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# Determination of $SU(4)_{TC}$ technicolor gauge group from embedding in extended technicolor

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In technicolor theories using a $SU(N_{TC})$ gauge group, the value of $N_{TC}$ is not, *a priori*, determined and is typically chosen by phenomenological criteria. Here we present a novel way to determine $N_{TC}$ from the embedding of a one-family technicolor model, with fermions in the fundamental represention of $SU(N_{TC})$, in an extended technicolor theory, and use it to deduce that $N_{TC} = 4$ in this framework.

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The possibility that electroweak symmetry breaking (EWSB) may occur dynamically, as in technicolor (TC) theories [1] remains, although the Higgs-like scalar boson discovered at the Large Hadron Collider (LHC) with mass $m_H \simeq 125$ GeV [2,3] is consistent with being the Standard Model (SM) Higgs. A TC theory features an asymptotically free, vectorial TC gauge symmetry and a set of massless TC-nonsinglet, SM-nonsinglet fermions, $\{F\}$. The TC theory becomes strongly coupled at the TeV scale, confining and producing technifermion condensates $\langle \bar{F}F \rangle$, with associated spontaneous chiral symmetry breaking (S$\chi$SB) and dynamical EWSB. Three of the resultant Nambu-Goldstone bosons (NGBs) are absorbed to give masses to the $W^{\pm}$ and $Z$. To give masses to SM fermions, one embeds the TC theory in a larger, extended technicolor (ETC) theory [4]. Reasonably ultraviolet (UV)-complete ETC theories have been constructed as asymptotically free chiral gauge theories that self-break [5] in stages down to the (vectorial) TC subsector, yielding the generational hierarchy of SM fermion masses, including light neutrino masses [6–8].

Viable TC theories exhibit a squared gauge coupling $\alpha_{TC}(\mu) = g_{TC}(\mu)^2/(4\pi)$ that grows to $O(1)$, but runs very slowly (walks) over a substantial interval of Euclidean momenta $\mu$ and an associated large anomalous dimension $\gamma_m \sim O(1)$ for the bilinear technifermion operator [9–10]. These properties of a walking TC (WTC) theory follow naturally if the theory has an approximate infrared (IR) zero in the TC beta function $\beta_{TC}$ at a value $\alpha_{IR}$ that is slightly larger than the critical minimal value, $\alpha_{cr,\bar{F}F}$, for the formation of the $\langle \bar{F}F \rangle$ condensates [11–12]. Since $\alpha_{cr,\bar{F}F} \sim O(1)$, it is useful to calculate $\beta_{TC}$ and $\alpha_{IR}$ to higher-loop order [13]. Indeed, lattice studies have shown that walking behavior can occur nonperturbatively even if the perturbative beta function does not exhibit an IR zero [14,15]. These $\langle \bar{F}F \rangle$ condensates spontaneously break the approximate scale invariance of the TC theory, giving rise to a light pseudo-NGB (PNGB), the technidilaton (TD), $\phi$ [9,16,17]. Using holographic methods, it has been shown that WTC theories may yield a light TD [18,19]. These holographic studies extend earlier analyses using Schwinger-Dyson and Bethe-Salpeter equations [20]. Recent lattice studies of (vectorial) SU(3) gauge theories with $N_f = 8$ Dirac fermions [which is the value of $N_f$ in the one-family TC (1FTC) model discussed here] have observed walking behavior and a light composite TD-like scalar [14,21] (see also [22,23]). The technidilaton in a WTC theory appears to be consistent, to within theoretical and experimental uncertainties, with currently measured properties of the Higgs-like scalar discovered at the LHC [18,19], [24–27] (although these properties are also consistent with the SM Higgs). Future data from the LHC will constrain the TC/TD scenario for the Higgs-like scalar further.

In addition to the requirement that the composite TD-like scalar in technicolor must be consistent with the observed Higgs-like scalar, TC/ETC theories are also subject to a number of phenomenological constraints, including those from precision electroweak data [28], limits on flavor-changing neutral current (FCNC) processes, etc. Both continuum (e.g., [18,29,30]) and lattice studies [15], [21–23] have shown that TC corrections gto the $W$ and $Z$ propagators (in particular, the $S$ parameter [28]) can be substantially reduced in a theory with walking and, moreover, via ETC effects [30]. Further, explicit calculations in a reasonably UV-complete ETC theory showed that residual approximate generational symmetries suppress FCNC processes [8].

The simplest embedding of the TC theory in ETC is obtained if one takes the technifermions to comprise one SM family [31], since in this case the ETC gauge bosons are SM-singlets and $[G_{ETC}, G_{SM}] = \emptyset$. A common choice for the TC gauge group is $SU(N_{TC})$. Further, the simplest models in this class of TC/ETC theories have technifermions transforming according to the fundamental representation, $\Box$, of the $SU(N_{TC})$ TC gauge group, since in this case one just extends the TC gauge indices on various fields to be ETC gauge indices [see Eq. (2)]. Therefore, we shall consider here a 1FTC model with TC gauge group $\mathrm{SU}(N_{\mathrm{TC}})$ and technifermions in the fundamental representation. (We do not consider topcolor or higher-dimensional TC fermion representations.) The value of $N_{\mathrm{TC}}$ is typically determined by phenomenological criteria, such as minimizing technifermion loop corrections to the $W$ and $Z$ propagators or fitting the properties of the Higgs-like scalar.

Here we determine $N_{\mathrm{TC}}$ from the embedding of a 1FTC theory, with technifermnions in the fundamental representation of $\mathrm{SU}(N_{\mathrm{TC}})$, in a specific ETC theory. We find $N_{\mathrm{TC}} = 4$ in this framework, which agrees with the value preferred by the fit to the Higgs-like scalar in this type of theory [18,24]. In addition to its phenomenological application, this provides a novel example of how the structure of a low-energy effective field theory is determined by its ultraviolet completion [32].

The natural embedding of the TC theory in ETC is

$$\mathrm{SU}(N_{\mathrm{TC}}) \subset \mathrm{SU}(N_{\mathrm{ETC}}). \tag{1}$$

The ETC theory gauges the SM generation index and combines it with the TC gauge index in such a way that each generational multiplet of SM fermions of a given type is combined with the 1FTC fermions with the same SM quantum numbers into a fundamental representation of $\mathrm{SU}(N_{\mathrm{ETC}})$. Hence, with $N_{\mathrm{gen}}$ denoting the number of SM fermion generations,

$$N_{\mathrm{ETC}} = N_{\mathrm{gen}} + N_{\mathrm{TC}}. \tag{2}$$

This determines $N_{\mathrm{ETC}}$ in terms of the known $N_{\mathrm{gen}} = 3$ and a given value of $N_{\mathrm{TC}}$ based on the embedding (1). This procedure was used with the minimal value, $N_{\mathrm{TC}} = 2$, to construct SU(5) ETC theories in [6–8]. In [6–8], given the value $N_{\mathrm{TC}} = 2$, the fermion content of the ultraviolet completion was chosen to yield the known $N_{\mathrm{gen}} = 3$ [33].

We proceed to show how the embedding (1) of a 1FTC model, with technifermions in the fundamental representation of the TC gauge group, in a specific ETC theory with Eq. (2) determines that $N_{\mathrm{TC}} = 4$. The key to our result is the specification of the fermion content of the ETC theory and the use of the requirement that the ETC theory must be free of chiral gauge anomalies. Interestingly, our result is independent of $N_{\mathrm{gen}}$ [where it is implicitly understood that $N_{\mathrm{gen}}$ is small enough so that $\mathrm{SU}(3)_c$ and $\mathrm{SU}(N_{\mathrm{ETC}})$ are asymptotically free]. To make this manifest, we will keep $N_{\mathrm{gen}}$ general in our derivation.

The gauge group at the ETC scale is $\mathrm{SU}(N_{\mathrm{ETC}}) \otimes G_{\mathrm{SM}}$, where $G_{\mathrm{SM}} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ is the SM gauge group, with $Q_{\mathrm{em}} = T_{3L} + (Y/2)$. The fermion content of the ETC theory is composed of two sectors, namely SM nonsinglets and SM singlets. The SM-nonsinglet sector is determined by our 1FTC theory and the embedding (1) with (2). The fields are indicated below in Young tableaux notation, where $a$ is a color $\mathrm{SU}(3)_c$ index, $i = 1, \ldots, N_{\mathrm{ETC}}$ is the ETC index, and the representation is $R = (R_{\mathrm{ETC}}, R_c, R_I)_Y$, with $I$ and $Y$ the weak isospin and hypercharge:

$$Q_L^{ai} = \begin{pmatrix} u^{ai} \\ d^{ai} \end{pmatrix}_L : \ (\square, \square, \square)_{1/3}, \tag{3}$$

$$u_R^{ai}: \ (\square, \square, 1)_{4/3}, \qquad d_R^{ai}: \ (\square, \square, 1)_{-2/3}, \tag{4}$$

$$L_L^i = \begin{pmatrix} \nu^i \\ \ell^i \end{pmatrix}_L : \ (\square, 1, \square)_{-1}, \tag{5}$$

$$\nu_R^i: \ (\square, 1, 1)_0, \qquad \ell_R^i: \ (\square, 1, 1)_{-2}. \tag{6}$$

The indices $i = 1, \ldots, N_{\mathrm{gen}}$ are SM generation indices and $N_{\mathrm{gen}} + 1 \le i \le N_{\mathrm{ETC}}$ are TC indices; the $a = 1, 2, 3$ are color $\mathrm{SU}(3)_c$ indices. Thus, for example, $u^{a1} \equiv u^a$, $u^{a2} \equiv c^a$, $u^{a3} \equiv t^a$ for the charge 2/3 quarks, and so forth for the other SM fermions.

The minimal set of SM-singlet, ETC-nonsinglet fermions (written as right handed) is as follows:

$$\psi_R^{ij} = \psi_R^{[ij]} : \ \left(\begin{smallmatrix}\square\\\square\end{smallmatrix}, 1, 1\right)_0 \tag{7}$$

and

$$\chi_{i,s,R}: \ (\bar{\square}, 1, 1)_0, \qquad 1 \le s \le N_{\mathrm{ETC}} - 4, \tag{8}$$

where $s$ labels each of the $N_{\mathrm{ETC}} - 4$ copies (flavors) of $\chi_{i,s,R}$. We denote the contribution of a chiral fermion of representation $R$ of $\mathrm{SU}(N)$ to the triangle anomaly as $\mathrm{Anom}(R)$. Since

$$Anom\left(\begin{smallmatrix}\square\\\square\end{smallmatrix}\right) = (N-4)\, Anom(\square) , \tag{9}$$

our ETC theory is anomaly free. One can also add a vectorlike SM-singlet, ETC-nonsinglet fermion subsector; here we restrict ourselves to the minimal version of the model.

For a given gauge group $G$, we denote the beta function $\beta_G = d\alpha_G/d\ln\mu = -2\alpha_G \sum_{\ell=1}^{\infty} b_\ell^{(G)} (\alpha_G/(4\pi))^\ell$. The ETC theory is asymptotically free, with one-loop beta function coefficient

$$b_1^{\mathrm{SU}(N_{\mathrm{ETC}})} = \frac{1}{3}(9N_{\mathrm{ETC}} - 10), \tag{10}$$

so as the scale $\mu$ decreases from the UV toward the IR, the ETC squared coupling $\alpha_{\mathrm{ETC}}(\mu)$ grows. As $\mu$ decreases through a scale that we denote $\Lambda_1$, $\alpha_{\mathrm{ETC}}(\mu)$ increases through the minimal critical value for the formation of a bilinear fermion condensate. Using a vacuum alignment argument [34], we infer that this forms in the channel

$$\begin{smallmatrix}\square\\\square\end{smallmatrix}\times\bar{\square}\to\square\,, \tag{11}$$

breaking $\mathrm{SU}(N_{\mathrm{ETC}})$ to $\mathrm{SU}(N_{\mathrm{ETC}}-1)$. The associated condensate is

$$\left\langle\sum_{j=2}^{N_{\mathrm{ETC}}}\psi_R^{1jT}C\chi_{j,1,R}\right\rangle, \tag{12}$$

where $C$ is the Dirac charge-conjugation matrix, and, by convention, we have taken the ETC gauge index $i=1$ in $\psi_R^{ij}$ and the copy index $s=1$ in $\chi_{j,s,R}$. The fermions $\psi_R^{1j}$ and $\chi_{j,1,R}$ with $2\le j\le N_{\mathrm{ETC}}$ involved in this condensate gain dynamical masses of order $\Lambda_1$ and are integrated out of the low-energy effective theory (LEET) applicable at scales $\mu<\Lambda_1$. Of the fermions in Eqs. (7) and (8), the remaining nonsinglet ones in this $\mathrm{SU}(N_{\mathrm{ETC}}-1)$ LEET are $\psi_R^{2j}$ with $3\le j\le N_{\mathrm{ETC}}$; and $\chi_{j,s,R}$ with $2\le j\le N_{\mathrm{ETC}}$ and $2\le s\le N_{\mathrm{ETC}}-4$. The $(2N_{\mathrm{ETC}}-1)$ ETC gauge bosons in the coset $\mathrm{SU}(N_{\mathrm{ETC}})/\mathrm{SU}(N_{\mathrm{ETC}}-1)$ gain masses of order $g_{\mathrm{ETC}}\Lambda_1\simeq\Lambda_1$. Diagrams involving exchanges of these massive ETC vector bosons connecting SM fermions with technifermions produce masses for the first generation ($i=1$) of SM fermions [35].

This $\mathrm{SU}(N_{\mathrm{ETC}}-1)$ theory is again asymptotically free [with $b_1^{\mathrm{SU}(N_{\mathrm{ETC}}-1)}=(1/3)(9N_{\mathrm{ETC}}-19)$] so the gauge coupling [inherited at $\Lambda_1$ from the $\mathrm{SU}(N_{\mathrm{ETC}})$ theory] grows, and we infer that at a somewhat lower scale, $\Lambda_2$, there is again condensation in the channel (11), breaking $\mathrm{SU}(N_{\mathrm{ETC}}-1)$ to $\mathrm{SU}(N_{\mathrm{ETC}}-2)$. The associated condensate is

$$\left\langle\sum_{j=3}^{N_{\mathrm{ETC}}}\psi_R^{2jT}C\chi_{j,2,R}\right\rangle, \tag{13}$$

where, by convention, we have taken the gauge index $i=2$ in $\psi_R^{ij}$ and the copy index $s=2$ in $\chi_{j,s,R}$. The fermions $\psi_R^{2j}$ and $\chi_{j,2,R}$ with $3\le j\le N_{\mathrm{ETC}}$ involved in this condensate gain dynamical masses of order $\Lambda_2$ and are integrated out of the $\mathrm{SU}(N_{\mathrm{ETC}}-2)$ LEET operative at $\mu<\Lambda_2$. Of the fermions in Eqs. (7) and (8), the remaining ones that are nonsinglets in the $\mathrm{SU}(N_{\mathrm{ETC}}-2)$ LEET are $\psi_R^{3j}$ with $4\le j\le N_{\mathrm{ETC}}$; and $\chi_{j,s,R}$ with $3\le j\le N_{\mathrm{ETC}}$ and $3\le s\le N_{\mathrm{ETC}}-4$. The $(2N_{\mathrm{ETC}}-3)$ ETC gauge bosons in the coset $\mathrm{SU}(N_{\mathrm{ETC}}-1)/\mathrm{SU}(N_{\mathrm{ETC}}-2)$ gain masses of order $\Lambda_2$. Diagrams involving exchanges of these massive vector bosons connecting SM fermions with technifermions produce masses for the second generation of SM fermions [35].

This sequential self-breaking of the $\mathrm{SU}(N_{\mathrm{ETC}})$ theory continues iteratively in $N_{\mathrm{ETC}}-4$ stages, using the $N_{\mathrm{ETC}}-4$ copies of $\chi_{j,s,R}$ fermions, so that the original $\mathrm{SU}(N_{\mathrm{ETC}})$ (chiral) gauge symmetry is finally reduced to the (vectorial) $\mathrm{SU}(N_{\mathrm{TC}})$ subgroup, with the indices corresponding to the broken ETC symmetries being the SM generation indices. Hence,

$$N_{\mathrm{gen}}=N_{\mathrm{ETC}}-4. \tag{14}$$

Substituting this expression for $N_{\mathrm{gen}}$ into Eq. (2), we obtain the result

$$N_{\mathrm{TC}}=4. \tag{15}$$

This is our main result. We have determined $N_{\mathrm{TC}}$ from the structure of the specific ETC theory in which our 1FTC theory is embedded. A particularly intriguing aspect of our result is that, although Eq. (2) connects $N_{\mathrm{ETC}}$ and $N_{\mathrm{TC}}$, it does so in a manner that involves $N_{\mathrm{gen}}$, but our result is actually independent of $N_{\mathrm{gen}}$, provided that $N_{\mathrm{gen}}$ is sufficiently small that the ETC theory is asymptotically free and breaks in the indicated manner, and also that $\mathrm{SU}(3)_c$ is asymptotically free. Very interestingly, our resulting value $N_{\mathrm{TC}}=4$ agrees with the value inferred from a fit to the properties of the Higgs-like scalar in a 1FTC theory [18,24,32,36].

Henceforth, we set $N_{\mathrm{gen}}$ equal to the known value, $N_{\mathrm{gen}}=3$. Combining this with Eqs. (2) and (15), we infer that the ETC gauge group is $\mathrm{SU}(7)_{\mathrm{ETC}}$. As discussed above, this breaks in three stages to the (vectorial) $\mathrm{SU}(4)_{\mathrm{TC}}$ group: $\mathrm{SU}(7)_{\mathrm{ETC}}\to\mathrm{SU}(6)_{\mathrm{ETC}}\to\mathrm{SU}(5)_{\mathrm{ETC}}\to\mathrm{SU}(4)_{\mathrm{TC}}$. The theory naturally accounts for the mass hierarchy in the SM fermion generations, since the SM fermion masses in the $i$th generation result from exchange of ETC vector bosons with mass $\Lambda_i$ and, in the ETC boson propagators, $\Lambda_1^{-2}\ll\Lambda_2^{-2}\ll\Lambda_3^{-2}$.

The fermion content in the $\mathrm{SU}(4)_{\mathrm{TC}}$ theory consists of the SM-nonsinglet fermions in Eqs. (3)–(6) and the SM-singlet fermions in Eqs. (7)–(8) with $4\le i,\ j\le 7$. This TC theory is again asymptotically free (with $b_1^{(\mathrm{TC})}=26/3$). Hence, the TC coupling $\alpha_{\mathrm{TC}}(\mu)$ inherited from the lowest ETC theory, $\mathrm{SU}(5)_{\mathrm{ETC}}$ at $\Lambda_3$, continues to grow as $\mu$ decreases below $\Lambda_3$ [37].

For a fermion condensation channel ($Ch$) $R_1\times R_2\to R_{Ch}$, a measure of the attractiveness is $(\Delta C_2)_{Ch}=C_2(R_1)+C_2(R_2)-C_2(R_{Ch})$, where $C_2(R)$ is the quadratic Casimir invariant [5]. A rough estimate of $\alpha_{\mathrm{cr},Ch}$ is $\alpha_{\mathrm{cr},Ch}\simeq 2\pi/[3(\Delta C_2)_{Ch}]$. The $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ field is self-conjugate in $\mathrm{SU}(4)_{\mathrm{TC}}$ and, at a scale $\Lambda_{AA}$ (where $A$ denotes the antisymmetric rank-2 tensor, $\begin{smallmatrix}\square\\\square\end{smallmatrix}$) forms a condensate in the most attractive channel $\begin{smallmatrix}\square\\\square\end{smallmatrix}\times\begin{smallmatrix}\square\\\square\end{smallmatrix}\to 1$, of the form $\langle\sum_{i,j,k,\ell=4}^{7}\epsilon_{ijk\ell}\psi_R^{ijT}C\psi_R^{k\ell}\rangle$, with $(\Delta C_2)_{AA}=2C_2(\begin{smallmatrix}\square\\\square\end{smallmatrix})=5$. This is invariant under both $\mathrm{SU}(4)_{\mathrm{TC}}$ and $G_{\mathrm{SM}}$. The next most attractive channel is $\square\times\bar{\square}\to 1$ in TC, with $(\Delta C_2)_{\bar{F}F}=15/4$, forming at the scale $\Lambda_{\bar{F}F}$ and involving the condensates $\langle\bar{F}F\rangle=\langle\bar{F}_LF_R\rangle+\langle\bar{F}_RF_L\rangle$ with the $F$ technifermions in (3)–(6).

The $\langle \bar{F}F \rangle$ condensates produce EWSB. One has the rough estimate

$$\frac{\Lambda_{AA}}{\Lambda_{\bar{F}F}} \simeq \exp\left[\frac{2\pi}{b_1^{(\mathrm{TC})}}(\alpha_{\mathrm{cr},AA}^{-1} - \alpha_{\mathrm{cr},\bar{F}F}^{-1})\right]. \qquad (16)$$

This yields $\Lambda_{AA}/\Lambda_{\bar{F}F} \simeq 1.5$. In general, a TD-like scalar in this theory contains $\bar{F}F$, $AA$, a techniglueball component, etc.; the $\bar{F}F$ component plausibly dominates because of the $\Lambda_{AA}/\Lambda_{\bar{F}F}$ ratio and the walking behavior, which implies that the TD mass is $\ll m_{\mathrm{TC,had}}$, where $m_{\mathrm{TC,had}}$ denotes the mass scale of technivector mesons and techniglueballs.

Neglecting ETC and SM gauge interactions, the 1FTC theory has a (nonanomalous) global flavor symmetry involving SM-nonsinglet fermions, $\mathrm{SU}(8)_{F_L} \otimes \mathrm{SU}(8)_{F_R} \otimes \mathrm{U}(1)_V$. This is broken to $\mathrm{SU}(8)_V \otimes \mathrm{U}(1)_V$ by the $\langle \bar{F}F \rangle$ condensate. In addition to the three NGBs absorbed by the $W^{\pm}$ and $Z$, this yields 60 PNGBs. Taking account of walking and the strong ETC interactions, it appears possible that their masses could be $\gtrsim \mathrm{O}(1)$ TeV, above current LHC limits [38].

Although 1FTC with $N_{\mathrm{TC}} = 4$ has a large perturbative value of $S$, viz., $S_{\mathrm{pert}} = 8/(3\pi)$, it is well known that the perturbative estimate of $S$ is not reliable because TC is strongly coupled at the scale of $m_W$ and $m_Z$. Here, motivated by the results of [18,29,30], we will assume that the walking and ETC effects can suppress $S$ sufficiently to obey experimental constraints. Ongoing and future lattice calculations will further test this assumption. A constraint on TC/ETC models is that the spectrum of technihadrons must be consistent with current limits from the LHC. We have already commented on the PNGBs. It also appears to be possible that the 1FTC technivector meson masses may lie above the LHC limits of a few TeV [39]. Additional ingredients are needed to fully explain the spectrum of quark and lepton masses, in particular, $t$-$b$ mass splitting.

In summary, we have presented a novel way to determine $N_{\mathrm{TC}}$ from the embedding of a one-family $\mathrm{SU}(N_{\mathrm{TC}})$ technicolor theory having technifermions in the fundamental representation, in a particular $\mathrm{SU}(N_{\mathrm{ETC}})$ extended technicolor theory, with the SM fermions combined with technifermions into fundamental representations of $\mathrm{SU}(N_{\mathrm{ETC}})$ as specified in Eqs. (3)–(6) and have shown that this yields the value $N_{\mathrm{TC}} = 4$. This value is the same as one inferred from a fit to the 125 GeV scalar boson in TC [18,24]. Our result motivates lattice studies of SU(4) gauge theory with $N_f = 8$ Dirac fermions. Future LHC data will yield stringent tests of this model. In addition to this phenomenological application, our result is of general interest for the insight that it provides on how the structure of a low-energy effective field theory—here the TC theory—is determined by its embedding in an ultraviolet completion, the ETC theory.

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[36] If one were to assume a type of unification of the TC and SM sectors in the deep UV, one might naively be led to infer a value of NTC larger than 3. However, TC is embedded in ETC far below a hypothetical TC-SM unification; see N. D. Christensen and R. Shrock, Phys. Rev. D **72**, 035013 (2005).

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