

Massive neutrinos and invisible axion minimally connectedStefano Bertolini,^{1,*} Luca Di Luzio,^{2,†} Helena Kolečová,^{3,4,5,‡} and Michal Malinsky^{4,§}¹*INFN, Sezione di Trieste, SISSA, Via Bonomea 265, 34136 Trieste, Italy*²*Dipartimento di Fisica, Università di Genova and INFN, Sezione di Genova, Via Dodecaneso 33, 16159 Genova, Italy*³*Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague, Břehová 7, 115 19 Praha 1, Czech Republic*⁴*Institute of Particle and Nuclear Physics, Faculty of Mathematics and Physics, Charles University in Prague, V Holešovičkách 2, 180 00 Praha 8, Czech Republic*⁵*SISSA/ISAS, Via Bonomea 265, 34136 Trieste, Italy*

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We survey a few minimal scalar extensions of the standard electroweak model that provide a simple setup for massive neutrinos in connection with an invisible axion. The presence of a chiral $U(1)$ à la Peccei-Quinn drives the pattern of Majorana neutrino masses while providing a dynamical solution to the strong CP problem and an axion as a dark matter candidate. We paradigmatically apply such a renormalizable framework to type-II seesaw and to two viable models for neutrino oscillations where the neutrino masses arise at one and two loops, respectively. We comment on the naturalness of the effective setups as well as on their implications for vacuum stability and electroweak baryogenesis.

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I. INTRODUCTION

The first LHC run has led to the discovery of a scalar particle that looks much like the Higgs boson of the $SU(2)_L \otimes U(1)_Y$ electroweak standard model (SM). The raising limits on exotic physics scales set a challenge to the popular issue of naturalness [1], a paradigm that has guided much of the beyond the SM modeling in the last decades. This notwithstanding, neutrino oscillations and dark matter call for physics beyond the standard scenario. Baryon asymmetry calls for it as well while electroweak vacuum stability may not be an issue in minimally extended scenarios [2]. We aim at discussing a class of minimal extensions of the SM that account for the aforementioned open issues. To this end we choose to maintain the fermionic SM content as it stands and consider only extensions of the scalar sector. Advantages of this choice will be clear in the following. According to that, the only tree-level realization of the dimension-five Weinberg operator $(LLHH)/M$ for Majorana neutrino masses is via the mediation of an $SU(2)_L$ scalar triplet of hypercharge one. This is commonly known as the type-II (TII) seesaw [3–7], Fig. 1(a).

At the radiative level an elegant and simple realization of the same was provided long ago by Zee [8]; the Weinberg operator is there obtained at one loop from the dimension-seven effective operator $(LLL e^c H)/M^3$ [9–11] when L and e^c are connected by the H Yukawa coupling (giving

rise to a chiral suppression), as shown in Fig. 1(b). The model requires one additional weak doublet and a weak scalar singlet of hypercharge one. In order to avoid Higgs mediated flavor changing neutral currents a Z_2 symmetry is called for [12]. Such a model, however, is not consistent with the neutrino oscillation data [13–15]. Recently, Babu and Julio (BJ) [16] presented a variant of the Zee model with a Z_4 discrete family symmetry that restores consistency with the observed neutrino mixing pattern. The model yields an inverted neutrino mass hierarchy and is highly predictive for neutrinoless double beta decay and lepton flavor violation (LFV).

At two loops, a popular realization of the Weinberg operator is given by the Zee-Babu (ZB) model [17,18]. In this setting, the neutrino mass matrix is obtained by dressing the dimension-nine effective operator $(LLL e^c Le^c)/M^5$ [10,19] and it requires two weak scalar singlets with hypercharge one and two, respectively [Fig. 1(c)]. It is a very simple extension of the SM that leads to calculable neutrino masses and mixings in agreement with all present oscillation and lepton flavor phenomenology (for a recent reappraisal see [20,21]).

Our interest is to discuss simple renormalizable extensions of the standard scenario that are effective at the TeV scale and lead to testable signals at the available energy and foreseen intensity facilities. There is one inherent large scale involved that is linked to the presence of a spontaneously broken Peccei-Quinn (PQ) symmetry [22,23] and the related axion [24,25]. As we shall discuss, it is noteworthy that the presence of such a large scale (above 10^9 GeV) does not endanger the radiative stability of the setup. While the anomalous $U(1)_{PQ}$ gives an elegant

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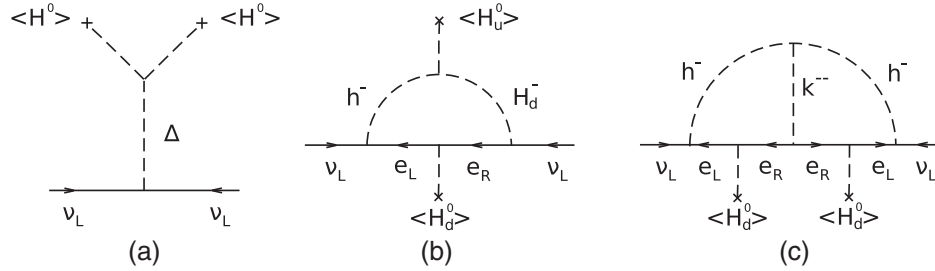


FIG. 1. Sample diagrams leading to the $\Delta L = 2$ Weinberg operator at the tree level (a), one loop (b), and two loops (c) in the type-II seesaw, Zee, and Zee-Babu models, respectively.

solution to the so-called strong CP problem in QCD [26–30], the axion provides a viable dark matter candidate (see [31] for a recent review). We find it appealing and intriguing that a simple renormalizable framework can be conceived where the origin of neutrino masses and the solution of the strong CP problem are fundamentally related and where the requirement of naturalness and stability of the scalar sector is tightly linked to the light neutrino scale.

The idea of connecting massive neutrinos with the presence of a spontaneously broken $U(1)_{PQ}$ comes a long way [32–46]. Considering only scalar extensions of the SM, a simple setup based on the Zee model for radiative neutrino masses was discussed in [39,40]. The model features a Dine, Fischler, Srednicki, and Zhitnitsky (DFSZ)¹ invisible axion [49,50], with a tiny coupling to neutrinos. The need for two different Higgs doublets and the role of the related Z_2 symmetry is there a free benefit of the minimal implementation of the anomalous PQ symmetry. Two additional neutral and two singly charged scalars remain naturally light (TeV scale). In spite of the presence of the large PQ scale the model is shown to exhibit a radiatively stable hierarchy. In all analogy with the Zee model, a simple Majorana neutrino mass matrix with vanishing diagonal entries arises at one loop, whose structure is determined by three parameters. As already mentioned such a structure is shown to exhibit nearly bimaximal mixing and it is ruled out by oscillation data.

In this paper we show how this setup can work in general. We discuss three explicit viable schemes: the paradigmatic low-scale TII seesaw, the one-loop BJ model, and the two-loop ZB model. In the extended BJ model a lepton-family-dependent PQ symmetry plays the role of the original Z_4 symmetry. In all cases one obtains a DFSZ invisible axion with a tiny coupling to neutrinos. In the BJ case the axion exhibits flavor violating couplings to the leptons of the same size of the diagonal ones. Such flavor violating couplings are not directly constrained by astrophysical processes and future laboratory tests of LFV might

even provide competitive bounds on the PQ scale [51]. In addition to a heavy neutral scalar (mainly) singlet the physical scalar spectrum exhibits in the three models two singly charged and two additional neutral states. In the case of TII and ZB a doubly charged scalar is present as well with a distinctive role in LFV phenomenology.

Stability of the scalar sector demands tiny interactions between the PQ heavy state and the remaining scalars. Because of an enhanced symmetry in the vanishing interaction limit, the smallness of the relevant couplings is preserved at higher orders. Remarkably, such a setup allows for naturally light neutrinos together with a rich scalar spectrum at the TeV scale. The possible presence of an exotic TeV-scale scalar sector is not yet excluded by collider searches and it is among the priorities in the coming years.

A fringe benefit of such an extension of the standard scalar sector is to improve the electroweak vacuum stability. On the other hand, the sizable interactions among the “light” scalar states open a possibility for the realization of a first-order electroweak phase transition. This is one of the requirements for electroweak baryogenesis [52]. However, no new sources of CP violation arise from the minimal scalar sectors featured in the considered setups. We shall comment on the possibility of addressing baryogenesis within such a framework.

In the next three sections we detail the extended TII, BJ, and ZB setups and discuss their generic features and shortcomings in Sec. V.

II. PQ-EXTENDED TYPE-II SEESAW

On top of the usual SM field content, the scalar sector of the PQ-extended type-II seesaw model features two Higgs doublets, an isospin triplet with hypercharge one and a SM singlet (cf. Table I). The PQ-charge assignments are displayed in Table I, where the presence of Yukawa interactions for quarks is already taken into account. Recall that the PQ current is axial, thus proportional to the difference between the charges of the left- and right-handed (colored) fermions. Hence, without loss of generality, we can always set $X_q = 0$. In this way, the color anomaly of the PQ current is proportional to $X_u + X_d$ (see, e.g., [53]).

¹No extension of the matter sector is needed at variance with the class of invisible axion models proposed by Kim, Shifman, Vainshtein and Zakharov [47,48] that feature a vectorlike quark.

TABLE I. Field content and charge assignment of the PQ-extended type-II seesaw model.

Field	Spin	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{PQ}$
q_L	$\frac{1}{2}$	3	2	$+\frac{1}{6}$	0
u_R	$\frac{1}{2}$	3	1	$+\frac{2}{3}$	X_u
d_R	$\frac{1}{2}$	3	1	$-\frac{1}{3}$	X_d
ℓ_L	$\frac{1}{2}$	1	2	$-\frac{1}{2}$	X_ℓ
e_R	$\frac{1}{2}$	1	1	-1	X_e
H_u	0	1	2	$-\frac{1}{2}$	$-X_u$
H_d	0	1	2	$+\frac{1}{2}$	$-X_d$
Δ	0	1	3	+1	X_Δ
σ	0	1	1	0	X_σ

A. Lagrangian

The only two sectors which are sensitive to the assignment of the PQ charges are the Yukawa Lagrangian and the scalar potential that we discuss in turn. The former reads

$$\begin{aligned}
 V_{\text{TH}} = & -\mu_1^2 |H_u|^2 + \lambda_1 |H_u|^4 - \mu_2^2 |H_d|^2 + \lambda_2 |H_d|^4 + \lambda_{12} |H_u|^2 |H_d|^2 + \lambda_4 |H_u^\dagger H_d|^2 - \mu_3^2 |\sigma|^2 + \lambda_3 |\sigma|^4 + \lambda_{13} |\sigma|^2 |H_u|^2 \\
 & + \lambda_{23} |\sigma|^2 |H_d|^2 + \text{Tr}(\Delta^\dagger \Delta) [\mu_\Delta^2 + \lambda_{\Delta 1} |H_u|^2 + \lambda_{\Delta 2} |H_d|^2 + \lambda_{\Delta 3} |\sigma|^2 + \lambda_{\Delta 4} \text{Tr}(\Delta^\dagger \Delta)] + \lambda_7 H_u^\dagger \Delta \Delta^\dagger H_u \\
 & + \lambda_8 H_d^\dagger \Delta \Delta^\dagger H_d + \lambda_9 \text{Tr}(\Delta^\dagger \Delta)^2 + (\lambda_5 \sigma^2 \tilde{H}_u^\dagger H_d + \lambda_6 \sigma H_u^\dagger \Delta^\dagger H_d + \text{H.c.}),
 \end{aligned} \tag{3}$$

where we employed the notation $\tilde{H}_u = i\tau_2 H_u^*$. Notice that terms like $\tilde{H}_u^\dagger H_d \text{Tr}(\Delta^\dagger \Delta)$ or $\tilde{H}_u^\dagger \Delta \Delta^\dagger H_d$ are not allowed since the QCD anomaly of the PQ current requires $X_u + X_d \neq 0$. Moreover, $H_{u,d}^\dagger (\Delta^\dagger \Delta + \Delta \Delta^\dagger) H_{u,d} = |H_{u,d}|^2 \text{Tr}(\Delta^\dagger \Delta)$, so that only two invariants out of three are linearly independent.

The terms $\lambda_5 \sigma^2 \tilde{H}_u^\dagger H_d$ and $\lambda_6 \sigma H_u^\dagger \Delta^\dagger H_d$ are needed in order to assign a nonvanishing PQ charge to the singlet σ and to generate neutrino masses. Notice that the simultaneous presence of λ_5 , λ_6 , and Y_Δ is needed to explicitly break lepton number. If any of the couplings are missing, either the lepton number is exact and neutrinos are massless or the lepton number is spontaneously broken and the vacuum exhibits a Majoron together with a Wilczek-Weinberg axion [39]. As shown next, the potential in Eq. (3) corresponds to a unique PQ-charge assignment that forbids among else the presence of trilinear interaction terms. The absence of cubic scalar interactions, which characterizes the three models here discussed, paves the way to their embedding in a classically scale invariant setup spontaneously broken *à la* Coleman-Weinberg. We shall comment on that in Sec. VI.

Finally, the couplings λ_5 and λ_6 can be set real by two independent rephasings of the fields.

B. PQ charges

The invariants in Eq. (1) and Eq. (3) enforce the following constraints on the PQ charges:

$$\begin{aligned}
 -\mathcal{L}_Y^{\text{TH}} = & Y_u \bar{q}_L u_R H_u + Y_d \bar{q}_L d_R H_d + Y_e \bar{\ell}_L e_R H_d \\
 & + \frac{1}{2} Y_\Delta \ell_L^T C i \tau_2 \Delta \ell_L + \text{H.c.},
 \end{aligned} \tag{1}$$

where the flavor contractions are understood (e.g., $Y_\Delta^T = Y_\Delta$), C is the charge conjugation matrix in the spinor space, and

$$\Delta \equiv \frac{\vec{\tau} \cdot \vec{\Delta}}{\sqrt{2}} = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}. \tag{2}$$

In Eq. (2), $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ are the Pauli matrices and $\vec{\Delta} = (\Delta_1, \Delta_2, \Delta_3)$ are the $SU(2)_L$ components of the scalar triplet. The electric charge eigenstates are obtained by the action of $Q = T_3 + Y$ on Eq. (2).

The scalar potential can be written as [39,54]

$$-X_\ell + X_e - X_d = 0, \tag{4}$$

$$2X_\ell + X_\Delta = 0, \tag{5}$$

$$2X_\sigma - X_u - X_d = 0, \tag{6}$$

$$X_\sigma + X_u - X_\Delta - X_d = 0. \tag{7}$$

Solving in terms of X_u and X_d we get

$$\begin{aligned}
 X_\ell = & -\frac{3X_u}{4} + \frac{X_d}{4}, & X_e = & -\frac{3X_u}{4} + \frac{5X_d}{4}, \\
 X_\Delta = & \frac{3X_u}{2} - \frac{X_d}{2}, & X_\sigma = & \frac{X_u}{2} + \frac{X_d}{2}.
 \end{aligned} \tag{8}$$

Following [39,50], we require the orthogonality of the hypercharge and axion currents. This leads to the relation

$$X_u v_u^2 = X_d v_d^2, \tag{9}$$

where $v_u = \langle H_u \rangle$ and $v_d = \langle H_d \rangle$. Adding this condition to Eq. (8), we can determine all the PQ charges up to an overall normalization. We choose this normalization by the condition

$$X_\sigma = 1. \tag{10}$$

By defining $x \equiv v_u/v_d$ the remaining charges in Eq. (8) read

$$\begin{aligned}
 X_u &= \frac{2}{x^2 + 1}, & X_d &= \frac{2x^2}{x^2 + 1}, & X_\ell &= \frac{x^2 - 3}{2(x^2 + 1)}, \\
 X_e &= \frac{5x^2 - 3}{2(x^2 + 1)}, & X_\Delta &= \frac{3 - x^2}{x^2 + 1}.
 \end{aligned}
 \quad (11)$$

C. Scalar spectrum

To compute the scalar spectrum we expand the scalar fields around the chargeless and CP -conserving vacuum configuration

$$H_u = \begin{pmatrix} v_u + \frac{h_u^0 + ih_u^+}{\sqrt{2}} \\ h_u^- \end{pmatrix}, \quad (12)$$

$$H_d = \begin{pmatrix} h_d^+ \\ v_d + \frac{h_d^0 + ih_d^-}{\sqrt{2}} \end{pmatrix}, \quad (13)$$

$$\Delta = \begin{pmatrix} \frac{\delta^+}{\sqrt{2}} & \delta^{++} \\ v_\Delta + \frac{\delta^0 + ih_\Delta^0}{\sqrt{2}} & -\frac{\delta^+}{\sqrt{2}} \end{pmatrix}, \quad (14)$$

$$\sigma = V_\sigma + \frac{\sigma^0 + ih_\sigma^0}{\sqrt{2}}, \quad (15)$$

with v_u , v_d , v_Δ , and V_σ denoting the relevant (real) vacuum expectation values (VEVs).²

The scalar spectrum of the model is detailed in the Appendix and the main features are discussed in Sec. V. Here we just anticipate that the model features a DFSZ invisible axion, with a tiny coupling to neutrinos, and its SM singlet companion with a PQ-scale mass. By invoking a technically natural ultraweak limit (see the discussion in Sec. VA) such a heavy scalar is sufficiently decoupled from all the other physical scalar states that are requested to live at the TeV scale, thus preserving the radiative stability of the light scalar spectrum. At the weak scale the model allows for a SM-like Higgs boson; this, together with a brief account of the relevant phenomenological constraints, shall be discussed in Sec. VC.

D. Neutrino masses

In the TII model, the neutrino masses are generated through the tree-level diagram in Fig. 2.

Their expression is conveniently obtained by computing the (induced) VEV of the triplet. Let us hence consider the projection of the scalar potential along the neutral VEV components of Eqs. (12)–(15)

²While it is assumed that there exists a region of the scalar potential parameters for which the absolute minimum preserves the electric charge, it can be shown (see Sec. VD) that the potential of Eq. (3) does not lead to spontaneous CP violation.

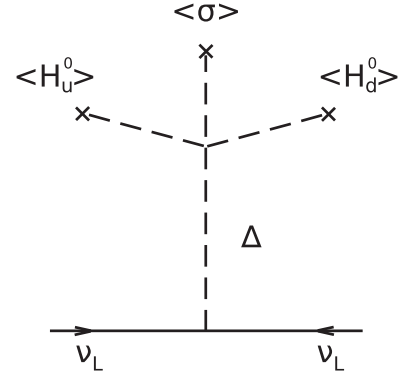


FIG. 2. The tree-level “hug” diagram responsible for the Majorana neutrino mass in the PQ-extended type-II seesaw model.

$$\begin{aligned}
 \langle V_{\text{TII}} \rangle &= (\mu_\Delta^2 + \lambda_{\Delta 3} V_\sigma^2 + \lambda_{\Delta 1} v_u^2 + (\lambda_{\Delta 2} + \lambda_8) v_d^2) v_\Delta^2 \\
 &\quad + 2\lambda_6 V_\sigma v_u v_d v_\Delta + \mathcal{O}(v_\Delta^4) + v_\Delta\text{-indep. terms.}
 \end{aligned}
 \quad (16)$$

Given the phenomenological hierarchy $V_\sigma \gg v_{u,d} \gg v_\Delta$, the stationary condition with respect to v_Δ is well approximated by

$$2M_\Delta^2 v_\Delta + 2\lambda_6 V_\sigma v_u v_d \approx 0, \quad (17)$$

where we defined the effective mass parameter

$$M_\Delta^2 = \mu_\Delta^2 + \lambda_{\Delta 3} V_\sigma^2 + \lambda_{\Delta 1} v_u^2 + (\lambda_{\Delta 2} + \lambda_8) v_d^2. \quad (18)$$

In the decoupling limit $v_{u,d}/V_\sigma \rightarrow 0$, this coincides with the triplet mass in the PQ-broken phase [cf. Eq. (A4)]. Hence, from Eq. (17), the induced VEV of Δ reads

$$v_\Delta \approx \frac{\lambda_6 V_\sigma v_u v_d}{M_\Delta^2}. \quad (19)$$

Since the triplet VEV breaks the SM custodial symmetry, it is bounded by the electroweak precision observables to $v_\Delta \lesssim 1$ GeV. This, in turn, implies the following bound on the scalar potential coupling λ_6 :

$$\lambda_6 \lesssim 10^{-9} \left(\frac{10^9 \text{ GeV}}{V_\sigma} \right) \left(\frac{M_\Delta^2}{v_u v_d} \right). \quad (20)$$

Finally, from the Yukawa Lagrangian in Eq. (1) we obtain

$$M_\nu^{\text{TII}} = Y_\Delta v_\Delta \approx \frac{Y_\Delta \lambda_6 V_\sigma v_u v_d}{M_\Delta^2}, \quad (21)$$

as diagrammatically represented by the graph in Fig. 2, and the bound on the heaviest neutrino $m_{\nu_3} \lesssim 1$ eV translates into the constraint

$$\lambda_6 Y_\Delta \lesssim 10^{-18} \left(\frac{10^9 \text{ GeV}}{V_\sigma} \right) \left(\frac{M_\Delta^2}{v_u v_d} \right). \quad (22)$$

The smallness of the absolute neutrino mass scale may have different sources. In this paper we take the point of view of building low-energy renormalizable setups that are technically natural. In this respect, the lightness of M_Δ (in the vicinity of the electroweak scale) and the smallness of the couplings of the SM singlet σ with the doublet and triplet states (among which is λ_6) are a required prerequisite (see Sec. VA). The triplet Yukawa coupling Y_Δ is also constrained by tree-level LFV (see Sec. VC).

III. PQ-EXTENDED BABU-JULIO MODEL

We shall here introduce a simple PQ extension of the model of Ref. [16], which is a special case of the general Zee model [8]. For convenience, we display the field content and the relative PQ charges in Table II, where $\alpha = 1, 2, 3$ denotes the family index and $X_u + X_d \neq 0$ in order to obtain a nonvanishing QCD anomaly. The non-universal assignment of the PQ charges in the leptonic sector replaces the role of the Z_4 symmetry employed in [16].

A. Lagrangian

The relevant Yukawa Lagrangian reads [16]

$$-\mathcal{L}_Y^{\text{BJ}} = Y_u \bar{q}_L u_R H_u + Y_d \bar{q}_L d_R H_d + Y_{\beta\alpha} \bar{\ell}_L^\beta e_R^\alpha H_d + Y_{1\alpha} \bar{\ell}_L^1 e_R^\alpha \tilde{H}_u + f_{23} (\ell_L^2)^T i\tau_2 C \ell_L^3 h^+ + \text{H.c.}, \quad (23)$$

where $\alpha = 1, 2, 3$ and $\beta = 2, 3$ label interaction eigenstates. The analogous contractions in the quark sector are understood.

In the scalar potential, the terms $\sigma^2 \tilde{H}_u^\dagger H_d$ and $\sigma h^- H_u^\dagger H_d$ are required. The former is needed to assign a nonvanishing PQ charge to the singlet σ , while the latter is needed to generate neutrino masses at one loop. Notice that the

TABLE II. Field content and charge assignment of the PQ-extended Babu-Julio model.

Field	Spin	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{\text{PQ}}$
q_L^α	$\frac{1}{2}$	3	2	$+\frac{1}{6}$	0
u_R^α	$\frac{1}{2}$	3	1	$+\frac{2}{3}$	X_u
d_R^α	$\frac{1}{2}$	3	1	$-\frac{1}{3}$	X_d
ℓ_L^1	$\frac{1}{2}$	1	2	$-\frac{1}{2}$	X_{ℓ_1}
$\ell_L^{2,3}$	$\frac{1}{2}$	1	2	$-\frac{1}{2}$	$X_{\ell_{2,3}}$
e_R^α	$\frac{1}{2}$	1	1	-1	X_e
H_u	0	1	2	$-\frac{1}{2}$	$-X_u$
H_d	0	1	2	$+\frac{1}{2}$	$-X_d$
h^+	0	1	1	+1	X_h
σ	0	1	1	0	X_σ

simultaneous presence of these scalar interactions, together with f_{23} , breaks the lepton number explicitly.

The remaining part of the scalar potential contains only moduli terms and coincides with that of Ref. [39], namely,

$$V_{\text{BJ}} = -\mu_1^2 |H_u|^2 + \lambda_1 |H_u|^4 - \mu_2^2 |H_d|^2 + \lambda_2 |H_d|^4 + \lambda_{12} |H_u|^2 |H_d|^2 + \lambda_4 |H_u^\dagger H_d|^2 - \mu_3^2 |\sigma|^2 + \lambda_3 |\sigma|^4 + \lambda_{13} |\sigma|^2 |H_u|^2 + \lambda_{23} |\sigma|^2 |H_d|^2 + |h|^2 (\mu_4^2 + \lambda_{41} |H_u|^2 + \lambda_{42} |H_d|^2 + \lambda_{43} |\sigma|^2 + \lambda_{44} |h|^2) + (\lambda_5 \sigma^2 \tilde{H}_u^\dagger H_d + \lambda_6 \sigma h^- H_u^\dagger H_d + \text{H.c.}). \quad (24)$$

The couplings λ_5 and λ_6 can be set real by two independent rephasings of the fields. For a detailed discussion of the symmetry breaking patterns and the scalar spectrum we refer the reader directly to Sec. 3 of [39].

The Higgs content of the original model is extended by just the SM singlet σ . The discussion here is rather similar to that in Sec. II C with the only difference that, given the lepton family dependence of the PQ symmetry, the axion here exhibits flavor nondiagonal couplings to leptons; for more details see Sec. V.

B. PQ charges

The devised $U(1)_{\text{PQ}}$ invariance of the Lagrangian leads to the following constraints on the PQ charges:

$$-X_{\ell_{2,3}} + X_e - X_d = 0, \quad (25)$$

$$-X_{\ell_1} + X_e + X_u = 0, \quad (26)$$

$$2X_{\ell_{2,3}} + X_h = 0, \quad (27)$$

$$2X_\sigma - X_u - X_d = 0, \quad (28)$$

$$X_\sigma - X_h + X_u - X_d = 0. \quad (29)$$

Solving these in terms of X_u and X_d one obtains

$$X_{\ell_1} = \frac{X_u}{4} + \frac{5X_d}{4}, \quad X_{\ell_{2,3}} = -\frac{3X_u}{4} + \frac{X_d}{4}, \\ X_e = -\frac{3X_u}{4} + \frac{5X_d}{4}, \quad X_h = \frac{3X_u}{2} - \frac{X_d}{2}, \\ X_\sigma = \frac{X_u}{2} + \frac{X_d}{2}. \quad (30)$$

As before, cf. Sec. II B, we require the orthogonality of the hypercharge and axion currents and fix the overall normalization of the charges by the condition $X_\sigma = 1$, which yields

$$\begin{aligned}
 X_{\ell_1} &= \frac{5x^2 + 1}{2(x^2 + 1)}, & X_{\ell_{23}} &= \frac{x^2 - 3}{2(x^2 + 1)}, \\
 X_e &= \frac{5x^2 - 3}{2(x^2 + 1)}, & X_h &= \frac{3 - x^2}{x^2 + 1}.
 \end{aligned} \quad (31)$$

The X_u and X_d charges are identical to those of the TII model and, as such, they are given in Eq. (11).

C. Neutrino masses

The radiatively induced neutrino mass matrix is found to be [16]

$$M_\nu^{\text{BJ}} = \kappa(\hat{f}M_\ell^{\text{diag}}\hat{Y}^T + \hat{Y}M_\ell^{\text{diag}}\hat{f}^T), \quad (32)$$

where M_ℓ^{diag} is the diagonal charged-lepton mass matrix and \hat{f} and \hat{Y} are the Yukawa coupling matrices transformed into the mass basis of the fields running in the loop (cf. Fig. 3). The main difference with respect to the Z_2 -assisted Zee model [8,12] is that \hat{Y} is nondiagonal, so that the anti-symmetric texture of \hat{f} is not transmitted into the neutrino mass matrix. This allows for nonvanishing diagonal entries at the leading order, a key feature for the consistency of the BJ model with the neutrino oscillation data. The overall factor κ involves the loop function and is given by [16]

$$\kappa = \frac{\sin 2\gamma}{16\pi^2} \log\left(\frac{M_1^2}{M_2^2}\right), \quad (33)$$

where $M_{1,2}$ are the masses of the physical charged scalar states and γ denotes the mixing angle between h^- and H^- obeying (cf. [39])

$$\sin 2\gamma = \frac{2\lambda_6 V_\sigma \sqrt{v_u^2 + v_d^2}}{M_1^2 - M_2^2}. \quad (34)$$

Interestingly, the structure of the neutrino mass matrix in Eq. (32) is very constrained. Albeit with nonvanishing

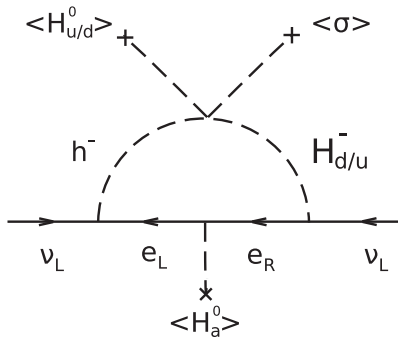


FIG. 3. Sample one-loop diagram responsible for the Majorana neutrino mass in the PQ-extended Babu-Julio model. Because of the family dependence of the PQ charges, both Higgs doublets couple to the first leptonic family.

diagonal entries, the mass matrix turns out to be traceless and real so that all the neutrino oscillation data can be described in terms of four real parameters [16]. This leads to several predictions: the neutrino mass hierarchy is predicted to be inverted, the Dirac CP -violating phase is fixed to $\delta_{CP} = \pi$ and, moreover, there is a relation among the three mixing angles, namely, $|U_{\tau 1}| = |U_{\tau 2}|$, allowing one of them to be expressed in terms of the other two. The consequences for neutrinoless double beta decay and LFV processes have been systematically worked out in Ref. [16].

As in the TII case, the smallness of neutrino masses can have different sources. If the charged scalar states are not far from the electroweak scale, as suggested by the naturalness arguments, the suppression must come from the scalar potential coupling λ_6 and/or the Yukawa matrices \hat{Y} and \hat{f} . Remarkably, the smallness of the coupling λ_6 is a necessary condition for a technically natural spectrum (see Sec. VA), while the Yukawa couplings \hat{Y} and \hat{f} are sharply constrained by neutrino oscillation data and LFV processes [16].

IV. PQ-EXTENDED ZEE-BABU MODEL

The last case we are going to consider is the PQ extension of the ZB model. The field content and the PQ charges are collected for convenience in Table III.

A. Yukawa interactions

The Yukawa Lagrangian of the ZB model reads [20]

$$\begin{aligned}
 -\mathcal{L}_Y^{\text{ZB}} &= Y_u \bar{q}_L u_R H_u + Y_d \bar{q}_L d_R H_d + Y_e \bar{\ell}_L e_R H_d \\
 &\quad + f \ell_L^T C i \tau_2 \ell_L h^+ + g e_R^T C e_R k^{++} + \text{H.c.},
 \end{aligned} \quad (35)$$

where Y_u , Y_d , Y_e , f , and g are matrices in the generation space (flavor indexes are understood). In particular, f is antisymmetric while g is symmetric. Equation (35) yields the following relations among the PQ charges of the fields involved:

TABLE III. Field content and charge assignment of the PQ-extended Zee-Babu model.

Field	Spin	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{\text{PQ}}$
q_L	$\frac{1}{2}$	3	2	$+\frac{1}{6}$	0
u_R	$\frac{1}{2}$	3	1	$+\frac{2}{3}$	X_u
d_R	$\frac{1}{2}$	3	1	$-\frac{1}{3}$	X_d
ℓ_L	$\frac{1}{2}$	1	2	$-\frac{1}{2}$	X_ℓ
e_R	$\frac{1}{2}$	1	1	-1	X_e
H_u	0	1	2	$-\frac{1}{2}$	$-X_u$
H_d	0	1	2	$+\frac{1}{2}$	$-X_d$
h^+	0	1	1	+1	X_h
k^{++}	0	1	1	+2	X_k
σ	0	1	1	0	X_σ

$$\begin{aligned}
 X_e - X_\ell - X_d &= 0, \\
 2X_\ell + X_h &= 0, \\
 2X_e + X_k &= 0.
 \end{aligned} \tag{36}$$

B. Scalar potential

As before, the scalar potential is restricted by the requirement of the $U(1)_{\text{PQ}}$ invariance. However, different assignments of the PQ charges allow for different terms. In particular, we consider

$$\begin{aligned}
 V_{\text{ZB}} &= -\mu_1^2 |H_u|^2 + \lambda_1 |H_u|^4 - \mu_2^2 |H_d|^2 + \lambda_2 |H_d|^4 \\
 &\quad + \lambda_{12} |H_u|^2 |H_d|^2 + \lambda_4 |H_u^\dagger H_d|^2 \\
 &\quad - \mu_3 |\sigma|^2 + \lambda_3 |\sigma|^4 + \lambda_{13} |\sigma|^2 |H_u|^2 + \lambda_{23} |\sigma|^2 |H_d|^2 \\
 &\quad + |h|^2 (\mu_4^2 + \lambda_{41} |H_u|^2 + \lambda_{42} |H_d|^2 + \lambda_{43} |\sigma|^2 + \lambda_{44} |h|^2) \\
 &\quad + |k|^2 (\mu_5^2 + \lambda_{51} |H_u|^2 + \lambda_{52} |H_d|^2 + \lambda_{53} |\sigma|^2 \\
 &\quad + \lambda_{54} |h|^2 + \lambda_{55} |k|^2) + (V_{\text{var}} + \text{H.c.}),
 \end{aligned} \tag{37}$$

with two different forms of the V_{var} part therein:

$$(i) \quad V_{\text{var}} = \lambda_5 \sigma^2 H_d^\dagger \tilde{H}_u + \lambda_6 \sigma h^- H_u^\dagger H_d + \lambda_7 \sigma (h^+)^2 k^{--}$$

In this case the additional equations restricting the PQ charges read

$$\begin{aligned}
 2X_\sigma + X_u + X_d &= 0, \\
 X_\sigma - X_h + X_u - X_d &= 0, \\
 X_\sigma + 2X_h - X_k &= 0.
 \end{aligned} \tag{38}$$

These, together with Eq. (36), represent a system of six equations for six variables (letting, e.g., X_d to be a free parameter): X_σ , X_h , X_k , X_e , X_ℓ , and X_u . Were these equations linearly independent, there would be a unique solution (up to the normalization of X_d) proportional to the SM hypercharge and, hence, $X_\sigma = 0$. Consequently, in order for σ to get a nontrivial PQ charge, the system of Eqs. (36) and (38) must be linearly dependent, which is indeed the case. Hence, we are allowed to set X_σ nonvanishing and we get

$$\begin{aligned}
 X_h &= -2X_d - X_\sigma, & X_k &= -4X_d - X_\sigma, \\
 X_e &= 2X_d + \frac{1}{2}X_\sigma, & X_\ell &= X_d + \frac{1}{2}X_\sigma, \\
 X_u &= -X_d - 2X_\sigma.
 \end{aligned} \tag{39}$$

Notice that the shape of V_{var} in case (i) is, in a sense, exceptional as it yields a linearly dependent set of equations for the PQ charges. Were we to replace $\sigma \rightarrow \sigma^*$ in any one (or any two) of the terms in V_{var} above (which is allowed by SM symmetries)

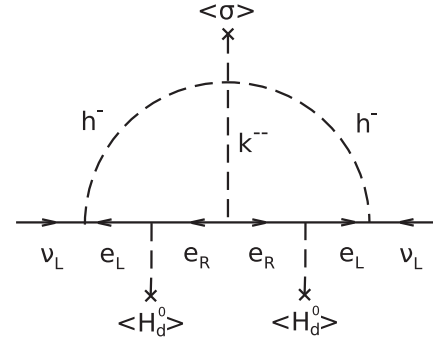


FIG. 4. Two-loop diagram responsible for the neutrino Majorana mass in the PQ-extended Zee-Babu model.

the resulting system of equations would be linearly independent, leaving as the only solution the one with the PQ charges proportional to the SM hypercharge. In such a case, in order to obtain a nontrivial assignment of the PQ charges, the number of conditions must be reduced by setting one of the couplings $\lambda_{5,6,7}$ to zero.

Unlike the term proportional to λ_7 which is required by the two-loop ZB diagram (see Fig. 4) the term proportional to λ_6 is, strictly speaking, not necessary (as a matter of fact, it induces an additional one-loop contribution to neutrino masses as in the Zee model³). Hence, in the following we will focus on an alternative PQ-charge assignment where $\lambda_6 = 0$.

$$(ii) \quad V_{\text{var}} = \lambda_5 (\sigma^*)^2 H_d^\dagger \tilde{H}_u + \lambda_7 \sigma (h^+)^2 k^{--}$$

In this case the equations relating the PQ charges read

$$\begin{aligned}
 -2X_\sigma + X_u + X_d &= 0, \\
 X_\sigma + 2X_h - X_k &= 0.
 \end{aligned} \tag{40}$$

By solving Eqs. (36) and (40) in terms of X_u and X_d we get

$$\begin{aligned}
 X_\ell &= \frac{X_u}{4} + \frac{5X_d}{4}, & X_e &= \frac{X_u}{4} + \frac{9X_d}{4}, \\
 X_h &= -\frac{X_u}{2} - \frac{5X_d}{2}, & X_k &= -\frac{X_u}{2} - \frac{9X_d}{2}, \\
 X_\sigma &= \frac{X_u}{2} + \frac{X_d}{2}.
 \end{aligned} \tag{41}$$

If we again require the orthogonality of the hypercharge and axion currents and choose the normalization $X_\sigma = 1$, we obtain the following expressions in terms of $x = v_u/v_d$

³Notice that this term is not present in the original ZB model, which features only one Higgs doublet [17,18].

$$\begin{aligned} X_e &= \frac{5x^2 + 1}{2(x^2 + 1)}, & X_e &= \frac{9x^2 + 1}{2(x^2 + 1)}, \\ X_h &= \frac{-5x^2 - 1}{x^2 + 1}, & X_k &= \frac{-9x^2 - 1}{x^2 + 1}, \end{aligned} \quad (42)$$

with X_u and X_d given as in Eq. (11).

From now on we will only consider the shape of the scalar potential in case (ii) (that leads to the extended ZB model), and we will briefly comment on the differences with respect to the case (i) in Sec. IV D.

C. Scalar spectrum

Adopting the notation of Eqs. (12), (13), and (15) for the relevant fields, the stationarity conditions read

$$-\lambda_5 v_d V_\sigma^2 + v_u (\lambda_{12} v_d^2 + \lambda_{13} V_\sigma^2 + 2\lambda_1 v_u^2 - \mu_1^2) = 0, \quad (43)$$

$$-\lambda_5 v_u V_\sigma^2 + v_d (\lambda_{12} v_u^2 + \lambda_{23} V_\sigma^2 + 2\lambda_2 v_d^2 - \mu_2^2) = 0, \quad (44)$$

$$V_\sigma (\lambda_{13} v_u^2 + \lambda_{23} v_d^2 + 2\lambda_3 V_\sigma^2 - 2\lambda_5 v_d v_u - \mu_3^2) = 0. \quad (45)$$

Using these and expanding around the vacuum configuration, the mass matrix of the neutral scalar fields in the $\{h_u^0, h_d^0, \sigma^0\}$ basis turns out to be

$$M_S^2 = V_\sigma^2 \times \begin{pmatrix} \lambda_5 \frac{v_d}{v_u} + 4\lambda_1 \frac{v_u^2}{V_\sigma^2} & -\lambda_5 + 2\lambda_{12} \frac{v_d v_u}{V_\sigma^2} & 2 \frac{\lambda_{13} v_u - \lambda_5 v_d}{V_\sigma} \\ -\lambda_5 + 2\lambda_{12} \frac{v_d v_u}{V_\sigma^2} & \lambda_5 \frac{v_u}{v_d} + 4\lambda_2 \frac{v_d^2}{V_\sigma^2} & 2 \frac{\lambda_{23} v_d - \lambda_5 v_u}{V_\sigma} \\ 2 \frac{\lambda_{13} v_u - \lambda_5 v_d}{V_\sigma} & 2 \frac{\lambda_{23} v_d - \lambda_5 v_u}{V_\sigma} & 4\lambda_3 \end{pmatrix}. \quad (46)$$

Assuming $V_\sigma \gg v \equiv \sqrt{v_u^2 + v_d^2}$ and keeping only $\mathcal{O}(V_\sigma^2)$ terms, the set of eigenvalues of M_S^2 reads $\{0, \frac{v^2}{v_d v_u} \lambda_5 V_\sigma^2, 4\lambda_3 V_\sigma^2\}$. The $\mathcal{O}(v V_\sigma)$ perturbations do not affect this result at the first order and, hence, we conclude that the eigenvalues of M_S^2 are

$$\left\{ \mathcal{O}(v^2), \frac{v^2}{v_d v_u} \lambda_5 V_\sigma^2 + \mathcal{O}(v^2), 4\lambda_3 V_\sigma^2 + \mathcal{O}(v^2) \right\}. \quad (47)$$

Next, the pseudoscalar mass matrix has two eigenvalues equal to zero: the combination $-\frac{v_u}{v} \eta_u^0 + \frac{v_d}{v} \eta_d^0$ corresponds to the goldstone boson (GB) “eaten” by the Z, whereas $\frac{v_u}{V_\sigma} \frac{v_u v_d}{v^2} \eta_u^0 + \frac{v_d}{V_\sigma} \frac{v_u v_d}{v^2} \eta_d^0 + \eta_\sigma^0$ is the axion—the GB corresponding to the breaking of the PQ symmetry. The third pseudoscalar eigenstate $-\frac{v_d}{v} \eta_u^0 - \frac{v_u}{v} \eta_d^0 + \frac{2v_u v_d}{v V_\sigma} \eta_\sigma^0$ acquires mass

$$m_{\text{PS}}^2 = \lambda_5 \left(4v_u v_d + \frac{v^2}{v_u v_d} V_\sigma^2 \right). \quad (48)$$

Among the singly charged scalars, the GB eaten by the W corresponds to the combination $\frac{1}{v}(-v_u h_u^+ + v_d h_d^+)$, whereas the orthogonal combination $\frac{1}{v}(v_d h_u^+ + v_u h_d^+)$ acquires mass

$$m_{H^+}^2 = \lambda_4 v^2 + \lambda_5 \frac{v^2}{v_u v_d} V_\sigma^2. \quad (49)$$

For $\lambda_6 = 0$ the singlet scalar h^+ does not mix with H^+ and its mass reads

$$m_{h^+}^2 = \mu_4^2 + \lambda_{41} v_u^2 + \lambda_{42} v_d^2 + \lambda_{43} V_\sigma^2. \quad (50)$$

Finally, the mass of the doubly charged scalar k^{++} is given by

$$m_{k^{++}}^2 = \mu_5^2 + \frac{1}{2} (\lambda_{51} v_u^2 + \lambda_{52} v_d^2) + \lambda_{53} V_\sigma^2. \quad (51)$$

Let us note that if $\lambda_{i3}, \lambda_5 \lesssim \mathcal{O}(\frac{v^2}{V_\sigma^2})$ (with i running over all the states but the SM singlet σ), all the scalars besides one [the σ -dominated state with eigenvalue proportional to λ_3 in Eq. (47)], have tree-level masses of the order of the electroweak scale. As a matter of fact when $\lambda_{i3}, \lambda_5, \lambda_7 \ll 1$ the singlet σ is only weakly coupled to the other fields. As we shall discuss in Sec. VA this is a technically natural limit associated with the emergence of an extra Poincaré symmetry in the action. Notice that even though λ_7 does not enter the spectrum at the tree level, it is expected to contribute at one loop (for instance, to the mass of k^{++}). This effect is estimated to be roughly $\frac{1}{16\pi^2} \lambda_7^2 V_\sigma^2$ and hence, the requirement that the mass of k^{++} is around the electroweak scale corresponds to $\lambda_7 \lesssim 4\pi \times \mathcal{O}(\frac{v}{V_\sigma})$ which is consistent with the decoupling of the heavy singlet.

D. Neutrino masses

Focusing on the case with $\lambda_6 = 0$, the only radiative contribution to the neutrino masses is a two-loop diagram *à la* ZB (see Fig. 4), which yields [20]

$$(M_\nu^{\text{ZB}})_{ij} = 16\lambda_7 V_\sigma f_{ia} m_a g_{ab}^* I_{ab} m_b f_{jb}, \quad (52)$$

with m_a denoting the a th charged-lepton mass. For $m_a \ll m_{h^+}, m_{k^{++}}$, the loop function reads

$$I_{ab} \simeq I = \frac{1}{(16\pi^2)^2} \frac{1}{M^2} \frac{\pi^2}{3} \tilde{I}(r), \quad (53)$$

with $M \equiv \max(m_{h^+}, m_{k^{++}})$ and

$$\tilde{I}(r) = \begin{cases} 1 + \frac{3}{\pi^2} (\log^2 r - 1) & r \gg 1 \\ 1 & r \rightarrow 0 \end{cases}, \quad (54)$$

where $r \equiv m_{k^{++}}^2 / m_{h^+}^2$. For the exact analytic result see [55].

An important feature of the ZB model is that the lightest neutrino is predicted to be massless. Indeed, since f is antisymmetric, $\det f = 0$ (for three generations) and, hence, $\det M_\nu^{\text{ZB}} = 0$.

As in the previous cases (TII and BJ), the smallness of neutrino masses can be due to different factors. Taking into account the strong bounds on the Yukawa couplings f and g coming from the LFV processes (see Sec. V C), it turns out that the assumption $\lambda_7 \lesssim 4\pi \times \mathcal{O}(\frac{v}{V_\sigma})$, tailored to keep the nonsinglet scalars at the electroweak scale, ensures also the correct absolute neutrino mass scale [20].

Finally, we briefly comment on the case $\lambda_6 \neq 0$. In such a setting there is an extra one-loop contribution to the neutrino masses, similar to the one in Fig. 3 [the relevant expression can be found in Eqs. (25) and (26) of [39]]. As already mentioned, the original Zee model is excluded by neutrino data and, in order to obtain a viable neutrino texture, the size of such a one-loop diagram must be comparable with the two-loop expression in Eq. (52), thus introducing a fine-tuning in the couplings. Let us also note that $\lambda_6 \neq 0$ introduces a tree-level mixing between the charged $SU(2)_L$ -doublet and singlet scalars that affects Eq. (53). At variance with the ZB model, the lightest

neutrino is no longer massless. In this study, we will not pursue the analysis of this hybrid model any further.

V. DISCUSSION

The three setups presented in the previous sections share a number of common features which we shall briefly summarize here. In particular, all three models contain a DFSZ invisible axion with a tiny coupling to neutrinos [39,40]. It is noticeable that, at variance with the TII and ZB extended models, the axion in the BJ case exhibits flavor violating couplings to the leptons of the same order of the flavor-diagonal ones:

$$\begin{aligned} \mathcal{L}_{a\ell\ell} = & -X_{\ell_{2,3}} \frac{\partial_\mu a}{f_a} [(\bar{e}_L^i \gamma^\mu e_L^i) + (\bar{\nu}_L^i \gamma^\mu \nu_L^i)] \\ & - X_e \frac{\partial_\mu a}{f_a} [(\bar{e}_R^i \gamma^\mu e_R^i)] \\ & - (X_{\ell_1} - X_{\ell_{2,3}}) \frac{\partial_\mu a}{f_a} [(\bar{e}_L^i \gamma^\mu (U_L^{e\dagger})^{i1} (U_L^e)^{1j} e_L^j)] \\ & + (\bar{\nu}_L^i \gamma^\mu (U_L^{\nu\dagger})^{i1} (U_L^\nu)^{1j} \nu_L^j), \end{aligned} \quad (55)$$

that, up to a total derivative, can be written as

$$\begin{aligned} \mathcal{L}_{a\ell\ell} = & i \frac{a}{f_a} [(X_e - X_{\ell_{2,3}}) m_i^e \bar{e}^i \gamma_5 e^i - X_{\ell_{2,3}} m_i^\nu \bar{\nu}^i \gamma_5 \nu^i] - i (X_{\ell_1} - X_{\ell_{2,3}}) \frac{a}{f_a} \left[(U_L^{e\dagger})^{i1} (U_L^e)^{1j} \bar{e}^i \left(\frac{m_j^e - m_i^e}{2} + \frac{m_j^e + m_i^e}{2} \gamma_5 \right) e^j \right. \\ & \left. + (U_L^{\nu\dagger})^{i1} (U_L^\nu)^{1j} \bar{\nu}^i \left(\frac{m_j^\nu - m_i^\nu}{2} + \frac{m_j^\nu + m_i^\nu}{2} \gamma_5 \right) \nu^j \right], \end{aligned} \quad (56)$$

where a denotes the axion field and $f_a = \sqrt{2} V_\sigma$. The mass eigenstates e_L^i, ν_L^i ($i = 1, 2, 3$) are connected to the interaction basis e_L^α, ν_L^α ($\alpha = 1, 2, 3$) via the relations $e_L^\alpha = (U_L^e)^{\alpha i} e_L^i$ and $\nu_L^\alpha = (U_L^\nu)^{\alpha i} \nu_L^i$. The equations of motion for Weyl fermions with a Majorana mass term are used and the axion neutrino couplings are written in terms of the Majorana mass eigenstates [56]. Present laboratory and astrophysical limits on flavor violating interactions do not seem to imply any constraints on the PQ scale stronger than those obtained from the diagonal interactions [51]. On the other hand, the presence of lepton flavor violating interactions of the axion in the extended BJ model deserves further detailed scrutiny.

The DFSZ invisible axion framework suffers from the domain wall problem (the nonperturbative instanton potential breaks the $U(1)_{\text{PQ}}$ explicitly to a Z_{N_q} discrete symmetry where N_q is the number of quark flavors). The standard cosmological solution is then the assumption of a low reheating temperature (see [57] for a comprehensive discussion).

A. Naturalness

An interesting feature of all the models considered in this study is the fact that the hierarchy between the electroweak

and the PQ scales can be made technically natural and stable against radiative corrections. Let us consider, for definiteness, the case of the PQ-extended TII model. At the tree level, the hierarchy between the PQ and the electroweak scale can be obtained without fine-tuning among the scalar potential parameters of Eq. (3) by requiring the ultraweak limit

$$\lambda_{i3}, \lambda_5 \sim \mathcal{O}\left(\frac{v^2}{V_\sigma^2}\right) \quad \text{and} \quad \lambda_6 \sim \mathcal{O}\left(\frac{v_\Delta}{V_\sigma}\right), \quad (57)$$

where the last equation is set by the stationarity condition (17) and i is running over all the scalar multiplets but the SM singlet (all nonsinglet mass parameters are taken at the weak scale). As a matter of fact, this guarantees that the heavy (PQ-scale) neutral singlet decouples from the rest of the spectrum (see the Appendix and Sec. IV C). It is noteworthy that the ultraweak limit $\lambda_{i3}, \lambda_5, \lambda_6 \ll 1$ is associated with the emergence of an additional Poincaré symmetry of the action [39,58] (see [59] for a recent discussion) which makes this limit perturbatively stable. It is readily verified that the renormalization of the couplings connecting the “light” and “heavy” sectors is as a set multiplicative (the relevant beta functions exhibit a fixed

point for vanishing couplings). The hierarchy among the ultraweak couplings in Eq. (57) is stable since $\lambda_6^2 \ll \lambda_{i3}$. The couplings λ_5 and λ_6 are themselves multiplicatively renormalized since lepton number is restored when one of them is vanishing. The naturalness requirement, together with the constraints coming from the LFV phenomenology, allows us to reproduce in all three setups above the correct neutrino mass scale together with a number of new scalar states in the reach of present collider searches.

Beyond the tree level, we can set naturalness bounds on the mass scale M of the non-SM scalar multiplets by requiring that the finite two-loop gauge corrections to the Higgs mass parameter m^2 do not exceed the Higgs pole mass. Adopting the notation of Ref. [1], each complex scalar multiplet with quantum numbers (n, Y) under $SU(2)_L \otimes U(1)_Y$ contributes by

$$\delta m^2(\bar{\mu}) = -\frac{nM^2}{(4\pi)^4} \left(\frac{n^2 - 1}{4} g_2^4 + Y^2 g_Y^4 \right) \times \left(\frac{3}{2} \ln^2 \frac{M^2}{\bar{\mu}^2} + 2 \ln \frac{M^2}{\bar{\mu}^2} + \frac{7}{2} \right), \quad (58)$$

where $\bar{\mu}$ is the renormalization scale in the $\overline{\text{MS}}$ scheme, to be identified with the cutoff of the effective theory Λ_{UV} (which in our setup is the onset of gravity, since we require the decoupling of the PQ-scale scalar singlet). The naturalness bounds are shown in Fig. 5, where we display the constraints on the individual contributions of the scalar triplet Δ , the extra Higgs doublet H' , the singly charged scalar h^+ , and the doubly charged scalar k^{++} , respectively. For a fully natural spectrum, the new states are expected to live not too far from the electroweak scale, say below 5 TeV to 200 GeV depending on the multiplet considered and the value of Λ_{UV} .

A few comments about gravity are in order. Even assuming for the time being no massive Planckian states, one should wonder whether the presence of the heavy SM singlet might give rise to gravity-mediated radiative corrections that destabilize the light Higgs mass. In

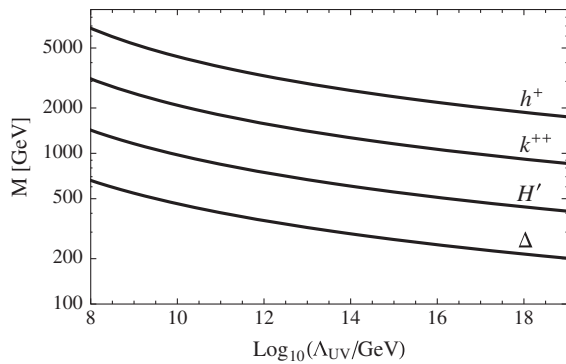


FIG. 5. Naturalness bounds on the mass of the exotic scalar multiplets present in the different PQ-extended neutrino mass models, as a function of the ultraviolet cutoff, see Eq. (58).

Refs. [1,60] it is pointed out that a finite gravity-mediated contribution arises at the three-loop order and it is estimated as $\delta m^2 \approx \frac{Y_t^2 G_N^2 M^6}{(4\pi)^6}$. This yields a naturalness bound on the singlet mass $M \lesssim 10^{14}$ GeV, which is compatible with the experimentally allowed range of the PQ scale. On the other hand, gravity-induced corrections to the Higgs mass are generally expected to arise at the two-loop level due to the breaking of the Higgs shift symmetry by SM interactions (one-loop contributions in the minimally coupled gravity are derivatively suppressed) and, on purely dimensional grounds, they can be estimated to be $\delta m^2 \approx \frac{G_N \Lambda_G^4}{(4\pi)^4}$. This raises an issue of naturalness already at $\Lambda_G \approx 10^{11}$ GeV, thus questioning the stability of the simple setups discussed here (as well as that of the SM). Softened gravity may be invoked, but still the appearance of Landau poles at trans-Planckian energies remains, in principle, an issue [61]. The authors of [61] invoke the total asymptotic freedom of the low-energy setup as a solution.

Another potential issue related to gravity is the possibility of an explicit breaking of the global $U(1)_{\text{PQ}}$ due to Planck-scale physics. This may induce a shift of the vacuum of the axion potential and, thus, endanger the PQ solution of the strong CP problem [62–64]. The authors of [65] argue that the presence of Majorana neutrinos may provide a protection for the PQ mechanism, which leads to a connection between the upper bound on neutrino masses and the onset of gravitational effects at the scale Λ_G . We plan to systematically address the potential issues related to gravity in a future work.

B. Electroweak vacuum stability

An added value of the models considered in this study is the fact that scalar extensions of the SM are tailor made to improve on the electroweak vacuum stability. This issue has been discussed at length in the literature (see, for instance, [2,66]), and we just briefly recall the argument here. The key effect is the contribution of the new scalars (through, e.g., the Higgs portal couplings) to the running of the Higgs quartic coupling λ_H (see, for example, Refs. [67–69] for analysis in the context of the type-II seesaw). These corrections contribute positively to the beta function of λ_H and, hence, they tend to stabilize the Higgs potential if they enter the running below the instability scale.⁴ Another interesting possibility for the cases at hand (pointed out in [2]) is that a heavy SM singlet σ may stabilize the Higgs potential through a large threshold effect, $\lambda_H \rightarrow \lambda_H - \lambda_{H\sigma}^2 / \lambda_\sigma$. In our case, however, we advocate the ultraweak limit $\lambda_{H\sigma} \lesssim v^2 / V_\sigma^2$ while leaving the σ self-interactions unconstrained. We are therefore bound to the running effect alone which

⁴The instability scale of the SM effective potential is a gauge dependent quantity [70]. A gauge-invariant criterium to include the effects of new physics can be devised [71].

suffices in the extended setups here considered. A detailed analysis of these matters is beyond the scope of the present paper and will be the subject of a future study.

Let us finally remark that even if the instability of the electroweak vacuum is, strictly speaking, not an issue as long as the lifetime of the false vacuum is long enough to comply with the age of the universe, the fate of the electroweak vacuum depends on the cosmological history and there are inflationary setups where the metastability of the vacuum may be an issue [72].

C. The extended scalar sector

The scalar sectors of the models here discussed are commonly extended to feature two Higgs doublets and a complex singlet (in addition to one $Y = 1$ triplet in the case of TII, one singly charged singlet in the BJ model and two singly and doubly charged singlets in the ZB setting). With the exception of the light axion field and its scalar partner whose masses are driven by the PQ-breaking scale, all other physical scalars may live at the TeV scale, yet be compatible with all the bounds from the low-energy phenomenology and collider physics.

Because of the hierarchy among the PQ, the electroweak and the triplet VEVs (following from the constrained form of the scalar potential), the weak scale neutral Higgs sector of the three models overlaps to a large extent with that of the two Higgs doublet extension of the SM (see the Appendix). Correspondingly, we may obtain one of the physical neutral scalars to behave as the SM Higgs by invoking the decoupling limit $m_A^2 \gg |\lambda_i|v^2$ [73] (where A is the pseudoscalar field and λ_i are the relevant scalar couplings). On the other hand, a SM-like Higgs does not necessarily imply the “true” decoupling of A (and the other physical doublet states), since the extra scalar couplings can be small enough to allow for a full spectrum at the TeV scale while retaining the $\alpha \approx \beta - \pi/2$ decoupling relation among the relevant diagonalization angles [73].

These considerations apply to any of the models discussed here. Since a detailed phenomenological study of the different scalar sectors is beyond the scope of the present work, we shall briefly comment upon some of the relevant common features focusing mainly on the lepton flavor phenomenology and collider signatures.

1. Lepton flavor violation

Unlike for the BJ case, in the TII and ZB models the tree-level flavor violation in neutral currents is forbidden by the presence of the PQ symmetry. On the other hand, in all three scenarios, the extended scalar sector allows for the tree-level flavor violation in the charged currents. In particular, the couplings of the charged scalars to leptons, closely related to the structure of the neutrino mass matrix, play a crucial role in the LFV phenomenology of the models.

In the case of the extended TII model, the LFV arises from the Yukawa coupling Y_Δ , which is directly

proportional to the neutrino mass matrix [cf. Eq. (21)]. As an example, in order to avoid the stringent constraints from the nonobservation of $\mu^- \rightarrow e^- e^+ e^-$, the relation $|(Y_\Delta)_{\mu e}|^2 |(Y_\Delta)_{ee}|^2 (\frac{200 \text{ GeV}}{m_{\Delta^{++}}})^4 \lesssim 10^{-12}$ must be satisfied [74]. This, in turn, considerably constrains the shape of the neutrino mass matrix in the case of our interest, i.e., when the triplet mass is in the LHC range. Taking into account these constraints, Ref. [74] predicts branching ratios for $\tau \rightarrow l_i^- l_i^+ l_i^-$ and $\mu \rightarrow e\gamma$ accessible to the upcoming experiments with implications for neutrino physics. The present bounds on these processes are weaker than those coming from the $\mu^- \rightarrow e^- e^+ e^-$ decay; for instance, the τ decay measurements yield the constraint $|(Y_\Delta)_{\tau i}|^2 |(Y_\Delta)_{jk}|^2 (\frac{200 \text{ GeV}}{m_{\Delta^{++}}})^4 \lesssim 10^{-7}$ [74]. Notice, however, that these bounds are, in general, more relevant for accessing the overall neutrino mass scale than the bound from $\mu^- \rightarrow e^- e^+ e^-$ which can be satisfied by tuning just one of the Y_Δ entries.

Similarly, in the extended ZB model, one can obtain strong bounds on the Yukawa couplings f and g in Eq. (35) from the LFV decays (together with the charged-currents universality and the lepton anomalous magnetic moments). The strongest bound is again due to the nonobservation of $\mu^- \rightarrow e^+ e^- e^-$ and it requires $|g_{e\mu} g_{ee}^*| (\frac{200 \text{ GeV}}{m_{k^{++}}})^2 \lesssim 10^{-6}$ [20]. The bounds on the f couplings from other LFV processes (such as $\mu \rightarrow e\gamma$) are somewhat weaker [20]. The tightest of them, induced by the nonobservation of $\mu \rightarrow e\gamma$ decay, reads $|f_{e\tau}^* f_{\mu\tau}|^2 (\frac{200 \text{ GeV}}{m_{h^+}})^4 + 16 |g_{ee}^* g_{e\mu} + g_{e\mu}^* g_{\mu\mu} + g_{e\tau}^* g_{\tau\mu}|^2 (\frac{200 \text{ GeV}}{m_{k^{++}}})^4 \lesssim 3 \times 10^{-9}$. Because of the complicated shape of these constraints, the correlations with the structure of the neutrino masses matrix (52) is not straightforward. On the other hand, a detailed numerical analysis [20] shows that combining these constraints with the neutrino oscillation data allows one to predict the shape of the g matrix and determine the branching ratios for the k^{++} decay. For $m_{k^{++}} \leq 400 \text{ GeV}$ and normal neutrino hierarchy the k^{++} scalar decays into ee or $\mu\mu$ only, whereas for the same mass range and inverse hierarchy the decay channel $\mu\tau$ is non-negligible and $k^{\pm\pm} \rightarrow h^\pm h^\pm$ opens. Consequently, the discovery of a doubly charged scalar would either rule out the ZB model or provide a testable information about the neutrino mass pattern.

The analysis of the extended BJ model must take into account that both Higgs doublets couple to leptons and, hence, the tree-level LFV processes can be mediated by the neutral scalars. On the other hand, this model is highly predictive and it is sharply constrained by the neutrino data. Remarkably, a thorough numerical analysis [16] shows that only two specific solutions for the lepton coupling matrix \hat{Y} are possible. We refer the reader to the original paper for a detailed discussion.

2. Collider physics

The direct production searches at LEP set a lower bound on the mass of the singly charged scalars (Δ^+ or h^+ in our

models) of about 90 GeV. For the case of the charged physical scalar of the two Higgs doublet extension of the SM the LHC does not add any new stringent constraint (see, e.g., [75]), while the signal of the charged $SU(2)_L$ singlet is expected to be yet weaker. On the other hand, in the TII case one has an extra constraint $|m_{\Delta^+} - m_{\Delta^{++}}| \lesssim 40$ GeV from the electroweak precision data [76] which slightly tightens the bound.

Searches for doubly charged scalars are performed both by ATLAS and CMS (see, e.g., [77,78]). These analyses strongly depend on the assumed branching ratios and only leptonic decays are considered. If, for instance, $\text{BR}(h^{\pm\pm} \rightarrow l_1^\pm l_2^\pm) = 1$ is assumed for different pairs of $l_1 l_2$, the bounds vary from 200 to 450 GeV, cf. [78] (note, however, that only the case of the type-II triplet was considered there). To this end, Ref. [77] sets a bound of 410 GeV for the triplet case and up to 320 GeV for a doubly charged $SU(2)_L$ singlet, again depending on the assumptions for the branching ratios.

In the case of the extended TII model, the branching ratios for the decay of Δ^{++} into leptons are directly connected to the entries of the neutrino mass matrix. However, depending on the shape of Y_Δ [see Eq. (1)], the leptonic Δ^{++} decays can be suppressed in favor of the decay into a pair of gauge bosons. Moreover, for nonzero mass splitting among the triplet components (which arises due to the electroweak symmetry breaking), the cascade decays $\Delta^{\pm\pm} \rightarrow \Delta^\pm W^{\pm*} \rightarrow \Delta^0 W^{\pm*} W^{\pm*}$ occur as well. For a complete “decay phase diagram” in the original TII model, see Ref. [79].

Recently, the TII model extended by an extra Higgs doublet has been studied in [80] with the conclusion that the limit on the mass splittings is relaxed by the mixing and the dominant decay channel is then $\Delta^{\pm\pm} \rightarrow h_{1,2}^{\pm*} W^{\pm*}$. The argument is that a $5\text{-}\sigma$ discovery of the doubly charged scalar is possible at LHC13 with an integrated luminosity of 40 fb^{-1} if its mass is lower than 330 GeV. On the other hand, this conclusion does not apply to our TII model where the form of the potential is constrained by the PQ symmetry [see Eq. (3)] and the size of the doublet-triplet mixing is always driven by the small triplet VEV (see the Appendix).

As far as the ZB model is concerned, the authors of [20] studied numerically the branching ratios of the doubly charged singlet k^{++} and they found a lower bound on its mass of about 310 GeV in the case of the inverse neutrino hierarchy and 200 GeV for the normal hierarchy.

D. Electroweak baryogenesis

An essential ingredient for the electroweak baryogenesis (see, e.g., [52] for a review) is a strong-enough first-order phase transition. The finite-temperature effective potential must contain a cubic term of the type $|\phi|^3 T$ which is enhanced with respect to that available in the SM. This may

be achieved if new light scalars with sizeable couplings to the Higgs doublet are present.

There are two qualitatively different ways the new scalars can affect the electroweak phase transition: (i) by contributing directly to the effective potential through the evolution of their field values in the early universe, (ii) by enhancing the cubic term in the effective potential at the one-loop level without developing a VEV. To this end, it is known, for instance (see, e.g., [81]), that the mechanism (i) works well when the new scalar developing a VEV is a gauge singlet S . In particular, for the mechanism to work, a crucial coupling to be present is the trilinear term $|H|^2 S$. Inspecting the different potentials studied in previous sections, the only term that satisfies the above conditions (in the PQ-broken phase) is $\lambda_6 V_\sigma H_u^\dagger \Delta H_d$ in Eq. (3) of the extended TII model. On the other hand, this is likely not enough since the neutrino masses [see Eq. (21)] require this coupling to be very small. Hence, we are left with option (ii) for which the role of the inert scalar running in the loop can be played by any of the two charged scalars h^+ and k^{++} present in the radiative neutrino mass models, as well as the TII triplet, whose VEV is negligible compared to the electroweak scale [82].

By denoting the new charged state X and by writing its interaction with the Higgs doublet as

$$V = M_X^2 |X|^2 + \lambda_{XH} |X|^2 |H|^2 + \dots, \quad (59)$$

the contribution to the finite-temperature effective potential due to the so-called daisy resummation takes the form [52]

$$\Delta V_{\text{eff}}(\phi, T) \supset -\frac{n_X T}{12\pi} \left[\Pi_X(T) + M_X^2 + \frac{\lambda_{XH}}{2} \phi^2 \right]^{3/2}, \quad (60)$$

where n_X is the number of degrees of freedom and $\Pi_X(T) = \kappa T^2$ is the thermal mass of X , with κ being a function of the scalar and gauge couplings. Hence, in order to maximize the contribution to the cubic term one needs a significant portal coupling λ_{XH} and/or a cancellation between the thermal mass and the potential mass parameter. While it is known that a strong first-order phase transition can be achieved in the case of a second inert scalar doublet [83] or an additional triplet [82], a detailed study of its feasibility in the three setups here discussed is left to further investigation.

Finally, let us comment on the additional sources of CP violation, another necessary ingredient for a successful baryogenesis. By taking advantage of the results of Ref. [84], it is readily shown that none of the potentials considered in our setups lead to spontaneous CP violation at the tree level. This statement is verified by a direct minimization of the relevant scalar potentials in the presence of complex VEVs. Hence, an extension of the minimal setups here discussed is required if the new source of CP violation has to come from the scalar potential.

The simplest option is adding a SM singlet [85] or to consider a PQ-extended three-Higgs-doublet model [36]. Another intriguing possibility is to identify the source of the extra CP violation with the very $\bar{\theta}_{\text{QCD}}$ -term which, in the early universe, has not yet relaxed to its minimum. Such a scenario can be realized in the context of the so-called cold electroweak baryogenesis where the electroweak phase transition is delayed to temperatures $T \lesssim \Lambda_{\text{QCD}}$ [86]. Further scrutiny on the matter is called for.

VI. CONCLUSIONS

Inspired by the present-day evidence for physics beyond the standard electroweak model, we have considered three simple setups that minimally extend the scalar sector of the SM. They feature massive Majorana neutrinos together with an invisible axion, thus interconnecting neutrino masses, dark matter, and the strong CP problem. In all cases, the presence of the PQ symmetry strongly constrains the relevant scalar potentials, replacing the role of the ad hoc discrete symmetries originally invoked in some of the models.

The Higgs sectors of the extended setups feature in all cases two Higgs doublets and one complex scalar singlet. Two widely different physical scales are connected in these setups: the PQ and the electroweak scale. It is noteworthy that the neutrino mass scale is fully compatible with the requirement of naturalness and stability of the scalar masses. The needed decoupling (ultra-weak) limit of the PQ singlet scalar is shown to be technically natural and allows for a plethora of physical scalar states at the TeV scale, within the reach of the collider searches. We commented upon the possible destabilizing role of gravity, but we feel the arguments do not compel us to abandon such a scenario.

The same extensions of the SM scalar sector allow for an improvement on the electroweak vacuum stability issue and, at the same time, they may trigger a strong-enough first-order electroweak phase transition to support the electroweak baryogenesis as the origin of the observed baryon asymmetry. Alas, the minimal scalar potentials here considered do not allow for additional sources of CP violation. The matter will be the subject of a further investigation.

Finally, it is intriguing that the hierarchy between the PQ and the electroweak scales may naturally arise as a consequence of the breaking of the classical scale invariance *à la* Coleman-Weinberg [87,88]. Because of the absence of scalar trilinear interactions which characterizes the neutrino-invisible axion models here discussed, they are naturally and readily embedded into such a context (the extended Poincaré invariance of the action in the ultra-weak limit is there replaced by a custodial shift symmetry of the singlet field). Again, this scenario will be scrutinized in a future work.

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APPENDIX: SCALAR SPECTRUM OF THE TII MODEL

In this appendix we detail the scalar spectrum of the extended type-II seesaw model. First, we consider the case where all the electroweak breaking VEVs are neglected, thus providing a very simple description of the spectrum in the PQ-broken phase. Later we discuss the general case by taking advantage of the large VEVs hierarchies.

1. $v_{u,d} = v_{\Delta} = 0$ case

With all the electroweak VEVs switched off ($v_{u,d} = v_{\Delta} = 0$), we only retain the PQ-symmetry-breaking VEV V_{σ} in Eq. (15), yielding the stationarity equation

$$0 = \frac{\partial \langle V_{\text{TII}} \rangle}{\partial V_{\sigma}} = 2V_{\sigma}(2\lambda_3 V_{\sigma}^2 - \mu_3^2). \quad (\text{A1})$$

Expanding the scalar potential up to the second order in the dynamical fields and using (A1), we obtain the following spectrum classified according to the unbroken SM symmetry:

- (i) A real scalar SM singlet σ^0 :

$$M_{\sigma^0}^2 = 4\lambda_3 V_{\sigma}^2. \quad (\text{A2})$$

- (ii) A real pseudoscalar SM singlet η_{σ}^0 :

$$M_{\eta_{\sigma}^0}^2 = 0, \quad (\text{A3})$$

which is the zero-mass mode of the PQ-breaking field corresponding to the axion.

(iii) A complex triplet Δ :

$$M_{\Delta}^2 = \lambda_{\Delta 3} V_{\sigma}^2 + \mu_{\Delta}^2. \quad (\text{A4})$$

(iv) Complex doublets H_u and H_d :

$$M_H^2 = \begin{pmatrix} \lambda_{13} V_{\sigma}^2 - \mu_1^2 & \lambda_5 V_{\sigma}^2 \\ \lambda_5 V_{\sigma}^2 & \lambda_{23} V_{\sigma}^2 - \mu_2^2 \end{pmatrix}; \quad (\text{A5})$$

here M_H^2 is written in the (H_u^*, H_d) basis (column indices). Equation (A5) is diagonalized by an orthogonal transformation

$$\begin{pmatrix} \hat{H}_u^* \\ \hat{H}_d \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} H_u^* \\ H_d \end{pmatrix}, \quad (\text{A6})$$

where

$$\tan 2\alpha = \frac{2\lambda_5 V_{\sigma}^2}{(\lambda_{13} - \lambda_{23})V_{\sigma}^2 - \mu_1^2 + \mu_2^2}. \quad (\text{A7})$$

The corresponding eigenvalues then read

$$2M_{u,d}^2 = (\lambda_{13} + \lambda_{23})V_{\sigma}^2 - \mu_1^2 - \mu_2^2 \pm \sqrt{((\lambda_{13} - \lambda_{23})V_{\sigma}^2 - \mu_1^2 + \mu_2^2)^2 + 4\lambda_5^2 V_{\sigma}^4}. \quad (\text{A8})$$

2. $v_{u,d,\Delta} \neq 0$ case

By plugging Eqs. (12)–(15) into the expression of the scalar potential in Eq. (3), we obtain the stationarity equations in the form

$$0 = \frac{\partial \langle V_{\text{III}} \rangle}{\partial v_u} = 2v_u(2\lambda_1 v_u^2 + \lambda_{12} v_d^2 + \lambda_{13} V_{\sigma}^2 + \lambda_{\Delta 1} v_{\Delta}^2 - \mu_1^2) + 2\lambda_6 v_{\Delta} V_{\sigma} v_d - 2\lambda_5 v_d V_{\sigma}^2, \quad (\text{A9})$$

$$0 = \frac{\partial \langle V_{\text{III}} \rangle}{\partial v_d} = 2v_d(2\lambda_2 v_d^2 + \lambda_{12} v_u^2 + \lambda_{23} V_{\sigma}^2 + (\lambda_{\Delta 2} + \lambda_8) v_{\Delta}^2 - \mu_2^2) + 2\lambda_6 v_{\Delta} V_{\sigma} v_u - 2\lambda_5 v_u V_{\sigma}^2, \quad (\text{A10})$$

$$0 = \frac{\partial \langle V_{\text{III}} \rangle}{\partial V_{\sigma}} = 2V_{\sigma}(2\lambda_3 V_{\sigma}^2 + \lambda_{13} v_u^2 + \lambda_{23} v_d^2 - 2\lambda_5 v_u v_d + \lambda_{\Delta 3} v_{\Delta}^2 - \mu_3^2) + 2\lambda_6 v_{\Delta} v_u v_d, \quad (\text{A11})$$

$$0 = \frac{\partial \langle V_{\text{III}} \rangle}{\partial v_{\Delta}} = 2v_{\Delta}(2(\lambda_{\Delta 4} + \lambda_9) v_{\Delta}^2 + \lambda_{\Delta 1} v_u^2 + (\lambda_{\Delta 2} + \lambda_8) v_d^2 + \lambda_{\Delta 3} V_{\sigma}^2 + \mu_{\Delta}^2) + 2\lambda_6 V_{\sigma} v_u v_d. \quad (\text{A12})$$

The spectrum is then obtained by expanding the scalar potential up to the second order in the fields of Eqs. (12)–(15), around the vacuum configuration given by the above stationarity equations. This yields

(i) Neutral scalars $(h_u^0, h_d^0, \sigma^0, \delta^0)$:

$$M_S^2 = \begin{pmatrix} 4\lambda_1 v_u^2 + (\lambda_5 V_{\sigma} - \lambda_6 v_{\Delta}) V_{\sigma} v_d / v_u & (\lambda_6 v_{\Delta} - \lambda_5 V_{\sigma}) V_{\sigma} + 2\lambda_{12} v_u v_d & -2\lambda_5 V_{\sigma} v_d + \lambda_6 v_{\Delta} v_d + 2\lambda_{13} V_{\sigma} v_u & \lambda_6 V_{\sigma} v_d + 2\lambda_{\Delta 1} v_{\Delta} v_u \\ (\lambda_6 v_{\Delta} - \lambda_5 V_{\sigma}) V_{\sigma} + 2\lambda_{12} v_u v_d & 4\lambda_2 v_d^2 + (\lambda_5 V_{\sigma} - \lambda_6 v_{\Delta}) V_{\sigma} v_u / v_d & -2\lambda_5 V_{\sigma} v_u + \lambda_6 v_{\Delta} v_u + 2\lambda_{23} V_{\sigma} v_d & \lambda_6 V_{\sigma} v_u + 2(\lambda_8 + \lambda_{\Delta 2}) v_{\Delta} v_d \\ -2\lambda_5 V_{\sigma} v_d + \lambda_6 v_{\Delta} v_d + 2\lambda_{13} V_{\sigma} v_u & -2\lambda_5 V_{\sigma} v_u + \lambda_6 v_{\Delta} v_u + 2\lambda_{23} V_{\sigma} v_d & 4\lambda_3 V_{\sigma}^2 - \lambda_6 v_{\Delta} v_u v_d / V_{\sigma} & \lambda_6 v_u v_d + 2\lambda_{\Delta 3} v_{\Delta} V_{\sigma} \\ \lambda_6 V_{\sigma} v_d + 2\lambda_{\Delta 1} v_{\Delta} v_u & \lambda_6 V_{\sigma} v_u + 2(\lambda_8 + \lambda_{\Delta 2}) v_{\Delta} v_d & \lambda_6 v_u v_d + 2\lambda_{\Delta 3} v_{\Delta} V_{\sigma} & 4(\lambda_9 + \lambda_{\Delta 4}) v_{\Delta}^2 - \lambda_6 V_{\sigma} v_d v_d / v_{\Delta} \end{pmatrix}, \quad (\text{A13})$$

where $\text{rank } M_S^2 = 4$. The exact form of the eigenvalues is quite cumbersome. However, the required hierarchy $v_{\Delta} \ll v_u$, $v_d \ll V_{\sigma}$ allows us to compute the eigenvalues perturbatively. Taking into account the scaling of the couplings in Eq. (57), we define $\lambda_6 \equiv c_6 \frac{v_{\Delta}}{V_{\sigma}}$, $\lambda_5 \equiv c_5 \frac{v_d^2}{V_{\sigma}^2}$, $\lambda_{i3} \equiv c_{i3} \frac{v_i^2}{V_{\sigma}^2}$ with c_5, c_6, c_{i3} being $\mathcal{O}(1)$ coefficients. Hence, the leading contribution to the neutral scalar mass matrix [given by the terms $\gtrsim \mathcal{O}(v^2)$] reads

$$M_S^{2(\text{LO})} = \begin{pmatrix} 4\lambda_1 v_u^2 + c_5 \frac{v_d v^2}{v_u} & c_5 v^2 + 2\lambda_{12} v_u v_d & 0 & 0 \\ c_5 v^2 + 2\lambda_{12} v_u v_d & 4\lambda_2 v_d^2 + c_5 \frac{v_u v^2}{v_d} & 0 & 0 \\ 0 & 0 & 4\lambda_3 V_{\sigma}^2 & 0 \\ 0 & 0 & 0 & -c_6 v_u v_d \end{pmatrix}.$$

It is now clear that, at the leading order in the VEV ratio expansion, the eigenvalues of the scalar mass matrix read

$$\{\mathcal{O}(v^2), \mathcal{O}(v^2), 4\lambda_3 V_\sigma^2, -v_u v_d c_6\} \quad (\text{A14})$$

and that there is no mixing of the singlet and triplet fields with the $SU(2)_L$ doublets. The first next-to-leading-order corrections to the mass matrix are of the order of $v_\Delta v$, which implies, for instance, that the mixing between the doublet and the triplet

components is of the order of v_Δ/v . One further finds that the first corrections to the eigenvalues are only of the order of v_Δ^2 and that the mixing with the singlet component is of the order of v/V_σ . For large $\tan\beta = v_u/v_d$ the two doublet eigenvalues are approximately $4\lambda_1 v^2$ and $c_5 v^2 \tan\beta$, while for the mixing angle α one obtains $\tan\alpha \approx \cot\beta \ll 1$. In this limit the lightest doublet scalar behaves as the standard model Higgs boson.

- (ii) Neutral pseudoscalars ($\eta_u^0, \eta_d^0, \eta_\sigma^0, \eta_\delta^0$):

$$M_{\text{PS}}^2 = \begin{pmatrix} (\lambda_5 V_\sigma - \lambda_6 v_\Delta) V_\sigma v_d / v_u & (\lambda_5 V_\sigma + \lambda_6 v_\Delta) V_\sigma & (2\lambda_5 V_\sigma + \lambda_6 v_\Delta) v_d & -\lambda_6 V_\sigma v_d \\ (\lambda_5 V_\sigma + \lambda_6 v_\Delta) V_\sigma & (\lambda_5 V_\sigma - \lambda_6 v_\Delta) V_\sigma v_u / v_d & (2\lambda_5 V_\sigma - \lambda_6 v_\Delta) v_u & \lambda_6 V_\sigma v_u \\ (2\lambda_5 V_\sigma + \lambda_6 v_\Delta) v_d & (2\lambda_5 V_\sigma - \lambda_6 v_\Delta) v_u & 4\lambda_5 v_u v_d - \lambda_6 v_\Delta v_u v_d / V_\sigma & \lambda_6 v_u v_d \\ -\lambda_6 V_\sigma v_d & \lambda_6 V_\sigma v_u & \lambda_6 v_u v_d & -\lambda_6 V_\sigma v_u v_d / v_\Delta \end{pmatrix} \quad (\text{A15})$$

is a rank = 2 matrix which implies the existence of two zero modes, one of them being the would-be Goldstone mode associated with the Z boson and the other corresponding to the axion that acquires mass by nonperturbative QCD effects. Even though the eigenvalues can be given in a closed form, it is sufficient to report the leading order (LO) result

$$\left\{ 0, \frac{v^4}{v_u v_d} c_5, 0, -v_u v_d c_6 \right\}, \quad (\text{A16})$$

where the entries correspond, consecutively, to the pair of $SU(2)_L$ (mostly) doublet components, the singlet and the triplet. The zeros are exact (at the perturbative level), while the other entries receive corrections at most of the order of v_Δ^2 . The mixing among the doublet and triplet components is again found to be of the order of v_Δ/v .

- (iii) Singly charged scalars: (h_u^+, h_d^+, δ^+)

$$M_{\text{+}}^2 = \begin{pmatrix} \lambda_4 v_d^2 + \lambda_7 v_\Delta^2 + (\lambda_5 V_\sigma - \lambda_6 v_\Delta) V_\sigma v_d / v_u & \lambda_5 V_\sigma^2 + \lambda_4 v_u v_d & \frac{\lambda_7 v_\Delta v_u - \lambda_6 V_\sigma v_d}{\sqrt{2}} \\ \lambda_5 V_\sigma^2 + \lambda_4 v_u v_d & \lambda_4 v_u^2 - \lambda_8 v_\Delta^2 + (\lambda_5 V_\sigma - \lambda_6 v_\Delta) V_\sigma v_u / v_d & \frac{\lambda_6 V_\sigma v_u + \lambda_8 v_\Delta v_d}{\sqrt{2}} \\ \frac{\lambda_7 v_\Delta v_u - \lambda_6 V_\sigma v_d}{\sqrt{2}} & \frac{\lambda_6 V_\sigma v_u + \lambda_8 v_\Delta v_d}{\sqrt{2}} & \frac{\lambda_7 v_u^2 - \lambda_8 v_d^2}{2} - \lambda_6 V_\sigma v_d v_u / v_\Delta \end{pmatrix} \quad (\text{A17})$$

is again of rank 2, which is related to the existence of a would-be Goldstone mode associated to the W boson. In analogy with the PS case, the eigenvalues read at LO

$$\left\{ 0, \lambda_4 v^2 + c_5 \frac{v^4}{v_u v_d}, -c_6 v_u v_d + \frac{1}{2} (\lambda_7 v_u^2 + \lambda_8 v_d^2) \right\}, \quad (\text{A18})$$

and the mixing of the doublet and triplet components is suppressed by the v_Δ/v ratio.

- (iv) Doubly charged scalar δ^{++} :

$$M_{\text{++}}^2 = \lambda_7 v_u^2 - \lambda_8 v_d^2 - 2\lambda_9 v_\Delta^2 - \lambda_6 v_u v_d V_\sigma / v_\Delta \approx \lambda_7 v_u^2 - \lambda_8 v_d^2 - c_6 v_u v_d. \quad (\text{A19})$$

By comparing (A19) with (A14), (A16), and (A18) one recognizes the weak mass splitting among the triplet components induced, at the leading order, by the λ_7 and λ_8 terms.

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