Measuring the speed of cosmological gravitational waves

Marco Raveri,[1,2] Carlo Baccigalupi,[1,2] Alessandra Silvestri,[1,2,3,4] and Shuang-Yong Zhou[1,2,5]

[1]*SISSA—International School for Advanced Studies, Via Bonomea 265, 34136, Trieste, Italy*
[2]*INFN, Sezione di Trieste, Via Valerio 2, I-34127 Trieste, Italy*
[3]*INAF—Osservatorio Astronomico di Trieste, Via G.B. Tiepolo 11, I-34131 Trieste, Italy*
[4]*Institute Lorentz, Leiden University, PO Box 9506, Leiden 2300 RA, The Netherlands*
[5]*Department of Physics, Case Western Reserve University, 10900 Euclid Avenue, Cleveland, Ohio 44106, USA*

In general relativity gravitational waves propagate at the speed of light; however, in alternative theories of gravity that might not be the case. We investigate the effects of a modified speed of gravity, c_T^2, on the B modes of the cosmic microwave background (CMB) anisotropy in polarization. We find that a departure from the light speed value would leave a characteristic imprint on the BB spectrum part induced by tensors, manifesting as a shift in the angular scale of its peaks which allows us to constrain c_T without any significant degeneracy with other cosmological parameters. We derive constraints from current data and forecast the accuracy with which c_T will be measured by the next generation CMB satellites. In the former case, using the available Planck and BICEP2 data sets, we obtain $c_T^2 = 1.30 \pm 0.79$ and $c_T^2 < 2.85$ at 95% C.L. by assuming a power law primordial tensor power spectrum and $c_T^2 < 2.33$ at 95% C.L. if the running of the spectral index is allowed. More interestingly, in the latter case we find future CMB satellites capable of constraining c_T^2 at percent level, comparable with bounds from binary pulsar measurements, largely due to the absence of degeneracy with other cosmological parameters.

Despite decades of intensive effort, gravitational waves (GWs) have yet to be observed directly. While the situation may change in the coming decade [1], a new window has been opened by cosmic microwave background (CMB) experiments that have recently detected the B modes of polarization [2,3], offering an indirect measurement of cosmological GWs (tensor modes). While on small angular scales the BB power spectrum is dominated by the lensing of the CMB, on larger scales, the B modes of polarization are primarily produced by tensor modes and give an insight into primordial GWs [4].

In general relativity, short-wavelength GWs follow the null geodesics of the background; thus, their propagation speed equals the speed of light on a flat background. However in models of modified gravity, this is often not the case. For instance, some of the generalized scalar-tensor models within the Horndeski family [5,6], like the covariant Galileon involving certain derivative couplings, are expected to modify the tensor propagation speed [7–10]; also, in Hořava-Lifshitz [11,12] gravity and in Lorentz-violating theories [13–15], the speed of gravity may deviate from that of light. Finally, quantum gravity effects may modify the dispersion relation of GWs [16], or the graviton may have a mass which prevents it from behaving lightlike [17]. Massive gravity has the added complication that diffeomorphism invariance is explicitly broken; therefore, we will not consider it in our analysis [17,18]. As we will discuss, in general, one can also have a modification of the Planck's mass in the equation for the propagation of tensor modes; however, such a modification is expected to leave a signature on the BB power spectrum, which is mild and

degenerate with other cosmological parameters [19]. We will focus on modifications of c_T which, as we will show, leave a distinct and potentially sizable signature on the BB spectrum.

A direct measurement of the speed of GWs could be achieved by comparing the arrival times of light and gravitational wave signals from a distant astronomical source [20–23]. This is not yet possible; however, indirect, local observations of gravitational radiation seem to suggest that its propagation speed, at the current epoch, is close to the speed of light. For instance, accurate measurements of binary pulsar timing indicate that the sound speed of GWs should not deviate from the general relativistic value by more than 1% [20]; the latter bound assumes that energy is lost via GWs; models of modified gravity might, however, imply also a loss of energy via scalar radiation associated with an additional degree of freedom, possibly modifying this bound [24–26]. The propagation speed of a GW can also be constrained from the observations of ultrahigh energy cosmic rays that would lose energy via Cherenkov radiation [27,28].

In this paper we focus on the B modes of CMB polarization and show how they offer a novel, *independent* way to measure the propagation speed of GWs at the time of recombination. As we will show, variations of c_T affect the BB power spectrum in a unique way which makes it orthogonal with other cosmological parameters. Interestingly, as we will discuss, the B modes are only sensitive to the modifications of the dispersion relation of the graviton around the time of recombination; therefore, they are complementary to bounds from binary pulsars, allowing

a combined constraint on the time variation of c_T. We derive bounds from current data, which we find to have limited constraining power, as well as forecasts from upcoming and future cosmic variance limited CMB experiments.

On a flat Friedmann-Robertson-Walker background, one can use the rotational and translational symmetries to decompose the metric perturbations into scalar, vector, and tensor components. We are only interested in the tensorial part:

$$ds^2 = -a(\tau)^2[d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j], \quad (1)$$

where h_{ij} satisfy $\partial_i h_{ij} = 0$ and $h_{ii} = 0$. We shall consider a linear perturbation theory that also satisfies the gauge symmetry $h_{ij} \rightarrow h_{ij} + \partial_{(i}\epsilon_{j)}$, where ϵ_i is a generic function of the coordinates.

Rather than restricting ourselves to some specific modified gravity models, we shall employ an effective field theory approach [29] to write down the most general quadratic action for h_{ij} that is ghost free and satisfies the symmetries mentioned above:

$$S_T^{(2)} = \frac{1}{8}\int d\tau d^3x a^2 M_P^2(\tau)(h'_{ij}h'_{ij} - c_T^2(\tau)\partial_k h_{ij}\partial_k h_{ij}), \quad (2)$$

where matter sources are assumed to be minimally coupled to the metric. Generally higher order gradient terms $h_{ij}\partial^{2n}h_{ij}(n > 1)$ should also be included in action (2), but we have neglected them as we are interested in the low energy phenomenology [18]. In addition, all of these operators would have relevant effects on small scales where the B-mode spectrum is dominated by lensing. The function $M_P^2(\tau)$ plays the role of the Planck mass, which is allowed by the above mentioned symmetries to vary in time. We fix its value to $M_P^2 \equiv (8\pi G)^{-1}$ since time variations of this quantity are constrained by a large number of complementary observations [20,30,31], while polarization observables are expected to depend weakly on this quantity [19]. We are then left with modifications of the

sound speed of the tensor modes, so that action (2) results in the following wave equation:

$$h''_{ij} + 2\frac{a'}{a}h'_{ij} - c_T^2\partial_k\partial_k h_{ij} = \frac{2}{M_P^2 a^2}S_{ij}, \quad (3)$$

where S_{ij} is the transverse and traceless component of the energy momentum tensor of the matter sector. This expression cannot be reduced to the general relativity (GR) case by means of a conformal/disformal transformation [32] because of the presence of matter. Unlike the case of inflation, in fact, this would introduce a nonminimal coupling between matter and gravity. This type of coupling will modify all of the matter equations and the physical effect we are identifying would show up in any case, with the final result not depending on the frame chosen.

Varying c_T^2 changes the relevant dynamical scale of tensor perturbations from the effective cosmological horizon, corresponding to the case $c_T^2 = 1$ (in units of the speed of light), to the sound horizon. For this reason the net effect on CMB spectra is a horizontal shift of the whole tensor induced component, whose main peak traces the angular scale of the GW sound horizon at recombination, as can be seen in Fig. 1. Notably, the sources of the E- and B-mode polarization spectra are peaked at the recombination epoch [4], thus making them dependent on the dynamics of tensor perturbations at earlier times but limiting the impact of a later evolution. We have studied this effect numerically and have found that a possible late time dependence of the GW's sound speed does not noticeably affect the polarization observables, which are in turn sensitive to its value around the recombination time. According to this result, we have assumed c_T^2 to be constant throughout all cosmological epochs. Changing the speed of GWs in principle also impacts the reionization bump at large scales in the polarization spectra. The effect is, however, less prominent than the shift of the recombination peak and its constraining power is also reduced by cosmic variance, which is stronger
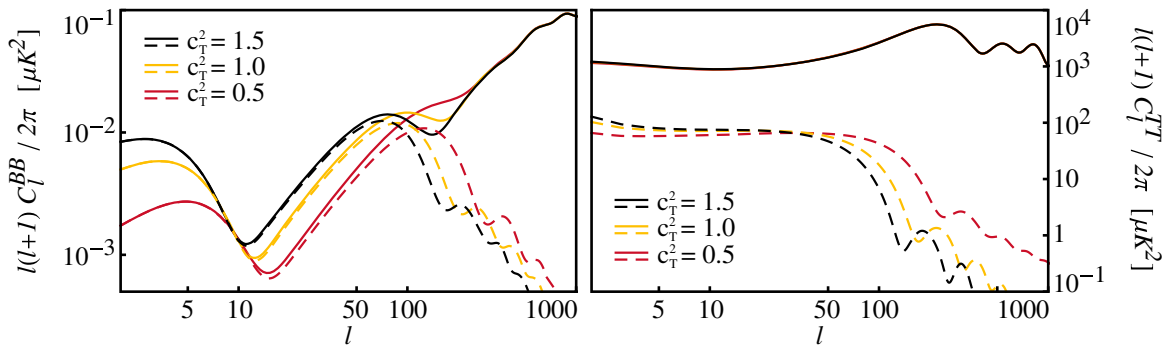

FIG. 1 (color online). (Left panel) The total B-mode polarization power spectrum (solid lines) and its component due to tensor perturbations (dashed lines). (Right panel) The total CMB temperature power spectrum (solid lines) and its tensor component (dashed lines). In both panels different colors correspond to different values of the speed of the GWs. The other cosmological parameters are fixed to the best fit of the Planck and BICEP2 data sets.

at those scales. The effect of horizontal shifting of the tensor component of the CMB spectra can, in principle, be mimicked by a change in the cosmological expansion history which is, however, tightly constrained by the scalar part of the CMB itself [33,34] and many other observations. Other cosmological parameters, especially those defining the primordial tensor power spectrum, are not expected to be degenerate with c_T^2. The tensor to scalar ratio, for example, vertically shifts the tensor part of the CMB spectra while the spectral index primarily changes its shape. More complicated models for the tensor primordial power spectrum could, in principle, be degenerate with c_T^2, but if inflationary consistency relations [35] are assumed, then the scalar sector is expected to break this degeneracy.

From Fig. 1 we can also see that the effect of changing c_T^2 weakly influences the CMB temperature power spectrum because the tensor induced component is several orders of magnitude smaller than the scalar one for values of the tensor to scalar ratio not yet excluded by observations. We have also investigated the influence of this effect on the E-mode polarization spectrum and have found it to be negligible. The B-mode spectrum is instead greatly influenced by changes in the speed of the GWs, thus making this CMB observable the most suited for these studies.

In addition, depending on the particular theory of gravity, a change in the speed of the GWs will be accompanied by an effect on the scalar sector of the theory in many different ways [36,37]. In general the scalar sector can exhibit a very rich and complex phenomenology while the tensor sector, on the contrary, depends on the key quantities that we described. For this reason, quantifying the effect on the scalar sector that corresponds to a modified speed of GWs in the tensor sector requires specifying to a particular model and is beyond the scope of this paper. However, if the modeling of the scalar sector is taken into account, we can identify two possible situations. In the first

case, the scalar sector will help in tightening the constraints on the speed of GWs if this quantity enters into the description of the scalar sector, as it happens in scalar tensor theories where a modified speed of GWs also modifies gravitational slip that is severely constrained at the time of recombination. On the other hand, if a modified speed of GWs has a limited impact on the scalar sector or the description of this sector is so complicated that the parameters describing it are degenerate, the constraining power of the tensor component of the CMB B-mode power spectrum, which we shall describe in this work, will help in breaking these degeneracies. In both cases it is clear that the constraints that can be obtained from these kinds of measurements will not be weaker than the one that we shall now describe.

We use the recently released BICEP2 [2] data, along with the Planck CMB temperature power spectrum measurements [38] and the WMAP low-ℓ polarization spectra [39], to constrain the speed of sound of cosmological GWs. To forecast the precision at which this quantity will be measured by the next generation of CMB experiments, we create simulated data sets adopting the specifications of the Cosmic Origins Explorer (COrE) [40] and the Polarized Radiation Imaging and Spectroscopy Mission (PRISM) [41] satellites. We perform a Markov chain Monte Carlo analysis of both the current data and the simulated data using the publicly available CosmoMC package [42]; in the case of the forecast, this allows us to have a good handle on the degeneracies among cosmological parameters. We allow variations of the six baseline cosmological parameters of the Λ Cold Dark Matter (ΛCDM) model, plus the running of the scalar spectral index and the amplitude of primordial cosmological GWs; we impose the single field inflationary scenarios to relate the spectral index of tensors to the scalar one.

The results of this analysis for both current data and the forecast are shown in Fig. 2, while in Fig. 3 we provide a
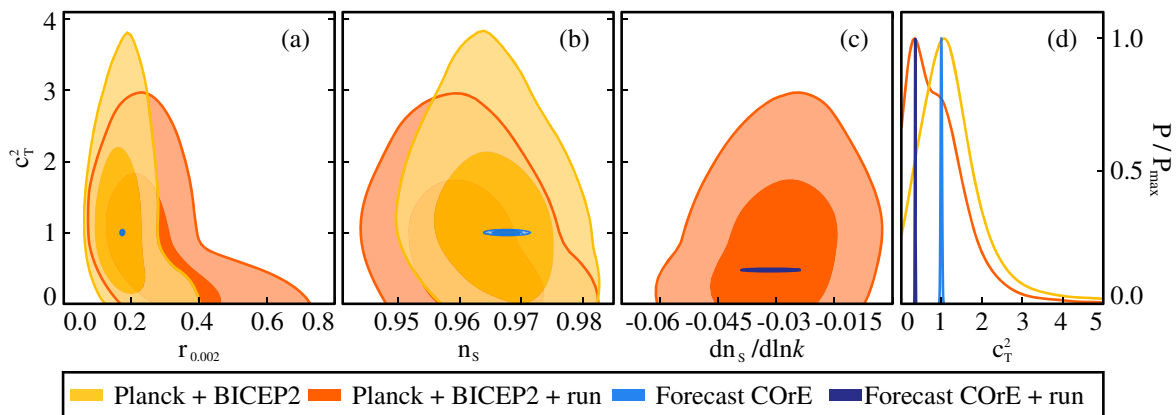

FIG. 2 (color online).   (Left panels) The marginalized joint likelihood for the GWs speed of sound c_T^2 and parameters defining the primordial tensor power spectrum: the tensor to scalar ratio $r_{0.002}$, the scalar perturbation spectral index n_s, and its running $dn_s/d\ln k$. Different colors correspond to different combinations of data sets and models, as shown in the legend. The two different shades indicate the 68% and 95% confidence regions. (Right panels) Marginalized likelihoods of the tensor perturbation sound speed for the considered data sets and models.
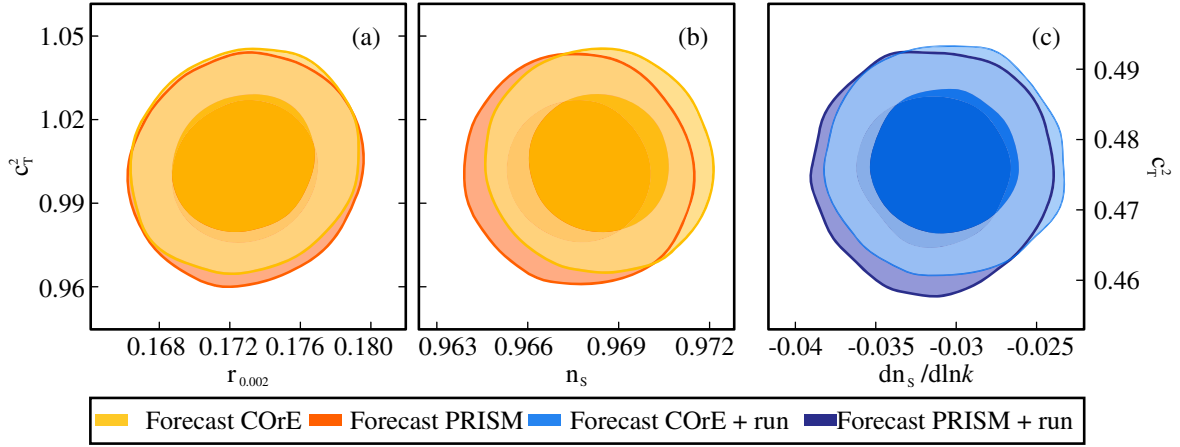
FIG. 3 (color online).   The marginalized joint likelihood for the GW's speed of sound c_T^2, the tensor to scalar ratio $r_{0.002}$, the scalar perturbation spectral index n_s, and its running $dn_s/d\mathrm{ln}k$. Different colors correspond to different instrumental specifications used in the forecast and different models, as shown in the legend. The two different shades indicate the 68% and 95% confidence regions.

zoom of the forecasts of the cosmological parameters most relevant for our analysis. In Fig. 2(d), we can see that the marginalized likelihood of c_T^2 is peaked at its GR value, i.e., $c_T^2 = 1$ (in units of the speed of light), when considering a model without the running of the spectral index. Very high values of c_T^2 are excluded since they would move the tensor component of the B-mode spectrum to large scales, resulting in a poor fit of the measured data points. In Fig. 2(a), we can notice that there is a degeneracy between c_T^2 and the tensor to scalar ratio at a pivot scale of $0.002 \ \mathrm{Mpc}^{-1}$. The reason for this degeneracy is that those values of c_T^2 shift the GW's contribution to the spectrum toward smaller scales so that the only way to fit the data points is to change the spectrum amplitude. We can also see in Fig. 2(b) that c_T^2 is weakly degenerate with the spectral index due to the poor constraining power of the BICEP2 measurements. From the combination of the Planck and BICEP2 data sets we obtain the marginalized bound: $c_T^2 = 1.30 \pm 0.79$ and $c_T^2 < 2.85$ at 95% C.L.

If we allow a running of the primordial tensor power spectrum index the situation changes slightly. From the marginalized joint likelihood of c_T^2, $r_{0.002}$, n_s, and $dn_s/d \ln k$ in Figs. 2(a)–2(c), we can see that c_T^2 is driven toward smaller values and that this is further confirmed by its marginalized distribution in Fig. 2(d). The peak of the probability distribution of c_T^2 is not found to be at its GR value, which is, however, not excluded. From the same figure we can see that as c_T^2 goes toward smaller values, its degeneracy with $r_{0.002}$ is enhanced while it is not so pronounced with respect to the running of the spectral index, shown in Fig. 2(c), and n_s itself, shown in Fig. 2(b). Given the skewness of the marginal distribution of c_T^2, which is also cut at $c_T^2 = 0$, we report here only its upper bound: $c_T^2 < 2.33$ at 95% C.L.

We now turn to the forecasts to further investigate these degeneracies and to evaluate our capability of constraining

the speed of cosmological GWs with future generation surveys. Our results do not include any forecast on delensing capability, and thus represent rather conservative bounds in the adopted forecast setup. Indeed, the CMB lensing signal represents the main contaminant for the measurement of primordial GWs from the BB spectrum, and the constraining power will improve according to the capability of tracing this signal. The results are shown in Fig. 3, with a fiducial model assumed to be the best fit one obtained with the Planck and BICEP2 data sets. We can clearly see that increasing the accuracy of B-mode polarization observations removes all of the degeneracy with the other cosmological parameters since the measurements would be able to disentangle the effect of horizontal shifting, due to changes in c_T^2, from the vertical shifting induced by varying $r_{0.002}$ or the shape changes due to n_s and the running of the spectral index. As a result, we can say that the speed of GWs at recombination is orthogonal to other cosmological parameters, as is theoretically expected. We can also notice that considering an instrument with higher precision like PRISM does not improve significantly on the determination of c_T^2 with respect to COrE since the effect is seen at degree angular scales where both of the considered experiments are cosmic variance limited.

Overall, we see that the next generation of CMB experiments will constrain the speed of cosmological GWs with a 1% accuracy, independent of the assumed shape of the primordial power spectrum.

In conclusion, we have considered a varying sound speed for cosmological GWs and its effects on the power spectrum of B modes of the CMB anisotropies polarization. We have derived the constraints on this quantity using the present CMB data from Planck and the BB power spectrum on the degree angular scale, as reported recently by the BICEP2 experiment. Moreover, we have derived the projected constraints which will be within reach of future generations, polarization dedicated CMB satellites.

We have found that a departure of c_T from the speed of light has an apparent projection effect on the characteristic peak of the BB power spectrum at the degree scale, which corresponds to the part produced by cosmological GWs. We have identified the origin of this feature in the effective "rescaling" of the gradient term of wave equation (3), affecting the horizon reentry time for the tensor component, and thus the location of the BB peak in the angular domain.

Despite the high constraining power of Planck on ΛCDM parameters, the claimed statistical significance in current measurements of B modes results in a broad constraint on the GW's sound speed, corresponding to $c_T^2 = 1.30 \pm 0.79$ and $c_T^2 < 2.85$ at 95% C.L. by assuming a power law primordial tensor power spectrum; $c_T^2 < 2.33$ at 95% C.L. is obtained if a running of the spectral index is allowed. Since the effect of c_T^2 is rather orthogonal with that of other cosmological parameters, we have established the ultimate constraining power by adopting the specifications of the future proposed CMB satellites. We have found that those are indeed capable of resolving the parameter space into a neat constraint on c_T, without degeneracies with other parameters, down to a percent level. We observe that such a constraining power is almost competitive with the one from observations of binary pulsar timing.

These results confirm the relevance of CMB polarization measurements in exploring fundamental physics. Current suborbital probes are expected to improve substantially on the present constraints on the propagation velocity of GWs. As we have shown, this quantity is independent from other ΛCDM parameters, and consequently the ultimate precision—within reach of future, polarization dedicated CMB satellites—will be at the level of the best probes which have been conceived so far. The Planck Collaboration is expected to publish results including polarization in the near future. At the same time, the progress in the observations at degree and arcminute (e.g., BICEP2 [2] and POLARBEAR [3]) scales, are expected to contribute substantially to the measurement of the tensor cosmological component constraining the effects which contribute to shaping the BB spectrum of CMB polarization anisotropies.

Note added.—Recently, a related paper [37] commented upon similar ideas. Our work focuses on the detectability of c_T while providing forecasts for next generation CMB experiments.

---

[1] Z. Marka and S. Marka, in *Proceedings of the 8th Edoardo Amaldi Conference*, Amaldi 8, New York, 2009.

[2] P. A. R. Ade *et al.* (BICEP2 Collaboration), BICEP2 I: Detection of *B*-Mode Polarization at Degree Angular Scales, Phys. Rev. Lett. **112**, 241101 (2014).

[3] P. A. R. Ade *et al.* (POLARBEAR Collaboration), A measurement of the cosmic microwave background *B*-mode polarization power spectrum at subdegree scales with POLARBEAR, Astrophys. J. **794**, 171 (2014).

[4] M. Zaldarriaga and U. Seljak, An all sky analysis of polarization in the microwave background, Phys. Rev. D **55**, 1830 (1997).

[5] G. W. Horndeski, Second-order scalar-tensor field equations in a four-dimensional space, Int. J. Theor. Phys. **10**, 363 (1974).

[6] C. Deffayet, X. Gao, D. A. Steer, and G. Zahariade, From *k*-essence to generalised Galileons, Phys. Rev. D **84**, 064039 (2011).

[7] L. Amendola, Cosmology with nonminimal derivative couplings, Phys. Lett. B **301**, 175 (1993).

[8] T. Kobayashi, M. Yamaguchi, and J.'i. Yokoyama, Generalized *G*-inflation: Inflation with the most general second-order field equations, Prog. Theor. Phys. **126**, 511 (2011).

[9] X. Gao and D. A. Steer, Inflation and primordial non-Gaussianities of "generalized Galileons," J. Cosmol. Astropart. Phys. 12 (2011) 019.

[10] A. De Felice and S. Tsujikawa, Inflationary non-Gaussianities in the most general second-order scalar-tensor theories, Phys. Rev. D **84**, 083504 (2011).

[11] P. Horava, Quantum gravity at a Lifshitz point, Phys. Rev. D **79**, 084008 (2009).

[12] C. Bogdanos and E. N. Saridakis, Perturbative instabilities in Horava gravity, Classical Quantum Gravity **27**, 075005 (2010).

[13] V. A. Rubakov, Lorentz-violating graviton masses: Getting around ghosts, low strong coupling scale and VDVZ discontinuity, arXiv:hep-th/0407104.

[14] S. L. Dubovsky, Phases of massive gravity, J. High Energy Phys. 10 (2004) 076.

[15] S. Liberati, Tests of Lorentz invariance: A 2013 update, Classical Quantum Gravity **30**, 133001 (2013).

[16] Y.-F. Cai and Y. Wang, Testing quantum gravity effects with latest CMB observations, Phys. Lett. B **735**, 108 (2014).

[17] S. Dubovsky, R. Flauger, A. Starobinsky, and I. Tkachev, Signatures of a graviton mass in the cosmic microwave background, Phys. Rev. D **81**, 023523 (2010).

[18] A. E. Gumrukcuoglu, S. Kuroyanagi, C. Lin, S. Mukohyama, and N. Tanahashi, Gravitational wave signal from massive gravity, Classical Quantum Gravity **29**, 235026 (2012).

[19] V. Pettorino and L. Amendola, Friction in gravitational waves: A test for early-time modified gravity, arXiv:1408.2224.

[20] C. M. Will, The confrontation between general relativity and experiment, Living Rev. Relativity **17**, 4 (2014).

[21] E. O. Kahya, A useful guide for gravitational wave observers to test modified gravity models, Phys. Lett. B **701**, 291 (2011).

[22] S. Desai, E. O. Kahya, and R. P. Woodard, Reduced time delay for gravitational waves with dark matter emulators, Phys. Rev. D **77**, 124041 (2008).

[23] A. Nishizawa and T. Nakamura, Measuring speed of gravitational waves by observations of photons and neutrinos from compact binary mergers and supernovae, Phys. Rev. D **90**, 044048 (2014).

[24] A. Silvestri, Scalar Radiation from Chameleon-Shielded Regions, Phys. Rev. Lett. **106**, 251101 (2011).

[25] C. de Rham, A. Matas, and A. J. Tolley, Galileon radiation from binary systems, Phys. Rev. D **87**, 064024 (2013).

[26] P. Brax, A.-C. Davis, and J. Sakstein, Pulsar constraints on screened modified gravity, Classical Quantum Gravity **31**, 225001 (2014).

[27] C. M. Caves, Gravitational radiation and the ultimate speed in Rosen's bimetric theory of gravity, Ann. Phys. (N.Y.) **125**, 35 (1980).

[28] G. D. Moore and A. E. Nelson, Lower bound on the propagation speed of gravity from gravitational Cherenkov radiation, J. High Energy Phys. 09 (2001) 023.

[29] C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan, and L. Senatore, The effective field theory of inflation, J. High Energy Phys. 03 (2008) 014.

[30] C. M. Will, The Confrontation between general relativity and experiment, Living Rev. Relativity **9**, 3 (2006).

[31] T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, Modified gravity and cosmology, Phys. Rep. **513**, 1 (2012).

[32] P. Creminelli, J. Gleyzes, J. Norea, and F. Vernizzi, Resilience of the Standard Predictions for Primordial Tensor Modes, Phys. Rev. Lett. **113**, 231301 (2014).

[33] P. A. R. Ade *et al.* (Planck Collaboration), Planck 2013 results. XVI. Cosmological parameters, Astron. Astrophys. **571**, A16 (2014).

[34] C. Antolini, M. Martinelli, Y. Fantaye, and C. Baccigalupi, Measuring primordial gravitational waves from CMB *B*-modes in cosmologies with generalized expansion histories, J. Cosmol. Astropart. Phys. 02 (2013) 024.

[35] A. R. Liddle and D. H. Lyth, COBE, gravitational waves, inflation and extended inflation, Phys. Lett. B **291**, 391 (1992).

[36] I. D. Saltas, I. Sawicki, L. Amendola, and M. Kunz, Anisotropic Stress as a Signature of Nonstandard Propagation of Gravitational Waves, Phys. Rev. Lett. **113**, 191101 (2014).

[37] L. Amendola, G. Ballesteros, and V. Pettorino, Effects of modified gravity on *B*-mode polarization, Phys. Rev. D **90**, 043009 (2014).

[38] P. A. R. Ade *et al.* (Planck Collaboration), Planck 2013 results. XV. CMB power spectra and likelihood, Astron. Astrophys. **571**, A15 (2014).

[39] G. Hinshaw *et al.* (WMAP Collaboration), Nine-year Wilkinson microwave anisotropy probe (WMAP) observations: Cosmological parameter results, Astrophys. J. Suppl. Ser. **208**, 19 (2013).

[40] C. Armitage-Caplan *et al.* (COrE Collaboration), COrE (Cosmic Origins Explorer) a white paper, arXiv:1102.2181.

[41] P. André *et al.* (PRISM Collaboration), PRISM (Polarized radiation imaging and spectroscopy mission): An extended white paper, J. Cosmol. Astropart. Phys. 02 (2014) 006.

[42] A. Lewis and S. Bridle, Cosmological parameters from CMB and other data: A Monte Carlo approach, Phys. Rev. D **66**, 103511 (2002).