

Modified Lyth bound and implications of BICEP2 resultsQing Gao (郜青),^{1,*} Yungui Gong (龚云贵),^{1,†} and Tianjun Li (李田军)^{2,3,‡}¹MOE Key Laboratory of Fundamental Quantities Measurement, School of Physics,

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To reconcile the BICEP2 measurement on the tensor-to-scalar ratio r with Planck constraint, a large negative running of scalar spectral index n_s is needed. So the inflationary observable such as n_s should be expanded at least to the second-order slow-roll parameters for single-field inflationary models. The large value of r and the Lyth bound indicate that it is impossible to obtain the sub-Planckian excursion for the inflaton. However, we derive an absolutely minimal bound $\Delta\phi/M_{\text{Pl}} > \sqrt{r/2}$ on the inflaton excursion for single-field inflationary models, which can be applied to non-slow-roll inflationary models as well. This bound excludes the possibility of the small-field inflation with $\Delta\phi < 0.1M_{\text{Pl}}$ if the BICEP2 result on r stands, and it opens the window of sub-Planckian excursion with $\Delta\phi < M_{\text{Pl}}$ even if r is as large as 0.1. To get the sub-Planckian excursion with $\Delta\phi < 0.1M_{\text{Pl}}$, our modified bound requires $r < 0.02$. Using a fifth-order polynomial potential as an explicit example, we show that it not only agrees with the observational results, but also violates the Lyth bound.

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I. INTRODUCTION

Inflation not only solves various problems in the standard big bang cosmology such as the flatness, horizon and monopole problems, etc., but also provides the seed of a large-scale structure by the quantum fluctuation of the inflaton [1–4]. Generic inflationary models predict that the spectrum of the density perturbation is Gaussian, adiabatic, and almost scale invariant. Besides the scalar perturbation, the tensor perturbation is generated as well, which gives the B-mode polarization as a signature of the primordial gravitational wave. The measurements of the cosmic microwave background radiation anisotropies provide a strong test of inflationary models. The Planck data of the temperature power spectrum [5] in combination with the nine years of WMAP polarization low-multipole likelihood data [6] and the high-multipole spectra data from the Atacama Cosmology Telescope [7] and the South Pole Telescope [8] (Planck + WP + highL) constrained the scalar spectral index to be $n_s = 0.960 \pm 0.014$ and the tensor-to-scalar ratio to be $r_{0.002} \leq 0.11$ at the 95% C.L. [9,10].

However, the ground-based the BICEP2 experiment has measured the tensor-to-scalar ratio to be $r = 0.20^{+0.07}_{-0.05}$ at the 68% C.L. for the lensed- Λ CDM model, with $r = 0$ disfavored at the 7.0σ level [11,12]. The BICEP2 result on

the tensor-to-scalar ratio is in tension with the Planck result. If the running $n'_s = d \ln n_s / d \ln k$ of the spectral index is included, then the results between the Planck and BICEP2 experiments can be consistent. With the running of spectral index, the Planck + WP + highL data give $n_s = 0.9570 \pm 0.0075$ and $n'_s = -0.022 \pm 0.010$ at the 68% C.L., and the tensor-to-scalar ratio is constrained to be $r_{0.002} < 0.26$ at the 95% C.L.. The combined Planck + WP + highL + BICEP2 data give the constraints $n_s = 0.9574^{+0.0073}_{-0.0074}$, $n'_s = -0.0292 \pm 0.0096$, and $r_{0.002} = 0.21^{+0.05}_{-0.06}$ at the 68% C.L.

The large value of tensor-to-scalar ratio excludes a large class of inflationary models. For example, the inflationary model with nonminimal coupling with gravity found that $n_s = 1 - 2/N$ and $r = 12/N^2$ [13]. If we take $N = 50$, then we get $r = 0.0048$, so the model is excluded by the BICEP2 result. The BICEP2 result also disfavors the small-field inflation such as the hilltop inflation with the potential $V(\phi) = V_0[1 - (\phi/\mu)^p]$ [4,14], the hybrid inflation [15–17], and many string-inspired models [18]. On the other hand, the running of spectral index is at the order of at most 10^{-3} for the simple single-field inflation because the scalar spectral index n_s deviates from 1 at the order of 10^{-2} . So the large negative running of spectral index becomes a big challenge to single-field inflationary models [19]. From the naive analysis of the Lyth bound $\Delta\phi/M_{\text{Pl}} > N\sqrt{r/8}$ [20] with $M_{\text{Pl}}^2 = (8\pi G)^{-1}$, generically we need large-field inflation if r is in the order of 0.1, and

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then the validity of effective field theory becomes an issue since higher-dimensional operators are not suppressed by the reduced Planck scale.

Therefore, a successful single-field inflationary model should satisfy the following three criteria:

- (i) C1: The spectral index n_s and the tensor-to-scalar ratio r are consistent with the observational values.
- (ii) C2: A large negative running of spectral index $n'_s \sim -0.03$ is required if the running is included.
- (iii) C3: The Lyth bound should be violated so that the sub-Planckian excursion of the inflaton can be realized unless the tensor-to-scalar ratio $r < 0.001$.

Inflationary models that satisfy the first condition C1 have been studied extensively [21–77]. Natural inflation models with sinusoidal potential can easily satisfy the first two conditions C1 and C2 [39,75]. With the help of two small decay constants, an effective large decay constant is realized for the natural inflation in string theory so that the condition C3 may be avoided [78,79]. For the inflationary model building, it seems that supergravity theory is a natural framework [80,81]. However, supersymmetry-breaking scalar masses in a generic supergravity theory are of the same order as the gravitino mass, inducing the so-called η problem [17,82,83], where all the scalar masses are of the order of the Hubble parameter due to the large vacuum energy density during inflation [84]. There are two elegant solutions: no-scale supergravity [85–91] and shift symmetry in the Kähler potential [92–101].

In this paper, we derive the modified Lyth bound $\Delta\phi/M_{\text{Pl}} > \sqrt{r/2}$ on the inflaton excursion for a single field inflation, which is applicable to any inflationary model. This bound excludes the possibility of small-field inflation with $\Delta\phi < 0.1M_{\text{Pl}}$ if the BICEP2 result on r stands, and it opens the window for the sub-Planckian excursion with $\Delta\phi < M_{\text{Pl}}$ even if r is as large as 0.1. The modified Lyth bound also tells us that the sub-Planckian excursion with $\Delta\phi < 0.1M_{\text{Pl}}$ requires $r < 0.002$. So the constraint on the field excursion with the condition C3 will be replaced by the modified Lyth bound. Then, we explicitly construct a polynomial inflationary model that satisfies all the conditions C1, C2, and C3. The model not only agrees with the observational results but also supports the modified Lyth bound.

II. POLYNOMIAL POTENTIAL AND SUPERGRAVITY MODEL BUILDING

For a given Kähler potential K and a superpotential W in the supergravity theory, we have the scalar potential

$$V = e^K ((K^{-1})^i_j D_i W D^{\bar{j}} \bar{W} - 3|W|^2), \quad (1)$$

where $(K^{-1})^i_j$ is the inverse of the Kähler metric $K^{\bar{j}}_i = \partial^2 K / \partial \Phi^i \partial \bar{\Phi}^{\bar{j}}$ and $D_i W = W_i + K_i W$. The kinetic term for the scalar field Φ^i is

$$\mathcal{L} = K^{\bar{j}}_i \partial_\mu \Phi^i \partial^\mu \bar{\Phi}^{\bar{j}}. \quad (2)$$

With the help of two superfields Φ and X , we consider the Kähler potential and superpotential

$$K = \frac{1}{2}(\Phi + \bar{\Phi})^2 + X\bar{X} - \delta(X\bar{X})^2, \quad (3)$$

$$W = Xf(\Phi), \quad (4)$$

so that the Kähler potential K is invariant under the shift symmetry [92–101]

$$\Phi \rightarrow \Phi + iCM_{\text{Pl}}, \quad (5)$$

with C a dimensionless real parameter. Because the Kähler potential K is a function of $\Phi + \Phi^\dagger$, it is independent of the imaginary part of Φ . Substituting the Kähler potential K (3) and superpotential W (4) into the scalar potential (1), the scalar potential becomes

$$V = e^K \left[\left| (\Phi + \bar{\Phi})Xf(\Phi) + X \frac{\partial f(\Phi)}{\partial \Phi} \right|^2 - 3|Xf(\Phi)|^2 + |(\bar{X} - 2\delta X\bar{X}^2)Xf(\Phi) + f(\Phi)|^2 \right]. \quad (6)$$

Because there is no imaginary component $\text{Im}[\Phi]$ of Φ in the Kähler potential due to the shift symmetry, the potential along $\text{Im}[\Phi]$ is very flat, and then $\text{Im}[\Phi]$ is a natural inflaton candidate. As we know from the previous studies [96,97,101], the real component $\text{Re}[\Phi]$ of Φ and X can be stabilized at the origin during inflation, i.e., $\text{Re}[\Phi] = 0$ and $X = 0$. Therefore, with $\text{Im}[\Phi] = \phi/\sqrt{2}$, we obtain the inflaton potential

$$V = |f(\phi/\sqrt{2})|^2. \quad (7)$$

Considering a polynomial function $f(\Phi) = \sum_{n=0} (-i\sqrt{2})^n a'_n \Phi^n$, we obtain the polynomial inflaton potential

$$V(\phi) = \left| \sum_{n=0} a'_n (\phi/M_{\text{Pl}})^n \right|^2. \quad (8)$$

To simplify the above potential, we define

$$V_0 \equiv |a'_0|^2, \quad a_n \equiv \frac{a'_n}{a'_0}. \quad (9)$$

The inflaton potential can be rewritten as follows:

$$V(\phi) = V_0 \left| a_0 + \sum_{n=1} a_n (\phi/M_{\text{Pl}})^n \right|^2. \quad (10)$$

In particular, we want to emphasize $a_0 = 1$. Let us consider the polynomial potential of the inflaton

$$V(\phi) = V_0 \left[1 + \sum_{m=1} \lambda_m \left(\frac{\phi - \phi_*}{M_{\text{Pl}}} \right)^m \right], \quad (11)$$

where the subscript $*$ means the value at the horizon crossing. Without loss of generality, we will take $\phi_* = 0$.

If the above polynomial potential is from the scalar potential in Eq. (10) for the supergravity model building, we have

$$\lambda_m = \sum_{i < m/2} 2a_i a_{m-i} \quad (12)$$

for odd m and

$$\lambda_m = \sum_{i < m/2} 2a_i a_{m-i} + a_{m/2}^2 \quad (13)$$

for even m .

III. LYTH BOUND IN SLOW-ROLL INFLATION

In terms of the slow-roll parameters

$$\epsilon(\phi) = \frac{M_{\text{Pl}}^2 V_\phi^2}{2V^2}, \quad (14)$$

$$\eta(\phi) = \frac{M_{\text{Pl}}^2 V_{\phi\phi}}{V}, \quad (15)$$

$$\xi^2(\phi) = \frac{M_{\text{Pl}}^4 V_\phi V_{\phi\phi\phi}}{V^2}, \quad (16)$$

the scalar spectral index and its running are given by [83,102]

$$n_s \approx 1 + 2\eta - 6\epsilon + 2 \left[\frac{1}{3} \eta^2 + (8C - 1)\epsilon\eta - \left(\frac{5}{3} + 12C \right) \epsilon^2 - \left(C - \frac{1}{3} \right) \xi^2 \right], \quad (17)$$

$$n'_s = 16\epsilon\eta - 24\epsilon^2 - 2\xi^2, \quad (18)$$

where $V_\phi = dV(\phi)/d\phi$, $V_{\phi\phi} = d^2V(\phi)/d\phi^2$, $V_{\phi\phi\phi} = d^3V(\phi)/d\phi^3$, and $C = -2 + \ln 2 + \gamma \simeq -0.73$ with γ the Euler–Mascheroni constant. The tensor spectral index and tensor-to-scalar ratio are [83,102]

$$n_t = -2\epsilon \left[1 + \left(4C + \frac{11}{3} \right) \epsilon - 2 \left(\frac{2}{3} + C \right) \eta \right] \approx -2\epsilon, \quad (19)$$

$$r = 16\epsilon \left[1 + 2 \left(C - \frac{1}{3} \right) (2\epsilon - \eta) \right] \approx 16\epsilon. \quad (20)$$

Note that the quantities ϵ , η , and ξ^2 in Eqs. (17)–(20) are evaluated at the horizon crossing $k = aH$ or ϕ_* for a given scale k . As discussed in Ref. [103], the observational result of large running requires us to consider the second-order correction to the scalar spectral index n_s in Eq. (17) because ξ^2 has the same order as ϵ and η and the main contribution to the running of spectral index comes from ξ^2 . The number of e -folds before the end of inflation is given by

$$N(\phi) = \int_t^{t_e} H dt \approx \frac{1}{M_{\text{Pl}}} \int_{\phi_e}^{\phi} \frac{d\phi}{\sqrt{2\epsilon(\phi)}}, \quad (21)$$

where the value ϕ_e of inflaton at the end of inflation is defined by $\epsilon(\phi_e) = 1$ or $\eta(\phi_e) = 1$. If $\epsilon(\phi)$ is a monotonic function of ϕ during inflation, we have $\epsilon(\phi) > \epsilon(\phi_*) = \epsilon = r/16$ and then get the Lyth bound [20]

$$\Delta\phi \equiv |\phi_* - \phi_e| > \sqrt{2\epsilon} N(\phi_*) M_{\text{Pl}} = \sqrt{r/8} N(\phi_*) M_{\text{Pl}}. \quad (22)$$

Note that the Lyth bound is not related to the spectral index n_s , and it holds for general slow-roll inflationary models whether they are consistent with observational constraints or not. Therefore, $r = 0.21$ requires large-field inflation due to $\Delta\phi > 9.72 M_{\text{Pl}}$ for $N = 60$. If we take $r = 0.1$, then the Lyth bound requires $\Delta\phi > 6.71 M_{\text{Pl}}$ for $N = 60$. To get the sub-Planckian excursion for the inflaton, the Lyth bound requires $r \lesssim 0.001$. Therefore, to reconcile the BICEP2 result and small-field inflation, the Lyth bound must be violated. In short, $\epsilon(\phi)$ should not be a monotonic function and needs to have at least one minimum between ϕ_* and ϕ_e [21,104], the inflaton value of which is defined as ϕ_{min} . It was argued in Ref. [45] that it is impossible to achieve $\Delta\phi < M_{\text{Pl}}$ for single-field inflationary models because $\Delta\phi/M_{\text{Pl}} \gtrsim \sqrt{r/8}/\langle\eta - 2\epsilon\rangle$, where $\langle\eta - 2\epsilon\rangle$ is the mean of $\eta - 2\epsilon$ between ϕ_{min} and ϕ_* . In Ref. [21], a large tensor-to-scalar ratio and large negative running were obtained by a single polynomial potential. In particular, they found that $n_s = 0.96$, $r = 0.1$, $n'_s = -0.07$, and $\Delta\phi(N = 60) \sim M_{\text{Pl}}$. Furthermore, it was argued that $\Delta\phi$ lies in a narrow range below M_{Pl} in Refs. [49,105]. In Ref. [106], it was derived that $\Delta\phi/M_{\text{Pl}} \approx 6r^{1/4}$. It is evident that the Lyth bound is not the lowest bound and a weaker bound exists; the main goal of this work is to derive the lowest bound and present some concrete examples.

To violate the Lyth bound and get the sub-Planckian excursion, the main contribution to N must come from $1/\sqrt{2\epsilon(\phi_{\text{min}})}$, so $\epsilon(\phi_{\text{min}}) < 1/(2N)^2 \sim 10^{-4} \ll \epsilon(\phi_*)$ and $\Delta\epsilon(\phi) = \epsilon(\phi_*) - \epsilon(\phi_{\text{min}}) \sim \epsilon$. Note that

$$M_{\text{Pl}} \left| \frac{d\epsilon(\phi)}{d\phi} \right| = \sqrt{2\epsilon(\phi)} |\eta(\phi) - 2\epsilon(\phi)|, \quad (23)$$

$$M_{\text{Pl}}^2 \frac{d^2 \epsilon(\phi)}{d\phi^2} = \eta^2(\phi) - 10\epsilon(\phi)\eta(\phi) + 12\epsilon^2(\phi) + \xi^2(\phi). \quad (24)$$

So for slow-roll inflation, before the scalar field reaches $\phi_{\min}$, both $\epsilon_\phi = d\epsilon(\phi)/d\phi$ and $\epsilon_{\phi\phi} = d^2\epsilon(\phi)/d\phi^2$ are small. If the excursion of scalar field $\Delta\phi/M_{\text{Pl}} < \sqrt{2\epsilon}$, then to get $\Delta\epsilon(\phi) \sim \epsilon$, the higher-order derivatives must be large and give the major contribution to $\Delta\epsilon(\phi)$. When the higher-order derivatives are large, the slow-roll condition may not be satisfied, and the higher-order corrections to the scalar spectral index cannot be neglected. For example, the third-order correction to n_s is [107,108]

$$\begin{aligned} & \left(-96C^2 - \frac{104}{3}C - \frac{3734}{9} + 44\pi^2 \right) \epsilon^3 \\ & + \left(96C^2 - \frac{4}{3}C + \frac{1190}{3} - 44\pi^2 \right) \epsilon^2 \eta \\ & + \left(-16C^2 + 12C - \frac{742}{9} + \frac{28\pi^2}{3} \right) \epsilon \eta^2 + \frac{4}{9} \eta^3 \\ & + \left(-12C^2 + 4C - \frac{98}{3} + 4\pi^2 \right) \epsilon \xi^2 \\ & + \left(C^2 - \frac{8}{3}C + \frac{28}{3} - \frac{13\pi^2}{12} \right) \eta \xi^2 \\ & + \left(C^2 - \frac{2}{3}C + \frac{2}{9} - \frac{\pi^2}{12} \right) \sigma^3, \end{aligned} \quad (25)$$

where $\sigma^3 = M_{\text{Pl}}^6 (V_\phi)^2 V_{\phi\phi\phi\phi}/V^3$. The last term may contribute to n_s if $\epsilon(\phi)$ changes fast. Therefore, it is impossible to get sub-Planckian excursion with $\Delta\phi < 0.1M_{\text{Pl}}$ for single-field slow-roll inflation if r is as large as 0.1. Integrating Eq. (23), we get

$$\sqrt{2\epsilon(\phi_*)} - \sqrt{2\epsilon(\phi_{\min})} = \int_{\phi_*}^{\phi_{\min}} d\phi [\eta(\phi) - 2\epsilon(\phi)]/M_{\text{Pl}}. \quad (26)$$

Since $N = 50-60$, $\epsilon(\phi_{\min}) \ll \epsilon(\phi_*)$, so we obtain [45]

$$\begin{aligned} \frac{\Delta\phi}{M_{\text{Pl}}} & \gtrsim \frac{\sqrt{2\epsilon}}{\langle \eta - 2\epsilon \rangle} = \frac{\sqrt{2\epsilon}}{\sqrt{2\epsilon} - \sqrt{2\epsilon(\phi_{\min})}} \frac{\phi_{\min} - \phi_*}{M_{\text{Pl}}} \\ & \approx \frac{\phi_{\min} - \phi_*}{M_{\text{Pl}}}. \end{aligned} \quad (27)$$

The above bound is trivial since it is just the difference between $\phi_{\min}$ and ϕ_* , and it is true only when a minimum for $\epsilon(\phi)$ exists in the region between ϕ_* and ϕ_e . The bound is also useless because it tells us nothing even if we have the observational information on n_s and r . To obtain the concrete value of $\langle \eta - 2\epsilon \rangle$, we need to calculate both $\phi_{\min}$ and ϕ_* , so it is not easy to calculate the average value of

$\eta - 2\epsilon$ and get the value of the bound (27) in general. In particular, for this bound, we still have no idea whether the sub-Planckian excursion can be realized when observational constraints on n_s and r are known. To get enough N , $\phi_{\min}$ is usually close to ϕ_e , and the potential changes rapidly after $\phi_{\min}$. The slow-roll parameters ϵ and η are smaller than 1, and $\phi_{\min}$ is not far away from ϕ_e , so we expect that $\langle \eta - 2\epsilon \rangle < 1/2$. Therefore, we propose the absolutely minimal modified Lyth bound on $\Delta\phi$,

$$\frac{\Delta\phi}{M_{\text{Pl}}} > \sqrt{8\epsilon} = \sqrt{\frac{r}{2}}. \quad (28)$$

Apparently the bound (28) is smaller than the bound (27), and it is more practical and useful since it depends only on the slow-roll parameter ϵ , or the observable r . The bound holds even if $\epsilon(\phi)$ has no minimum between ϕ_* and ϕ_e . The modified Lyth bound is lower, universal, and model independent in the sense that it involves the slow-roll parameter ϵ only. Although the number of e -folds $N(\phi_*)$ before the end of inflation is absent in the above bounds (27) and (28), the results are not independent of $N(\phi_*)$ because they hold under the assumption that $\epsilon(\phi_{\min}) \ll \epsilon(\phi_*)$, which is based on large N . The modified Lyth bound (28) holds for any single-field inflationary model and is independent of the conditions C1 and C2. With the modified Lyth bound, it is possible to get the sub-Planckian excursion for the inflaton with $\Delta\phi < M_{\text{Pl}}$ even though r is as large as 0.1. We want to emphasize that it is very difficult to saturate this bound. The modified Lyth bound (28) tells us that r should satisfy $r < 0.02$ to get $\Delta\phi < 0.1M_{\text{Pl}}$.

For a fifth polynomial potential with $\lambda_1 = -0.1581$, $\lambda_2 = 0.003433$, $\lambda_3 = -0.01054$, $\lambda_4 = 0.9115$, and $\lambda_5 = -0.97$, we get $\Delta\phi = \phi_e = 0.67M_{\text{Pl}}$ with $N = 57.5$ and $r = 0.2$. For this potential, $\langle \eta - 2\epsilon \rangle = 0.28$, and the bound (27) requires $\Delta\phi > 0.56M_{\text{Pl}}$, the Lyth bound requires $\Delta\phi > 9.1M_{\text{Pl}}$, and our modified bound requires $\Delta\phi > 0.32M_{\text{Pl}}$, which is lower than the bound (27). The result $\Delta\phi = 0.67M_{\text{Pl}}$ satisfies both the bounds (27) and (28) and violates the Lyth bound. If we take $\lambda_1 = -0.1581$, $\lambda_2 = 0.003433$, $\lambda_3 = -0.01054$, $\lambda_4 = 5.0$, and $\lambda_5 = -9.4728$, we get $\Delta\phi = 0.36M_{\text{Pl}}$ with $N = 57.1$ and $r = 0.2$. For this case, $\langle \eta - 2\epsilon \rangle = 0.4994$, both bounds (27) and (28) require $\Delta\phi > 0.32M_{\text{Pl}}$, and the Lyth bound requires $\Delta\phi > 9.03M_{\text{Pl}}$. The model also satisfies both the bounds (27) and (28) and violates the Lyth bound. If we take $r = 0.01$, $\lambda_1 = -0.0353553$, $\lambda_2 = -0.00802833$, $\lambda_3 = -0.00235702$, $\lambda_4 = 57.0$, and $\lambda_5 = -396.072$, we get $\Delta\phi = \phi_e = 0.099M_{\text{Pl}}$ and $N = 58.5$. The corresponding Lyth bound is $\Delta\phi > 2.1M_{\text{Pl}}$, the modified Lyth bound (28) is $\Delta\phi > 0.07M_{\text{Pl}}$, and the bound (27) is $\Delta\phi > 0.087M_{\text{Pl}}$ with $\langle \eta - 2\epsilon \rangle = 0.41$. The above result is derived based on the slow-roll approximation, and in general, we expect the inflaton excursion is larger than the modified Lyth bound (28) if we solve the exact dynamical equation either

analytically or numerically, especially for the nonstandard slow-roll potentials. For the fifth polynomial potential with $\lambda_1 = -0.1581$, $\lambda_2 = 0.003433$, $\lambda_3 = -0.01054$, $\lambda_4 = 0.9115$, and $\lambda_5 = -0.97$, the numerical result gives $\Delta\phi = \phi_e = 1.14M_{\text{Pl}}$ and $N = 59.2$. For the potential with $\lambda_1 = -0.1581$, $\lambda_2 = 0.003433$, $\lambda_3 = -0.01054$, $\lambda_4 = 5.0$, and $\lambda_5 = -9.4728$, the numerical result gives $\Delta\phi = \phi_e = 0.69M_{\text{Pl}}$ and $N = 59.2$. For the potential with $\lambda_1 = -0.0353553$, $\lambda_2 = -0.00802833$, $\lambda_3 = -0.00235702$, $\lambda_4 = 57.0$, and $\lambda_5 = -396.072$, the numerical result gives $\Delta\phi = \phi_e = 0.29M_{\text{Pl}}$ and $N = 59.8$.

IV. POLYNOMIAL POTENTIAL

Now, let us show how to construct an inflationary model that satisfies all three conditions C1–C3 with the polynomial potential as an example. For simplicity, we assume $M_{\text{Pl}} = 1$ and denote the magnitudes of the inflaton ϕ at the horizon crossing and the end of inflation as ϕ_* and ϕ_e , respectively. For the polynomial potential, the slow-roll parameters at the horizon crossing ϕ_* are

$$\epsilon = \frac{r}{16} = \frac{\lambda_1^2}{2}, \quad \eta = 2\lambda_2, \quad \xi^2 = 6\lambda_1\lambda_3. \quad (29)$$

From the observational constraint on r , we can get the coefficient λ_1 . As we discussed above, the main contribution to the running of the scalar spectral index comes from ξ^2 . So the observational constraints on ϵ and n'_s give the coefficient λ_3 [21],

$$\lambda_1 = -\sqrt{2\epsilon} = -\sqrt{\frac{r}{8}}, \quad \lambda_3 \approx \frac{n'_s}{3\sqrt{2r}}. \quad (30)$$

Once ϵ and ξ^2 are known, the slow-roll parameter η is determined from the scalar spectral index (17), and the coefficient λ_2 is

$$\lambda_2 \approx \frac{n_s - 1}{4} + \frac{3r}{32} - \frac{1}{4}(C - 1/3)n'_s. \quad (31)$$

For $m = 3$, if we take $\lambda_1 = -0.162$, $\lambda_2 = 0.0016$, and $\lambda_3 = -0.0132$, we get $n_s = 0.957$, $r = 0.21$, $n'_s = -0.0292$, and $\phi_e = 2.7M_{\text{Pl}}$. Because ϵ is a monotonic function, the number of e -folds before the end of inflation is $N = 9.12$, which is not large enough to solve the horizon problem. Therefore, we need to introduce a few more terms λ_m with $m > 3$ so that we have a high enough number of e -folds. If we add one more term and consider $m = 4$, we find that the slow-roll parameters are always smaller than 1 if λ_4 is too small and that ϕ_{min} decreases as λ_4 increases. Because the third-order slow-roll parameter σ^3 is proportional to λ_4 , the slow-roll condition requires that λ_4 be small. Therefore, λ_4 lies in a narrow region. However, for those values of λ_4 , $\epsilon(\phi_{\text{min}}) = 0$ and near ϕ_{min} , $\epsilon(\phi) \approx \epsilon_{\phi\phi}(\phi_{\text{min}})(\phi - \phi_{\text{min}})^2/2$, the integral $\int 1/\sqrt{2\epsilon(\phi)}d\phi$ is

logarithm divergent, and then we need to consider more terms. From the number of freedom counting, we might only need to introduce at most two terms, for example, the λ_4 and λ_5 terms. The coefficients λ_4 and λ_5 are then determined from $N(\phi = \phi_*) = 60$ and $\epsilon(\phi_e) = 1$ (or $\eta(\phi_e) = 1$) [21]. Additionally, we require that the potential has a flat plateau so that $\epsilon(\phi)$ has a minimum and $\epsilon(\phi_{\text{min}})$ is close to 0. At ϕ_{min} , $\eta \approx 0$, so we may express λ_5 in terms of ϕ_{min} and λ_4 , and then $\epsilon(\phi_{\text{min}})$ is a function of λ_4 . The number of e -folds before the end of inflation is usually between 50 and 60. To get enough e -folds, we require $\epsilon(\phi_{\text{min}})$ to be around 10^{-6} , and then λ_4 is determined once ϕ_{min} is given.

Following the procedure discussed above, we construct inflationary models that are consistent with the observational constraints. In particular, we consider the inflaton potential with $\lambda_1 = -0.162$, $\lambda_2 = 0.00161$, $\lambda_3 = -0.0132$, $\lambda_4 = 0.01$, and $\lambda_5 = -0.00146$. By using the slow-roll formula, we get $n_s = 0.957$ with the second-order correction (17), $r = 0.21$, $n'_s = -0.0292$, $N = 56.4$, and $\phi_e = 4.42$. So $\Delta\phi = 4.42 < N\sqrt{2\epsilon} = 9.13$. For the potential with $\lambda_1 = -0.0354$, $\lambda_2 = -0.0078$, $\lambda_3 = -0.00024$, $\lambda_4 = 1.398$, and $\lambda_5 = -2.766$, we get $n_s = 0.9655$, $r = 0.01$, $n'_s = -0.0003$, $N = 58.7$, and $\phi_e = 0.41$, which is much less than the Lyth bound $\Delta\phi > 2.08$.

To understand why the polynomial potential we constructed violates the Lyth bound but is consistent with the observational results, we plot the potential and the slow-roll parameters in Fig. 1. We also show the slow-roll results and the observational contours in Fig. 2. At the horizon crossing ϕ_* , the potential has a large slope, so the slow-roll parameter $\epsilon(\phi)$ is relatively large at ϕ_* , and the derived n_s , n'_s , and r are consistent with the observations. After the horizon crossing, the potential becomes very flat, and $\epsilon(\phi)$

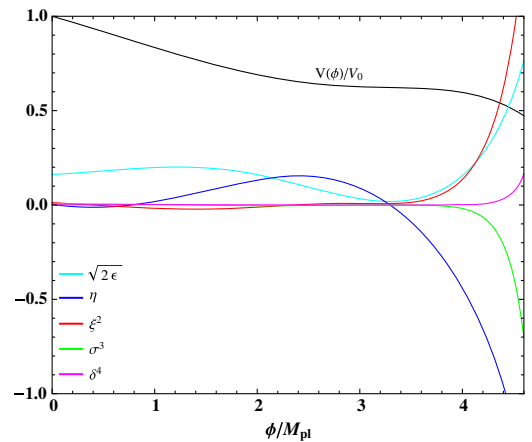


FIG. 1 (color online). The behavior of the potential $V(\phi)$ and the corresponding slow-roll parameters $\epsilon(\phi)$, $\eta(\phi)$, $\xi^2(\phi)$, $\sigma^3(\phi)$, and $\delta^4(\phi) = M_{\text{Pl}}^8 (V_\phi)^3 V_{\phi\phi\phi\phi\phi} / V^4(\phi)$ for the polynomial potential with the coefficients $\lambda_1 = -0.162$, $\lambda_2 = 0.00161$, $\lambda_3 = -0.0132$, $\lambda_4 = 0.01$, and $\lambda_5 = -0.00146$.

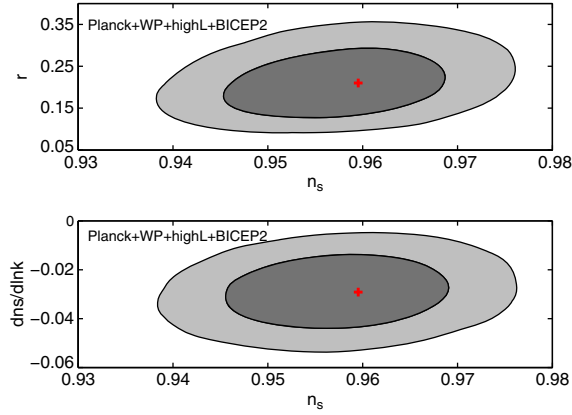


FIG. 2 (color online). The “+” stands for the results on n_s , r , and n'_s from the polynomial potential with $\lambda_1 = -0.162$, $\lambda_2 = 0.00161$, $\lambda_3 = -0.0132$, $\lambda_4 = 0.01$, and $\lambda_5 = -0.00146$. We also show the 68% and 95% contours constrained from the combination of Planck + WP + highL + BICEP2 data.

decreases to be very small. Near the end of inflation, the potential changes quickly, and the slow-roll parameter $\epsilon(\phi)$ or $\eta(\phi)$ quickly increases to 1. Therefore, in principle, all three conditions C1–C3 can be satisfied if the potential has the above property.

Here, we construct the potential by using the slow-roll conditions. If the slow-roll conditions are not satisfied, we need to solve the Mukhanov–Sasaki equation numerically [109,110],

$$\frac{d^2 v_k}{d\tau^2} + \left(k^2 - \frac{1}{z} \frac{d^2 z}{d\tau^2} \right) v_k = 0, \quad (32)$$

and calculate the power spectrum

$$\mathcal{P}_{\mathcal{R}} = \frac{k^3}{2\pi^2} \left| \frac{v_k}{z} \right|^2 = A_s \left(\frac{k}{k_*} \right)^{n_s - 1 + \frac{1}{2} n'_s \ln(k/k_*) + \dots}, \quad (33)$$

where $z = a\mathcal{H}^{-1}d\phi/d\tau$, $\mathcal{H} = a^{-1}da/d\tau$, and the conformal time $d\tau = dt/a$. By doing so, Ben–Dayan and Brustein obtained the sub-Planckian excursion with $\Delta\phi \sim 0.5M_{\text{Pl}}$ [21]. For the potential we considered above, the numerical solution to the Mukhanov–Sasaki equation gives $n_s = 0.9587$, $n'_s = -0.0297$, and $r = 0.2$ for $k_* = 0.002 \text{ Mpc}^{-1}$, which are consistent with the slow-roll results. For this model, we also get $\Delta\phi = 5.07M_{\text{Pl}}$ and $N = 59.1$. As expected, the numerical results on $\Delta\phi$ and N are larger than the slow-roll results.

V. DISCUSSIONS AND CONCLUSION

Whatever the conditions C1 and C2 are satisfied or not, both the Lyth bound and the modified Lyth bound hold for

simple slow-roll inflationary models with $N = 50$ –60. If r is at the order of 0.1, then the Lyth bound tells us that $\Delta\phi \gtrsim 5.6M_{\text{Pl}}$, and the sub-Planckian excursion for the inflaton is impossible. To get the sub-Planckian excursion, the Lyth bound requires that $r \lesssim 10^{-3}$. On the other hand, the Lyth bound is based on the assumption that $\epsilon(\phi)$ is a monotonic function, so it can be violated if $\epsilon(\phi)$ is not a monotonic function. We have derived the absolutely minimal modified Lyth bound $\Delta\phi/M_{\text{Pl}} > \sqrt{8\epsilon} = \sqrt{r/2}$ by requiring that $N(\phi_*)$ be large. With the modified Lyth bound, if $r = 0.2$, then it is possible to get the sub-Planckian excursion for the inflaton since $\Delta\phi > 0.32M_{\text{Pl}}$. For example, if we choose $\lambda_1 = -0.1581$, $\lambda_2 = 0.00343$, $\lambda_3 = -0.010541$, $\lambda_4 = 5.0$, and $\lambda_5 = -9.47$, we get $\Delta\phi = 0.36M_{\text{Pl}}$ with $N = 57.1$ while the Lyth bound requires that the model satisfies $\Delta\phi > 9.03M_{\text{Pl}}$. So the modified Lyth bound opens the possibility that all three conditions C1, C2, and C3 can be satisfied. We provided a procedure to construct models that satisfy the conditions C1 and C2 by using the polynomial potential as an example. The model we considered in this work gives $n_s = 0.957$, $n'_s = -0.0292$, $r = 0.21$, $\Delta\phi = 4.42M_{\text{Pl}}$, and $N = 56.4$ by using the slow-roll formulas, and the more accurate numerical calculation gives $n_s = 0.9587$, $n'_s = -0.0297$, $r = 0.2$, $N = 59.1$, and $\Delta\phi = 5.07M_{\text{Pl}}$ for $k_* = 0.002 \text{ Mpc}^{-1}$. So it satisfies the conditions C1–C3. Furthermore, the model supports the modified Lyth bound. The modified Lyth bound (28) is further supported by the work in Ref. [21] with $\Delta\phi \sim 0.5M_{\text{Pl}}$. Even if r is found to be much smaller and the inflaton is a small field, the modified Lyth bound still limits the inflaton excursion. For example, to get $\Delta\phi < 0.1M_{\text{Pl}}$, the modified Lyth bound (28) only requires $r < 0.002$. By using the slow-roll formulas, we constructed a polynomial potential with $\lambda_1 = -0.0354$, $\lambda_2 = -0.0078$, $\lambda_3 = -0.00024$, $\lambda_4 = 1.398$, and $\lambda_5 = -2.766$ to give $n_s = 0.9655$, $r = 0.01$, $n'_s = -0.0003$, and $N = 58.7$. The corresponding inflaton excursion is $\Delta\phi = 0.41M_{\text{Pl}}$.

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