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Rare meson decay through off-shell doubly charged scalars

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The presence of charged scalars is almost inevitable in most of the beyond standard model scenarios. They are expected to be detected easily if they are there due to their electromagnetic charge. These charged scalars can be produced at the LHC and their decays may lead to interesting signals—multilepton final states, displaced vertices, etc. These charged particles also play crucial roles in low energy rare processes. Thus apart from the collider searches in low energy rare processes their presence can be smelled. Here, we have noted the impact of doubly charged scalars in rare meson decays. As the mesons are lighter these heavy scalars always appear off shell. Due to their off-shell structure the phase space is relatively complicated to deal with. In this paper we have supplemented a general proposal to compute these decays that involve off-shell doubly charged scalars. We have argued that our prescription can be used for any process involving off-shell heavy scalars. Using the prescribed method we have computed two possible meson decays: $M^\pm \to l_i^\pm l_j^\pm M'^\mp$, $M^\pm \to l_i^\pm l_j^\pm l_m^\mp l_n^\mp M'^\pm$. We have also estimated the numerical values of the branching ratios in different channels for different charged mesons.

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I. INTRODUCTION

The discovery of Higgs boson at ATLAS [1] and CMS [2] has validated the standard model (SM) of particle physics. But there are already several credible experimental evidences which do not agree with the standard model predictions. The majority of the theoretical as well as the experimental community of particle physics is searching for beyond standard model (BSM) signals to explain many such issues, namely tiny nonzero masses of active neutrinos, gauge hierarchy, stabilization of Higgs boson mass, vacuum stability, dark matter, dark energy and so on.

Theoretical BSM models are in general constructed by extending SM by either adding some new particles and (or) by supplementing the gauge sector which can be originated from some unified scenario. These newly proposed particles can be fermions or scalar/gauge bosons which are expected to be quite heavy or extremely light and hence have remained elusive in the direct experimental searches. Thus the prime task for LHC, after the Higgs discovery, is to find out or at least obtain hints of this new physics.

While the LHC is trying its best to discover the BSM particles through their productions and decays, here we are interested to demonstrate their possible impact in rare meson decays. More precisely we will concentrate on lepton number and/or flavor violating decay channels in meson decays ($M^\pm \to l_i^\pm l_j^\pm M'^\mp$ [3–10]), neutrinoless double beta decay ($2n \to 2p + 2e^-$) [11–14] etc. These processes occur in BSM models developed to understand light neutrino masses. In these models, neutrino masses are being generated through higher dimensional operators which appear after integrating out the heavy particles. Many of these scenarios contain different representations of $SU(2)_L$ scalar multiplets [15] or heavy neutrinos. An important feature of these models is lepton number violation by two units ($\Delta L = 2$). This nontrivial lepton number violation along with lepton flavor violations lead to unique BSM signatures at the collider [16–23]. Phenomenologically, these doubly charged scalars can be produced at the LHC as a pair or associated with singly charged scalar. In these rare processes due to mass differences among mesons, neutrons and protons the intermediate BSM particles always appear off shell. Then their presence cannot be justified by invariant mass reconstruction and on-shell prescriptions also fail. Hence, in a long cascade decay due to these off-shell particles one may need to deal with a very complicated phase space.

In this paper, we also evaluate the branching ratios for the processes, $M^\pm \to l_i^\pm l_j^\pm M'^\mp$, $M^\pm \to l_i^\pm l_j^\pm l_m^\mp l_n^\mp M'^\pm$. Both of these decays can be possible through the mediation of Majorana neutrino as well as doubly charged scalars. In this paper, we have discussed these decays when the intermediate decaying particle is doubly charged scalar. The decay of a doubly charged scalar to a pair of same sign lepton is controlled by the Yukawa coupling which is flavor nonuniversal thus can lead to lepton flavor violating same sign dileptons. In the SM there is no lepton flavor violation, thus this feature can be a smoking gun to smell the presence of new physics. In the case of same sign but opposite flavor final state which is achieved through the exchange of

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heavy Majorana neutrinos (N), the effective amplitude is proportional to $|U^*_{\nu_i N_i} U_{\nu_i N_j}|$ ($i \neq j$). Now whether this process is suppressed or not will depend on specific neutrino mass models and controlled by the nonunitarity factors ($\propto U_{\nu_i N_j}$) allowed by experiments. This argument is also applicable for the four-lepton final state. In the second possible decay mode of meson though there are two different vertices which signify lepton number violation by two units but in the final state the lepton number is conserved. Here, also the possibility of lepton violation is still open like the previous case. We are proposing the above-mentioned second possibility of meson decay for the first time in this paper. Here, we also propose an alternate technique to compute the decays involving off-shell scalar particles. We have shown our prescription leads to the same result computed by using the general phase space method. But the advantage of this new method over the earlier one is notable: now one does not need to compute the full n-body amplitude for decay: $1 \to n$-particles. We have computed two decay processes to justify the credibility of our method: $M^\pm \to l_i^\pm l_j^\pm M'^\mp$, $M^\pm \to l_i^\pm l_j^\pm l_m^\mp l_n^\mp M'^\pm$. We have tested our method for off-shell scalar particles so far. We are convinced that our prescription cannot be used directly when the intermediate off-shell particles carry nonzero spin. Our future proposal along side this work is to generalize this procedure for all particles.

II. PROPOSAL TO DEAL WITH OFF-SHELL SCALARS

Many rare processes like meson decay and neutrinoless double beta decay can occur through intermediate heavy scalars [5,11]. The recent search for doubly charged scalar at the LHC put severe constraints on its mass. ATLAS has ruled out the mass of doubly charged scalar from $[200 \div 400]$ GeV depending on its leptonic branching fraction [24]. We have noted that respecting this exclusion limit, even the maximum mass difference between any two mesons (M, M') is much less than the mass of the doubly charged scalar. Thus, these charged scalars always appear off shell in these decays. Given that we cannot simply use branching ratios in our calculation, we need to deal with complicated phase factors for all the final state particles. The full amplitude involves hadronic and leptonic parts which can be dealt with independently of each other. The hadronic part will be discussed below Eqs. (5) and (6). In the existing method more attention is paid on factorizing the phase space, however in our analysis we have not followed this method exactly.

The general prescription of obtaining the three body decay rates in terms of two body decay rates: $X_1 \to X_2 X_3^* \to X_2 X_4 X_5$ will give us a good understanding of the basics of our proposal.

The decay width for this process can be written, using the standard phase space method, as

$$\Gamma^{X_1}_{X_2X_4X_5} = \int \frac{dm^2_{45}}{2\pi} \int d_{PS(X_1 \to X_2 X_3^*)} \int d_{PS(X_3^* \to X_4 X_5)} \times \frac{|A(X_1 \to X_2 X_4 X_5)|^2}{2m_{X_1}}, \quad (1)$$

where m_{45} is the invariant mass of the intermediate particle X_3^*. We note that this decay width of the parent particle X_1 to $X_2X_4X_5$ where X_4X_5 is the decay product of off-shell scalar X_Δ^* can also be expressed as

$$\Gamma^{X_1}_{X_2X_4X_5} = \int \frac{dm^2_{45}}{(m^2_{X_\Delta})\pi} \int d_{PS(X_1 \to X_2 X_\Delta^*)} \times \frac{|A(X_1 \to X_2 X_\Delta^*)|^2}{2m_{X_1}} \frac{\tilde{\Gamma}^{X_\Delta^*}_{X_4X_5}}{(m_{X_\Delta})}, \quad (2)$$

where

$$\tilde{\Gamma}^{X_\Delta^*}_{X_4X_5} = \left(1 - \frac{1}{2}\delta_{X_4X_5}\right) \int d_{PS(X_\Delta^* \to X_4X_5)} \frac{|A(X_\Delta^* \to X_4X_5)|^2}{2m_{X_\Delta}}. \quad (3)$$

m_{X_Δ} is the on-shell mass of the scalar X_Δ, while the phase space $\int d_{PS(X_\Delta^* \to X_4X_5)}$ is similar to $\int d_{PS(X_3^* \to X_4X_5)}$ appearing in Eq. (1). We know that if the particle X_Δ is produced on shell then we can plug in the branching fraction for decay $X_\Delta \to X_4X_5$ in our calculation. We have mimicked that idea to compute the decay when the particle, X_Δ, is off shell. We have replaced the branching ratio (BR), i.e., $\Gamma(X_\Delta \to X_4X_5)/\Gamma^{X_\Delta}_{\text{total}}$ [1] by $\tilde{\Gamma}(X_\Delta^* \to X_4X_5)/(m_{X_\Delta})^2$ to make this quantity dimensionless. As the off-shell particles might have a range of momenta depending on the kinematic availability the dimensionless integration, $\int \frac{dm^2_{45}}{(m^2_{X_\Delta})\pi}$, is added. Thus like the on-shell case, here, also we can cut the off-shell scalar propagators and add such terms every time. This will help to avoid the amplitude computation of $1 \to n$-particles through long cascade decays. After discussing the details of the calculation for $M^\pm \to l_i^\pm l_j^\pm M'^\mp$ in II A, a complete calculation using this prescription can be found in Appendix A.

We can extend our conjecture for long cascade decays, like $X_1 \to X_2 \Delta_1^* \Delta_2^* \to X_2 X_4 X_5 X_6 X_7$, where X_4X_5 and X_6X_7 are decay products of off-shell scalars Δ_1^* and Δ_2^* respectively. This can be found similarly as in II B followed by Appendix B 1.

The most general decay width involving multiple off-shell scalar can be written as

[1] $\Gamma^{X_\Delta}_{\text{total}}$ is the total decay width of X_Δ.
[2] Here, the definition of $\tilde{\Gamma}$ is slightly different than the usual decay width. As this is an off-shell particle its partial decay width is defined as $\tilde{\Gamma}(X_\Delta^* \to X_4X_5) = \int d_{PS(X_\Delta^* \to X_4X_5)} \frac{|A(X_\Delta^* \to X_4X_5)|^2}{m_{X_\Delta}}$.

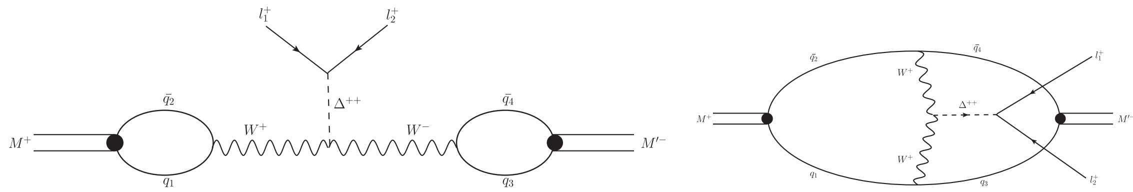

FIG. 1. Meson decay $M^{\pm} \to M'^{\mp} l_1^{\pm} l_2^{\pm}$ through the heavy doubly charged scalars: s-channel (left) and t-channel (right) diagrams.

$$
\begin{aligned}
\Gamma^{X_1}_{X_2(X_4X_5)(X_6X_7)\ldots(X_iX_j)} &= \Bigg[\int \frac{dm^2_{45}}{(m^2_{X_{\Delta_1}})\pi} \int \frac{dm^2_{67}}{(m^2_{X_{\Delta_2}})\pi} \cdots \\
&\times \int \frac{dm^2_{ij}}{(m^2_{X_{\Delta_n}})\pi} [\Gamma(X_1 \to X_2 X^*_{\Delta_1} X^*_{\Delta_2} \ldots X^*_{\Delta_n})] \\
&\times \left[\frac{\tilde{\Gamma}(X^*_{\Delta_1} \to X_4 X_5)}{(m_{X_{\Delta_1}})}\right] \left[\frac{\tilde{\Gamma}(X^*_{\Delta_2} \to X_6 X_7)}{(m_{X_{\Delta_2}})}\right] \cdots \\
&\times \left[\frac{\tilde{\Gamma}(X^*_{\Delta_n} \to X_i X_j)}{(m_{X_{\Delta_n}})}\right]\Bigg]. \qquad (4)
\end{aligned}
$$

$\Gamma(X_1 \to X_2 X^*_{\Delta_1} X^*_{\Delta_2} \ldots X^*_{\Delta_n})$ can be computed using the standard phase space method as has been used to write down Eq. (1). Here, each $X^*_{\Delta_n}$ is decaying to $(X_i X_j)$. In the commonly used phase space method one needs to compute the amplitude of the full process $(X_1 \to X_2 X_4 X_5)$ which might be cumbersome for a very long cascade. But following our proposal we need to compute only amplitude and decay of one to two body decays which have very standard well-known forms.

A. $M^{\pm} \to M'^{\mp} l_1^{\pm} l_2^{\pm}$

The meson decay, $M^{\pm} \to M'^{\mp} l_1^{\pm} l_2^{\pm}$, is mediated by doubly charged scalars as shown in Fig. 1. This doubly charged scalar can belong to different representations[3] and that controls the vertex factor. The vertex factors of couplings $W^{\mp}_{\mu} W^{\mu\mp} \Delta^{\pm\pm}$, $\Delta^{\pm\pm} l_1^{\mp} l_2^{\mp}$, and $\Delta^{\pm\pm}\Delta^{\mp\mp} W^{\pm}_{\mu} W^{\mu\mp}$ which are necessary for our analysis are given as $c_g v_\Delta g^2/2$, $y_{l_1 l_2}$, and $c_{ww} g^2/2$ respectively. The group theoretic factors are given as $c_g = \sqrt{(T_3+Y)(T_3+Y-1)(T_3-Y+2)(T_3-Y+1)}$, and $c_{ww} = 2[T(T+1) - (2-Y)^2]$ (with definition $Q = T_3 + Y$). Another coupling $y_{l_1 l_2}$ is the measure of interaction of two leptons with the doubly charged scalar, and is related to the light neutrino masses.

The hadronic contribution for the processes we have considered in this paper are the same. The hadronic part of the diagrams, Fig. 1, can be calculated using the Bethe-Salpeter amplitude method for mesons as bound states [4,25]. Since $M_\Delta \gg m_M$, the hadronic matrix element can be written as $\langle M'|\bar{q}_3 V_{q_3 q_4} \gamma^\mu (1-\gamma_5) q_4 \bar{q}_2 V_{q_2 q_1} \gamma_\mu (1-\gamma_5) q_1 |M\rangle$ for the s-channel process[4] and $\langle M'|\bar{q}_3 V_{q_3 q_1} \gamma^\mu (1-\gamma_5) \times q_1 \bar{q}_2 V_{q_2 q_4} \gamma_\mu (1-\gamma_5) q_4 |M\rangle$ for the t-channel process [4,6]. Using the vacuum saturation approximation, we can replace the s-channel matrix element by $\langle M'|\bar{q}_3 \gamma^\mu (1-\gamma_5) q_4 |0\rangle \times \langle 0|\bar{q}_2 \gamma_\mu (1-\gamma_5) q_1 |M\rangle$. The expression $\langle 0|\bar{q}_2 \gamma_\mu (1-\gamma_5) q_1 |M\rangle$ involves long distance QCD and cannot be calculated explicitly but it can be parametrized in terms of decay constants $(f_M, f_{M'})$ which can be measured experimentally [26]. Eventually we get

$$
\begin{aligned}
&\langle M'|\bar{q}_3 V_{q_3 q_4} \gamma^\mu (1-\gamma_5) q_4 \bar{q}_2 V_{q_2 q_1} \gamma_\mu (1-\gamma_5) q_1 |M\rangle \\
&\quad = V_{q_3 q_4} V_{q_2 q_1} f_{M'} f_M (P_M \cdot P_{M'}). \qquad (5)
\end{aligned}
$$

The t-channel matrix element can be converted to a similar expression by using Fierz transformation,

$$
\begin{aligned}
&\langle M'|\bar{q}_3 V_{q_3 q_1} \gamma^\mu (1-\gamma_5) q_1 \bar{q}_2 V_{q_2 q_4} \gamma_\mu (1-\gamma_5) q_4 |M\rangle \\
&\quad = V_{q_3 q_1} V_{q_2 q_4} f_{M'} f_M (P_M \cdot P_{M'})/N_c, \qquad (6)
\end{aligned}
$$

where $1/N_c$ is a result of the different color factors in the t-channel.

Now combining the hadronic and the leptonic contributions the partial decay width of the decay $M^{\pm} \to M'^{\mp} l_1^{\pm} l_2^{\pm}$ in the presence of the doubly charged scalar is given as

$$
\begin{aligned}
\Gamma^M_{M' l_1 l_2} &= \left(1 - \frac{1}{2}\delta_{l_1 l_2}\right) \frac{1}{2^8 \pi^3} G_F^4 K_{V1}^2 f_M^2 f_{M'}^2 \frac{1}{m_M^3} c_g^2 v_\Delta^2 \frac{1}{M_\Delta^4} y^2_{l_1 l_2} \\
&\times \int_{(m_{l_1}+m_{l_2})^2}^{(m_M - m_{M'})^2} \Bigg[dm^2_{X_1} \frac{\lambda^{1/2}(m_M^2, m_{M'}^2, m_{X_1}^2)}{m^2_{X_1}} \\
&\times \lambda^{1/2}(m^2_{X_1}, m^2_{l_1}, m^2_{l_2}) (m_M^2 + m_{M'}^2 - m^2_{X_1})^2 \\
&\times (m^2_{X_1} - m^2_{l_1} - m^2_{l_2}) \Bigg], \qquad (7)
\end{aligned}
$$

where $K_{V1} = V_{12} V_{43} + 1/N_c V_{13} V_{42}$ with V_{ij} being elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and $N_c = 3$.

[3]As the doubly charged scalar and gauge coupling vertex is of type $W - \Delta - W$, this interaction term is possible iff the Δ^{++} belongs to a multiplet which contains a neutral scalar field and that acquires a vacuum expectation value.

[4]Here, the considered hadronic process is $q_1 q_2 \to q_3 q_4$.

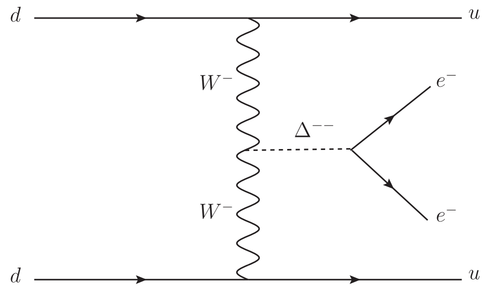

FIG. 2. Neutrinoless double beta decay ($0\nu\beta\beta$) diagram mediated by doubly charged scalar.

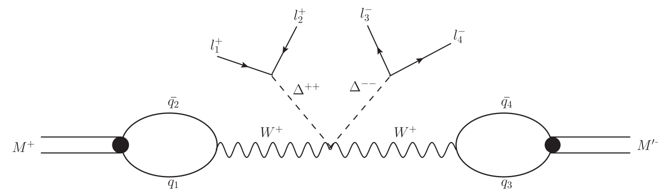

FIG. 3. Charged meson decay $M^\pm \to M'^\pm l_1^\pm l_2^\pm l_3^\mp l_4^\mp$ through the heavy doubly charged scalars.

Another important and interesting consequence of lepton number violation is neutrinoless double beta decay ($0\nu\beta\beta$) which can also be mediated by Majorana neutrino and doubly charged scalar [11,12,27,28], see Fig. 2. In passing we would like to mention that this process possesses a similar leptonic part ($W^+W^+ \to \Delta^{++*} \to l_1^+ l_2^+$). But one needs to keep in mind that the hadronic part in Eq. (7) should be replaced by the appropriate nuclear matrix element. While computing this process one must be careful to incorporate the correction due to the Coulomb attraction between the final state electrons and the nucleus as suggested in [27] and references therein, and thus can not implement this method blindly. We are not discussing it in detail as this is already discussed in [27]. In general, our proposal will be meaningful for the processes where one can deal with the leptonic part containing $\Delta - W - W$, $\Delta - \Delta - W - W$ vertices with off-shell Δ separately. But one needs to modify this proposal if these off-shell particles have nonzero spin or other quantum numbers, like color. We are working on that.

B. $M^\pm \to M'^\pm l_1^\pm l_2^\pm l_3^\mp l_4^\mp$

Here we have calculated an interesting possibility of meson decay: $M^\pm \to M'^\pm l_1^\pm l_2^\pm l_3^\mp l_4^\mp$. We have computed the decay width for this process when the decay is possible only through doubly charged scalars. But as we have mentioned this decay can be mediated by Majorana neutrinos also, one needs to take care of both contributions if both particles are present in that model. As we have mentioned earlier that the doubly charged scalars always appear off shell, this can be a very good smoking gun to note their presence. Also unlike the other decay here the doubly charged scalar can belong to any representation as the $W - W - \Delta - \Delta$ vertex is always there as an outcome of the scalar kinetic term.[5] The process $M^\pm \to M'^\pm l_1^\pm l_2^\pm l_3^\mp l_4^\mp$ is also possible through $\Delta^\pm$ instead of $W^\pm$ in Fig. 3. However, the strength of interactions $\Delta^{++} - \Delta^{--} - \Delta^+ - \Delta^-$ and $qq'\Delta^\pm$ is proportional to scalar quartic and Yukawa couplings respectively. These couplings in usual scenarios are much smaller than the gauge coupling and also suppressed by the scalar mixing angles. Thus these lead to negligible contributions to the actual processes.

Here, we have used our prescription, Appendix B 1, and also verified the result using the phase space technique B 2. Both procedures lead to same decay width. This verifies our proposal.

In passing we would like to note that, because of the coupling $W - W - Z - Z$ present in the SM, this four-lepton final state can also be mimicked. But there will be no possibility of flavor violating signal as the Z boson always decays to the same flavored charged lepton, like $e^+e^+e^-e^-$ or $\mu^+\mu^+\mu^-\mu^-$ or $e^\pm e^\mp \mu^\pm \mu^\mp$. Thus the asymmetry in lepton flavors among these four leptons, like $e^\pm e^\pm e^\mp \mu^\mp$ or $e^\pm \mu^\pm \mu^\mp \mu^\mp$ and final states like $e^\pm e^\pm \mu^\mp \mu^\mp$, will surely signify the presence of new particles beyond SM, like doubly charged scalars, heavy Majorana neutrino in theory.

The decay width of the following decay $M^\pm \to M'^\pm l_1^\pm l_2^\pm l_3^\mp l_4^\mp$ is given as

$$\Gamma^M_{M' l_1^+ l_2^+ l_3^- l_4^-} = \frac{K_{V2}^2 f_M^2 f_{M'}^2 G_F^4 c_g^2 y_{l_1 l_2}^2 y_{l_3 l_4}^2}{\pi^7 2^{16} M_{\Delta^{++}}^4 M_{\Delta^{--}}^4 m_M}\left(1 - \frac{1}{2}\delta_{l_3 l_4}\right)\left(1 - \frac{1}{2}\delta_{l_1 l_2}\right)\int dm_{X_1}^2 \frac{\lambda^{1/2}(m_M^2, m_{M'}^2, m_{X_1}^2)}{m_M^2}$$
$$\times \int dm_{12}^2 \int dm_{34}^2 \frac{\lambda^{1/2}(m_{X_1}^2, m_{\Delta^{++}}^2, m_{\Delta^{--}}^2)}{m_{X_1}^2}\frac{\lambda^{1/2}(m_{12}^2, m_{l_1}^2, m_{l_2}^2)}{m_{12}^2}\frac{\lambda^{1/2}(m_{34}^2, m_{l_3}^2, m_{l_4}^2)}{m_{34}^2}$$
$$\times (m_M^2 + m_{M'}^2 - m_{X_1}^2)^2 (m_{12}^2 - m_{l_1}^2 - m_{l_2}^2)(m_{34}^2 - m_{l_3}^2 - m_{l_4}^2), \qquad (8)$$

with $K_{V2} = V_{12} V_{43}$.

[5]Thus if the doubly charged scalar belongs to an $SU(2)$ multiplet which does not contain any neutral scalar field, the contribution to the decay width for $M^\pm \to M'^\mp l_1^\pm l_2^\pm$ vanishes. But the meson decay leading to a four-lepton final state is still present.

TABLE I. Properties of mesons.

Meson (M)	f_M (GeV)	Mass (GeV)	Lifetime τ (Sec)
$\pi^\pm$	130.7×10^{-3}	$(139.57018 \pm 0.00035) \times 10^{-3}$	$(2.6033 \pm 0.00035) \times 10^{-8}$
$K^\pm$	159.8×10^{-3}	$(493.677 \pm 0.016) \times 10^{-3}$	$(1.238 \pm 0.0021) \times 10^{-8}$
$D^\pm$	228×10^{-3}	$(1869.62 \pm 0.15) \times 10^{-3}$	$(1040 \pm 7) \times 10^{-15}$
$D_s^\pm$	251×10^{-3}	$(1968.50 \pm 0.32) \times 10^{-3}$	$(500 \pm 7) \times 10^{-15}$
$B^\pm$	200×10^{-3}	$(5279.26 \pm 0.17) \times 10^{-3}$	$(1.641 \pm 0.008) \times 10^{-12}$

The limits of the above integrals are as follows:

$$m_{12}^2 \in [(m_{l_1} + m_{l_2})^2, (m_{X_1} - (m_{l_3} + m_{l_4}))^2],$$
$$m_{34}^2 \in [(m_{l_3} + m_{l_4})^2, (m_{X_1} - m_{12})^2],$$
$$m_{X_1}^2 \in [(m_{l_1} + m_{l_2} + m_{l_3} + m_{l_4})^2, (m_M - m_{M'})^2]. \quad (9)$$

This type of meson decay is not only associated with doubly charged scalar. The heavy non-SM neutral scalar can have a vertex like $W - \Delta^0 - \Delta^0 - W$. Then these off-shell neutral scalars might have flavor violating but lepton number conserving decay to two opposite sign charged leptons: $\Delta^0 \to l_i^\pm l_j^\mp$. Then also the final state consists of $l_1^\pm l_2^\pm l_3^\mp l_4^\mp$. The decay width involving these neutral scalars will be the same as Eq. (8) where M_Δ will be the mass of heavy neutral scalar, with appropriate group theoretic factor c_g.

III. NUMERICAL RESULTS

In this paper, we have encapsulated the impact of a doubly charged scalar in rare meson decays. We have focused mainly on the charged mesons ($M^\pm$) and their decays to $l_i^\pm l_j^\pm M'^\mp$ and $l_i^\pm l_j^\pm l_m^\mp l_n^\mp M'^\pm$. Both of these final states carry the signatures of lepton number violation and/or lepton flavor violations. More precisely the decay of doubly charged scalar to a pair of charged leptons is lepton number violating and in general that coupling is proportional to the light neutrino mass. If these charged leptons are of different flavor then this decay leads to violation of lepton flavor also. We have enlisted the form factors (f_M), mass and the lifetime of the charged mesons that are involved in our analysis, see Table I [26]. Here we have taken central values of mass and lifetime from Table I to compute the decay branching ratios of different charged mesons. In Tables VI and VII we have provided the decay BRs of the charged mesons (K^+, D^+, D_s^+, B^+) to $M'^\mp l_i^\pm l_i^\pm$, where $M'^\mp$s are the respectively light mesons. There are already experimental (Exp.) upper bounds on these decay BRs, thus we can put some lower bound on $\frac{M_\Delta^2}{c_g v_\Delta y_{l_1 l_2}}$ in a model independent way. For specific models depending on the specific structures of the couplings the mass on doubly

TABLE II. Decay BR for $K^\pm$ meson decays leading to four-lepton final state.

Meson decay	$\mathrm{BR}/\left(\frac{c_g^2 y_{l_1 l_2}^2 y_{l_3 l_4}^2}{M_\Delta^8}\right)$ [GeV^8]
$K^\pm \to \pi^\pm e^+ e^+ e^- e^-$	1.355×10^{-27}
$K^\pm \to \pi^\pm e^+ e^+ e^- \mu^-$	4.479×10^{-28}
$K^\pm \to \pi^\pm e^+ e^+ \mu^- \mu^-$	1.335×10^{-29}
$K^\pm \to \pi^\pm e^+ \mu^+ e^- \mu^-$	5.339×10^{-29}
$K^\pm \to \pi^\pm \mu^+ \mu^+ \mu^- e^-$	2.360×10^{-32}

TABLE III. Decay BR for $D^\pm$ meson decay into four-lepton final state.

Meson decay	$\mathrm{BR}\left(\frac{c_g^2 y_{l_1 l_2}^2 y_{l_3 l_4}^2}{M_\Delta^8}\right)$ [GeV^8]
$D^\pm \to K^\pm e^+ e^+ e^- e^-$	7.204×10^{-25}
$D^\pm \to K^\pm e^+ e^+ e^- \mu^-$	1.229×10^{-24}
$D^\pm \to K^\pm e^+ e^+ \mu^- \mu^-$	5.206×10^{-25}
$D^\pm \to K^\pm e^+ \mu^+ e^- \mu^-$	2.082×10^{-24}
$D^\pm \to K^\pm \mu^+ \mu^+ \mu^- e^-$	8.745×10^{-25}
$D^\pm \to K^\pm \mu^+ \mu^+ \mu^- \mu^-$	3.638×10^{-25}
$D^\pm \to \pi^\pm e^+ e^+ e^- e^-$	2.486×10^{-23}
$D^\pm \to \pi^\pm e^+ e^+ e^- \mu^-$	4.412×10^{-23}
$D^\pm \to \pi^\pm e^+ e^+ \mu^- \mu^-$	1.949×10^{-23}
$D^\pm \to \pi^\pm e^+ \mu^+ e^- \mu^-$	7.797×10^{-23}
$D^\pm \to \pi^\pm \mu^+ \mu^+ \mu^- e^-$	3.420×10^{-23}
$D^\pm \to \pi^\pm \mu^+ \mu^+ \mu^- \mu^-$	1.502×10^{-23}

TABLE IV. Decay BR for $Ds^\pm$ meson decay into four-lepton final state.

Meson decay	$\mathrm{BR}/\left(\frac{c_g^2 y_{l_1 l_2}^2 y_{l_3 l_4}^2}{M_\Delta^8}\right)$ [GeV^8]
$Ds^\pm \to K^\pm e^+ e^+ e^- e^-$	1.712×10^{-23}
$Ds^\pm \to K^\pm e^+ e^+ e^- \mu^-$	2.977×10^{-23}
$Ds^\pm \to K^\pm e^+ e^+ \mu^- \mu^-$	1.286×10^{-23}
$Ds^\pm \to K^\pm e^+ \mu^+ e^- \mu^-$	5.145×10^{-23}
$Ds^\pm \to K^\pm \mu^+ \mu^+ \mu^- e^-$	2.209×10^{-23}
$Ds^\pm \to K^\pm \mu^+ \mu^+ \mu^- \mu^-$	9.417×10^{-24}
$Ds^\pm \to \pi^\pm e^+ e^+ e^- e^-$	5.320×10^{-22}
$Ds^\pm \to \pi^\pm e^+ e^+ e^- \mu^-$	9.565×10^{-22}
$Ds^\pm \to \pi^\pm e^+ e^+ \mu^- \mu^-$	4.278×10^{-22}
$Ds^\pm \to \pi^\pm e^+ \mu^+ e^- \mu^-$	1.711×10^{-21}
$Ds^\pm \to \pi^\pm \mu^+ \mu^+ \mu^- e^-$	7.625×10^{-22}
$Ds^\pm \to \pi^\pm \mu^+ \mu^+ \mu^- \mu^-$	3.385×10^{-22}

TABLE V. Decay BR for $B^{\pm}$ meson rare decay leading to four leptons in the final state.

Meson decay	$\mathrm{BR}/\left(\frac{c_g^2 y_{l_1 l_2}^2 y_{l_3 l_4}^2}{M_\Delta^8}\right)$ [GeV^8]
$B^\pm \to Ds^\pm e^+ e^+ e^- e^-$	2.279×10^{-21}
$B^\pm \to Ds^\pm e^+ e^+ e^- \mu^-$	4.431×10^{-21}
$B^\pm \to Ds^\pm e^+ e^+ \mu^- \mu^-$	2.153×10^{-21}
$B^\pm \to Ds^\pm e^+ \mu^+ e^- \mu^-$	8.612×10^{-21}
$B^\pm \to Ds^\pm \mu^+ \mu^+ \mu^- e^-$	4.183×10^{-21}
$B^\pm \to Ds^\pm \mu^+ \mu^+ \mu^- \mu^-$	2.031×10^{-21}
$B^\pm \to D^\pm e^+ e^+ e^- e^-$	1.261×10^{-22}
$B^\pm \to D^\pm e^+ e^+ e^- \mu^-$	2.454×10^{-22}
$B^\pm \to D^\pm e^+ e^+ \mu^- \mu^-$	1.194×10^{-22}
$B^\pm \to D^\pm e^+ \mu^+ e^- \mu^-$	4.777×10^{-22}
$B^\pm \to D^\pm \mu^+ \mu^+ \mu^- e^-$	2.324×10^{-22}
$B^\pm \to D^\pm \mu^+ \mu^+ \mu^- \mu^-$	1.130×10^{-22}
$B^\pm \to K^\pm e^+ e^+ e^- e^-$	4.065×10^{-22}
$B^\pm \to K^\pm e^+ e^+ e^- \mu^-$	7.999×10^{-22}
$B^\pm \to K^\pm e^+ e^+ \mu^- \mu^-$	1.574×10^{-21}
$B^\pm \to K^\pm e^+ \mu^+ e^- \mu^-$	4.777×10^{-22}
$B^\pm \to K^\pm \mu^+ \mu^+ \mu^- e^-$	7.740×10^{-22}
$B^\pm \to K^\pm \mu^+ \mu^+ \mu^- \mu^-$	3.806×10^{-22}
$B^\pm \to \pi^\pm e^+ e^+ e^- e^-$	5.672×10^{-21}
$B^\pm \to \pi^\pm e^+ e^+ e^- \mu^-$	1.117×10^{-20}
$B^\pm \to \pi^\pm e^+ e^+ \mu^- \mu^-$	5.496×10^{-21}
$B^\pm \to \pi^\pm e^+ \mu^+ e^- \mu^-$	2.199×10^{-20}
$B^\pm \to \pi^\pm \mu^+ \mu^+ \mu^- e^-$	1.082×10^{-20}
$B^\pm \to \pi^\pm \mu^+ \mu^+ \mu^- \mu^-$	5.324×10^{-21}

TABLE VI. Branching ratios for charged meson (K^+, D^+, D_s^+) rare decays leading to same sign dileptons. These are signatures of lepton number violations and lepton flavor violations too in a few cases. Here, we have taken the central values of mass and lifetime from Table I.

Meson decay mode	Exp. upper bound on BR	$\mathrm{BR}/\left(\frac{c_g^2 v_\Delta^2 y_{l_1 l_2}^2}{M_\Delta^4}\right)$ [GeV^2]	Lower bound on $\frac{M_\Delta^2}{c_g v_\Delta y_{l_1 l_2}}$ (GeV)
$K^+ \to \pi^- e^+ e^+$	6.4×10^{-10}	2.21×10^{-16}	587.6×10^{-6}
$K^+ \to \pi^- \mu^+ \mu^+$	3.0×10^{-9}	6.34×10^{-17}	145.4×10^{-6}
$K^+ \to \pi^- e^+ \mu^+$	5.0×10^{-10}	2.61×10^{-16}	722.5×10^{-6}
$D^+ \to \pi^- e^+ e^+$	9.6×10^{-5}	5.53×10^{-16}	2.4×10^{-6}
$D^+ \to \pi^- \mu^+ \mu^+$	1.7×10^{-5}	5.19×10^{-16}	5.5×10^{-6}
$D^+ \to \pi^- e^+ \mu^+$	5.0×10^{-5}	1.07×10^{-15}	4.6×10^{-6}
$D^+ \to K^- e^+ e^+$	1.2×10^{-4}	1.18×10^{-16}	1.0×10^{-6}
$D^+ \to K^- \mu^+ \mu^+$	1.2×10^{-4}	1.09×10^{-16}	1.0×10^{-6}
$D^+ \to K^- e^+ \mu^+$	1.3×10^{-4}	2.28×10^{-16}	1.3×10^{-6}
$D_s^+ \to \pi^- e^+ e^+$	6.9×10^{-4}	5.04×10^{-15}	2.7×10^{-6}
$D_s^+ \to \pi^- \mu^+ \mu^+$	8.2×10^{-5}	4.75×10^{-15}	7.6×10^{-6}
$D_s^+ \to \pi^- e^+ \mu^+$	7.3×10^{-4}	9.79×10^{-15}	3.7×10^{-6}
$D_s^+ \to K^- e^+ e^+$	6.3×10^{-4}	5.66×10^{-16}	0.9×10^{-6}
$D_s^+ \to K^- \mu^+ \mu^+$	1.8×10^{-4}	5.29×10^{-16}	1.7×10^{-6}
$D_s^+ \to K^- e^+ \mu^+$	6.8×10^{-4}	1.10×10^{-15}	1.3×10^{-6}

TABLE VII. Branching ratios for charged meson (B^+) rare decays leading to the same sign dileptons. These are signatures of lepton number violations and lepton flavor violations too in a few cases. Here, we have taken the central values of mass and lifetime from Table I.

Meson decay mode	Exp. upper bound on BR	$\mathrm{BR}/\left(\frac{c_g^2 v_\Delta^2 y_{l_1 l_2}^2}{M_\Delta^4}\right)$ [GeV^2]	Lower bound on $\frac{M_\Delta^2}{c_g v_\Delta y_{l_1 l_2}}$ (GeV)
$B^+ \to \pi^- e^+ e^+$	3.9×10^{-3}	2.36×10^{-16}	0.2×10^{-6}
$B^+ \to \pi^- \mu^+ \mu^+$	9.1×10^{-3}	2.34×10^{-16}	0.2×10^{-6}
$B^+ \to \pi^- e^+ \mu^+$	6.4×10^{-3}	4.7×10^{-16}	0.3×10^{-6}
$B^+ \to \pi^- \tau^+ \tau^+$	...	1.51×10^{-17}	...
$B^+ \to \pi^- e^+ \tau^+$	...	1.74×10^{-16}	...
$B^+ \to \pi^- \mu^+ \tau^+$	...	1.73×10^{-16}	...
$B^+ \to K^- e^+ e^+$	3.9×10^{-3}	1.85×10^{-17}	0.07×10^{-6}
$B^+ \to K^- \mu^+ \mu^+$	9.1×10^{-3}	1.84×10^{-17}	0.05×10^{-6}
$B^+ \to K^- e^+ \mu^+$	6.4×10^{-3}	3.69×10^{-17}	0.08×10^{-6}
$B^+ \to K^- \tau^+ \tau^+$	...	1.34×10^{-19}	...
$B^+ \to K^- e^+ \tau^+$	...	1.34×10^{-17}	...
$B^+ \to K^- \mu^+ \tau^+$	...	1.33×10^{-17}	...

charged scalars can be computed. We have checked that these bounds are severe only for the very light doubly charged scalars. We have also explored the possibility of other types of meson decays and computed their branching fractions in Tables II, III, IV, and V. These are more suppressed than the dilepton case mostly due to extended phase space.

IV. CONCLUSIONS

In this paper, we have discussed the impact of doubly charged scalars in rare meson decays in a model independent way. While computing the decay width for these processes, we noted that one can deal with the hadronic and the leptonic part independently. The processes under consideration are such that the involved doubly charged scalars appear always off shell. Thus the branching ratio technique does not work in this analysis. This makes the computation cumbersome and the phase spaces complicated as one cannot truncate the off-shell particles in principle. To make the computation easier we come up with an alternative proposal. Though we do not have a formal proof for this, we have understood how to achieve that from the idea of a phase space procedure for three body decay and then we have generalized that for n-body decay modes. Then we have shown explicitly that our proposal and the phase space method lead to the same result for five body decay. This method can be used for the similar leptonic parts for other decays, for example neutrinoless double beta decay via off-shell doubly charged scalar. In this paper we have estimated the decay branching ratios for processes like $M^\pm \to M'^\mp l_1^\pm l_2^\pm$, $M^\pm \to M'^\pm l_1^\pm l_2^\pm l_3^\mp l_4^\mp$ for different charged mesons ($B^\pm, D_s^\pm, K^\pm, D^\pm$). The decay branching ratios are very small compared to the present experimental bounds thus they cannot put significant constraints on the mass of the doubly charged scalars. The four-lepton final state is much more suppressed, mainly due to phase space, compare to the other two-lepton cases. We have checked that our proposal can be used for long cascades involving only off-shell scalar particles. This proposal needs further modifications to deal with off-shell particles with nonzero spins.

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APPENDIX A: MESON DECAY VIA HEAVY DOUBLY CHARGED SCALAR: $M^{\pm} \to M'^{\mp}\Delta^{\pm\pm} \to M'^{\mp} l_1^{\pm} l_2^{\pm}$

The decay width is proposed as

$$\Gamma(M^{\pm} \to M'^{\mp} l_1^{\pm} l_2^{\pm}) = \int \left[\frac{1}{\pi} d\left(\frac{m_{12}^2}{m_{\Delta^{++}}^2}\right)\right][\Gamma(M \to M'\Delta^{++*})] \times \left[\frac{\tilde{\Gamma}(\Delta^{++*} \to l_1 l_2)}{(m_{\Delta^{++}})}\right], \quad \text{(A1)}$$

$$[\Gamma(M^{+} \to M'^{-}\Delta^{++*})] = \int d_{PS(M\to M'\Delta^{++*})} \times \frac{|A_{2l}(M \to M'\Delta^{++*})|^2}{2m_M}, \quad \text{(A2)}$$

where m_{12} is the invariant mass of the intermediate particle Δ^{++*} which appears as off shell. Thus m_{12} is different from $m_{\Delta^{++}}$. Here, the hadronic contribution can be recollected from Fig. 1 and then the amplitude of this decay is given as

$$A_{2l}(M^{+} \to M'^{-}\Delta^{++*}) = \frac{1}{2^2} K_{V1} f_M f_{M'} (P_M.P_{M'}) L'(p_{l_1}, p_{l_2}), \quad \text{(A3)}$$

where, f_M and $f_{M'}$ are meson decay constants; $K_{V1} = V_{12}V_{43} + \frac{1}{N_c} V_{13}V_{42}$ with V_{ij} being elements of the CKM matrix and color-factor $N_c = 3$.

Here, leptonic contribution can be expressed as

$$L'(p_{l_1}, p_{l_2}) = \left(\frac{g}{\sqrt{2}}\right)^2 \frac{1}{M_W^4} \frac{c_g v_\Delta g^2}{2}. \quad \text{(A4)}$$

We can write the squared amplitude as

$$|A_{2l}|^2 = \frac{1}{2^4} K_{V1}^2 f_M^2 f_{M'}^2 (P_M.P_{M'})^2 \left(\left(\frac{g}{\sqrt{2}}\right)^2 \frac{1}{M_W^4} \frac{c_g v_\Delta g^2}{2}\right)^2. \quad \text{(A5)}$$

From momentum conservation equations ($P_M = P_{M'} + p_{l_1} + p_{l_2}$, $k = p_{l_1} + p_{l_2}$), and in rest frame of lepton pair $\vec{p}_{l_1} + \vec{p}_{l_2} = 0$ and $p_{l_1} + p_{l_2} = k = (m_{12}, 0, 0, 0)$, we find

$$(P_M.P_{M'}) = \frac{1}{2}(m_M^2 + m_{M'}^2 - m_{12}^2),$$
$$p_{l_2}.p_{l_1} = \frac{1}{2}(m_{12}^2 - m_{l_1}^2 - m_{l_2}^2),$$
$$E_{l_2} = \frac{m_{12}^2 + m_{l_2}^2 - m_{l_1}^2}{2m_{12}}. \quad \text{(A6)}$$

We also have

$$|\vec{p}_{l_2}| = \frac{1}{2m_{12}} \lambda^{1/2}(m_{12}^2, m_{l_1}^2, m_{l_2}^2),$$
$$|\vec{P}_{M'}| = \frac{1}{2m_M} \lambda^{1/2}(m_M^2, m_{M'}^2, m_{12}^2). \quad \text{(A7)}$$

The phase space is given as

$$d_{PS(M\to M'\Delta^{++*})} = \frac{1}{16\pi^2} \frac{\lambda^{1/2}(m_M^2, m_{M'}^2, m_{12}^2)}{2m_M^2} d\Omega, \quad \text{(A8)}$$

and the partial decay width is expressed as

$$\tilde{\Gamma}(\Delta^{++*} \to l_1 l_2) = \frac{1}{16\pi^2} \frac{\lambda^{1/2}(m_{12}^2, m_{l_1}^2, m_{l_2}^2)}{2m_{12}^2}(4\pi) \times \frac{1}{2m_{\Delta^{++}}} y_{l_1 l_2}^2 [m_{12}^2 - m_{l_1}^2 - m_{l_2}^2]. \quad \text{(A9)}$$

Now the decay width, Eq. (A1), can be written as

$$\Gamma_{M' l_1 l_2}^{M} = \left(1 - \frac{1}{2}\delta_{l_1 l_2}\right) \frac{1}{2^8 \pi^3} G_F^4 K_{V1}^2 f_M^2 f_{M'}^2 \frac{1}{m_M^3} c_g^2 v_\Delta^2 \frac{1}{M_\Delta^4} y_{l_1 l_2}^2 \times \int_{(m_{l_1}+m_{l_2})^2}^{(m_M - m_{M'})^2} dm_{12}^2 \frac{\lambda^{1/2}(m_M^2, m_{M'}^2, m_{12}^2)}{m_{12}^2} \times \lambda^{1/2}(m_{12}^2, m_{l_1}^2, m_{l_2}^2)(m_M^2 + m_{M'}^2 - m_{12}^2)^2 \times (m_{12}^2 - m_{l_1}^2 - m_{l_2}^2). \quad \text{(A10)}$$

APPENDIX B: DECAY WIDTH FOR MESON DECAY $M^{\pm} \to M'^{\pm} l_1^+ l_2^+ l_3^- l_4^-$

1. Using our prescription

We have proposed an alternative treatment to compute the decay width of this process: $M^{\pm} \to M'^{\pm} l_1^+ l_2^+ l_3^- l_4^-$.

The phase space for the case of five decay products is complicated. It is possible to simplify this by the introduction of intermediate particles so that our process can be written as

$$M^{\pm} \to M'^{\pm} X_1^* \to M'^{\pm} X_2^* X_3^* \to M'^{\pm} l_1^+ l_2^+ l_3^- l_4^-, \quad \text{with} \quad X_2^* \to l_1^+ l_2^+ \quad \text{and} \quad X_3^* \to l_3^- l_4^-. \tag{B1}$$

The decay width is proposed by following our prescription:

$$\Gamma(M^{\pm} \to M'^{\pm} l_1^+ l_2^+ l_3^- l_4^-) = \left[\int \left[\frac{1}{\pi} d\left(\frac{m_{12}^2}{m_{\Delta^{++}}^2}\right)\right] \int \left[\frac{1}{\pi} d\left(\frac{m_{34}^2}{m_{\Delta^{--}}^2}\right)\right] [\Gamma(M \to M' \Delta^{++*} \Delta^{--*})] \left[\frac{\tilde{\Gamma}(\Delta^{++*} \to l_1 l_2)}{(m_{\Delta^{++}})}\right] \times \left[\frac{\tilde{\Gamma}(\Delta^{--*} \to l_3 l_4)}{(m_{\Delta^{--}})}\right]\right] \tag{B2}$$

where $[\Gamma(M \to M' \Delta^{++*} \Delta^{--*})] = \int d_{PS(M \to M' \Delta^{++*} \Delta^{--*})} \frac{|A_{4l}(M \to M' \Delta^{++*} \Delta^{--*})|^2}{2m_M}$,

$$d_{PS(M \to M' \Delta^{++*} \Delta^{--*})} = \int \frac{dm_{X_1}^2}{2\pi} \frac{1}{16\pi^2} \frac{\lambda^{1/2}(m_M^2, m_{M'}^2, m_{X_1}^2)}{2m_M^2} (4\pi) \frac{1}{16\pi^2} \frac{\lambda^{1/2}(m_{X_1}^2, m_{12}^2, m_{34}^2)}{2m_{X_1}^2} (4\pi). \tag{B3}$$

The amplitude is given as

$$A_{4l}(M \to M' \Delta^{++*} \Delta^{--*}) = \frac{1}{2^2} K_{V2} f_M f_{M'} (P_M \cdot P_{M'}) \left(\frac{g}{\sqrt{2}}\right)^2 \frac{1}{M_W^4} \frac{c_g g^2}{2}, \tag{B4}$$

with $K_{V2} = V_{12} V_{34}$. Thus we have

$$|A_{4l}(M \to M' \Delta^{++*} \Delta^{--*})|^2 = \left[\frac{1}{2^2} K_{V2} f_M f_{M'} (P_M \cdot P_{M'}) \left(\frac{g}{\sqrt{2}}\right)^2 \frac{1}{M_W^4} \frac{c_g g^2}{2}\right]^2. \tag{B5}$$

Now Eq. (B2) can be expressed as

$$\begin{aligned}\Gamma^M_{M' l_1^+ l_2^+ l_3^- l_4^-} = {} & \int \frac{dm_{12}^2}{(\pi) m_{\Delta^{++}}^2} \int \frac{dm_{34}^2}{(\pi) m_{\Delta^{--}}^2} \left(1 - \frac{1}{2} \delta_{l_1} l_2\right) \left(1 - \frac{1}{2} \delta_{l_3} l_4\right) \\ & \times \int \frac{dm_{X_1}^2}{2\pi} \frac{1}{16\pi^2} \frac{\lambda^{1/2}(m_M^2, m_{M'}^2, m_{X_1}^2)}{2m_M^2} (4\pi) \frac{1}{16\pi^2} \frac{\lambda^{1/2}(m_{X_1}^2, m_{12}^2, m_{34}^2)}{2m_{X_1}^2} (4\pi) \\ & \times \frac{1}{2m_M} \left[\frac{1}{2^4} K_{V2}^2 f_M^2 f_{M'}^2 \frac{1}{4} [m_M^2 + m_{M'}^2 - m_{X_1}^2]^2 \frac{g^4}{4M_W^8} \frac{c_g^2 g^4}{4}\right] \\ & \times \frac{1}{m_{\Delta^{++}}} \left[\frac{1}{16\pi^2} \frac{\lambda^{1/2}(m_{12}^2, m_{l_1}^2, m_{l_2}^2)}{2m_{12}^2} (4\pi) \frac{1}{2m_{\Delta^{++}}} y_{l_1 l_2}^2 [m_{12}^2 - m_{l_1}^2 - m_{l_2}^2]\right] \\ & \times \frac{1}{m_{\Delta^{--}}} \left[\frac{1}{16\pi^2} \frac{\lambda^{1/2}(m_{34}^2, m_{l_3}^2, m_{l_4}^2)}{2m_{34}^2} (4\pi) \frac{1}{2m_{\Delta^{--}}} y_{l_3 l_4}^2 [m_{34}^2 - m_{l_3}^2 - m_{l_4}^2]\right].\end{aligned} \tag{B6}$$

Simplifying the expression we will get

$$\Gamma^{M}_{M' l_1^+ l_2^+ l_3^- l_4^-} = \frac{K_{V2}^2 f_M^2 f_{M'}^2 G_F^4 c_g^2 y_{l_1 l_2}^2 y_{l_3 l_4}^2}{\pi^7 2^{16} M_{\Delta^{++}}^4 M_{\Delta^{--}}^4 m_M}\left(1-\frac{1}{2}\delta_{l_3 l_4}\right)\left(1-\frac{1}{2}\delta_{l_1 l_2}\right)\int dm_{X_1}^2 \frac{\lambda^{1/2}(m_M^2, m_{M'}^2, m_{X_1}^2)}{m_M^2}\int dm_{12}^2 \int dm_{34}^2$$
$$\times \frac{\lambda^{1/2}(m_{X_1}^2, m_{12}^2, m_{34}^2)}{m_{X_1}^2}\frac{\lambda^{1/2}(m_{12}^2, m_{l_1}^2, m_{l_2}^2)}{m_{12}^2}\frac{\lambda^{1/2}(m_{34}^2, m_{l_3}^2, m_{l_4}^2)}{m_{34}^2}$$
$$\times (m_M^2 + m_{M'}^2 - m_{X_1}^2)^2 (m_{12}^2 - m_{l_1}^2 - m_{l_2}^2)(m_{34}^2 - m_{l_3}^2 - m_{l_4}^2). \tag{B7}$$

The limits of the above integrals are as follows:

$$m_{12}^2 \in [(m_{l_1}+m_{l_2})^2, (m_{X_1} - (m_{l_3}+m_{l_4}))^2],$$
$$m_{34}^2 \in [(m_{l_3}+m_{l_4})^2, (m_{X_1} - m_{12})^2],$$
$$m_{X_1}^2 \in [(m_{l_1}+m_{l_2}+m_{l_3}+m_{l_4})^2, (m_M - m_{M'})^2]. \tag{B8}$$

2. Using phase space method

The decay width for the process depicted in Eq. (B1) can also be computed using the phase space method. The expression for the decay width is then given by

$$\Gamma^{M}_{M' l_1^+ l_2^+ l_3^- l_4^-} = \int \frac{dm_{X_1}^2}{2\pi}\int d_{PS(M\to M' X_1^*)}\int \frac{dm_{X_2}^2}{2\pi}\int \frac{dm_{X_3}^2}{2\pi}\int d_{PS(X_1^*\to X_2^* X_3^*)}$$
$$\times \int d_{PS(X_2^*\to l_1^+ l_2^+)}\int d_{PS(X_3^*\to l_3^- l_4^-)}\frac{|A(M\to M' l_1 l_2 l_3 l_4)|^2}{2m_M}. \tag{B9}$$

In order to evaluate this expression, we need to express everything in terms of the masses. The phase space can be plugged into the above expression.

The amplitude for the case of meson decaying to four leptons is depicted as

$$A(M\to M' l_1 l_2 l_3 l_4) = \frac{1}{2^2} K_{V2} f_M f_{M'} (P_M . P_{M'}) L''(p_{l_1}, p_{l_2}, p_{l_3}, p_{l_4}). \tag{B10}$$

The leptonic part is given as

$$L''(p_{l_1}, p_{l_2}) = \left(\frac{g}{\sqrt{2}}\right)^2 \frac{1}{M_W^4}\frac{c_g g^2}{2}\frac{1}{M_{\Delta^{++}}^2}\frac{1}{M_{\Delta^{--}}^2} y_{l_1 l_2} y_{l_3 l_4} [\overline{l_{1L}}(p_{l_1}) l_{2R}(p_{l_2})][\overline{l_{3L}}(p_{l_3}) l_{4R}(p_{l_4})]. \tag{B11}$$

The amplitude squared becomes

$$|A(M\to M' l_1 l_2 l_3 l_4)|^2 = \left(\frac{1}{2^2} K_{V2} f_M f_{M'} (P_M . P_{M'})\right)^2 \left(\left(\frac{g}{\sqrt{2}}\right)^2 \frac{1}{M_W^4}\frac{c_g g^2}{2}\frac{1}{M_{\Delta^{++}}^2}\frac{1}{M_{\Delta^{--}}^2} y_{l_1 l_2} y_{l_3 l_4}\right)^2 2(p_{l_1}\cdot p_{l_2}) 2(p_{l_3}\cdot p_{l_4}). \tag{B12}$$

The dot products $(P_M \cdot P_{M'})$, $(p_{l_1}\cdot p_{l_2})$ and $(p_{l_3}\cdot p_{l_4})$ can be expressed in terms of masses using the same technique used in Eqs. (A6) and (A7) and using the momentum conservation equations $p_M = p_{M'} + p_{X_1}$, $p_{X_2} = p_{l_1} + p_{l_2}$ and $p_{X_3} = p_{l_3} + p_{l_4}$.

The final expression for the decay width is

$$\Gamma^{M}_{M' l_1^+ l_2^+ l_3^- l_4^-} = \frac{K_{V2}^2 f_M^2 f_{M'}^2 G_F^4 c_g^2 y_{l_1 l_2}^2 y_{l_3 l_4}^2}{\pi^7 2^{16} M_{\Delta^{++}}^4 M_{\Delta^{--}}^4 m_M} \left(1 - \frac{1}{2}\delta_{l_1 l_2}\right)\left(1 - \frac{1}{2}\delta_{l_3 l_4}\right)$$
$$\times \int dm_{X_1}^2 \frac{\lambda^{1/2}(m_M^2, m_{M'}^2, m_{X_1}^2)}{m_M^2} \int dm_{X_2}^2 \int dm_{X_3}^2 \frac{\lambda^{1/2}(m_{X_1}^2, m_{X_2}^2, m_{X_3}^2)}{m_{X_1}^2} \frac{\lambda^{1/2}(m_{X_2}^2, m_{l_1}^2, m_{l_2}^2)}{m_{X_2}^2} \frac{\lambda^{1/2}(m_{X_3}^2, m_{l_3}^2, m_{l_4}^2)}{m_{X_3}^2}$$
$$\times (m_M^2 + m_{M'}^2 - m_{X_1}^2)^2 (m_{X_2}^2 - m_{l_1}^2 - m_{l_2}^2)(m_{X_3}^2 - m_{l_3}^2 - m_{l_4}^2). \quad \text{(B13)}$$

The limits of the above integrals are as follows:

$$m_{X_1}^2 \in [(m_{l_1} + m_{l_2} + m_{l_3} + m_{l_4})^2, (m_M - m_{M'})^2],$$
$$m_{X_2}^2 \in [(m_{l_1} + m_{l_2})^2, (m_{X_1} - m_{X_3})^2],$$
$$m_{X_3}^2 \in [(m_{l_3} + m_{l_4})^2, (m_{X_1} - m_{l_1} - m_{l_2})^2]. \quad \text{(B14)}$$

We find that both procedures lead to the same results for $\Gamma^{M}_{M' l_1^+ l_2^+ l_3^- l_4^-}$. To compare Eqs. (B13) and (B7), make the following replacements $m_{X_2} \leftrightarrow m_{12}$ and $m_{X_3} \leftrightarrow m_{34}$.

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