Chiral phase transition with a chiral chemical potential in the framework of Dyson-Schwinger equations

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Within the framework of Dyson-Schwinger equations , we discuss the chiral phase transition of QCD with a chiral chemical potential μ_5 as an additional scale. We focus especially on the issues related to the widely accepted as well as interesting critical end point (CEP). With the help of a scalar susceptibility, we find that there might be no CEP_5 in the $T-\mu_5$ plane, and the phase transition in the $T-\mu_5$ plane might be totally crossover when $\mu < 50$ MeV, which has apparent consistency with the lattice QCD calculation. Our study may also provide some useful hints to some other studies related to μ_5.

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I. INTRODUCTION

Quantum chromodynamics (QCD), which describes the interactions between quarks and gluons, is already commonly accepted as an essential part of the Standard Model of particle physics. Dynamical chiral symmetry breaking (DCSB) and quark color confinement are two fundamental features of QCD, and there are also many laboratories and experiments in this field, such as the famous Relativistic Heavy Ion Collider and Large Hadron Collider (LHC). However, thanks to the complicated non-Abelian feature of QCD itself, it is so difficult to have a thorough understanding of the mechanisms of DCSB and confinement, especially in the interesting nonperturbative region, where quarks and gluons are strongly coupled to each other and the related processes have small momentum transfer (or, equivalently, the coupling constant becomes large and running). In this case nowadays, people often in some sense have to resort to various effective models to study them phenomenologically, such as the chiral perturbation theory [1–5], the global color symmetry model [6–10], the quasiparticle model [11–18], the QCD sum rules [19–22], the Nambu–Jona-Lasinio (NJL) model and the related Polyakov-loop-extended Nambu–Jona-Lasinio (PNJL) model [23–34], lattice QCD [35–37], and the Dyson-Schwinger equations (DSEs) [38–45]. Through these studies, people hope to get profound insight of our nature as well as the early Universe.

Generally speaking, chiral symmetry is an exact global symmetry only when the current quark mass m is zero (the chiral limit case). In the low temperature (T) and low chemical potential (μ) phase (the hadronic phase, often referred to as Nambu-Goldstone phase or Nambu phase), this symmetry is spontaneously broken, and as a consequence there exist N_f^2-1 (N_f is the number of flavor) pseudoscalar Nambu-Goldstone bosons; meanwhile, the QCD vacuum hosts a chiral condensate (two-quark condensate) $\langle\bar{q}q\rangle$ which can actually act as an order parameter for chiral phase transition. At present, it is commonly accepted that when temperature and/or quark chemical potential are high enough, the strongly interacting hadronic matter will undergo a phase transition to some new phase, where the chiral symmetry is restored for the chiral limit case or partially restored for the $m \neq 0$ case. This new phase is usually called the Wigner phase and, in some sense, is related to the famous quark gluon plasma, which is expected to appear in the ultrarelativistic heavy ion collisions or the inner core of compact stars. As for the nature of the chiral phase transition when $m \neq 0$, a popular scenario favors a crossover at small chemical potential and then a first-order chiral transition for larger chemical potential at a critical end point (CEP) [46]. Such a picture is consistent with most lattice QCD simulations and various QCD-inspired models, as listed in the last paragraph; however, it is not yet clarified directly from the first principles of QCD. The search for such a CEP is also one of the main goals in the high-energy physics experiments, such as the beam energy scan program at Relativistic Heavy Ion

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Collider [47–50]. Unfortunately, lattice Monte Carlo simulations cannot be used to resolve this issue directly due to the "sign problem" [51–56], and there is still no firm theoretical evidence for the existence of such a CEP, so the calculations based on some effective QCD models are irreplaceable.

In Ref. [57], K. Fukushima *et al.* first introduce the chiral chemical potential (also called axial chemical potential in other work, such as Ref. [58]), μ_5, which is conjugated to chiral charge density; and in Ref. [59], M. Ruggieri suggests that the CEP of the chiral phase diagram can be detected by means of lattice QCD simulations of grand-canonical ensembles with this chiral chemical potential. By concrete calculations within some chiral models, the author shows that a continuation of the CEP at finite temperature and finite chemical potential to a possible "CEP_5" in the $T-\mu_5$ plane is reachable, which is then helpful in determining the CEP in the $T-\mu$ plane from lattice QCD. The existence of such a possible CEP_5 is also confirmed in some other chiral model studies, for example, Refs. [60,61]. In Ref. [58], the authors investigated the effect of the vector interaction as well as the finite current quark mass on the location of the CEP. In this paper, we will discuss related topics within the framework of Dyson-Schwinger equations, which is widely used and has been proved successful in hadron physics and phase transitions of strongly interacting matter. This paper is organized as follows. In Sec. II we give a basic introduction to the DSEs at finite temperature and nonzero chemical potential as well as an effective model gluon propagator, and with the help of a scalar susceptibility we also discuss the nature of the chiral phase transition within this framework. In Sec. III, we discuss in detail the influence of the chiral chemical potential on the chiral phase transition of QCD and focus primarily on the behavior of the CEP, not only the algebra but also the numerical results. Finally, we give a brief summary in Sec. IV.

II. DYSON-SCHWINGER EQUATIONS AND AN EFFECTIVE GLUON PROPAGATOR

In this section, we briefly review the formula of the Dyson-Schwinger equations, which is widely used ont only in the nonperturbative region of QCD but also in some other fields like quantum electrodynamics in $(2+1)$ dimensions (QED_3) [38,62–65], etc. At zero temperature and zero chemical potential, the DSE for the quark propagator reads [38] (We always work in Euclidean space and take the number of flavors $N_f=2$ and number of colors $N_c=3$ throughout this paper. Moreover, as we employ an ultraviolet finite model, renormalization is actually unnecessary.)

$$S(p)^{-1}=S_0(p)^{-1}+\frac{4}{3}\int\frac{\mathrm{d}^4q}{(2\pi)^4}g^2D_{\mu\nu}(p-q)\gamma_\mu S(q)\Gamma_\nu, \tag{1}$$

where $S(p)$ is the dressed quark propagator,

$$S_0(p)^{-1}=i\gamma\cdot p+m, \tag{2}$$

is the inverse of the free quark propagator, g is the strong coupling constant, $D_{\mu\nu}(p-q)$ is the dressed gluon propagator, and $\Gamma_\nu=\Gamma_\nu(p,q)$ is the dressed quark-gluon vertex. According to the Lorentz structure analysis, we have

$$S(p)^{-1}=i\not{p}A(p^2)+B(p^2), \tag{3}$$

where $A(p^2)$ and $B(p^2)$ are scalar functions of p^2. After the gluon propagator and the quark-gluon vertex are specified, people can then solve this equation numerically.

The extension of the above quark DSE to its nonzero temperature and nonzero quark chemical potential version is systematically accomplished by transcription of the quark four-momentum via $p\to p_k=(\vec{p},\tilde{\omega}_k)$, where $\tilde{\omega}_k=\omega_k+i\mu$ with $\omega_k=(2k+1)\pi T$, $k\in\mathbb{Z}$ are the fermion Matsubara frequencies, and no new parameters are introduced [39],

$$S(p_k)^{-1}=S_0(p_k)^{-1}+\frac{4}{3}T\sum\!\!\!\!\!\!\int g^2D_{\mu\nu}(p_k-q_n)\gamma_\mu S(q_n)\Gamma_\nu, \tag{4}$$

where

$$S_0(p_k)^{-1}=i\gamma\cdot p_k+m, \tag{5}$$

and $\sum\!\!\!\!\!\!\int$ denotes $\sum_{l=-\infty}^{+\infty}\int\frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}$. Nevertheless, its solution now should have four independent amplitudes due to the breaking of $O(4)$ symmetry down to $O(3)$ symmetry [39],

$$\begin{aligned}S(p_k)^{-1}=&\,i\vec{\not{p}}A(\vec{p}^2,\tilde{\omega}_k^2)+\mathbf{1}B(\vec{p}^2,\tilde{\omega}_k^2)\\&+i\gamma_4\tilde{\omega}_kC(\vec{p}^2,\tilde{\omega}_k^2)+\vec{\not{p}}\gamma_4\tilde{\omega}_kD(\vec{p}^2,\tilde{\omega}_k^2),\end{aligned} \tag{6}$$

where $\vec{\not{p}}=\vec{\gamma}\cdot\vec{p}$, $\vec{\gamma}=(\gamma_1,\gamma_2,\gamma_3)$, and the four scalar functions $\mathcal{F}=A,B,C,D$ are complex and satisfy

$$\mathcal{F}(\vec{p}^2,\tilde{\omega}_k^2)^*=\mathcal{F}(\vec{p}^2,\tilde{\omega}_{-k-1}^2). \tag{7}$$

But as discussed in Ref. [39], the dressing function D is power-law suppressed in the ultraviolet region, so it actually does not contribute to the cases investigated in our work. At zero temperature but in the nonzero chemical potential case, D vanishes exactly since the corresponding tensor structure has the wrong transformation properties under time reversal [66]. For these reasons, in most cases we can just neglect D, and get the commonly used general structure of the inverse of the quark propagator as

$$S(p_k)^{-1} = i\vec{\not{p}}A(\vec{p}^2,\tilde{\omega}_k^2) + \mathbf{1}B(\vec{p}^2,\tilde{\omega}_k^2) + i\gamma_4\tilde{\omega}_k C(\vec{p}^2,\tilde{\omega}_k^2). \tag{8}$$

For the dressed gluon propagator, the general form is

$$g^2 D_{\mu\nu}(k_{nl}) = P^T_{\mu\nu} D_T(\vec{k}^2,\omega_{nl}^2) + P^L_{\mu\nu} D_L(\vec{k}^2,\omega_{nl}^2), \tag{9}$$

where $k_{nl} = (\vec{k},\omega_{nl}) = (\vec{p}-\vec{q},\omega_n-\omega_l)$, and $P^{T,L}_{\mu\nu}$ are transverse and longitudinal projection operators, respectively. And for the domain $T < 0.2$ GeV, with which we are concerned in this work, the authors of Ref. [67] have proved that $D_T = D_L$ is a good approximation. For the in-vacuum interaction, in this work we will adopt the following form of ansatz as in Ref. [68],

$$D_T = D_L = D_0 \frac{4\pi^2}{\sigma^6} k_{nl}^2 e^{-k_{nl}^2/\sigma^2}, \tag{10}$$

which is a simplified version of the famous as well as widely used one in Refs. [69,70]. It can be proved that this dressed gluon propagator at $T = 0$ violates the axiom of reflection positivity [71] and is, therefore, not observable; i.e., the excitation it describes is confined. The same is true of the dressed quark propagator which is also not positive definite and, hence, is confined (Actually, we can take the gluon propagator as input, and the quark propagator can then be solved numerically. The results show that there is no singularity on the real, positive, i.e., timelike, p^2 axis, which implies that quarks are confined).

Concerning the quark-gluon vertex, in this work we will take the rainbow truncation, which means a simple but symmetry-preserving bare vertex is adopted,

$$\Gamma_\nu(p_n,q_l) = \gamma_\nu. \tag{11}$$

The status of propagator and vertex studies can be tracked from Ref. [72].

Now let us fix the related parameters and then show some of the numerical results. D_0 and σ are usually fixed by fitting the observables, such as the two-quark condensate, the pion decay constant ($f_\pi = 131$ MeV), and the pion mass ($m_\pi = 138$ MeV). In this work we adopt the parameters from Ref. [70], that $D_0 = 9.3\times10^5$ MeV^2 and $\sigma = 400$ MeV. For the current quark mass, we will use $m = 5$ MeV. Then substituting Eqs. (5), (8), (10), and (11) into Eq. (4), we can solve the quark DSE for each value of temperature and chemical potential by means of numerical iteration. As an example, we show $B(0,\tilde{\omega}_0^2)$ as a function of μ for different T in Fig. 1, and the corresponding chiral susceptibility with respect to m, which is defined as

$$\chi_m(T,\mu) = \frac{\partial B(0,\tilde{\omega}_0^2)}{\partial m}, \tag{12}$$

in Fig. 2.

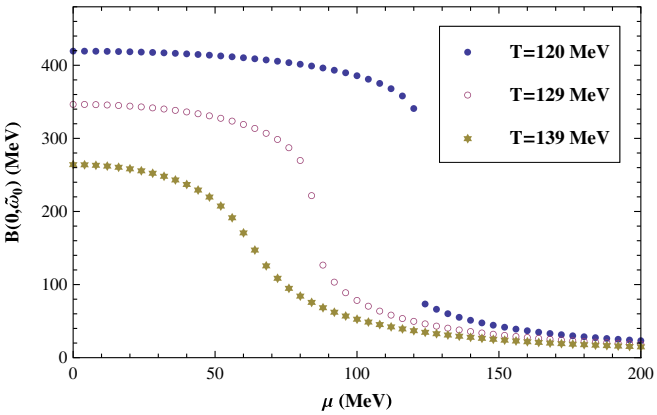

FIG. 1 (color online). $B(0,\tilde{\omega}_0^2)$ as a function of μ for three different T.

In general, we can see from Fig. 1 that the scalar function $B(0,\tilde{\omega}_0^2)$ will decrease when the chemical potential μ increases; this phenomenon holds true for the temperature T and momentum $\vec{p}^2$ as well. It is known that the scalar part $B(\vec{p}^2,\tilde{\omega}_k^2)$ of the quark propagator Eq. (4) in some sense reflects the dressing effect of the quark, so the results show that the dressing effect becomes weaker and weaker for higher T,μ, and $\vec{p}^2$. We can also see from Fig. 1 that for different values of T, $B(0,\tilde{\omega}_0^2)$ may behave differently: for T larger than a critical $T_c = 129$ MeV, $B(0,\tilde{\omega}_0^2)$ changes gradually but continuously from the Nambu solution to the Wigner solution; while for T smaller than T_c, there will appear a sudden discontinuity at some critical μ.

To study the nature of the chiral phase transition, especially to determine the critical value of μ_c at T_c, people often employ various susceptibilities of QCD [73–77]. We can see from Fig. 2 that, for $T \geq T_c$, the susceptibility χ_m indicates a crossover from the Nambu phase to the Wigner phase, and the peak grows higher and higher when T approaches T_c. At T_c, χ_m shows a sharp and narrow divergent peak, and the value of this peak turns out

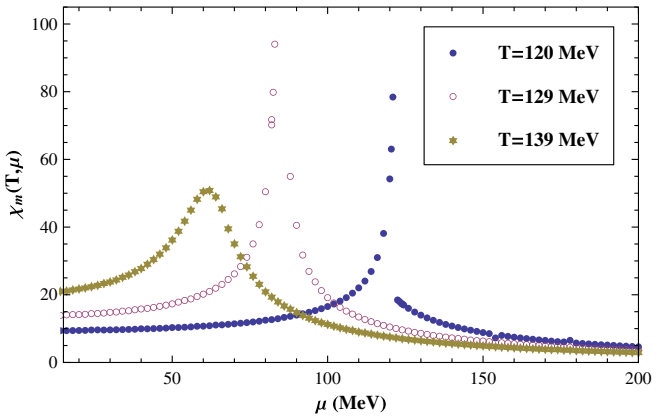

FIG. 2 (color online). $\chi_m(T,\mu)$ as a function of μ for three different T.

to be ∞, which demonstrates that this is a second-order phase transition, corresponding to the CEP. And for $T \leq T_c$, an obvious first-order phase transition will occur. According to these results, we can move on to study the chiral phase transition, especially the behavior of the CEP. In Sec. III, we will focus on the variance of the CEP when the chiral chemical potential is considered as an additional scale.

III. INFLUENCES OF THE CHIRAL CHEMICAL POTENTIAL ON THE CHIRAL PHASE TRANSITION

The concept of chiral chemical potential was first proposed by K. Fukushima *et al.* in a study related to the external magnetic field [57]. Since topological-charge-changing transitions can induce an asymmetry between the number of right- and left-handed quarks due to the axial anomaly, they introduce the chiral chemical potential μ_5, which couples to the difference between the number of right- and left-handed fermions. The chirality is also expected to be produced in the high-temperature phase of QCD [58]. Many researchers argue that although μ_5 is a mere mathematical artifice instead of a true chemical potential,[1] it has the advantage that it can be simulated on the lattice QCD with $N_c = 3$; hence, it is likely to provide some useful information for the studies of the CEP and even for inhomogeneous phases or the inner structure of compact stellar objects. One of the most interesting features of the introduction of μ_5 is that it makes possible the continuation of the CEP to a possible "CEP_5" in the $T-\mu_5$ plane, which is of course helpful in determining the CEP from lattice QCD. Some other researchers also confirm such a possible CEP_5 in related chiral model studies [60,61]. In this part, we discuss related topics within the framework of the Dyson-Schwinger equations.

To be specific, in order to study the effects of μ_5, people should add the following term to the Lagrangian density:

$$\mu_5\bar{\psi}\gamma_4\gamma_5\psi. \tag{13}$$

In our work, the quark propagator and its inverse can now include, at most, eight components according to the Lorentz structure analysis, namely,

$$1, \vec{\not{p}}, \gamma_4, \vec{\not{p}}\gamma_4, \gamma_5, \vec{\not{p}}\gamma_5, \gamma_4\gamma_5, \vec{\not{p}}\gamma_4\gamma_5. \tag{14}$$

So now the general inverse form of the dressed quark propagator is

$$\begin{aligned} S(p_n,\mu_5)^{-1} = {} & i\vec{\not{p}}A + B + i\gamma_4\tilde{\omega}_n C + \vec{\not{p}}\gamma_4\tilde{\omega}_n D \\ & + (i\vec{\not{p}}A_5 + B_5 + i\gamma_4\tilde{\omega}_n C_5 + \vec{\not{p}}\gamma_4\tilde{\omega}_n D_5)\gamma_5. \end{aligned} \tag{15}$$

The eight scalar functions $\mathcal{F} = A, B, C, D, A_5, B_5, C_5, D_5$ denote $\mathcal{F} = \mathcal{F}(\vec{p}^2, \tilde{\omega}_n^2, \mu_5)$, which are all complex and satisfy the following equation:

$$\mathcal{F}(\vec{p}^2, \tilde{\omega}_k^2, \mu_5)^* = \mathcal{F}(\vec{p}^2, \tilde{\omega}_{-k-1}^2, \mu_5). \tag{16}$$

Now the quark DSE at nonzero temperature and nonzero chemical potential is

$$\begin{aligned} S(p_n,\mu_5)^{-1} = {} & S_0(p_n,\mu_5)^{-1} \\ & + \frac{4}{3}T\sum\!\!\!\!\!\!\int g^2 D_{\mu\nu}(p_n - q_l)\gamma_\mu S(q_l,\mu_5)\Gamma_\nu(p_n,q_l), \end{aligned} \tag{17}$$

where

$$S_0(p_n,\mu_5)^{-1} = \vec{i\not{p}} + m + i\gamma_4\tilde{\omega}_n - \mu_5\gamma_4\gamma_5. \tag{18}$$

For the details of $S(q_l,\mu_5)$, please see the Appendix.

Substituting Eqs. (15), (9), and (11) into Eq. (17), we find that the solution is the following coupled integral equations [for the sake of brevity, here all $\mathcal{F}(p)$ means $\mathcal{F}(\vec{p}^2, \tilde{\omega}_n^2, \mu_5)$]:

$$\begin{aligned} A(p) &= 1 + c(T)\sum\!\!\!\!\!\!\int k_{nl}^2 e^{-k_{nl}^2/\sigma^2} \times \mathcal{K}_A, \\ B(p) &= m + c(T)\sum\!\!\!\!\!\!\int k_{nl}^2 e^{-k_{nl}^2/\sigma^2} \times \mathcal{K}_B, \\ C(p) &= 1 + c(T)\sum\!\!\!\!\!\!\int k_{nl}^2 e^{-k_{nl}^2/\sigma^2} \times \mathcal{K}_C, \\ D(p) &= c(T)\sum\!\!\!\!\!\!\int k_{nl}^2 e^{-k_{nl}^2/\sigma^2} \times \mathcal{K}_D, \\ A_5(p) &= c(T)\sum\!\!\!\!\!\!\int k_{nl}^2 e^{-k_{nl}^2/\sigma^2} \times \mathcal{K}_{A_5}, \\ B_5(p) &= c(T)\sum\!\!\!\!\!\!\int k_{nl}^2 e^{-k_{nl}^2/\sigma^2} \times \mathcal{K}_{B_5}, \\ C_5(p) &= i\mu_5/\omega_n c(T)\sum\!\!\!\!\!\!\int k_{nl}^2 e^{-k_{nl}^2/\sigma^2} \times \mathcal{K}_{C_5}, \\ D_5(p) &= c(T)\sum\!\!\!\!\!\!\int k_{nl}^2 e^{-k_{nl}^2/\sigma^2} \times \mathcal{K}_{D_5}, \end{aligned} \tag{19}$$

in which

[1]The reason for this is easy to understand since the difference in densities of the right- and left-handed quarks, $n_5 = n_R - n_L$, does not conserve.

$$
\begin{aligned}
c(T) &= \frac{16\pi^2 T}{3\sigma^6},\\
\mathcal{K}_A &= -[(\vec{p}\cdot\vec{q}k_{nl}^2 + 2\vec{k}\cdot\vec{p}\,\vec{k}\cdot\vec{q})\sigma_A + 2\vec{k}\cdot\vec{p}\omega_{nl}\omega_l\sigma_C]/(\vec{p}^2k_{nl}^2),\\
\mathcal{K}_B &= 3\sigma_B,\\
\mathcal{K}_C &= -\omega_l\sigma_C/\omega_n - 2\omega_{nl}(\vec{k}\cdot\vec{q}\sigma_A + \omega_{nl}\omega_l\sigma_C)/(\omega_n k_{nl}^2),\\
\mathcal{K}_D &= [\vec{p}\cdot\vec{q}(2\omega_{nl}^2 - k_{nl}^2) + 2\vec{k}\cdot\vec{p}\,\vec{k}\cdot\vec{q}]\omega_l\sigma_D/(\vec{p}^2\omega_n k_{nl}^2),\\
\mathcal{K}_{A_5} &= [(\vec{p}\cdot\vec{q}k_{nl}^2 + 2\vec{k}\cdot\vec{p}\,\vec{k}\cdot\vec{q})\sigma_{A_5} + 2\vec{k}\cdot\vec{p}\omega_{nl}\omega_l\sigma_{C_5}]/(\vec{p}^2k_{nl}^2),\\
\mathcal{K}_{B_5} &= -3\sigma_{B_5},\\
\mathcal{K}_{C_5} &= \omega_l\sigma_C/\omega_n + 2\omega_{nl}(\vec{k}\cdot\vec{q}\sigma_{A_5} + \omega_{nl}\omega_l\sigma_{C_5})/(\omega_n k_{nl}^2),\\
\mathcal{K}_{D_5} &= [\vec{p}\cdot\vec{q}(2\omega_{nl}^2 - k_{nl}^2) + 2\vec{k}\cdot\vec{p}\,\vec{k}\cdot\vec{q}]\omega_l\sigma_{D_5}/(\vec{p}^2\omega_n k_{nl}^2).
\end{aligned}
\tag{20}
$$

Now we can solve Eq. (19) numerically for specific chiral chemical potential μ_5, as well as for the temperature T and normal chemical potential μ. The critical $T_c[\mu_5]$ and $\mu_c[\mu_5]$, which are coordinates of the new "critical end point" at a specific μ_5 (CEP[μ_5]), are determined by the scalar susceptibility that is defined in Eq. (12).[2] In this work, we will concentrate on the behavior of the CEP[μ_5], which is expected to be linked to a possible CEP_5 in the $T-\mu_5$ plane [59]. In Figs. 3 and 4, we plot the relations between μ_5 and $T_c[\mu_5]$ as well as μ_5 and $\mu_c[\mu_5]$, respectively, which are obtained by seeking the corresponding CEP[μ_5] for different μ_5. Therefore, each point in these two lines means a "CEP' in the $T-\mu$ plane for the corresponding μ_5.

We can see in Fig. 3 that $T_c[\mu_5]$ increases slowly when μ_5 is smaller than approximately 300 MeV, and it starts to increase quickly for larger μ_5. Nevertheless, the whole increase is smooth and monotonic. The most interesting thing is shown in Fig. 4: $\mu_c[\mu_5]$ will decrease first when μ_5 is smaller than about 400 MeV and then increase for larger μ_5. The behavior of $\mu_c[\mu_5]$ for different μ_5 is quite different from the previous results from some chiral models, such as Fig. 4 of Ref. [59], that at some critical value of μ_5, $\mu_c[\mu_5]$ will decrease to 0, where a CEP_5 is expected to exist. When μ_5 is not very large, the qualitative properties of the results in Fig. 4 are similar to those from chiral models, but the decrease of $\mu_c[\mu_5]$ is much slower. Our results also indicate there might be no CEP_5 in the $T-\mu_5$ plane, and the phase transition in the $T-\mu_5$ plane might be totally crossover when $\mu < 50$ MeV, which then has apparent consistency with the lattice QCD calculation [53]. Furthermore, our studies may also provide some hints for recent studies related to the chiral chemical potential, such as Ref. [78].

[2]Here we summarize that in this work "CEP" means the critical end point in the $T-\mu$ plane with $\mu_5 = 0$, while CEP_5 denotes the possible one in the $T-\mu_5$ plane with $\mu = 0$ [59], and CEP[μ_5] is generally the similar critical end point in the $T-\mu$ plane for a specific μ_5.

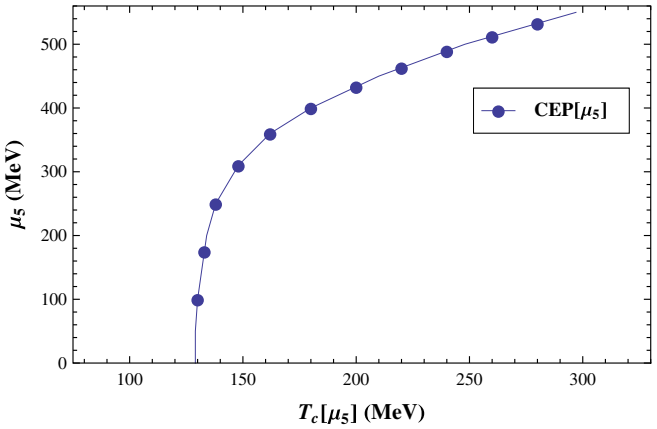

FIG. 3 (color online). The relation between μ_5 and the corresponding $T_c[\mu_5]$ in the $T-\mu$ plane.

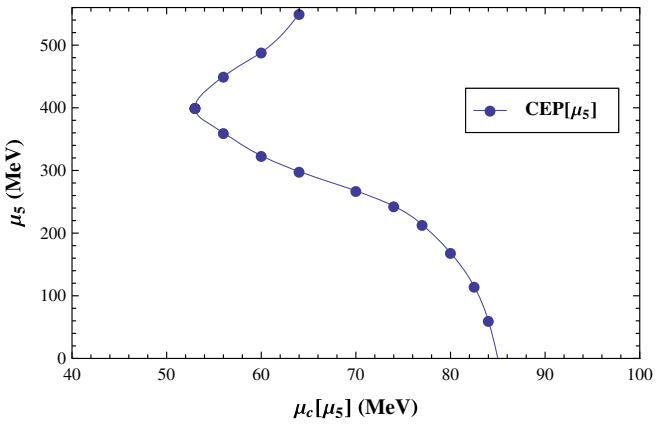

FIG. 4 (color online). The relation between μ_5 and the corresponding $\mu_c[\mu_5]$ in the $T-\mu$ plane.

Since comparing with the chiral models, DSEs are renormalizable and include the effects of color confinement as well as DCSB simultaneously; accordingly, the DSEs approach is commonly accepted to be closer to QCD itself [38–41,43]. In the calculations within the framework of DSEs, it is not necessary to introduce any annoying parameters, such as the momentum cutoff scale which destroys some basic symmetries of QCD, and thus more reliable results can be achieved. In other words, although the chiral models have been found to give reasonable phase diagrams in the $T-\mu$ plane, they cannot guarantee the robustness when μ_5 acts as an additional scale and makes applicable the calculations $T+\mu+\mu_5 < \Lambda$ (Λ is some cutoff scale).

IV. SUMMARY

Thanks to the complicated non-Abelian feature of quantum chromodynamics (QCD) itself, its two fundamental features, namely, dynamical chiral symmetry breaking (DCSB) and quark color confinement, have to be studied

phenomenologically through various effective models at present, especially in the most interesting nonperturbative region. In this work, we discuss the chiral phase transition of QCD within the framework of Dyson-Schwinger equations, with a chiral chemical potential μ_5 as an additional scale other than the normal temperature T and quark chemical potential μ. We first give a basic introduction to the DSEs at finite temperature and nonzero chemical potential as well as an effective model gluon propagator and then focus primarily on the calculations related to the famous critical end point in the $T-\mu$ plane, which is predicted by many model studies and has created much interest on both the experimental side (one of the main goals in some high-energy physics experiments) and the theoretical side. With the help of a scalar susceptibility, which often acts as an order parameter of a chiral phase transition, we find that there might be no CEP_5 in the $T-\mu_5$ plane, which is thought to exist by some chiral model calculations, and the phase transition in the $T-\mu_5$ plane might be totally crossover when $\mu < 50$ MeV, which has apparent consistency with the lattice QCD calculation. DSEs are widely used and have been proved successful in hadron physics and phase transitions of strongly interacting matter, so our study may also provide some useful hints to other studies related to μ_5. Last but not least, we would like to say that the related issues deserve further study.

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APPENDIX: STRUCTURE OF THE QUARK PROPAGATOR

Using Eq. (15), after some algebra we find that the quark propagator can be written as [note that here $\mathcal{F} = \mathcal{F}(\vec{q}^2, \tilde{\omega}_l^2, \mu_5)$]

$$S(q_l,\mu_5) = \vec{i\not{q}}\sigma_A + \sigma_B + i\gamma_4\tilde{\omega}_l\sigma_C + \vec{\not{q}}\gamma_4\tilde{\omega}_l\sigma_D + (\vec{i\not{q}}\sigma_{A_5} + \sigma_{B_5} + i\gamma_4\tilde{\omega}_l\sigma_{C_5} + \vec{\not{q}}\gamma_4\tilde{\omega}_l\sigma_{D_5})\gamma_5, \tag{A1}$$

where

$$\begin{aligned}
\sigma_A &= (-At_1 + C_5\omega_l t_2)/t_3,\\
\sigma_B &= (Bt_1 - D_5\omega_l\vec{q}^2 t_2)/t_3,\\
\sigma_C &= (-Ct_1 - A_5\vec{q}^2 t_2/\omega_l)/t_3,\\
\sigma_D &= (-Dt_1 - B_5 t_2/\omega_l)/t_3,\\
\sigma_{A_5} &= (A_5 t_1 - C\omega_l t_2)/t_3,\\
\sigma_{B_5} &= (-B_5 t_1 + D\omega_l\vec{q}^2 t_2)/t_3,\\
\sigma_{C_5} &= (-C_5 t_1 - A\vec{q}^2 t_2/\omega_l)/t_3,\\
\sigma_{D_5} &= (-D_5 t_1 - Bt_2/\omega_l)/t_3,
\end{aligned} \tag{A2}$$

and

$$\begin{aligned}
t_1 &= B^2 - B_5^2 + (C^2 - C_5^2)\omega_l^2 + [A^2 - A_5^2 + (D^2 - D_5^2)\omega_l^2]\vec{q}^2,\\
t_2 &= 2\omega_l(A_5 C + B_5 D - AC_5 - BD_5),\\
t_3 &= t_1^2 + \vec{q}^2 t_2^2.
\end{aligned} \tag{A3}$$

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