\epsilon}_{\rm sr}$ as

$$r = 16\bar{\epsilon}_{\rm sr}, \tag{34}$$

which does not happen in the MBS, because there the tensor/scalar ratio depends on the whole background dynamics, and not solely on those corresponding to quasimatter domination.

B. Power law expansion

As an example, we will choose the following potential [17]:

$$V(\varphi) = V_0 e^{-\sqrt{3(1+\omega)}|\varphi|}, \tag{35}$$

which leads to the power law expansion

$$a \propto t^{\frac{2}{3(1+\omega)}}. \tag{36}$$

An easy calculation yields for the MBS

$$n_s - 1 = 12\omega. \tag{37}$$

In contrast, in the case of slow-roll inflation, for the same potential (35) one gets

$$n_s - 1 = -3(1 + \omega) \quad \text{and} \quad r = 24(1 + \omega). \tag{38}$$

For either of these theories—MBS or inflation—to be viable, they have to match astronomical data that is more and more accurate. Focusing in particular on Planck data, the resulting spectral index is given by $n_s = 0.9603 \pm 0.0073$; specifically, this means that:

(1) In the MBS, in order for the potential (35) to match with observations, one needs to choose $\omega = -0.0033 \pm 0.0006$.
(2) In power law inflation, the potential (35) turns out to be in agreement with the observational value of the spectral index provided $\omega = -0.9867 \pm 0.0024$. Moreover, since the tensor/scalar ratio is given by $r = 24(1+\omega)$, for this potential to fit well with Planck data one has to impose $\omega \leq -0.9954$, which is not compatible with the previous number, $\omega = -0.9867 \pm 0.0024$. On the other hand, to match the ratio of tensor-to-scalar perturbations with the BICEP2 data, one has to choose $\omega \in [-0.9937, -09887]$; this, together with the condition $\omega = -0.9867 \pm 0.0024$, restricts the value of the parameter ω to be $\omega = -0.9890^{+0.0001}_{-0.0003}$.

This calculation clearly shows that, in order to match with current observational data, the parameter ω that appears in both theories must be conveniently tuned.

Finally, the power law expansion given by the potential (35) has no running, which is in contradiction with the latest Planck data [3] that provides the experimental value $\alpha_s = -0.0134 \pm 0.009$. For this reason, other models must be considered as alternatives.

III. QUASIMATTER DOMINATION POTENTIALS OBTAINED FROM THE EQUATION OF STATE

We now continue once again with the parametrization of the scale factor in the contracting phase given by $a(N) = a_f e^N$, where a_f is the value of the scale factor at the end of the quasimatter domination period. We will assume, as in inflation [18], an equation of state (EoS) of the form $\frac{P}{\rho} = \frac{\beta}{(N+1)^\alpha}$ where $\alpha > 0$ and $\beta < 0$ (the fluid has negative pressure) are both of order 1. This particular dependence between $\frac{P}{\rho}$ and the number of e-folds will allow us to obtain, in a simple way, potentials that lead to a quasimatter domination. Effectively, the conservation equation reads

$$\frac{d\ln\rho}{dN} = -3\left(1+\frac{P}{\rho}\right) = -3 - \frac{3\beta}{(N+1)^\alpha}, \qquad (39)$$

and the solution of this equation is given by

$$\rho(N) = \begin{cases} \rho_f e^{-3N}(N+1)^{-3\beta}, & \alpha = 1, \\ \rho_0 e^{-3N} e^{\frac{3\beta}{(\alpha-1)(N+1)^{\alpha-1}}}, & \alpha \neq 1. \end{cases} \qquad (40)$$

On the other hand, in the contracting phase,

$$\frac{d\varphi}{dN} = \frac{\dot\varphi}{H} = -\sqrt{3}\sqrt{1+\frac{P}{\rho}} = -\sqrt{3}\sqrt{1+\frac{\beta}{(N+1)^\alpha}}, \qquad (41)$$

where we have used $\dot\varphi^2 = \rho + P$. This equation could be explicitly integrated for $\alpha = 1, 2$; for example, when $\alpha = 1$, one has

$$\varphi(N) = -\sqrt{3\left(1+\frac{\beta}{(N+1)}\right)}(N+1) + \frac{\sqrt{3}\beta}{2}\ln\left(\frac{\sqrt{1+\frac{\beta}{(N+1)}}-1}{\sqrt{1+\frac{\beta}{(N+1)}}+1}\right). \qquad (42)$$

Here, however, we will make the approximation $\sqrt{1+\frac{\beta}{(N+1)^\alpha}} = 1 + \frac{\beta}{2(N+1)^\alpha}$ for large values of N. Then, one obtains

$$\varphi(N) \cong \begin{cases} -\sqrt{3}(N + \ln(N+1)^{\frac{\beta}{2}}), & \alpha = 1, \\ -\sqrt{3}\left(N - \frac{\beta}{2(\alpha-1)(N+1)^{\alpha-1}}\right), & \alpha \neq 1. \end{cases} \qquad (43)$$

Finally, introducing the approximation of quasimatter domination, $\rho(N) \cong 2V(N)$, we obtain, for $N \geq 1 \Leftrightarrow \varphi \to -\infty$,

$$V(\varphi) \cong \begin{cases} V_0 e^{\sqrt{3}\varphi}(N(\varphi)+1)^{-\frac{3\beta}{2}}, & \alpha = 1 \\ V_0 e^{\sqrt{3}\varphi} e^{\frac{3\beta}{2(\alpha-1)(N(\varphi)+1)^{\alpha-1}}}, & \alpha \neq 1, \end{cases} \qquad (44)$$

where $N(\varphi)$ is obtained by solving for N in (43).

A. Viability of the models

The spectral index and the running parameter for the EoS $\frac{P}{\rho} = \frac{\beta}{(N+1)^\alpha}$, and, thus, for potentials of the form (44), can be easily obtained from Eq. (28) by using the relation $\frac{d}{Hdt} = \frac{d}{dN}$, which yields

$$n_s - 1 = \frac{12\beta}{(N+1)^\alpha}, \qquad \alpha_s \cong \frac{24\alpha\beta}{(N+1)^{\alpha+1}}. \qquad (45)$$

Note that in the MBS, the ratio of tensor-to-scalar perturbations is not related to the quasimatter domination parameters and has to be calculated using Eq. (30). This calculation can be carried out numerically for the solution of the conservation equation

$$\ddot\varphi + 3H(\varphi)\dot\varphi + V_\varphi = 0, \qquad (46)$$

corresponding to a universe that takes N e-folds to leave the quasimatter domination epoch, i.e., for the solution that satisfies the initial conditions

$$\varphi_i = \varphi(N), \qquad \dot\varphi_i = H\frac{d\varphi}{dN} = \sqrt{\rho(N)}\sqrt{1+\frac{\beta}{(N+1)^\alpha}}, \qquad (47)$$

where $\varphi(N)$ and $\rho(N)$ are given by (43) and (40), respectively.

However, it is important to realize that the constraint of the tensor/scalar ratio provided by the Wilkinson MIcrowave Anisotropy Probe and Planck projects ($r \leq 0.11$) is obtained indirectly, assuming the *consistency* slow-roll relation $r = 16\bar{\epsilon}_{\rm sr}$ [19], because gravitational waves are not detected by those projects. This means that the slow-roll inflationary models must satisfy this constraint, but the bouncing models, where there is not any consistency relation, do not need to satisfy it. This point is very important because some very complicated mechanisms are sometimes implemented in the MBS in order to enhance the power spectrum of scalar perturbations to achieve the observational bound provided by Planck [20]. Moreover, numerical calculations have been performed for holonomy-corrected and teleparallel loop quantum cosmology [21], and those theoretical values of the tensor/scalar ratio have been compared with the corresponding observational values provided by the Planck and BICEP2 projects. In fact, in the MBS, to check if the models provide a viable value of the tensor/scalar ratio, gravitational waves must first be clearly detected in order to determine the observed value of this ratio. We hope that more accurate unified Planck-BICEP2 data (the B2P Collaboration), which will be issued soon, may address this point. In contrast, the spectral index of scalar perturbations and its running could be calculated independently of the theory [22]; this means that in order to check bouncing models in the absence of evidence of gravitational waves, one has to work in the space (n_s, α_s).

1. Example 1

As a first example, we can compare our results relative to the MBS with those for chaotic inflation given by the potential $V(\varphi) = \lambda\varphi^{2n}$. In this case, one has [23]

$$n_s - 1 = -\frac{2(n+1)}{2N+n}, \qquad r = \frac{16n}{2N+n}. \tag{48}$$

We can see that, for the same number of e-folds, one obtains the same spectral index in both the MBS and chaotic inflation, after choosing $\alpha = 1$ and $\beta = -\frac{1}{4}$ in the MBS and a quartic potential for inflation. That is, one obtains the same spectral index for these parameters ($\alpha = 1$, $\beta = -\frac{1}{4}$, and $n = 2$) for modes that leave the Hubble radius about a number N of e-folds before the end of the corresponding period (quasimatter domination in bouncing cosmologies and the slow-roll phase in inflation).

From (48) we can see that in order to achieve the observed value of the spectral index one has to choose $N \in [62.829, 91.592]$. On the other hand, in slow-roll inflation one also has the constraint $r = \frac{16}{N+1}$, which, compared with the Planck constraint $r \leq 0.11$, implies $N \geq 144.454$. This means, however, that the chaotic quartic potential is ruled out by Planck data. However, if one considers the BICEP2 data $r = 0.20^{+0.07}_{-0.05}$, one obtains $N \in [58.259, 105.666]$, which means that the quartic potential fits well with BICEP2 data for $N \in [62.829, 91.592]$.

In our bouncing model, with the aim of obtaining the theoretical value of the tensor/scalar ratio, one has to use the formula (30). Thus, one needs to calculate this quantity for the solution of the conservation equation with initial conditions

$$\varphi_i = \varphi(N), \qquad \dot{\varphi}_i = H\frac{d\varphi}{dN} = \sqrt{\rho(N)}\sqrt{1 - \frac{1}{4(N+1)}}, \tag{49}$$

where $\varphi(N)$ is given by (42), $\rho(N)$ by (40), and $N \in [62.829, 91.592]$.

2. Example 2

As a second example we will deal with R^2 gravity, where [24]

$$n_s - 1 = -\frac{2}{N}, \qquad r = \frac{12}{N^2}. \tag{50}$$

The same spectral index could be obtained from (45) choosing $\alpha = 1$, $\beta = -\frac{1}{6}$ and considering $N - 1$ e-folds instead of N. In this case the correct power spectrum is obtained by choosing $N \in [42.553, 61.728]$.

In inflationary cosmology the model matches correctly with Planck data, because the constraint $r \leq 0.11$ is equivalent to $N \geq 10.44$. However, the model is incompatible with the BICEP2 data, because it implies $N \in [6.66, 8.94]$.

B. Compatibility between the spectral index and the running parameter

Here we will compare the compatibility of the spectral index and the running parameter in both the inflation and MBSs. In the slow-roll regime, both the spectral index and the running parameter for a perfect fluid can be easily calculated from the equations

$$\begin{aligned} n_s - 1 &= -3\left(1 + \frac{P}{\rho}\right) + \frac{d}{dN}\ln\left(1 + \frac{P}{\rho}\right), \\ \alpha_s &= \frac{\dot{n}_s}{H} = -\frac{dn_s}{dN}, \end{aligned} \tag{51}$$

and if one considers a fluid satisfying the condition

$$\left|\frac{d}{dN}\ln(\rho + P)\right| \ll \left|1 + \frac{P}{\rho}\right|, \tag{52}$$

one gets [25]

$$n_s - 1 \cong -6\left(1+\frac{P}{\rho}\right), \qquad \alpha_s \cong -18\left(1+\frac{P}{\rho}\right)^2. \quad (53)$$

From these equations one obtains the relation $\alpha_s = -\frac{1}{2}(1-n_s)^2$. Now, inserting the observed value for the spectral index, $n_s = 0.9603 \pm 0.0073$, this yields $\alpha_s \in (-5.2\times 10^{-4}, -1.1\times 10^{-3})$, which is in clear contradiction with the observed value, $\alpha_s = -0.0134 \pm 0.009$. As a consequence, inflation corresponding to this kind of perfect fluid is less favored by the current observational data.

For a fluid with an EoS $1+\frac{P}{\rho} = \frac{\beta}{(N+1)^\alpha}$, with both α and β positive and of order 1 [18], which does not satisfy the condition (52), one has

$$\alpha_s = \begin{cases} -\frac{1}{\alpha}(n_s-1)^2 & \alpha > 1 \\ -\frac{1}{3\beta+1}(n_s-1)^2 & \alpha = 1 \\ \frac{\alpha}{N+1}(n_s-1) & \alpha < 1. \end{cases} \quad (54)$$

Then:

(1) For $\alpha \geq 1$ one has

$$|\alpha_s| \leq (n_s-1)^2 = (0,0397 \pm 0.0073)^2 < |-0.0134 \pm 0.009|,$$

ruling out, at 1σ confidence level, this kind of model.

(2) For $\alpha < 1$, to match the theoretical values with the experimental ones, the parameters must satisfy

$$2\alpha \leq N+1 \leq 11\alpha \quad \text{and}$$
$$-\frac{3\beta}{(N+1)^\alpha} = -0.0397 \pm 0.0073.$$

However, since $\alpha < 1$, the number of e-folds before the end of inflation satisfies $N+1 < 11$, which is incompatible with the minimum number of e-folds (for the most general models $N \geq 50$ [3]) to solve both the horizon and flatness problems in general relativity.

The problem with slow-roll inflation is that, in general, the spectral index is of order N^{-1}, while the running parameter is of order N^{-2} ; consequently, one has $\alpha_s \sim (1-n_s)^2$, which in most cases is incompatible with Planck data, because the observed value of the running is not small enough [26]. Moreover, the constraint of the tensor/scalar ratio provided by the WMAP and Planck projects ($r \leq 0.11$) is obtained indirectly assuming the *consistency* slow-roll relation $r = 16\bar{\epsilon}_{\rm sr}$ [19], because gravitational waves are not detected by those projects. This means that the slow-roll inflationary models must satisfy this constraint, but the bouncing ones, where there is not any consistency relation, do not need to satisfy it. It is the combination of the three data (n_s, α_s, r) that rules out, at 1σ confidence level for the running, all the standard slow-roll inflationary models.

For instance, we consider the $\Lambda\mathrm{CDM} + r + \alpha_s$ model from Planck combined with WP and BAO data, which gives the following results: $n_s = 0.9607 \pm 0.0063$, $r \leq 0.25$ at 95% C.L., and $\alpha_s = -0.021^{+0.012}_{-0.010}$ (see Table 5 of [3]). In slow-roll inflation, a simple calculation leads to the relation

$$\alpha_s = \frac{1}{2}(n_s-1)r + \frac{3}{32}r^2 - 2\bar{\xi}^2_{\rm sr}. \quad (55)$$

Thus, considering n_s at 2σ confidence level and taking the conservative bound $r \leq 0.32$ (see Fig. 4 of [3]), the minimum of the function $\frac{1}{2}(n_s-1)r + \frac{3}{32}r^2$ is bigger than -0.0018, which provides the bound

$$\alpha_s \geq -0.0018 - 2\bar{\xi}^2_{\rm sr}. \quad (56)$$

This means that potentials such as $V(\varphi) = V_0(1-\frac{\varphi^2}{\mu^2}+\ldots)$ (hilltop), $V(\varphi) = V_0(1-\frac{\varphi^2}{\mu^2})^2$ (plateau) [27], or $V(\varphi) = V_0(1+\cos(\frac{\varphi}{\mu}))$ (natural) [28], when one considers values of the running at 1σ confidence level ($\alpha_s = -0.021^{+0.012}_{-0.010} \Leftrightarrow -0.031 \leq \alpha_s \leq -0.009$), are disfavored by Planck data because $\bar{\xi}^2_{\rm sr} \leq 0$ for all of them.

Dealing with the monomial potential $V(\varphi) = V_0\varphi^p$, one obtains

$$n_s - 1 = -\frac{p(p+2)}{\varphi^2}, \qquad \alpha_s = -\frac{2p^2(p+2)}{\varphi^4}, \quad (57)$$

which means that p must be positive in order to have a spectral index with a red tilt and a negative running. As a first consequence, inverse power law potentials [29] are disfavored.

For $p = 1, 2$ one has $\bar{\xi}^2_{\rm sr} = 0$; thus, one can apply the bound (56) to disfavor these models. In general, since for this monomial potential one has $\bar{\xi}^2_{\rm sr} = \frac{p-2}{p-1}\bar{\eta}^2_{\rm sr}$, one can obtain the following exact formula:

$$\alpha_s = \frac{p+2}{8(p-1)}(n_s-1)r + \frac{3(p+2)}{128(p-1)}r^2 - \frac{p-2}{2(p-1)}(n_s-1)^2. \quad (58)$$

For $p \geq 3$, using $\frac{p-2}{p-1} \leq 2$ and the fact that $\frac{p+2}{p-1}$ increases as a function of p, one gets the bound

$$\alpha_s \geq \frac{5}{16}(n_s-1)r - (n_s-1)^2 \geq -0.0084, \quad (59)$$

which is incompatible with the running provided by Planck at 1σ confidence level.

Finally, for a general hilltop potential $V(\varphi) = V_0(1 - \frac{\varphi^p}{\mu^p} + \ldots)$ with $p \geq 3$ [30], one also has the relation $\bar{\xi}^2_{\rm sr} = \frac{p-2}{p-1}\bar{\eta}^2_{\rm sr}$; thus, one can apply the same reasoning as in the previous case for monomial potentials.

One way to solve this problem is to break the slow-roll approximation for a short while—for example, by including a quickly oscillating term in the potential. In this case the theoretical value of the running parameter gets larger and could match well with experimental data [31].

On the other hand, in the MBS, when dealing with a perfect fluid with EoS $\frac{p}{\rho} = \frac{\beta}{(N+1)^\alpha}$, one obtains from (45) the following relation:

$$\alpha_s = \frac{2\alpha}{N+1}(n_s - 1), \tag{60}$$

which is perfectly compatible with the experimental data. In fact, for instance, if one takes $\alpha = 2$ and $N = 12$ (note that in bouncing cosmologies a large number of e-folds is *not* required, because the horizon problem does not exist—at the bounce all parts of the Universe are already in causal contact—and the flatness problem is also improved [10]), one obtains, for $n_s = 0.9603 \pm 0.0073$, the value of the running parameter $\alpha_s = 0.0122 \pm 0.0022$, which is compatible with the Planck data. Effectively, for these values of α and N one gets $n_s - 1 = \frac{12}{13^2}\beta \cong 0.071\beta$, which is indeed compatible with its observed value, by choosing $\beta \cong -\frac{1}{2}$.

IV. CONCLUSIONS

In this paper we have introduced, at early times in the contracting phase of bouncing cosmologies, a quasimatter domination regime controlled by some convenient small parameters that we have defined here. This has allowed us to obtain theoretical values of the spectral index and the running parameter, which are in perfect agreement with the most recent and accurate observational data from the Planck satellite.

We have shown in detail, and with the help of several simple examples, the viability of our bouncing models for isotropic fluids with an equation of state that depends on the number of e-folds occurring before the end of the quasimatter domination epoch. We have also demonstrated that, in contrast to these results, slow-roll inflationary models are generically less favored by the most recent Planck observational data; this is due, in particular, to the rather small value of the running parameter predicted by all these slow-roll theories.

We expect that more precise unified Planck-BICEP2 data (the B2P Collaboration), which will be issued soon, may fit even better the bouncing cosmologies under consideration here.

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