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Is the $X(3915)$ the $\chi_{c0}(2P)$?

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The Particle Data Group Meson Summary Table lists the $X(3915)$ meson, an $\omega J/\psi$ mass peak seen in $B\to K\omega J/\psi$ decays and $\gamma\gamma\to\omega J/\psi$ two-photon fusion reactions, as the $\chi_{c0}(2P)$, the 2^3P_0 charmonium state. Here, with some reasonable assumptions, it is shown that if the $X(3915)$ is the $\chi_{c0}(2P)$, the measured strength of the $\gamma\gamma\to X(3915)$ signal implies an *upper* limit on the branching fraction $\mathcal{B}(\chi_{c0}(2P)\to\omega J/\psi)$ that is below a *lower* limit inferred for the same quantity from the $B\to KX(3915)$ decay rate. Also, the absence any signal for $X(3915)\to D^0\bar{D}^0$ in $B^+\to K^+D^0\bar{D}^0$ decays is used to infer the limit $\mathcal{B}(X(3915)\to D^0\bar{D}^0)<1.2\times\mathcal{B}(X(3915)\to\omega J/\psi)$. This contradicts expectations that $\chi_{c0}(2P)$ decays to $D^0\bar{D}^0$ should be a dominant process, while decays to $\omega J/\psi$, which are Okubo-Zweig-Iizuka suppressed, should be relatively rare. These, plus reasons given earlier by Guo and Meissner, raise concerns about the $X(3915)=\chi_{c0}(2P)$ assignment.

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I. INTRODUCTION

A number of meson candidates, dubbed the XYZ mesons, that contain charmed- and anticharmed-quark ($c\bar{c}$) pairs but do not match expectations for any of the unassigned levels of the $c\bar{c}$ charmonium spectrum have been observed in recent experiments. Some have nonzero electric charge [1] and cannot be accommodated in the spectrum of the electrically neutral charmonium mesons. Others are neutral and have quantum numbers that are accessible by $c\bar{c}$ systems, but have properties that fail to match the tightly constrained expectations of any of the unassigned charmonium states [2]. To date, there is no compelling theoretical explanation for these XYZ mesons. Experimental observations of additional states and more measurements of properties of existing states may eventually reveal patterns that give clues to their underlying structure. Also important is the careful distinction between new states that are conventional charmonium mesons from those that are not.

The $X(3915)$ was observed by both Belle [3] and *BABAR* [4] as a near-threshold peak in the $\omega J/\psi$ invariant mass distribution in exclusive $B\to K\omega J/\psi$ decays. Belle [5] and *BABAR* [6] also reported an $\omega J/\psi$ mass peak with similar mass and width in the two-photon fusion process $\gamma\gamma\to\omega J/\psi$. The similar masses and widths of the peaks seen in B decay and in two-photon fusion processes suggest that these are two different production mechanisms for the same state. The Particle Data Group's (PDG) average values of the mass and width measurements from both production channels are [7]

$$M(X(3915)) = 3918.4\pm 1.9\ \text{MeV}$$
$$\Gamma(X(3915)) = 20.0\pm 5.0\ \text{MeV}. \tag{1}$$

The weighted average of the Belle [3] and *BABAR* [4] product branching fraction measurements for $X(3915)$ production in B decay is

$$\mathcal{B}(B^+\to K^+X(3915))\times\mathcal{B}(X(3915)\to\omega J/\psi) = 3.0^{+0.6+0.5}_{-0.5-0.3}\times 10^{-5}. \tag{2}$$

The PDG average of Belle and *BABAR* measurements of the two-photon production rate (using $J^{PC}=0^{++}$) is [7]

$$\Gamma^{\gamma\gamma}_{X(3915)}\times\mathcal{B}(X(3915)\to\omega J/\psi) = 54\pm 9\ \text{eV}, \tag{3}$$

where $\Gamma^{\gamma\gamma}_{X(3915)}$ is the partial width for $X(3915)\to\gamma\gamma$ and the error is dominantly statistical.

A *BABAR* study of angular correlations among the final-state particles in $\gamma\gamma\to X(3915)\to\omega J/\psi$ events established the J^{PC} quantum numbers of the $X(3915)$ to be 0^{++} [6]. The presence of a J/ψ among its decay products indicate that the $X(3915)$ contains a $c\bar{c}$ quark pair. Since the only unassigned 0^{++} $c\bar{c}$ charmonium level in the vicinity of the $X(3915)$ mass is the $\chi_{c0}(2P)$, the first radial excitation of the χ_{c0} charmonium state, the *BABAR* paper identified the $X(3915)$ as the $\chi_{c0}(2P)$, a classification that the PDG adopted. [In the following, the $\chi_{c0}(2P)$ is referred to as the χ'_{c0}.]

Concerns about this assignment were raised by Guo and Meissner [8], primarily because

(i) the partial width for $X(3915)\to\omega J/\psi$ is too large for a decay process that is Okubo-Zweig-Iizuka (OZI)-suppressed for a charmonium state;
(ii) the lack of evidence for $X(3915)\to D\bar{D}$ decays, which are expected to be dominant χ'_{c0} decay modes;
(iii) the small $\chi_{c2}(2P)$-$\chi_{c0}(2P)$ mass splitting.

These concerns were echoed by Wang, Yang and Ping in a paper in which they presented theoretical calculations that predict a χ'_{c0} mass (3868 MeV) that is substantially lower than that of the $X(3915)$ [9].

If the $X(3915)$ is not conventional charmonium but, instead, another XYZ meson, it would be the lightest observed scalar and one of the narrowest of these new states. As such, it would likely play a key role in attempts to understand their underlying nature. Thus, the question of whether or not the $X(3915)$ is the χ'_{c0} is a critical issue that needs to be carefully addressed. In this paper I amplify some of the Guo-Meissner and Wang-Yang-Ping points and identify some other concerns with the $X(3915) = \chi'_{c0}$ assignment.

II. THE $\chi_{c2}(2P)$ CHARMONIUM STATE

The expected properties of the χ'_{c0} are constrained by measurements of its $J = 2$ multiplet partner, the $\chi_{c2}(2P)$, or χ'_{c2}, that was seen by both Belle [10] and *BABAR* [11] as a distinct $M(D\bar{D})$ peak in the two-photon fusion process $\gamma\gamma \to D\bar{D}$. Both groups see a clear $\sin^4\theta^*$ production angle dependence that is characteristic of a $J = 2$ charmonium state, and there are no reasons to question the χ'_{c2} assignment. The Belle $M(D\bar{D})$ and $dN/d|\cos\theta^*|$ distributions are shown in Fig. 1. Belle and *BABAR* measurements for the mass and width are in good agreement; the PDG average values are [7]

$$M(\chi'_{c2}) = 3927.2 \pm 2.6\ \text{MeV}$$
$$\Gamma(\chi'_{c2}) = 24.0 \pm 6.0\ \text{MeV}. \tag{4}$$

Belle and *BABAR* measurements of its two-photon production rate are also in good agreement and are characterized by the product [7]

$$\Gamma^{\gamma\gamma}_{\chi'_{c2}} \times \mathcal{B}(\chi'_{c2} \to D\bar{D}) = 210 \pm 40 \times\ \text{eV}. \tag{5}$$

III. CONSEQUENCES OF $X(3915) = \chi'_{c0}$

A. The $\chi_{c2}(2P)$-$\chi_{c0}(2P)$ mass splitting

As pointed out by Guo and Meissner, the $X(3915) = \chi'_{c0}$ assignment implies an anomalously small $\chi_{c2}(2P)$-$\chi_{c0}(2P)$ mass splitting. Current measurements put it at $\Delta M(2P) = 8.8 \pm 3.2$ MeV, in which case

$$r_c \equiv \frac{\Delta M(2P)}{\Delta M(1P)} = 0.06 \pm 0.02. \tag{6}$$

This is much smaller than potential-model predictions for charmonium that are in the range $0.6 < r_c < 0.9$ [12]. Studies of modifications to potential-model mass calculations caused by couplings to open-charmed mesons predict values for r_c that range from $r_c \simeq 0.8$ [13] for a screened potential approach [14], $r_c \simeq 0.72$ [9] for a version of the $^3P_0$ model [15], and $r_c \simeq 0.35$ [16] for the Cornell coupled-channel model [17]. These are all well above the value given in Eq. (6). Moreover, the latter two studies find χ'_{c0}-$D\bar{D}$ coupling strengths that cannot be supported by measured $X(3915)$ data: the predicted $\chi'_{c0} \to D\bar{D}$ partial widths are 48 MeV for the $^3P_0$ approach and 61.5 MeV for the Cornell coupled-channel model, values that exceed the $X(3915)$ total width and are substantially larger than a partial-width upper limit that is presented below.

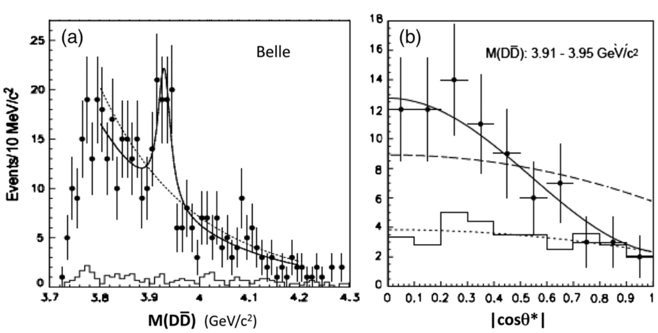

FIG. 1. (a) The $M(D\bar{D})$ distributions for $\gamma\gamma \to D\bar{D}$ decays from Ref. [10]. The open histogram shows the background level determined from D mass sidebands. The solid (dashed) curve shows the result of a fit that includes (excludes) a χ'_{c2} signal. (b) The $dN/d|\cos\theta^*|$ distribution for events in the peak region. The solid (dashed) curve shows expectations for $J = 2$ ($J = 0$). The histogram shows the nonresonant contribution.

B. Limits on $\mathcal{B}(\chi'_{c0} \to \omega J/\psi)$

Using measured numbers and some reasonable assumptions, I infer an upper limit on $\mathcal{B}(\chi'_{c0} \to \omega J/\psi)$ from the two-photon fusion production rate that is incompatible with a lower limit on the same quantity determined from the rate for χ'_{c0} production in $B^+ \to K^+\chi'_{c0}$ decays.

a. From $\gamma\gamma \to \chi'_{c0} \to \omega J/\psi$: From Eq. (3) it is clear that an upper limit on $\mathcal{B}(\chi'_{c0} \to \omega J/\psi)$ can be inferred from a lower limit on $\Gamma^{\gamma\gamma}_{\chi'_{c0}}$. In potential models, the dependence of the ratio $\Gamma^{\gamma\gamma}_{\chi_{c0}(nP)}/\Gamma^{\gamma\gamma}_{\chi_{c2}(nP)}$ on the radial quantum number n is limited to effects of changes in the QCD coupling strength α_s with increasing mass [18]; between the $1P$ and $2P$ levels, these effects are a few percent. Ignoring these α_s variations, one can infer[1]

$$\frac{\Gamma^{\gamma\gamma}_{\chi'_{c0}}}{\Gamma^{\gamma\gamma}_{\chi'_{c2}}} \simeq \frac{\Gamma^{\gamma\gamma}_{\chi_{c0}}}{\Gamma^{\gamma\gamma}_{\chi_{c2}}} = 4.4 \pm 0.6, \tag{7}$$

where the numerical result comes from PDG average values for $\Gamma^{\gamma\gamma}_{\chi_{c0}}$ and $\Gamma^{\gamma\gamma}_{\chi_{c2}}$ [7]. While this relation is based on potential models and ignores the influence of couplings to open-charmed meson pairs, it agrees, to within 10%, with results from a screened potential calculation that considers these effects [13]. Since $\mathcal{B}(\chi'_{c2} \to D\bar{D})$ is necessarily less than unity, Eq. (5) implies a lower bound on $\Gamma^{\gamma\gamma}_{\chi'_{c2}}$. This, together with Eqs. (7) and (3), translates into the 90% confidence level (CL) upper limit

[1] In this paper I assume all errors are Gaussian and, when required, combine statistical and systematic errors in quadrature.

$$\mathcal{B}(\chi'_{c0} \to \omega J/\psi) < 8.1\%. \tag{8}$$

The errors on the measured quantities that go into this relation are mostly statistical. Here, and in other results reported in this section, any positively correlated systematic error components would tend to cancel in the ratios used to form the limits. Moreover, the presence of any substantial anticorrelated systematic error component is unlikely since the measured values that go into the numerator and denominator mostly come from the same experiments. If the effects of a decreasing α_s value between the $1P$ and $2P$ levels are included, the Eq. (8) limit decreases and becomes more stringent.

b. From $B^+ \to K^+\chi'_{c0}$; $\chi'_{c0} \to \omega J/\psi$: A lower limit of $\mathcal{B}(\chi'_{c0} \to \omega J/\psi)$ can be deduced from Eq. (2) if an upper limit on $\mathcal{B}(B^+ \to K^+\chi'_{c0})$ can be established. Here I assume that the $\mathcal{B}(B^+ \to K^+\chi'_{c0})$ is less than or equal to $\mathcal{B}(B^+ \to K^+\chi_{c0})$, where the PDG average of measurements of the latter is [7]

$$\mathcal{B}(B^+ \to K^+\chi_{c0}) = 1.50^{+0.15}_{-0.14} \times 10^{-4}. \tag{9}$$

This average is dominated by *BABAR* [19] measurements with errors that are almost entirely statistical. The $\mathcal{B}(B \to K\chi'_{c0}) < \mathcal{B}(B \to K\chi_{c0})$ assumption is reasonable for a few reasons, including the available phase space for $B \to K\chi'_{c0}$ is significantly smaller than that for $B \to K\chi_{c0}$; the B-meson decay rate to P-wave charmonium mesons is expected to be proportional to the derivative of the $c\bar{c}$ radial wave function at the origin [20], which decreases with increasing n. Moreover, measured B-meson branching fractions to excited charmonium states, where they exist, are all smaller than those to the ground states.[2] With this assumption, Eq. (2) implies a 90% CL *lower* limit

$$\mathcal{B}(\chi'_{c0} \to \omega J/\psi) > 14.6\%. \tag{10}$$

The compatibility of this $\mathcal{B}(\chi'_{c0} \to \omega J/\psi)$ lower limit with the upper limit given in Eq. (8), determined from the central values used to generate them, is $\chi^2/\mathrm{dof} = 5.8/1$, with a corresponding probability of 2%.

C. Limits on $\mathcal{B}(\chi'_{c0} \to D\bar{D})$

Although $\chi'_{c0} \to D\bar{D}$, the only kinematically allowed open-charmed "fall-apart" mode, is expected to be the dominant χ'_{c0} decay mode, there are no signs of the $X(3915)$ in either $\gamma\gamma \to D\bar{D}$ or $B \to KD\bar{D}$. The authors of Refs. [8] and [9] cite this as an argument against the $X(3915) = \chi'_{c0}$ assignment. Here I quantify these concerns with limit estimates on $\mathcal{B}(\chi'_{c0} \to D\bar{D})$ that are derived from each of the production processes.

c. From $\gamma\gamma \to X(3915) \to D\bar{D}$: The possibility that an $X(3915) \to D\bar{D}$ signal is lurking in the $M(D\bar{D})$ and $dN/d|\cos\theta^*|$ distributions for $\gamma\gamma \to D\bar{D}$ from Belle and *BABAR* was examined by Chen, He, Liu and Matsuki [21]. Based on fits to Belle's measurements (Fig. 1), they claim a signal for $\gamma\gamma \to X(3915) \to D\bar{D}$ with marginal significance at a strength that is 69% of that for the χ'_{c2}. I use this result to conclude that $\Gamma^{\gamma\gamma}_{X(3915)} \times \mathcal{B}(X(3915) \to D\bar{D}) < \Gamma^{\gamma\gamma}_{\chi'_{c2}} \times \mathcal{B}(\chi'_{c2} \to D\bar{D})$. If $X(3915) = \chi'_{c0}$, this, together with the ratio given in Eq. (7), implies a 90% CL upper limit

$$\mathcal{B}(\chi'_{c0} \to D\bar{D}) < 0.25\mathcal{B}(\chi'_{c2} \to D\bar{D}) < 25\%, \tag{11}$$

which is well below the expectation of nearly 100%. This upper limit corresponds to an upper limit on the partial width for $\chi'_{c0} \to D\bar{D}$ of 7 MeV.

d. From $B^+ \to K^+X(3915)$, $X(3915) \to D^0\bar{D}^0$: The $D^0\bar{D}^0$ invariant mass distribution for $B^+ \to K^+D^0\bar{D}^0$ decays from the Belle experiment [22] is shown in Fig. 2, where a strong, 68 ± 15 event signal for $B^+ \to K^+\psi(3770)$, $\psi(3770) \to D^0\bar{D}^0$ is evident. The measured product branching fraction for this process is [7]

$$\mathcal{B}(B^+ \to K^+\psi(3770)) \times \mathcal{B}(\psi(3770) \to D^0\bar{D}^0) = 1.6 \pm 0.6 \times 10^{-4}. \tag{12}$$

There is no sign in Fig. 2 of a peak near $M(D^0\bar{D}^0) \sim 3.92$ GeV that would correspond to the decay chain $B^+ \to K^+X(3915)$, $X(3915) \to D^0\bar{D}^0$. In fact, the Belle analysis attributes most of the events that are seen in the 3.92 GeV mass region to the process $B^+ \to D_{sJ}(2700)^+\bar{D}^0$; $D_{sJ}(2700)^+ \to K^+D^0$. Ignoring this possibility and attributing all of the 8 ± 5 events in the 20 MeV-wide bin centered at 3.917 GeV to the $X(3915)$, then scaling this to the $\psi(3770)$ signal (assuming constant acceptance) and comparing the result with the measured rate for $X(3915) \to \omega J/\psi$ production in B decays [Eq. (2)], gives the 90% CL limit

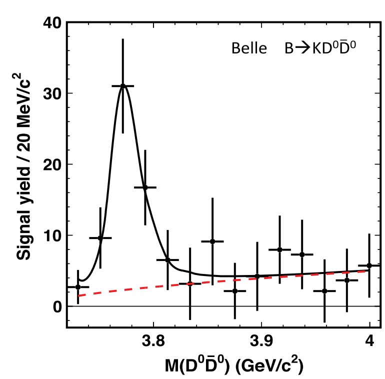

FIG. 2 (color online). The $M(D^0\bar{D}^0)$ distribution for $B \to KD^0\bar{D}^0$ decays from Ref. [22]. The peak near 3.77 GeV is due to the $\psi(3770)$.

[2]For example [7], $\mathcal{B}(B^+ \to K^+\psi')/\mathcal{B}(B^+ \to K^+J/\psi) = 0.63 \pm 0.04$, $\mathcal{B}(B^+ \to K^+\eta_c(2S))/\mathcal{B}(B^+ \to K^+\eta_c) = 0.35 \pm 0.19$ and $\mathcal{B}(B^+ \to K^*(890)^+\psi')/\mathcal{B}(B^+ \to K^*(890)^+J/\psi) = 0.47 \pm 0.10$.

$$\mathcal{B}(X(3915) \to D^0\bar{D}^0) < 1.2 \times \mathcal{B}(X(3915) \to \omega J/\psi), \quad (13)$$

which is independent of the $X(3915) = \chi'_{c0}$ assumption or any properties of the charmonium model. This strongly conflicts with expectations that $\chi'_{c0} \to D^0\bar{D}^0$ should be a dominant decay mode with a branching fraction of ~ 0.5, while $\chi'_{c0} \to \omega J/\psi$ would be an OZI-suppressed decay mode. Measured OZI-suppressed charmonium decays have partial widths of order 100 keV or less. A partial width of this magnitude would correspond to a $\chi'_{c0} \to \omega J/\psi$ branching fraction that is below 1%.

IV. DISCUSSION

Since the $X(3915)$ is above the open-charmed threshold, charmonium potential-model expectations for the χ'_{c2}-χ'_{c0} mass splittings can be modified by the effects of on-shell $D\bar{D}^{(*)}$ loops, which are not calculable from basic principles. Theoretical estimates of these effects [9,13,16] give values for the χ'_{c2}-χ'_{c0} mass splitting that differ by a much as a factor of 2, and although none are nearly sufficient to accommodate the splitting that would be associated with $X(3915) = \chi'_{c0}$, they are all model dependent. Similar effects can modify the potential-model-based assumption on the $\gamma\gamma$ partial-width ratio given in Eq. (7) in a poorly understood way and, thereby, invalidate the limit given in Eq. (8). The decays $B \to K\chi'_{c0}$ and $B \to K\chi_{c2}$ are non-factorizable [23], which is not the case for the examples given in footnote 2. Thus, the assumption that $\mathcal{B}(B^+ \to K^+\chi'_{c0}) \leq \mathcal{B}(B^+ \to K^+\chi_{c0}$ and the lower limit given in Eq. (10) might not be valid. On the other hand, the limit given in Eq. (13) is independent of any assumptions about charmonium properties.

Given our current state of theoretical understanding, it is possible that some, if not all, of the above-mentioned objections might be evaded. Nevertheless, the combination of all of these concerns raises some doubts about the validity of the $X(3915) = \chi'_{c0}$ assignment.

If the $X(3915)$ is not the χ'_{c0}, what is it? Also, where is the real χ'_{c0}? In the following, I briefly discuss these issues.

A. Where is the χ'_{c0}?

Gou and Meissner suggested that the "nonresonant" events seen by Belle and *BABAR* in the $\gamma\gamma \to D\bar{D}$ distribution [see Fig. 1(a)] are, in fact, due to $\chi'_{c0} \to D\bar{D}$. Since these events possibly include feed down from $\chi'_{c2} \to D\bar{D}^*$, $\bar{D}^* \to D\pi(\gamma)$, where the π or γ is undetected, and contributions from nonresonant $\gamma\gamma \to D\bar{D}$ events, the extraction of a $\chi'_{c0} \to D\bar{D}$ signal from these data might be difficult, especially if it is wide. However, with sufficient statistics, the strength of the $D\bar{D}^*$ contribution could be determined from the number of $D^+\bar{D}^0$ events in the data sample and a χ'_{c0} signal might be distinguishable from a nonresonant contribution by the mass dependence of the remaining distribution [24]. This could be done at Belle II [25].

A Belle study of the annihilation processes $e^+e^- \to J/\psi(c\bar{c})$, where $(c\bar{c})$ represents charmonium states, found significant cross sections only for cases where $(c\bar{c})$ has zero spin [26]. This suggests that a signal for the χ'_{c0} might show up in $e^+e^- \to J/\psi D\bar{D}$ events. Figure 3 shows the $M(D\bar{D})$ distribution for this process [27], where there is a clear excess of events above the non-J/ψ and/or non-$D\bar{D}$ backgrounds that are reliably determined from J/ψ and D mass sideband data. Chao suggested that this excess might be due to the χ'_{c0} [28]. The Belle fit to this excess returned a signal with a statistical significance of 3.8σ and a mass and width of $M = 3878 \pm 48$ MeV and $\Gamma = 347^{+316}_{-143}$ MeV, which are consistent with theoretical expectations for the χ'_{c0}. However, since the Belle fit was unstable under variations of the background parametrization and bin width, they made no claims for an observation. A reanalysis of this channel with a larger data sample by the Belle group is currently in progress [29].

B. What is the $X(3915)$?

A variety of interpretations for the XYZ peaks have been suggested, including: quark-antiquark-gluon hybrids [30], tightly bound QCD tetraquarks in colored diquark-diantiquark configurations [31], molecule-like structures formed from mesons bound by nuclear-like meson-exchange forces [32], hadrocharmonium in which $c\bar{c}$ states are bound to light quarks and/or gluons via chromo-electric dipole forces [33], cusps produced by near-threshold dynamics involving open-charmed mesons [34].

The $X(3915)$ mass is well below the lattice QCD calculated values for the lightest 0^{++} charmonium hybrid, which are around 4450 MeV [35]. It is also far from any relevant open-charmed-meson threshold. Thus, hybrid and cusp interpretations for the $X(3915)$ can probably be ruled out. The decay $X(3915) \to \omega J/\psi$ would not be OZI suppressed in a four-quark system. Both the QCD-tetraquark picture [36] and the hadrocharmonium model [37] predict large partial widths for hadronic decays to hidden-charm states for the $Z_c(3900)$. One could expect similar results from a corresponding analysis of the $X(3915)$.

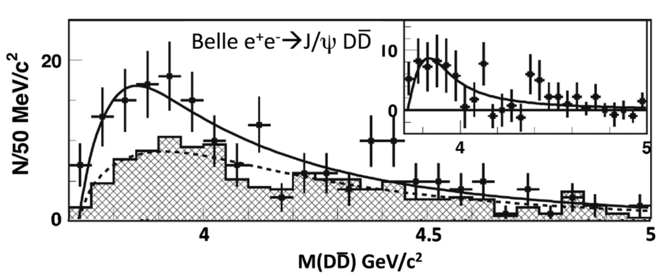

FIG. 3. The $M(D\bar{D})$ distribution for $e^+e^- \to J/\psi D\bar{D}$ from Ref. [27]. The histogram is the non-J/ψ plus non-$D\bar{D}$ backgrounds. The inset shows the background-subtracted distribution with a curve showing results of the fit discussed in the text.

V. SUMMARY

In this paper I reiterate comments from Refs. [8] and [9] about discrepancies between the mass of the $X(3915)$ and theoretical expectations for the χ'_{c0}, quantify concerns about the lack of any evidence for $X(3915) \to D\bar{D}$ decays at anywhere near the level expected for the χ'_{c0} [Eqs. (11) and (13)], and present an apparent inconsistency between limits on $\mathcal{B}(\chi'_{c0} \to \omega J/\psi)$ inferred from $X(3915)$ production rates in $\gamma\gamma$ fusion reactions and in B-meson decays [Eqs. (8) and (10)]. These concerns suggest that the assignment of the $X(3915)$ as the χ'_{c0} charmonium state is not a settled issue and remains a subject of further investigation and debate.

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