# $c\bar{c}$ interaction above threshold and the radiative decay $X(3872) \to J/\psi\gamma$

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Radiative decays of $X(3872)$ are studied in single-channel approximation (SCA) and in the coupled-channel (CC) approach, where the decay channels $D\bar{D}^*$ are described with the string-breaking mechanism. In SCA the transition rate $\tilde{\Gamma}_2 = \Gamma(2^3P_1 \to \psi\gamma) = 71.8$ keV and large $\tilde{\Gamma}_1 = \Gamma(2^3P_1 \to J/\psi\gamma) = 85.4$ keV are obtained, giving for their ratio the value $\tilde{R}_{\psi\gamma} = \frac{\tilde{\Gamma}_2}{\tilde{\Gamma}_1} = 0.84$. In the CC approach, three factors are shown to be equally important: First, the admixture of the $1^3P_1$ component in the normalized wave function of $X(3872)$ due to the CC effects. Its weight $c_X(E_R) = 0.200 \pm 0.015$ is calculated. Second, the use of the multipole function $g(r)$ instead of $r$ in the overlap integrals, determining the partial widths. Third, the choice of the gluon-exchange interaction for $X(3872)$, as well as for other states above threshold. If for $X(3872)$ the gluon-exchange potential is taken to be the same as for low-lying charmonium states, then in the CC approach $\Gamma_1 = \Gamma(X(3872) \to J/\psi\gamma) \sim 3$ keV is very small, giving the large ratio $R_{\psi\gamma} = \frac{\mathcal{B}(X(3872)\to\psi(2S)\gamma)}{\mathcal{B}(X(3872)\to J/\psi\gamma)} \gg 1.0$. Arguments are presented why the gluon-exchange interaction may be suppressed for $X(3872)$ and in this case $\Gamma_1 = 42.7$ keV, $\Gamma_2 = 70.5$ keV, and $R_{\psi\gamma} = 1.65$ are predicted for the minimal value $c_X(\min) = 0.185$, while for the maximal value $c_X = 0.215$ we obtain $\Gamma_1 = 30.8$ keV, $\Gamma_2 = 73.2$ keV, and $R_{\psi\gamma} = 2.38$, which agrees with the LHCb data.

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## I. INTRODUCTION

In 2003, the Belle Collaboration discovered the $X(3872)$ as a narrow peak in the $J/\psi\pi\pi$ invariant mass distribution in the decays $B \to J/\psi\pi\pi K$ [1]. Now its characteristics, like the mass, the strict restriction on the width, $\Gamma \lesssim 1.2$ MeV, and the charge parity $C = +$, are well established [2–5]. In recent CDF and LHCb experiments, the quantum numbers of $X(3872)$ were determined to be $J^{PC} = 1^{++}$ [6,7]. Still, discussions about the nature of $X(3872)$ continue, and to understand its exotic properties, one must understand the special role played by the radiative decays, $X(3872) \to J/\psi\gamma$ and $X(3872) \to \psi(3686)\gamma$, which are sensitive to the behavior of the $X(3872)$ wave function (w.f.) at medium and large distances.

The first evidence for the decay $X(3872) \to J/\psi\gamma$ was obtained by the Belle Collaboration [8] and confirmed by the *BABAR* Collaboration [9]. Later, *BABAR* has also observed the radiative decay $X(3872) \to \psi(3686)\gamma$ and determined the branching ratio fraction $R_{\psi\gamma} = \frac{\mathcal{B}(X(3872)\to\psi(3686)\gamma)}{\mathcal{B}(X(3872)\to J/\psi\gamma)} = 3.4 \pm 1.4$ [10]. However, Belle has not found evidence for the radiative decay $X(3872) \to \psi(3686)\gamma$ and has set an upper limit for the ratio [11],

$$R_{\psi\gamma} < 2.1. \tag{1}$$

Recently the LHCb group [12] has observed the decay $X(3872) \to \psi(3686)\gamma$ with good statistics and determined its value to be

$$R_{\psi\gamma} = 2.46 \pm 0.64 \pm 0.29, \tag{2}$$

while in Ref. [13] the weighted average over three groups of measurements was determined to be $\bar{R}_{\psi\gamma} = 2.31 \pm 0.57$.

Unfortunately, theoretical predictions for the partial widths $\Gamma_1$ and $\Gamma_2$ of the radiative decays $X(3872) \to J/\psi\gamma$ and $X(3872) \to \psi(3686)\gamma$, respectively, vary widely in different models [14–22] (see also the recent reviews [23,24]). If $X(3872)$ is considered as a pure $2^3P_1$ charmonium state, then a large value $R_{\psi\gamma} \simeq 5$ is obtained as shown in Refs. [16–19], exceeding the LHCb result. On the contrary, in a molecular picture the transition $X(3872) \to \psi(3686)\gamma$ is suppressed and the ratio $R_{\psi\gamma}$ should be much smaller [14,15]. Also, in many-channel models, both in the $^3P_0$ model [17,18] and in the Cornell model [19], large values $R_{\psi\gamma} \geq 5.0$ were predicted.

It is surprising that the predicted widths $\Gamma_1$ and $\Gamma_2$ strongly differ even within the single-channel approximation (SCA), i.e., when $X(3872)$ is supposed to be the $2^3P_1$ charmonium state [16–19]. It occurs because different

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parameters in the potentials and different kinematics are used. Therefore, it remains unclear to what degree the theoretical predictions refer to specific features of the radiative decays or to the choice of fitting parameters and kinematics. Meanwhile, during the last decade within the field correlator method [25–27] and in lattice QCD [28,29] the static potential was shown to be universal in the region $r \lesssim 1.5$ fm, where there is no creation of light $q\bar{q}$ pairs. Here we shall use this information and perform a parameter-free analysis of the charmonium states with the use of the relativistic string Hamiltonian (RSH) [25].

It was already underlined by Li and Chao [16] and in Ref. [24] that the use of relativistic kinematics is very important for the radiative decays, and in nonrelativistic approximation the overlap integrals, determining the partial widths, may significantly differ from those in relativistic calculations.

In our previous paper [21], the rates and the ratio $R_{\psi\gamma}$ were calculated considering $X(3872)$ (with $J^{PC} = 1^{++}$) in the coupled-channels (CC) approach, where the coupling to the $D\bar{D}^*$ channels is determined by the relativistic string-breaking mechanism [20–22]. This mechanism was successfully applied to $X(3872)$, explaining it as the $2^3P_1$ charmonium state shifted down due to CC effects and appearing as a sharp peak just at the $D^0\bar{D}^{*0}$ threshold [20]. Owing to such CC effects, the w.f. of $X(3872)$ acquires an admixture $c_X(E_R)$ from the $1^3P_1$ component, equal to ∼0.20. Although this mixing parameter is not very large, it nevertheless strongly affects the value of the partial width $\Gamma_1$, in which the overlap integral is small and very sensitive to different corrections.

However, in Ref. [21] in the overlap integral $K_1 = \langle\varphi(2P)|g(r)|\varphi(1S)\rangle$, the multipole moment $g(r)$ was replaced by $r$, which is a common practice. In this approximation $\tilde{K}_1 = \langle\varphi(2P)|r|\varphi(1S)\rangle = 0.21$ has a small value, close to those given in Refs. [18,24]. It appears that the magnitude of $K_1$, defined using the multipole function $g(r)$, is a factor of 2 larger than $\tilde{K}_1$, increasing the partial width $\Gamma_1$ by about 4 times. Thus, the replacement of $g(r)$ by $r$ cannot be applied to this radiative decay, proceeding with a large photon energy.

We also show that the integral $K_1$ strongly depends on the gluon-exchange (GE) part of the static potential and consider two possibilities: First, when in $X(3872)$, the GE potential is taken to be the same as for the low-lying states (below threshold). In this case, the results are different in the SCA and in the CC approach. The second possibility implies that in higher charmonium states (above threshold), including $X(3872)$, the GE interaction is suppressed. For such a $c\bar{c}$ dynamics the partial width $\Gamma_1$ increases, while the partial width $\Gamma_2 = \Gamma(X(3872) \to \psi(3686)\gamma)$ weakly depends on the GE interaction.

We use here the CC approach, where the admixture from the continuum to the w.f. of $X(3872)$ is defined by the mixing parameter $c_X(E_R = 3.872$ GeV), which is calculated here with a good accuracy: $c_X = 0.200 \pm 0.015$. Although its value is not large, it nevertheless gives an important contribution to $\Gamma_1$.

Also in our analysis, we lay the stress on the choice of the $c$-quark mass, the QCD constant $\Lambda(n_f = 3)$, and the string tension, which cannot be arbitrary parameters lest the physical picture would be distorted.

## II. THE $c\bar{c}$ INTERACTION BELOW AND ABOVE THE THRESHOLD

Before studying the $X(3872)$ in the many-channel approach, one needs to know its w.f. in the SCA. The rate of the electric dipole ($E1$) transition between an initial state $i$: $n^3P_1$ and a final state $f$: $m^3S_1$ is given by the expression [23,24]

$$\Gamma(i \xrightarrow{\text{E1}} f + \gamma) = \frac{4}{3}\alpha e_Q^2 k_\gamma^3 (2J_f + 1) \mathrm{S}^{\mathrm{E}}_{if} |\mathcal{E}_{if}|^2, \qquad (3)$$

which contains the overlap integral $\mathcal{E}_{if}$ (called also the m.e.), calculated here via the relativistic w.f. The photon energy $k_\gamma$ is defined as $k_\gamma = \frac{M_i^2 - M_f^2}{2M_i}$, where $M_i(M_f)$ is the mass of the initial (final) state. In Eq. (3) the statistical factor $\mathrm{S}^{\mathrm{E}}_{if} = \mathrm{S}^{\mathrm{E}}_{fi}$ is given by

$$\mathrm{S}^{\mathrm{E}}_{if} = \max(l, l') \begin{Bmatrix} J & 1 & J' \\ l' & s & l \end{Bmatrix}^2, \qquad (4)$$

which for the $E1$ transitions between the $n^3P_J$ and $m^3S_1(m^3D_1)$ states, with the same spin $S = 1$, is equal to $\frac{1}{9}(\frac{1}{18})$.

The overlap integral $\mathcal{E}_{if}$, given by

$$\mathcal{E}_{if} = \int dr r^2 R_{n_i,l_i}(r) g(r) R_{n_f,l_f}(r) = \langle n_i, l_i|g(r)|n_f, l_f\rangle, \qquad (5)$$

is determined by the relativistic radial w.f.'s $R_{n_i,l_i}(r)$ and $R_{n_f,l_f}(r)$. The function $g(r)$ [24] is

$$g(r) = \frac{3}{k_\gamma}[y j_0(y) - j_1(y)] = \frac{3}{2}\frac{r}{y}\left\{\sin y\left(1 - \frac{1}{y^2}\right) + \frac{1}{y}\cos y\right\}, \qquad (6)$$

where $j_n(y)(n = 0, 1)$ are the spherical Bessel functions and the variable $y = k_\gamma r/2$. It can be easily shown that the replacement of $g(r)$ by $r$ ($g(r) < r$), used in many analyses, is a good approximation for $y \le 1.0$, while in the range $1.1 \le y \le 1.3$, the difference $g(r) - r$ can already reach $\le 30\%$. The zero of the function $g(r)$ occurs at $y_0 = 2.74$, corresponding to very large $r$, $r \gtrsim 10$ GeV$^{-1}$, even for the photon energy $k_\gamma = 0.697$ GeV for the $X(3872) \to J/\psi\gamma$

radiative decay. In $r$-space, the difference $g(r) - r$ remains small for photon energies $k_\gamma \lesssim 0.40$ GeV.

The overlap integrals, involving the $1P$ and $2P$ states, are denoted below as

$$I_m \equiv I_{1P,mS} = \int drr^2 R_{1P}(r) g(r) R_{mS}(r), \qquad (m = 1, 2),$$
$$K_m \equiv K_{2P,mS} = \int drr^2 R_{2P}(r) g(r) R_{mS}(r), \qquad (m = 1, 2). \tag{7}$$

In Eq. (7), the relativistic radial w.f.'s are calculated with the use of the RSH $H_0$, derived in Ref. [25] and applied many times to different effects in quarkonia [30,31]. For heavy quarkonia, $H_0$ has a very simple form:

$$H_0 = 2\sqrt{\boldsymbol{p}^2 + m_c^2} + V_{\rm B}(r), \tag{8}$$

where its kinetic term is similar to that in the relativized quark model (RQM) [32]. However, in $H_0$, by derivation, the fundamental value of the $c$-quark mass is equal to its pole mass, $m_c \simeq 1.42$ GeV. [It takes into account corrections perturbative in $\alpha_s(m_c)$ and corresponds to the conventional current mass $\bar{m}_c(\bar{m}_c) \sim 1.23$ GeV [33]]. In the RQM, the $c$-quark mass is considered as a fitting parameter, e.g., $m_c = 1.628$ GeV is used in Ref. [32] and $m_c = 1.562$ GeV in Ref. [17].

In the RSH, the universal static potential $V_{\rm B}(r)$ contains the confining and GE terms,

$$V_{\rm B}(r) = \sigma r - \frac{4\alpha_{\rm B}(r)}{3r}, \tag{9}$$

where the string tension $\sigma = 0.18$ GeV$^2$ is fixed by the slope of the Regge trajectories for light mesons and therefore cannot be considered as a fitting parameter. As shown in lattice QCD and the field correlator method, the value of the string tension is determined via the vacuum correlation function and does not depend on the quark mass [26].

An important step is to make the correct choice of the vector coupling $\alpha_{\rm B}(r)$ [34]. To this end, the parameters defining the coupling are here taken in correspondence to the existing data for the strong coupling $\alpha_s(r)$ in the $\overline{\rm MS}$ scheme in perturbative QCD [35]. The asymptotic-freedom behavior of $\alpha_{\rm B}$ is determined by the "vector" QCD constant $\Lambda_{\rm B}$, which, however, is not a new parameter but is expressed through $\Lambda_{\overline{\rm MS}}$. In particular, for $n_f = 3$ the relation $\Lambda_{\rm B}(n_f = 3) = 1.4753\Lambda_{\overline{\rm MS}}(n_f = 3)$ is valid. In pQCD,

$$\Lambda_{\overline{\rm MS}}(n_f = 3) = 339 \pm 10 \text{ MeV} \tag{10}$$

was extracted with the use of the matching procedure at the quark-mass thresholds [35]. This value $\Lambda_{\overline{\rm MS}}(n_f = 3) = (339 \pm 10)$ MeV gives a rather large value of $\Lambda_{\rm B}(n_f = 3) = (500 \pm 15)$ MeV, while in Ref. [34] from the analysis of the bottomonium spectrum, a smaller $\Lambda_{\rm B}(n_f = 3) = (465 \pm 20)$ MeV was shown to be preferable; its value corresponds to $\Lambda_{\overline{\rm MS}}(n_f = 3) = (315 \pm 14)$ MeV, in agreement with the lower limit in Eq. (10).

Also in Ref. [34], the infrared regulator $M_{\rm B}$ was introduced, since for $n_f = 3$ the characteristic momenta $q^2$ are rather small and nonperturbative effects become important. As shown in Ref. [36], the regulator $M_{\rm B}$ is not a new parameter, but expressed via the string tension $\sigma$, according to the relation $M_{\rm B}^2 = 2\pi\sigma$, derived with an accuracy ~10%. Then for $\sigma = 0.18$ GeV$^2$, one finds that $M_{\rm B} = 1.06 \pm 0.10$ GeV; here the value $M_{\rm B} = 1.10$ GeV is used.

Thus, the potential $V_{\rm B}(r)$ contains only fundamental quantities: the current (pole) $c$-quark mass, the string tension from the Regge trajectories, and $\Lambda_{\overline{\rm MS}}(n_f = 3)$. In our calculations the following set of parameters is used:

$$m_c = 1.425 \text{ GeV}, \qquad \sigma = 0.18 \text{ GeV}^2,$$
$$M_{\rm B} = 1.10 \text{ GeV}, \qquad \Lambda_{\rm B}(n_f = 3) = 465 \text{ MeV}. \tag{11}$$

For these values of $\Lambda_{\rm B}(n_f = 3)$ and $M_{\rm B}$ we find the frozen (asymptotic) value of the coupling $\alpha_{\rm crit} = 0.6086$.

A remarkable property of the RSH is that in heavy quarkonium the centroid masses $M_{\rm cog}(nl)$ of the $nl$ multiplet exactly coincide with the eigenvalues of the spinless Salpeter equation (SSE):

$$H_0 \varphi_{nl} = M_0(nl) \varphi_{nl}. \tag{12}$$

In the charmonium spectrum, not many levels are below the open charm threshold, where the universal static potential $V_{\rm B}(r)$ may be tested. However, above the $D\bar{D}$ threshold the $c\bar{c}$ interaction may differ from that for low-lying states, due to open channels or light-quark pair creation. In the string-breaking picture it is manifested as a flattening of the static potential at relatively large distances, and this phenomenon is seen in lattice QCD [37] and also observed in the radial excitations of light mesons [38]. For that reason, for higher states (above threshold) we consider two possibilities: the first case, when the GE potential is taken to be the same as in Eq. (9); and the second case, when the GE interaction is supposed to be suppressed for higher states, so that the $c\bar{c}$ dynamics is totally determined by the confining potential. There are several arguments in favor of the latter assumption:

(1) Suppression of the GE interaction follows from the analysis of the orbital Regge trajectories in charmonium [39,40], which are not compatible with a significant GE contribution to the masses of higher states.

(2) The fine-structure effects, caused by the GE interaction, are not seen for the $2P$ charmonium multiplet. For example, the measured mass difference

$\delta M(2P) = M(\chi_{c2}(2P)) - M(\chi_{c0}(2P)) \leq 10$ MeV is very small compared to the same mass difference for the $1P$ multiplet: $\delta M(1P) = M(\chi_{c2}(1P)) - M(\chi_{c0}(1P)) = 142$ MeV [33], being smaller by an order of magnitude. It is difficult to explain such a suppression of the fine-structure effects, if a universal GE potential is used for both high- and low-lying states. (The suppression of the fine-structure effects does not contradict possible hyperfine splittings of the higher $S$–wave charmonium states due to a short-range hyperfine potential [27]).

(3) Our calculations of the charmonium spectrum show that the masses of higher states are practically the same for the linear and linear + GE potentials, if the correct choice of the $c$-quark mass is made (see below Table I).

The confining potential used,

$$V_C(r) = \sigma r, \tag{13}$$

has the same string tension $\sigma = 0.18$ GeV$^2$. However, for the linear potential the $c$-quark mass may be different, since for a totally suppressed GE interaction there are no perturbative corrections to the $c$-quark mass, and in this case $m_c$ is equal to the current mass [here $m_c(\bar{m}_c) = (1.27 - 1.29)$ GeV is used].

In Table I we give the centroid masses $M_{\rm cog}(nl)$ for both potentials, Eqs. (7) and (9).

From Table I, one can see that both spectra coincide within $\lesssim$20 MeV accuracy. Moreover, the masses $M(1D)$, $M(2D)$, $M(2P)$, and $M(4S)$, calculated in SCA, are close to the experimental masses of $\psi(3770)$, $\psi(4160)$, $\chi_{c2}(2P)$, and $\psi(4420)$, respectively, and one can expect that their hadronic shifts are not large, $\lesssim$20 MeV. It is not so for the $3S$ state, where $M(3S)$ is about $(60 \pm 10)$ MeV larger than $M(\psi(4040))$, indicating the importance of the CC effects for $\psi(4040)$.

Thus, our analysis of the charmonium spectrum does not allow us to draw a definite conclusion on what type of the $c\bar{c}$ interaction is preferable above threshold. Therefore, other phenomena in charmonium have to be studied. One of them is just the radiative decay $X(3872) \to J/\psi\gamma$, which, as was already underlined by E. Swanson [15], "is particularly sensitive to model details."

TABLE I. The centroid masses $M_{\rm cog}(nl)$ (in MeV) of higher multiplets for the potentials $V_B(r)$ and $V_C(r)$.

| State | Potential $V_B(r)$ $m_c = 1.425$ GeV | Potential $V_C(r)$ $m_c = 1.290$ GeV | Experiment |
|---|---|---|---|
| 3S | 4092 | 4112 | $4039 \pm 1$ |
| 4S | 4447 | 4448 | $4424 \pm 4$ |
| 2P | 3949 | 3949 | $3927 \pm 3$[a] |
| 3P | 4319 | 4298 | Absent |
| 1D | 3802 | 3788 | $3773 \pm 3$ |
| 2D | 4187 | 4155 | $4153 \pm 3$ |

[a]For $M_{\rm cog}(2P)$, the experimental mass $M(\chi_{c2}(2^3P_2))$ is given.

## III. THE RADIATIVE DECAYS OF $X(3872)$ IN SINGLE-CHANNEL APPROXIMATION

Here we calculate the partial widths of the radiative transitions, $X(2^3P_1) \to J/\psi\gamma$ and $X(2^3P_1) \to \psi(2S)\gamma$ in SCA, i.e., assuming that $X(3872)$ is the $2^3P_1$ state. Then the widths (in units GeV) can be written as

$$\Gamma_1^0 = 1.4418 \times 10^{-3} k_{1\gamma}^3 |K_1|^2,$$
$$\Gamma_2^0 = 1.4418 \times 10^{-3} k_{2\gamma}^3 |K_2|^2, \tag{14}$$

where the number $1.4418 \times 10^{-3}$ is the product of known factors in Eq. (3), $k_{1\gamma} = 0.6974$ GeV, $k_{2\gamma} = 0.1815$ GeV, and for $X(3872)$ as the $2^3P_1$ charmonium state, the integrals $K_1$, $K_2$ (in the units GeV$^{-1}$) are defined in Eq. (7).

From Eq. (7) and for the potential $V_B(r)$ we obtained

$$K_1 = K_{2P,1S} = -0.418 \text{ GeV}^{-1}, \tag{15}$$

which in SCA gives a large partial width:

$$\Gamma_1 = 85.4 \text{ keV}. \tag{16}$$

This integral $K_1$, defined with $g(r)$, has a magnitude that is 2 times larger than $\tilde{K}_1 = \langle R_{2P}|r|R_{1S}\rangle$, where $g(r)$ is approximated by $r$:

$$\tilde{K}_1 = -0.210 \text{ GeV}^{-1}, \qquad \tilde{\Gamma}_1 = 21.6 \text{ keV}. \tag{17}$$

Notice that even smaller magnitudes of $\tilde{K}_1$ were obtained in other models [17,21] (see Table II). (The choice of the negative sign of $K_1$ will be explained in Sec. V). Thus, in the overlap integral determining the radiative decay $X(2^3P_1) \to J/\psi\gamma$ with rather large photon energy, $k_\gamma = 0.697$ GeV, the replacement of $g(r)$ by $r$ has a poor accuracy. On the contrary, the differences between the overlap integral $K_2$ and the width $\Gamma_2$,

$$K_2 = \langle 2P|g(r)|2S\rangle = 2.886 \text{ GeV}^{-1}, \qquad \Gamma_2 = 71.8 \text{ keV}, \tag{18}$$

and the approximate quantities $\tilde{K}_2$ and $\tilde{\Gamma}_2$,

$$\tilde{K}_2 = \langle 2P|r|2S\rangle = 3.017 \text{ GeV}^{-1}, \qquad \tilde{\Gamma}_2 = 78.5 \text{ keV}, \tag{19}$$

are rather small, $\lesssim$8%. Then, because of the small value of $\tilde{\Gamma}_1$, the ratio of the partial widths $\tilde{\Gamma}_2$ to $\tilde{\Gamma}_1$ is large,

$$\tilde{R}_{\psi\gamma} = 3.6. \tag{20}$$

TABLE II. The partial widths of the $X(3872)$ radiative decays (in keV) in relativistic models.[a]

| Transition | $k_\gamma$ | [41] | [16] | [17] | [18] | Our paper | Our paper |
|---|---|---|---|---|---|---|---|
| Method | in GeV | SCA | CCM | SCA | SCA | SCA | CCM |
| $2^3P_1 \to 1^3S_1 + \gamma$ | 0.697 | 33 | 45 | 11 | 11 | 85.4 | $37 \pm 6$ |
| $2^3P_1 \to 2^3S_1 + \gamma$ | 0.181 | 146 | 60 | 70 | 63.9 | 71.8 | $72 \pm 1$ |
| $2^3P_1 \to 1^3D_1 + \gamma$ | 0.098 | 7.9 | | 4.0 | 3.7 | 5.8 | |

[a]The partial widths are calculated either in single-channel approximation (SCA) or within a coupled-channels method (CCM).

TABLE III. The overlap integrals and the partial widths of the radiative decays $\chi_{cJ}(1P) \to J/\psi\gamma$.

| Transition | $k_\gamma$ (GeV) | $\langle r \rangle$ (GeV$^{-1}$) | $\langle g(r) \rangle^{a}$ (GeV$^{-1}$) | $\Gamma$(th) | $\Gamma$(exp) |
|---|---|---|---|---|---|
| $1^3P_{c2} \to J/\psi + \gamma$ | 0.429 | 1.976 | 1.849 | 390 | $386 \pm 17$ |
| $1^3P_{c1} \to J/\psi + \gamma$ | 0.389 | 1.976 | 1.849 | 290 | $296 \pm 13$ |
| $1^3P_{c0} \to J/\psi + \gamma$ | 0.303 | 1.976 | 1.849 | 137 | $133 \pm 10$ |

[a]Calculations refer to the variant with $m_c = 1.425$ GeV, $\Lambda_{\rm B}(n_f = 3) = 465$ MeV, and $M_{\rm B} = 1.1$ GeV.

On the contrary, if the integrals $K_1$ and $K_2$ are calculated with $g(r)$: $\Gamma_1 = 85.4$ keV, $\Gamma_2 = 71.8$ keV, then

$$R_{\psi\gamma} = 0.84 \tag{21}$$

is 4 times smaller. This ratio is not compatible with the *BABAR* [9] and the LHCb results in Eq. (2), but it does not contradict the Belle restriction, $R_{\psi\gamma} \lesssim 2.1$.

In Table II we give the partial widths $\Gamma_1$ and $\Gamma_2$, predicted in several relativistic models. One can see that in the SCA, even with relativistic kinematics, the predicted values of $\Gamma_1$ vary in a wide range, (11–85) keV.

Surprisingly, the m.e.'s $K_1$ in Eq. (15) and $\Gamma_1$ in Eq. (16) calculated here with a relativistic Hamiltonian, practically coincide with the approximate m.e.'s $\langle 2P|r|1S\rangle = 0.416\ \mathrm{GeV}^{-1}$ and $\Gamma_1 = 84.6$ keV, calculated in a nonrelativistic model with the logarithmic potential in Ref. [42]; the predicted value found there, $R_{\psi\gamma} = 1.1$, is also close to our number in Eq. (21).

As a test of the w.f., defined by the RSH and used here, we also calculated the partial widths of the $E1$ transitions: $\chi_{cJ}(1P) \to J/\psi\gamma (J = 0, 1, 2)$, where all three states lie far below the $D\bar{D}$ threshold and are described by the linear + GE potential. For $\chi_{c1}(1P) \to J/\psi\gamma$ and $\chi_{c2}(1P) \to J/\psi\gamma$, the calculated widths appeared to be in precise agreement with the experimental data. For these radiative decays the calculated integral $I_{1P,1S} \equiv I(1^3P_{cJ}, 1S) = 1.849\ \mathrm{GeV}^{-1}$ is taken to be the same for $J = 0, 1, 2$. For $\psi(3686)$ its w.f. contains an admixture from the $1D$ state [43], and therefore this radiative decay cannot be used as a test.

For all three radiative transitions the photon energies are not large, $k_\gamma \lesssim 0.40$ GeV, and therefore the differences between the m.e.'s $\langle g(r)\rangle$ and $\langle r\rangle$ are rather small, $\lesssim 7\%$. Nevertheless, with the use of the function $g(r)$ the partial widths of those radiative decays decrease by $\sim 10\%$, improving the agreement with experiment.

From Table III one can see that the calculated values $\Gamma(\chi_{c0}(1P) \to J/\psi\gamma) = 137$ keV, $\Gamma(\chi_{c1}(1P) \to J/\psi\gamma) = 290$ keV, and $\Gamma(\chi_{c2}(1P) \to J/\psi\gamma) = 390$ keV are in precise agreement with the experimental widths: $\Gamma_{\rm exp}(\chi_{c0} \to J/\psi\gamma) = 133 \pm 10$ keV, $\Gamma_{\rm exp}(\chi_{c1} \to J/\psi\gamma) = 296 \pm 13$ keV, and $\Gamma_{\rm exp}(\chi_{c2} \to J/\psi\gamma) = 386 \pm 17$ keV [33].

## IV. DYNAMICS OF COUPLED CHANNELS AND AN ADMIXTURE OF NEIGHBORING STATES

In the CC approach, $X(3872)$ is treated as the original $2^3P_1(Q\bar{Q})$ state, shifted down by the CC interaction with the $D\bar{D}^*(\bar{D}D^*)$ channel. Moreover, in Ref. [20] explicit parameters of this interaction were found, which put the pole of $X(3872)$ in the multichannel Green's function exactly at the position of the open-channel threshold. It is also important that within our approach the parameters are calculated and found to lie in a narrow range, i.e., they are not fitted. Below we use these fixed parameters to predict the admixture generated by the CC interaction, to the original $2^3P_1$ state.

Leaving the details of the CC formalism to the Appendix and the original papers [20–22], we recapitulate briefly the essence of the mixing problem. We define the extra (open)-channel interaction by the function $V_{\rm CC}(\boldsymbol{q}, \boldsymbol{q}', E)$ in momentum space, which depends on the total energy $E$ and contains all thresholds of the open channels. Then we are writing the full Green's function $G^{(BB)}_{Q\bar{Q}}$, which describes the evolution of a $Q\bar{Q}$ pair (produced in the $e^+e^-$ annihilation or the $B$ decay process), interacting in the intermediate states with $D\bar{D}^*(\bar{D}D^*)$ and finally ending up again as a $Q\bar{Q}$ pair (in a similar $e^+e^-$ annihilation or the $B$ decay process), as

$$G^{(BB)}_{Q\bar{Q}} = \sum_{n,m} (\hat{B}\psi_n) \frac{1}{\hat{E} - E + \hat{w}} (\hat{B}\psi_m)^\dagger, \quad (22)$$

where the matrices are defined as $\hat{E} = E_n \delta_{nm}$, and $\hat{w} = w_{nm}$ with

$$w_{nm}(E) = \int \psi^\dagger_n(\boldsymbol{q}) V_{\rm CC}(\boldsymbol{q}, \boldsymbol{q}', E) \psi_m(\boldsymbol{q}') \frac{d^3\boldsymbol{q}}{(2\pi)^3} \frac{d^3\boldsymbol{q}'}{(2\pi)^2}. \quad (23)$$

Here the symbol $\hat{B}$ in $(\hat{B}\psi_n)$ stands for an operator, defining the production (annihilation) process. In the case when the pure $Q\bar{Q}$ state is already formed, one can put $\hat{B} \equiv 1$, while in the $e^+e^-$ annihilation process the operator $\hat{B}$ is proportional to $\gamma_\mu$.

Note that $G^{(BB)}_{Q\bar{Q}}$ contains all CC contributions in the intermediate states, but the experimental conditions require these contributions to be projected onto the $Q\bar{Q}$ states. We now include in $G^{(BB)}_{Q\bar{Q}}$ the possibility to emit any particle $(\gamma, \pi, \ldots)$ in the intermediate state. As shown in Ref. [21] (see Appendix), this can be done by inserting the self-energy part with the photon loop to $\hat{E}$ and $\hat{w}$: $\hat{E} + \hat{w} \to \hat{E}(\gamma) + \hat{w}(\gamma)$, so that the probability of this emission is given by the discontinuity of the self-energy part of $G^{(BB)}_{Q\bar{Q}}$, e.g.,

$$\begin{aligned} Y_\gamma(E) &= \frac{1}{2} \Delta G^{(BB)}_{Q\bar{Q}} \\ &= \sum_{n,m,l} (\hat{B}\psi_n) \left(\frac{1}{\hat{E} - E + \hat{w}}\right)_{nm} \\ &\quad \times \frac{1}{2i} \Delta(\hat{E}(\gamma) + \hat{w}(\gamma))_{ml} \left(\frac{1}{\hat{E} - E + \hat{w}}\right)_{lq} (\hat{B}\psi_q)^\dagger \\ &= \sum_{k,n} a_k \psi_k \frac{1}{2i} \Delta(\hat{E} + \hat{w})_{kn} \psi^\dagger_n a^\dagger_n. \end{aligned} \quad (24)$$

In this way one can start in $e^+e^-$, or the $B$ decay, with the creation of any combination $\{\hat{B}\psi_n\}$, which finally, at the moment of the $\gamma$ detection, becomes the set of the states $(\psi_k + \gamma)$ with the probability amplitude $a_k(\hat{B}\psi_n)$.

The amplitude $a_k$ is given by

$$a_k = \sum_l \left(\frac{1}{\hat{E} - E + \hat{w}}\right)_{kl} (B\psi_l). \quad (25)$$

Since $\hat{E}$ is diagonal, $\hat{E}_{ik} = E_i \delta_{ik}$, all matrix structure is due to $\hat{w}$. This matrix was calculated in Ref. [21] for the $1^{++}$ states, giving the values shown in Table IV. Here one can see that only two $1^{++}$ states are strongly connected by $w_{nm}$, namely $1^3P_1$ and $2^3P_1$, while higher states yield very small admixtures.

TABLE IV. The m.e. $w_{nm}$ (in GeV) between $n^3P_1$ and $m^3P_1$ states for two approximations of the exact w.f.

| $n$ $m$ | 1 1 | 1 2 | 2 2 | 3 2 |
|---|---|---|---|---|
| $w_{nm}$ for 5 HO | −0.320 | 0.122 | −0.099 | −0.0003 |
| $w_{nm}$ for 3 HO | −0.319 | 0.121 | −0.098 | −0.0011 |

Defining the amplitudes $a_1$ and $a_2$ for the $1^3P_1$ and $2^3P_1$ states, respectively, one has

$$\frac{a_1}{a_2} = \frac{(E_2 + w_{22} - E)\frac{B\psi_1}{B\psi_2} - w_{21}}{E_1 + w_{11} - E - w_{12}\frac{B\psi_1}{B\psi_2}}. \quad (26)$$

With the use of the RSH we have $E_2 = 3949 \pm 10$ MeV, $E_1 = 3523 \pm 10$ MeV, and then from Table IV

$$\begin{aligned} E_2 + w_{22} &= (3850 \pm 10)\ \text{MeV}, \\ E_1 + w_{11} &= (3203 \pm 10)\ \text{MeV}. \end{aligned} \quad (27)$$

From Eq. (26) one can see that the solution for $a_1/a_2$ is sensitive to the original value of $E_2$ but depends weakly on $B\psi_1/B\psi_2$, and we take for this ratio the same value as for the $e^+e^-$ process, namely $|\psi_1(0)|^2/|\psi_2(0)|^2 \approx 0.85$. Inserting these numbers into Eq. (26) and taking $E_2 =$ 3949 MeV from Table I (the contribution from the $D^*\bar{D}^*$ channel is neglected), one obtains the value of $c_{\rm X}$ at the resonance energy, $E_{\rm R} = 3872$ MeV $(B\psi_1/B\psi_2 = 0)$:

$$c_{\rm X}(E_{\rm R}) = \frac{a_1}{a_2}(E = 3872\ \text{MeV}) = 0.183. \quad (28)$$

However, this number can be considered as a lower limit, since we have not accounted for the contribution of the closed $D^*\bar{D}^*$ channel to the original levels $E_1$ and $E_2$. The estimate of these $D^*\bar{D}^*$ contributions to the $2^{++}(2^3P_2)$ and $0^{++}(2^3P_0)$ states was made in Ref. [20] for the same CC parameters as for $X(3872)$. It yields a shift around −30 MeV, in good agreement with the $Z(3930)$ as the $2^3P_2$ state. In this case in Eq. (28), $E_2 = 3920$ MeV and a larger value of $c_{\rm X}$ is obtained,

$$c_{\rm X} = \frac{a_1}{a_2}(E = 3872\ \text{MeV}) \approx 0.215. \quad (29)$$

As a result, we choose to accept the value of $c_{\rm X}$ from the range, defined by Eqs. (28) and (29),

$$c_{\rm X} = 0.200 \pm 0.015. \quad (30)$$

In our convention $c_X = -\sin\theta$ is positive, and therefore the calculated mixing angle is negative, $\theta \approx -13°$. Although this admixture of the continuum component is not large, it is very important for the X(3872) $\to J\psi\gamma$

transition and also for radiative E1 decays of higher charmonium states, like $\mathrm{Y}(4260) \to \gamma \mathrm{X}(3872)$ [44].

## V. THE COUPLED-CHANNELS CORRECTIONS TO THE $X(3872)$ WAVE FUNCTION

In the previous section, the effects from open $D\bar{D}^*$ channels were calculated, resulting in the contribution from the $1^3P_1$ state to the normalized w.f. of $X(3872)$, which can be represented as

$$\varphi(X(3872)) = \sqrt{1-c_{\mathrm{X}}^2}\varphi(2^3P_1) + c_{\mathrm{X}}\varphi(1^3P_1). \quad (31)$$

As seen from Eq. (30), the mixing parameter $c_{\mathrm{X}}$ at the energy $E_{\mathrm{R}} = 3.782$ GeV lies in a narrow range; this admixture was not fitted, but calculated within the string-breaking mechanism.

Moreover, in the many-channels mechanism, the sign of the continuum component cannot be arbitrary anymore, and here we choose positive $c_{\mathrm{X}}$: it takes place if the radial w.f. $R_{2P}(r)$ is chosen to be negative at small distances (for $r \lesssim 0.5$ fm), while the radial w.f. of the ground state $R_{1P}(r)$ is positive in the whole region. Just for this choice, a negative value for the overlap integral in Eqs. (15), (17) is obtained.

Due to the CC admixture in the $X(3872)$ w.f., the m.e. in the radiative decays of $X(3872)$ can be presented as a superposition,

$$\langle X(3872)|g(r)|J/\psi\rangle = \sqrt{1-c_{\mathrm{X}}^2}K_1 + c_{\mathrm{X}}I_1,$$
$$\langle X(3872)|g(r)|\psi(2S)\rangle = \sqrt{1-c_{\mathrm{X}}^2}K_2 + c_{\mathrm{X}}I_2, \quad (32)$$

where $I_n$ and $K_n (n=1,2)$ are defined in Eq. (7) and the radial w.f. of $\psi(3686)$ is taken as $R(2S)(r)$, i.e., without mixing with the $1D$ state.

For $X(3872)$, two different $c\bar{c}$ potentials are used below. In the first case, the w.f.'s of all states involved in the radiative decays of $X(3872)$ are defined by the RSH with the linear + GE potential, $V_{\mathrm{B}}(r)$, given in Eq. (9). Then in the m.e. Eq. (32), the two terms have different signs, and therefore the magnitude of this m.e. is smaller as compared to the number given in Eq. (15). Due to this cancellation, this m.e. appears to be very small, even for the minimal value of $c_{\mathrm{X}}(\min) = 0.185$ from Eq. (28):

$$|\langle X(3872, 2^3P_1))|g(r)|J/\psi\rangle| \leq 0.08\ \mathrm{GeV}^{-1}, \quad (33)$$

giving a small partial width $\Gamma_1 = 3.0$ keV.

On the contrary, for the transition $X(3872) \to \psi(2S)\gamma$, its m.e. increases to 3.30 $\mathrm{GeV}^{-1}$, and $\Gamma_2 = 94$ keV is large. Thus, within the CC approach, taking the linear + GE potential for $X(3872)$, one obtains a small partial width $\Gamma_1 \lesssim 3$ keV and a very large value for the ratio $R_{\psi\gamma} \gg 1.0$.

The situation is different if we assume that the GE interaction is totally suppressed for higher charmonium states, i.e., the $c\bar{c}$ dynamics is determined by the confining potential. [Note that such a suppression does not exclude the use of the hyperfine interaction, having a very small range [28], for the higher $S$-wave charmonium states, like $\psi(4040)$, concentrated near the origin.] As a result, the m.e. of the radiative transitions, $X(3872) \to J/\psi\gamma$ and $X(3872) \to \psi(3686)\gamma$ are defined by the "mixed" m.e., in which the $2P$ w.f. is calculated with the confining potential, while the w.f. of $J/\psi$ and $\psi(3686)$ are calculated with the linear + GE potential.

These m.e.'s are denoted as

$$K_1(\mathrm{mix}) = \int drr^2 R_{2P}(\mathrm{lin})|g(r)|R_{1S}(\mathrm{lin}+\mathrm{GE}),$$
$$K_2(\mathrm{mix}) = \int drr^2 R_{2P}(\mathrm{lin})|g(r)|R_{2S}(\mathrm{lin}+\mathrm{GE}). \quad (34)$$

It appears that the magnitude of the m.e. $K_1(\mathrm{mix})$ is larger compared to the number given in Eq. (15), and for the unregularized w.f. [the solutions of the SSE in Eq. (12)], the m.e.'s are

$$K_1(\mathrm{mix}) = -0.693\ \mathrm{GeV}^{-1},$$
$$K_2(\mathrm{mix}) = 2.053\ \mathrm{GeV}^{-1}. \quad (35)$$

Instead, below we use the m.e. $K_i(\mathrm{mix}) (i=1,2)$, calculated for the regularized w.f. with the use of the Einbein approximation:

$$K_1(\mathrm{mix}) = -0.664\ \mathrm{GeV}^{-1},$$
$$K_2(\mathrm{mix}) = 2.475\ \mathrm{GeV}^{-1}. \quad (36)$$

For the calculated $K_1(\mathrm{mix})$, the partial width $\Gamma_1$ strongly depends on the value of the mixing parameter $c_{\mathrm{X}}$, even if it is taken from the narrow range given in Eq. (30). Using the m.e. $I_m$ (which refers to the $1P$ state) from Table III and $K_1(\mathrm{mix})$ from Eq. (36), one has

$$\Gamma_1 = \Gamma(X(3872) \to J/\psi\gamma) = (37 \pm 6)\ \mathrm{keV}$$
$$\text{for } c_{\mathrm{X}} = 0.200 \pm 0.015. \quad (37)$$

Owing to the CC effects, the value of $\Gamma_1$ appears to be ~2.2 times smaller than in SCA. While changing the admissible value of $c_{\mathrm{X}}$ from the minimal to the maximum values, i.e., by 15%, the partial width $\Gamma_1$ increases from 31 keV to 43 keV, i.e., 1.4 times. It means that a precise knowledge of $\Gamma_1$ from experiment would put a strong restriction on the mixing parameter $c_{\mathrm{X}}$ at the resonance energy $E_{\mathrm{R}} = 3.872$ GeV.

For the radiative decay $X(3872) \to \psi(3686)\gamma$, we have used the radial w.f. $R_{2S}(r)$ for $\psi(3686)$. Then, taking the m.e. $I_{2S,1P}$ from Table III and $K_2(\mathrm{mix})$ from Eq. (36), one obtains

$$\Gamma_2 = (72 \pm 1.0)\ \text{keV} \quad \text{for } c_X = 0.200 \pm 0.015. \quad (38)$$

Its value weakly depends on the mixing parameter $c_X$. However, this width may be smaller, if the $2S - 1D$ mixing is taken into account in the w.f. of $\psi(3686)$ [43].

Thus, in the CC approach with the minimal value of the mixing parameter, $c_X = 0.185$, we have

$$\Gamma_1 = 42.7\ \text{keV}, \qquad \Gamma_2 = 70.5\ \text{keV}, \qquad R_{\psi\gamma} = 1.65; \quad (39)$$

while for the maximal value, $c_X = 0.215$,

$$\Gamma_1 = 30.8\ \text{keV}, \qquad \Gamma_2 = 73.2\ \text{keV}, \qquad R_{\psi\gamma} = 2.38; \quad (40)$$

i.e., this partial width $\Gamma_1$ is by 40% smaller than for $c_X(\text{min}) = 0.185$. The ratio of the widths also changes in a rather wide range:

$$R_{\psi\gamma}(\text{th}) = 2.02 \pm 0.36, \quad (41)$$

being 2.4 times larger than that in the SCA Eq. (20). This value is in a good agreement with the LHCb data given in Eq. (2). Also, its lower limit (for $c_X = 0.185$) does not contradict the Belle result, $R_{\psi\gamma} < 2.1$.

Thus, in our analysis the agreement with the LHCb result was obtained under two conditions: when the GE interaction is suppressed and the CC admixture to the w.f. of $X(3872)$ is not small, ~20%. For the partial width $\Gamma_1$ we predict the value $37 \pm 6$ keV.

## VI. CONCLUSIONS

The $c\bar{c}$ dynamics in $X(3872)$ with $J^{PC} = 1^{++}$ was studied within the CC approach, where a coupling to the $D\bar{D}^*$ channels is determined by the string-breaking mechanism. We show that the conventional $c\bar{c}$ component dominates in X(3872), and the admixture of the continuum is calculated to be $c_X(E = E_R) = 0.200 \pm 0.015$. The experimental value of the ratio $R_{\psi\gamma}$ puts serious restrictions on the $c\bar{c}$ dynamics in X(3872) and two variants of the $c\bar{c}$ interaction are considered:

(1) The linear + GE potential used is the same for the lower and the higher charmonium states. In this case the partial width $\Gamma_1 = \Gamma(X(3872) \to J/\psi\gamma) \sim 3.0$ keV is small and gives rise to a very large value of the ratio $R_{\psi\gamma} \gg 1.0$ in the CC approach. On the contrary, in SCA the ratio $R_{\psi\gamma} = 0.84$ is not large.

(2) The GE interaction is supposed to be suppressed for higher states. For this dynamics, the charmonium spectrum above the $D\bar{D}$ threshold can be described with a good accuracy, if perturbative corrections in the $c$-quark mass are neglected.

If the suppressed GE potential is used, then the partial width $\Gamma_1$ increases, being very sensitive to the value of the mixing parameter $c_X$. For $c_X = 0.215$, we find $\Gamma_1 = 30.8$ keV, $\Gamma_2 = 73.2$ keV, and the calculated ratio of the partial widths $R_{\psi\gamma}(\text{max}) = 2.38$ is in good agreement with the LHCb result. The predicted minimal value, $R_{\psi\gamma}(\text{min}) = 1.65$, does not contradict the Belle measurements with $\psi_\gamma(\text{Belle}) < 2.1$.

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## APPENDIX: THE COUPLED-CHANNEL MECHANISM

Here we essentially use the formalism, suggested in Ref. [45] and developed further in Refs. [20,21], and below we partly reiterate the material from Ref. [21] for the convenience of the reader.

We use here the string-decay Lagrangian of the $^3P_0$ type for the decay $c\bar{c} \to (c\bar{q})(\bar{c}q)$ [20]:

$$\mathcal{L}_{\text{sd}} = \int d^4x \bar{\psi}_q M_\omega \psi_q, \quad (\text{A1})$$

where the light quark bispinors are treated in the limit of large $m_c$, which allows us to go over to the reduced $(2 \times 2)$ form of the decay matrix elements. Moreover, to simplify the calculations, the actual w.f.'s of the $c\bar{c}$ states, calculated in Ref. [21] with the use of the RSH, are fitted here by five (or three) oscillator w.f.'s (HO), while the $D$ meson w.f. is described by a single HO term, which provides a few percent accuracy with the parameter $\beta_2 \simeq 0.48$. In this case the factor $M_\omega$ in (A1) is calculated to be $M_\omega = 4\sqrt{2}\sigma/\sqrt{\pi}\beta_2 \simeq 0.8$ GeV (see below), which produces the correct total width of $\psi(3770)$. The transition m.e.'s for the decays $(c\bar{c})_n \to (D_{n_2}\bar{D}_{n_3})$ are denoted here as $n \to n_2, n_3$, and in the $2 \times 2$ formalism this m.e. reduces to

$$J_{nn_2n_3}(\boldsymbol{p}) = \frac{\gamma}{\sqrt{N_c}} \int \bar{y}_{123} \frac{d^3\boldsymbol{q}}{(2\pi)^3} \Psi_n^+(\boldsymbol{p}+\boldsymbol{q}) \psi_{n_2}(\boldsymbol{q}) \psi_{n_3}(\boldsymbol{q}). \quad (\text{A2})$$

The factor $\bar{y}_{123}$ contains a trace of the spin-angular variables (for details, see Refs. [20,21]), and $\gamma = \frac{2M_\omega}{\langle m_q + U_S - V_D + \varepsilon_0 \rangle}$, where the average over the Dirac denominator contains the light quark mass $m_q$, the scalar $U_S = \sigma r$ and the vector $V_D(r) = -\frac{4\alpha}{3r}$ potentials, and $\varepsilon_0$, which is the Dirac eigenvalue in the static potential, created by the heavy quark:

$$\vec{\alpha} \cdot \vec{p} + \beta(m_q + U_S(r))\psi(r) = (\varepsilon_0 - V_D(r))\psi(r). \quad (\text{A3})$$

Knowing the string tension $\sigma = 0.18\ \mathrm{GeV}^2$ and the averaged momentum distribution $\beta_2$ in the w.f. of a heavy-light meson, the value $M_\omega = \frac{4\sqrt{2}\sigma}{\sqrt{\pi}\beta_2}$ was calculated in Ref. [20]. For the 1$S$, 2$S$, and 3$S$ charmonium states $M_\omega$ is different and equal to 0.65, 0.80, and 1.10, respectively. Using the averaged values of the scalar and vector potentials, one obtains $\gamma \approx 1.4$.

The intermediate decay channels, like $DD^*$, induce an additional interaction "potential" $V_{\rm CC}(\boldsymbol{q}, \boldsymbol{q}', E)$ (the quotation marks imply the nonlocality and energy dependence of this potential):

$$V_{\rm CC}(\boldsymbol{q}, \boldsymbol{q}', E) = \sum_{n_2 n_3} \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{X_{n_2 n_3}(\boldsymbol{q}, \boldsymbol{p}) X^\dagger_{n_2 n_3}(\boldsymbol{q}', \boldsymbol{p})}{E - E_{n_2 n_3}(\boldsymbol{p})}, \tag{A4}$$

$$X_{n_2 n_3}(\boldsymbol{q}, \boldsymbol{p}) = \frac{\gamma}{\sqrt{N_c}} \bar{y}_{123}(\boldsymbol{q}, \boldsymbol{p}) \psi_{n_2}(\boldsymbol{q} - \boldsymbol{p}) \psi_{n_3}(\boldsymbol{q} - \boldsymbol{p}). \tag{A5}$$

The new Hamiltonian with account of the CC interaction is

$$H = H_0 + V_{\rm CC}, \tag{A6}$$

where $H_0$ is given in Eqs. (8) and (9). At this point it is important to stress that $V_{\rm CC}$ contains all thresholds of open channels, and to treat the spectrum of the Hamiltonian $H$ rigorously, one should exploit the Weinberg eigenvalue formalism, developed for this purpose in Ref. [20], second reference. However, for the states below all thresholds (or neglecting the open channel amplitudes in first approximation), one can use an expansion in the complete set of the eigenvalues of $H_0$. One can show, as in Ref. [20], that the exact Weinberg formalism reduces to this expansion for the real eigenvalues at or below thresholds. Using that, one obtains for the Green function an expansion,

$$G_{Q\bar{Q}}(1, 2; E) = \sum_{n,m} \psi_n(1) (\hat{E} - E + \hat{w})^{-1}_{nm} \psi_m(2), \tag{A7}$$

where the matrix element of $\hat{w}$ is denoted by $w_{nm}$ and given by

$$\begin{aligned} w_{nm}(E) &= \int \frac{d^3\boldsymbol{q}}{(2\pi)^3} \frac{d^3\boldsymbol{q}'}{(2\pi)^3} \psi_n(\boldsymbol{q}) V_{\rm CC}(\boldsymbol{q}, \boldsymbol{q}', E) \psi_m(\boldsymbol{q}') \\ &= \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \sum_{n_2 n_3} \frac{J_{n n_2 n_3}(\boldsymbol{p}) J^\dagger_{m n_2 n_3}(\boldsymbol{p})}{E - E_{n_2 m_3}(\boldsymbol{p})}, \end{aligned} \tag{A8}$$

and the energy eigenvalues are obtained from the determinant condition

$$\det(E - \hat{E} - \hat{w}) = 0, \tag{A9}$$

where $(\hat{E})_{nm} = E_n \delta_{nm}$.

Defining as in Eq. (22) the effective $Q\bar{Q}$ w.f.,

$$\psi^{(\mathcal{B})}_{Q\bar{Q}} = \sum_{k,l} \psi_k \left( \frac{1}{\hat{E} - E + \hat{w}} \right)_{kl} (\hat{B}\psi_l) \equiv \sum_k a_k \psi_k, \tag{A10}$$

one has to calculate the resulting amplitudes $a_k$ in the expansion of X(3872) in terms of the $Q\bar{Q}$ states. The values of $w_{nm}$ are given in Table IV, and one can see that only two states, $|1P\rangle$ and $|2P\rangle$, are important.

Keeping only two $Q\bar{Q}$ eigenfunctions, one has

$$a_1 = \frac{E_2 - E + w_{22}}{\det} (\hat{B}\psi_1) - \frac{w_{21}}{\det} (\hat{B}\psi_2), \tag{A11}$$

$$a_2 = \frac{E_1 - E + w_{11}}{\det} (\hat{B}\psi_2) - \frac{w_{12}}{\det} (\hat{B}\psi_1), \tag{A12}$$

where $\det \equiv \det(\hat{E} - E + \hat{w})$. Then one obtains the ratio

$$\frac{a_1}{a_2} = \frac{-w_{21} + (E_2 + w_{22} - E) \frac{(\hat{B}\psi_1)}{(\hat{B}\psi_2)}}{E_1 + w_{11} - E - w_{12} \frac{(\hat{B}\psi_1)}{(\hat{B}\psi_2)}}. \tag{A13}$$

We calculate this ratio for the energy $E_{\rm R} = 3872$ MeV, taking the values of $w_{ik}$ from Table IV. Note that we have corrected the sign of $w_{12}$ in accordance with the relative signs of the wave functions $|1P\rangle$ and $|2P\rangle$. The resulting values of $c_{\rm X} \equiv \frac{a_1}{a_2}(3872\ \mathrm{MeV})$ are given in Eqs. (28)–(30).

From Table IV, one can see that the $3^3P_1$ state gives a negligible admixture. The values of $w_{mn}$ for $E = E_R$ are computed according to Eq. (A8) with the w.f., calculated in Ref. [20], and are given in Table IV.

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