Direct photon production as a probe of quarks chromoelectric and chromomagnetic dipole moments at the LHC

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In this paper, we show that the γ + jet invariant mass distribution in proton-proton collisions at the LHC is significantly sensitive to the quark chromomagnetic (κ) and chromoelectric ($\tilde{\kappa}$) dipole moments. It is shown that the presence of κ or $\tilde{\kappa}$ leads to an increment of the cross section of the γ + jet process, in particular, in the tail of the γ + jet invariant mass distribution. Using the measured γ + jet invariant mass distribution by the CMS experiment at the center-of-mass energy of 8 TeV, we derive bounds on the quark chromoelectric and chromomagnetic dipole moments. In the extraction of the limits, we consider both theoretical and systematic uncertainties. The uncertainties originating from variation of the renormalization or factorization scales and the choice of proton parton distribution functions are taken into account as a function of the γ + jet invariant mass. We exclude κ or $\tilde{\kappa}$ above 10⁻⁵ at 95% confidence level. This is the most stringent direct upper limit on κ or $\tilde{\kappa}$.

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I. INTRODUCTION

The Standard Model (SM) is a framework that describes our present understanding of fundamental constituents of matter and their interactions. The SM predictions are found to be well compatible with the experimental data up to the scale a few TeV. While already the information which has come out of the LHC and Tevatron experiments shows consistency with the SM expectations with good precisions, the LHC future runs provide the possibility to achieve more precise determination of the SM particle properties. Although no significant indication for new physics beyond the SM has been found up to now, more precise measurements of SM predictions can reveal tracks of the effects of new physics beyond the SM. A systematic and powerful way to parametrize the new physics effects is to utilize the effective Lagrangian approach, which is a modelindependent technique in the probe of new physics effects. In this approach, the effective Lagrangian follows the same symmetries as the SM and is constructed from the existing SM fields. The leading terms in \mathcal{L}_{eff} are the SM terms, and the effects of new physics are parametrized by the coefficients of higher dimension operators. The effective Lagrangian for exploring the new interactions has been provided in Refs. [1,2]. The lowest order couplings between a gluon and a quark are dimension four and five operators with the following form:

$$\mathcal{L}_{\rm eff} = g_s \bar{q} T^a \bigg[-\gamma^{\mu} G^a_{\mu} + \frac{\kappa}{4m_q} \sigma^{\mu\nu} G^a_{\mu\nu} - \frac{i\tilde{\kappa}}{4m_q} \sigma^{\mu\nu} \gamma^5 G^a_{\mu\nu} \bigg] q,$$
(1)

where G^a_{μ} denotes the gluon field and $\kappa/2m_q$ and $\tilde{\kappa}/2m_q$ correspond to chromomagnetic (CMDM) and chromoelectric (CEDM) dipole moments of a quark. It should be noted that within the SM these couplings are zero at tree level and are induced at loop level. The chromomagnetic and chromoelectric dipole moments κ and $\tilde{\kappa}$ can be considerable, since they appear as dimension five operators and are suppressed only by one power of Λ (a new physics scale). In the denominator, the mass factors are taken conventionally to be the quark mass m_q to express these terms as quark dipole moments. The effective Lagrangian \mathcal{L}_{eff} is valid for all quark flavors within the vast range of quark masses. There is much interest to measure CEDM, since a nonzero value of CEDM indicates a new source of CP violation. In the SM, the number of quark chromoelectric moments (CEDM) is very small. For example, the CEDM of the heaviest quark, i.e., top quark, is at the order of $10^{-17} g_s$ cm [3]. Several extensions of the SM such as the minimal supersymmetric Standard Model, grand unified theories, the two Higgs doublet model, the Higgs triplet model, the left-right symmetric model, and extra dimensions can generate sizable (chromo)electric and (chromo)magnetic dipole moments [4–10]. So far, there are several studies on constraining the top quark CEDM and CMDM in the literature using different methods. In Ref. [11], we have used a single top in the tW channel to probe the dipole moments. In Refs. [12-21], the total and the differential cross sections of top pair production at the LHC and Tevatron have been used to constrain the dipole moments. In hadron colliders, direct photon production $[pp(\bar{p}) \rightarrow \gamma + \text{jet}]$ provides very useful information to search for new physics beyond the SM as well as increasing our knowledge of SM. For example, within the SM the total and differential γ + jet cross sections are used to

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understand the proton parton distribution functions (PDFs) and even are used for testing the perturbative QCD [22-24]. From the experimental point of view, γ + jet events are clean and are well measured with the electromagnetic calorimeters which are used in improving photon energy resolution. More importantly, γ + jet is the final state of many new physics signatures such as quantum black holes [25–27], excited quarks [28–30], quirks [31–33], and Regge excitations of string theory [34-36]. Indeed, this is not the complete list of new physics models that can be explored with photons at hadron colliders; however, this shows the ability of the photon final state, which covers a broad range of theoretical models beyond the SM. The results of the experimental searches based on the γ + jet final state at the LHC and Tevatron can be found in Refs. [37-44].

In Ref. [38], the authors have studied the influence of anomalous CMDM (κ) and CEDM ($\tilde{\kappa}$) of light quarks on the direct photon production at the Tevatron. It has been shown that the γ + jet rate is sensitive to anomalous interactions of quarks to gluons, in particular, the differential cross section $d\sigma/dp_{T\gamma}$. The transverse momentum spectrum has been found to be sensitive to CEDM and CMDM. Nonzero values of CEDM or CMDM enhance the cross section in the photon high transverse momentum region. The upper bound of 0.0027 has been set on κ by using 0.1 fb⁻¹ of CDF and D0 data.

In this paper, we show that the presence of anomalous CEDM or CMDM of quarks increases the rate of photon production in the tail of γ + jet invariant mass distribution at the LHC. Then by using the recent γ + jet spectrum measurement at the center-of-mass energy of 8 TeV with 19.7 fb⁻¹ of data, upper bounds are set on CEDM and CMDM at 95% confidence level. In the limit setting process, special attention is paid to the uncertainties. We calculate the uncertainty originating from variation of renormalization or factorization scales and the uncertainty coming from the limited knowledge of proton PDFs as functions of the γ + jet invariant mass. It should be mentioned that the analysis is performed with the assumption of flavor universality on the anomalous couplings of CEDM or CMDM of the quarks.

This paper is organized as follows. Section II is dedicated to theoretical calculations of the cross section of γ + jet. Section III describes the sensitivity estimate using the measured mass spectrum of the photon jet and presents the results. The results are compared with the ones obtained with previous works and the future expected bounds. Finally, Sec. IV concludes the paper.

II. EFFECT OF THE ANOMALOUS COUPLINGS κ AND $\tilde{\kappa}$ ON THE CROSS SECTION

In this section, we present the theoretical calculation of the γ + jet production cross section at the LHC in the presence of chromoelectric and chromomagnetic dipole



FIG. 1. The representative Feynman diagrams for production of γ + jet at leading order in proton-proton collisions at the LHC.

moments of the quarks. The main contribution to γ + jet final state in proton-proton collisions comes when a hadronic jet and a photon are produced in a hard scattering. This can be achieved by the Compton scattering of the quark gluon $(gq \rightarrow \gamma q)$ and quark-antiquark annihilation $(q\bar{q} \rightarrow \gamma g)$ at leading order. In Fig. 1, the representative Feynman diagrams for production of γ + jet are depicted. As mentioned previously, the effective interaction of qqgg which is absent in the SM appears in the effective Lagrangian to ensure the gauge invariance [1,45]. This new four-point interaction does not affect the γ + jet production.

Based on the effective Lagrangian introduced in Eq. (1), the new Feynman rule describing the interaction of a quark with a gluon has the following form:

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \mathcal{L}_{q_i q_j g}$$

= $-g_s \bar{q}_j T^a_{ji} \bigg[\gamma^{\mu} + \frac{i}{2m_q} \sigma^{\mu\nu} q_{\nu} (\kappa - i\tilde{\kappa}\gamma^5) \bigg] q_i G^a_{\mu}, \quad (2)$

where q_i and q_j are the quark spinors and q_{ν} is the fourmomentum of the gluon. It is notable that, in general, the anomalous couplings could be dependent on the gluon momentum transfer. However, the transfer momentum is much smaller than the new physics scale; therefore, the dependency is neglected. The color and spin averaged amplitude for $q(p)g(p') \rightarrow \gamma(k)q(k')$ is found as

$$\overline{\sum} |M|^2 = \frac{16\pi^2 \alpha_s \alpha_{\rm em} e_q^2}{3} \left[-\frac{\hat{s}^2 + \hat{t}^2}{\hat{s}\,\hat{t}} - \frac{\hat{u}}{2m_q^2} (\kappa^2 + \tilde{\kappa}^2) \right],\tag{3}$$

where *s*, *t*, and *u* are the Mandelstam variables defined as $\hat{s} = (p + p')^2$, $\hat{t} = (p - k)^2$, and $\hat{u} = (p - k')^2$, respectively, and e_q is the electric charge of the quark *q* in units of

the proton charge. The color and spin averaged amplitude for $q(p)\bar{q}(p') \rightarrow \gamma(k)g(k')$ has the following form:

$$\overline{\sum} |M|^2 = \frac{128\pi^2 \alpha_s \alpha_{\rm em} e_q^2}{9} \left[\frac{\hat{t}^2 + \hat{u}^2}{\hat{u}\,\hat{t}} + \frac{\hat{s}}{2m_q^2} (\kappa^2 + \tilde{\kappa}^2) \right].$$
(4)

All calculations are consistent with Ref. [38]. Now, the hadronic cross section is obtained by convoluting the parton level cross section with the proton parton density functions:

$$d\sigma(pp \to \gamma + \text{jet}) = \sum_{ij=qg} \int_0^1 dx_1 \int_0^1 dx_2 f_i(x_1, Q^2) f_j(x_2, Q^2) d\hat{\sigma}_{ij}, \quad (5)$$

where $f_i(x, Q^2)$ denotes the PDFs. We use a recently released CT10 [46] set for PDFs to calculate the cross section. In this analysis, the nominal renormalization and factorization scales have been set to the photon transverse momentum $\mu_R = \mu_F = Q = p_{T,\gamma}$. It should be mentioned here that there are higher order processes contributing to the total cross section like $gg \rightarrow \gamma + g$. The theoretical prediction for the higher order corrections leads to a kfactor of 1.3 [47,48]. This is applied to include the next-toleading order effects for the γ + jet cross section. We emphasize here that in reality the k factor varies in different bins of the photon-jet invariant mass and is not fixed. In Ref. [48], the authors have calculated the k factor as a function of photon p_T for the Tevatron and for the LHC at the center-of-mass energy of 14 TeV. Since the k factor as a function of $M_{\gamma-\text{iet}}$ at the center-of-mass energy of 8 TeV is not available, we use the fixed value similar to other analyses [39].

The dependence of the cross section to κ and $\tilde{\kappa}$ is similar, as the amplitudes of two subprocesses are proportional to $\kappa^2 + \tilde{\kappa}^2$. Therefore, the effects because of nonzero CEDM are quite similar to nonzero CMDM in the total cross section. If the γ + jet total cross section is measured at the LHC8 (LHC14) compatible with the SM prediction with a relative uncertainty of 5%, an upper bound of 8.5×10^{-5} (7×10^{-5}) on κ or $\tilde{\kappa}$ will be obtained. However, in the next section, we will show that the γ + jet mass spectrum $(d\sigma/dM_{\gamma-jet})$ is more sensitive to CEDM and CMDM and gives stringent limits.

III. PHOTON-JET MASS SPECTRUM SENSITIVITY ESTIMATE

In this section, we concentrate on the measured γ + jet mass spectrum to probe the chromoelectric and chromomagnetic dipole moments of the quarks. In Ref. [38], it has been shown that the prompt photon transverse momentum spectrum is sensitive to CEDM and CMDM in proton-antiproton collisions at the Tevatron. Then, by using the CDF and D0 data, any value of κ or $\tilde{\kappa}$ above 0.0027 has been excluded.

Recently, the CMS experiment has measured the γ + jet mass spectrum in proton-proton collisions at the center-ofmass energy of 8 TeV with an integrated luminosity of 19.7 fb^{-1} of data [39]. Then the invariant mass spectrum has been used to look for a signature of new physics (excited quarks) after implementing the fiducial requirements on both the photon and jet. The mass spectrum is well fitted to a parameterization which describes the SM prediction, and no significant excess over the SM expectation has been observed. In Fig. 2, the measured and SM expected distributions of the invariant mass of γ + jet are shown after applying similar kinematic requirements as the CMS experiment. The transverse momenta of the photon and jet are required to be greater than 170 GeV. The photon is restricted to be in the pseudorapidity range of $|\eta_{\gamma}| < 1.44$, and the jet is required to be in $|\eta_{jet}| < 3$. The angular separation of the jet and photon is required to be larger than 1.5, while the differences of pseudorapidities are required to be less than 2.0. The effects of the presence of κ or $\tilde{\kappa}$ are also shown in the plot. As can be seen, the photon-jet mass spectrum is affected by the presence of κ or $\tilde{\kappa}$. Nonzero values of κ or $\tilde{\kappa}$ lead to a significant increase in the cross section, in particular, in the high mass region. In this analysis, all quark flavors except for the top quark are included in the calculations. We have assumed that the CEDM and CMDM of these quarks are the same.

Now, in order to constrain the quark CEDM and CMDM, we combine the information of all bins of the photon-jet mass spectrum as shown in Fig. 2 into a global χ^2 fit. The χ^2 is defined as



FIG. 2 (color online). The measured and SM expected distributions of the invariant mass of jet + γ at the LHC with the center-of-mass energy of 8 TeV. The spectrum is weighted to the integrated luminosity of 19.7 fb⁻¹. The expectation of the new effective Lagrangian for two values of κ is also presented.



FIG. 3 (color online). Left: The absolute value of the relative uncertainty due to variation of the factorization or renormalization scales. Right: The absolute value of the relative uncertainty due to the choice of PDF obtained by using the PDF4LHC recommendations.

$$\chi^2 = \sum_{i=\text{bins}} \frac{(N_i^{\text{obs}} - N_i^{\text{th}})^2}{\Delta_i^2},\tag{6}$$

where the sum is over the bins of the photon-jet invariant mass, N_i^{th} is the number of expected events in a given theory, defined by the values of κ and $\tilde{\kappa}$ in each mass bin, and N_i^{obs} is the observed number of events in each bin. The uncertainty of the expectation in each bin is denoted by Δ_i that covers both theoretical and experimental uncertainties. We have included 40 bins of the photon-jet invariant mass in χ^2 . In order to have a realistic estimation of the upper limits on the anomalous couplings κ and $\tilde{\kappa}$, all sources of uncertainties must be taken into account. In the denominator of χ^2 , Δ_i contains all the uncertainties. There are several sources of uncertainties: statistical, systematic, and theoretical uncertainties. The main sources of theoretical uncertainties are the uncertainty due to variation of factorization or renormalization scales, the uncertainty originating from our limited knowledge of parton distribution functions, and the uncertainty on the value of strong coupling constant α_s . To estimate the uncertainty from variation of factorization or renormalization scales, the scales are varied by a factor of 0.5 and 2. The absolute values of the resulting differences with the nominal scale are shown in Fig. 3 (left). According to Fig. 3, the uncertainty increases with the photon-jet invariant mass. At the invariant mass around 3 TeV, the uncertainty varies between 25% and 35%. In each bin, we take the average of uncertainty from $Q = 2p_{T\gamma}$ and $Q = p_{T\gamma}/2$ variations. The uncertainty originating from the choice of parton distribution functions is calculated in bins of the photonjet invariant mass according to the PDF4LHC recommendations [49]. The results are shown in the right side of Fig. 3. The uncertainty varies from 3% to 7% when the photon-jet invariant mass varies from 600 to 3000 GeV. For the sake of considering the uncertainty on the value of strong coupling constant α_s , we vary the value of α_s around the nominal value $\alpha_s = 0.118$ by ± 0.0002 . The small dependence of the strong coupling constant on $M_{\gamma-\text{jet}}$ is neglected. There are several sources of instrumental uncertainties coming from jet energy and photon energy resolutions. Conservatively, an overall value of 5% in each bin of invariant mass is considered. All uncertainties are considered as a quadratic sum of each uncertainty in each bin of the photon-jet invariant mass: $\Delta^2 = \sigma_{\text{stat}}^2 + \sigma_{\text{theory}}^2 + \sigma_{\text{syst}}^2$.

Now, we set upper limits on κ or $\tilde{\kappa}$ at the 95% confidence level. Similar to Ref. [38], the results are also presented in terms of $\Lambda = \frac{2m_q}{\kappa}$. Including only statistical uncertainties, the upper limit of 8.27×10^{-6} is obtained. This upper limits gets looser and reaches to 2.5×10^{-5} after considering all systematic and theoretical uncertainties. This is an improvement on the previous direct limits on CEDM or CMDM (2.7×10^{-3}) with 2 orders of magnitude. We have put $\tilde{\kappa} = 0$ and obtained an upper limit on κ . Since the cross section is proportional to $\kappa^2 + \tilde{\kappa}^2$, the same upper bound is obtained on $\tilde{\kappa}$. In Table I, the results are presented in terms of $\Lambda = \frac{2m_q}{\kappa}$ as well as κ . The results are compared with the results that have been obtained in Ref. [38] for the future run of LHC at 14 TeV center-of-mass energy. As has been mentioned in the past, the analysis of Ref. [38] is based on

TABLE I. Comparison of upper limits on κ or $\tilde{\kappa}$ and also on Λ from this analysis and from Ref. [38].

Experiment	\sqrt{s} (TeV)	$\int \mathcal{L} (fb^{-1})$	Limit on κ	Λ (TeV)
Tevatron $(p_{T\gamma})$	1.8	0.1	2.7×10^{-3}	0.7
LHC $(p_{T\gamma})$	14	10	1.3×10^{-4}	4.5
LHC $(p_{T\gamma})$	14	100	$9.5 imes 10^{-5}$	6.3
LHC $(M_{\gamma-\text{jet}})$	8	19.7	$2.5 imes 10^{-5}$	24

the effect of CEDM and CMDM on the photon transverse momentum.

IV. CONCLUSIONS

We have shown that the photon-jet invariant mass distribution in proton-proton collisions at the LHC receives significant contributions from the quark chromomagnetic (κ) and chromoelectric ($\tilde{\kappa}$) dipole moments, in particular, at large values of the γ + jet invariant mass. We use the measured γ + jet mass spectrum by the CMS experiment at the center-of-mass energy of 8 TeV to derive upper limits on quark chromoelectric and chromomagnetic dipole moments. All theoretical and systematic uncertainties are included in the limit setting. We have calculated the uncertainty originating from variation of factorization or renormalization scales as a function of the photon-jet invariant mass. The uncertainty from variation of factorization or renormalization scales increases with increasing the photon-jet invariant mass and reaches up to 35% at an invariant mass of 3 TeV. The uncertainty due to the choice of PDF has been calculated by using the PDF4LHC recommendations. We exclude κ or $\tilde{\kappa}$ above 10⁻⁵ at 95% confidence level. The sensitivity is also presented in terms of an energy scale parameter $\Lambda = 2m_q/\kappa$. Any value of Λ below 24 TeV has been excluded by using 19.7 fb⁻¹ of LHC data at the center-of-mass energy of 8 TeV.

- [1] W. Buchmuller and D. Wyler, Nucl. Phys. B268, 621 (1986).
- [2] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, J. High Energy Phys. 10 (2010) 085.
- [3] K.-M. Cheung, Phys. Rev. D 53, 3604 (1996).
- [4] T. Fukuyama, Int. J. Mod. Phys. A 27, 1230015 (2012).
- [5] R. Martinez, M. A. Perez, and N. Poveda, Eur. Phys. J. C 53, 221 (2008).
- [6] L. Ding and C.-X. Yue, Commun. Theor. Phys. 50, 441 (2008).
- [7] D. McKeen, M. Pospelov, and A. Ritz, Phys. Rev. D 87, 113002 (2013).
- [8] Z. Hioki and K. Ohkuma, Eur. Phys. J. C 71, 1 (2011).
- [9] T. Ibrahim and P. Nath, Phys. Rev. D 82, 055001 (2010).
- [10] T. Ibrahim and P. Nath, Phys. Rev. D 84, 015003 (2011).
- [11] S. Y. Ayazi, H. Hesari, and M. M. Najafabadi, Phys. Lett. B 727, 199 (2013).
- [12] H. Hesari and M. M. Najafabadi, Mod. Phys. Lett. A 28, 1350170 (2013).
- [13] K. m. Cheung, Phys. Rev. D 53, 3604 (1996); K. Cheung, Phys. Rev. D 55, 4430 (1997).
- [14] Z. Hioki and K. Ohkuma, Phys. Rev. D 88, 017503 (2013).
- [15] W. Bernreuther and Z.-G. Si, Phys. Lett. B 725, 115 (2013).
- [16] S. S. Biswal, S. D. Rindani, and P. Sharma, Phys. Rev. D 88, 074018 (2013).
- [17] C. Englert, A. Freitas, M. Spira, and P. M. Zerwas, Phys. Lett. B 721, 261 (2013).
- [18] D. Atwood, A. Kagan, and T. G. Rizzo, Phys. Rev. D 52, 6264 (1995).
- [19] T. G. Rizzo, arXiv:hep-ph/9902273.
- [20] Z. Hioki and K. Ohkuma, Phys. Lett. B 716, 310 (2012).
- [21] R. Martinez and J. A. Rodriguez, Phys. Rev. D 55, 3212 (1997).
- [22] A. Kumar, M. K. Jha, B. M. Sodermark, A. Bhardwaj, K. Ranjan, and R. Shivpuri, Phys. Rev. D 67, 014016 (2003).
- [23] M. Fontannaz and G. Heinrich, Eur. Phys. J. C 34, 191 (2004).
- [24] F. Halzen and D. Scott, Phys. Rev. D 21, 1320 (1980).
- [25] P. Meade and L. Randall, J. High Energy Phys. 05 (2008) 003.
- [26] X. Calmet, W. Gong, and S. D. Hsu, Phys. Lett. B 668, 20 (2008).

- [27] D. M. Gingrich, J. Phys. G 37, 105008 (2010).
- [28] S. Bhattacharya, S. S. Chauhan, B. C. Choudhary, and D. Choudhury, Phys. Rev. D 80, 015014 (2009).
- [29] U. Baur, M. Spira, and P. M. Zerwas, Phys. Rev. D 42, 815 (1990).
- [30] O. Cakir and R. Mehdiyev, Phys. Rev. D 60, 034004 (1999).
- [31] J. Kang and M. A. Luty, J. High Energy Phys. 11 (2009) 065.
- [32] S. P. Martin, Phys. Rev. D 83, 035019 (2011).
- [33] L. B. Okun, Nucl. Phys. B173, 1 (1980).
- [34] L. A. Anchordoqui, H. Goldberg, S. Nawata, and T. R. Taylor, Phys. Rev. D 78, 016005 (2008).
- [35] L. A. Anchordoqui, H. Goldberg, D. Lust, S. Nawata, S. Stieberger, and T. R. Taylor, Phys. Rev. Lett. 101, 241803 (2008).
- [36] P. Nath *et al.*, Nucl. Phys. B, Proc. Suppl. **200–202**, 185 (2010).
- [37] F. Abe et al., Phys. Rev. Lett. 72, 3004 (1994).
- [38] K.-m. Cheung and D. Silverman, Phys. Rev. D 55, 2724 (1997).
- [39] CMS Collaboration, Report No. CMS PAS EXO-13-003.
- [40] ATLAS Collaboration, Phys. Lett. B 728, 562 (2014).
- [41] ATLAS Collaboration, Phys. Rev. Lett. 110, 011802 (2013).
- [42] ATLAS Collaboration, Report No. ATL-PHYS-PROC-2013-316.
- [43] ATLAS Collaboration, Report No. ATLAS-CONF-2012-147.
- [44] ATLAS Collaboration, Report No. ATLAS-CONF-2013-022.
- [45] D. Atwood, S. Bar-Shalom, G. Eilam, and A. Soni, Phys. Rep. 347, 1 (2001).
- [46] J. Gao, M. Guzzi, J. Huston, H.-L. Lai, Z. Li, P. Nadolsky, J. Pumplin, D. Stump, and C.-P. Yuan, Phys. Rev. D 89, 033009 (2014).
- [47] W. T. Giele, E. W. N. Glover, and D. A. Kosower, Nucl. Phys. B403, 633 (1993).
- [48] T. P. Stavreva and J. F. Owens, Phys. Rev. D 79, 054017 (2009).
- [49] M. Botje et al., arXiv:1101.0538.