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Probing Wilson loops in AdS/QCD

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We use AdS/QCD to analyze the quark and gluon scalar and pseudoscalar condensates around static color sources described by a circular Wilson loop. We also derive the static dipole-dipole interactions between rectangular Wilson loops in AdS/QCD and discuss their relevance for static string interactions in QCD at strong coupling.

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I. INTRODUCTION

QCD is the fundamental theory of strong interactions. It has proven challenging in the infrared as its fundamental constituents are confined. In this regime, the quarks and gluons interact strongly and form strongly interacting hadrons. Holographic QCD is an attempt to solve QCD at a large number of colors N_c and strong t' Hooft coupling $\lambda = g^2 N_c$, guided by the gauge/gravity duality observed in string theory [1].

The gauge/gravity principle states that in the double limits of a a large number of colors or strong coupling ($N_c \gg \lambda \gg 1$) supersymmetric gauge theory is equivalent to a one-higher dimensional gravity theory coupled to some bulk fields that are dual to gauge invariant operators of QCD. The conformal nature of the gauge theory is encoded in an anti-de Sitter (AdS) space for gravity. Each of the propagating field in bulk AdS is in one-to-one correspondence with an operator in the field theory. Although the correspondence is established for type IIB superstring theory in $AdS_5 \times S_5$, it is believed to hold for string theory in a general background.

The string theory in curved backgrounds is in general difficult to solve. However, at strong coupling the string theory turns to a weakly coupled classical supergravity which is tractable. The best established gauge/gravity dual correspondence is for $\mathcal{N} = 4$ super-Yang—Mills (YM) theory, which on the gravity side is described by a stack of N_c D3 branes sourcing an AdS_5 metric in bulk. So far, there is no exact gravity dual candidate of pure Yang—Mills or QCD. The closest dual proposal from string theory is the Witten—Sakai—Sugimoto model [2,3] based on a stack of D4 branes with probe D8 branes using the so-called top-down approach. A more phenomenological or bottom-up approach was originally suggested by Erlich, Katz, Son, and Stephanov (EKSS) [4] and others [5]. We will refer to it as AdS/QCD.

In this paper we will use the bottom-up approach to analyze the gluonic and fermionic condensates around static color sources as circular Wilson loops. We will also derive an explicit static dipole-dipole interactions between rectangular Wilson loops. The study of the spatial distribution of the quark condensates around rectangular Wilson loops has been carried recently on the lattice [6] to understand the faith of the chiral pairing in the vicinity of a flux tube. These studies aim at probing the interplay between the spontaneous breaking of chiral symmetry and confinement, two key aspects of the QCD vacuum. Recent phenomenological analyses of Ref. [6] suggest the presence of a σ-meson cloud around the QCD string [7]. A model analysis in the Schwinger model was also suggested in Ref. [8].

Finally, the nature and strength of two small dipole-dipole interactions in QCD may shed light on the character of the static interactions between QCD strings. These interactions are important in the QCD string-black hole duality whether in thermal equilibrium or in high-energy collisions [9] (and references therein). Static dipole-dipole interactions have been discussed using YM instantons [10,11] and $\mathcal{N} = 4$ SYM [12].

The outline of this paper is as follows. In Sec. 2, we formulate the model. In Sec. 3, we define the minimal circular loops and their coupling to the lowest dimensional scalar. In Secs. 4 and 5, we analyze the scalar and pseudoscalar quark condensates, the scalar and pseudoscalar gluonic condensates around a heavy quark described by a circular Wilson loop. Some critical remarks regarding our analysis are given in Sec. 6. In Sec. 7, we construct the static dipole-dipole potential in the confined phase and discuss its qualitative structure in the Coulomb phase. Our conclusions are in Sec. 8.

II. MODEL

The soft-wall version of the EKSS model consists of a five-dimensional (flavor) gauge theory in a slice of AdS_5 space-time. Assuming the gauge/gravity correspondence, one introduces bulk fields dual to the gauge-singlet operators in QCD. Specifically [13],

$$S = \int d^4x dz \sqrt{-g} e^{-\phi} \left[-|DX|^2 - m_5^2 |X|^2 - \frac{1}{2g_5^2} (F_L^2 + F_R^2) \right], \quad (1)$$

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with the dilaton background $\phi \equiv k^2 z^2$, $DX = dX - iA_L X + iXA_R$, and the AdS_5 gravity background metric

$$ds^2 = \frac{1}{z^2}(-dt^2 + dx^2 + dz^2) \tag{2}$$

in units where the AdS radius is set to 1. The soft scale k is set by the rho meson trajectory $m_n^2 = 4(n+1)k^2$ after fitting the rho mass or $k = m_\rho/2 = 385$ MeV for $n = 1$ [14]. Throughout, this scale will be set to 1 and restored when needed by inspection. The gravity dual of any confining gauge theory should have a bulk geometry that caps off at a finite distance in the holographic direction as first suggested by Polshinski and Strassler [15] in the so-called hard-wall model. Here, the dilaton background enforces that softly through a quadratic profile or soft wall.

The bulk fields correspond to the following QCD operators at the boundary

$$\begin{aligned} X_{ij} &\to \bar{q}_{Li} q_{Rj} \\ A^a_{L,R\mu} &\to \bar{q}\gamma_\mu(1 \pm \gamma_5)\tau^a q, \end{aligned} \tag{3}$$

which are the flavor scalars and left-right vector fields. A comparison of the correlators on the boundary and in the bulk shows that the bulk mass is related to the scaling dimension Δ and spin p of the boundary operators. Specifically,

$$m_5^2 = (\Delta - p)(\Delta + p - 4). \tag{4}$$

For the scalar quark operator $\bar{q}q$ in QCD, $\Delta = 3$ and $m_5^2 = -3$. For both the scalar and pseudoscalar gluon operators F^2 and $F\tilde{F}$ in QCD, $\Delta = 4$ and $m_5^2 = 0$ (see below). We note that the use of a warped background that accounts for the breaking of conformality in modified AdS/QCD changes the field strengths anomalous dimensions [16]. It will not be pursued here.

III. HEAVY COLOR SOURCE AS A CIRCULAR WILSON LOOP

We model a heavy color source on the boundary through a circular Wilson loop of radius a. In the gauge/gravity correspondence, the Wilson loop is described by a minimal Nambu—Goto (NG) string in bulk with a circle as a boundary. For small a in units of the AdS radius (set to 1 here), the minimal surface is

$$x = \sqrt{a^2 - z^2}\cos\varphi \tag{5}$$

$$y = \sqrt{a^2 - z^2}\sin\varphi. \tag{6}$$

The surface area in form notation is

$$dA = \frac{a\,dz\,d\varphi}{z^2}. \tag{7}$$

The coupling of the NG string as a circular loop of size a to the flavor scalar is not known in the soft-wall model. Since operators in QCD are only identified in bulk by their anomalous dimension Δ and spin p, we suggest that the bulk scalar-to-Wilson-loop coupling in the soft-wall model can be borrowed from the analogous coupling in $\mathcal{N} = 4$ SYM. In the latter, various bulk fields follow from the Kaluza-Klein reduction of type IIB supergravity on $AdS_5 \times S_5$. In particular the trace of the graviton (the dilaton) contributes a scalar X in bulk with the same dimension and free mass $m_5^2 = -3$ [12]. Since the graviton couples naturally to the string world sheet, so does its trace through [12]

$$\frac{1}{2\pi\alpha N_c}\int dA(-6X)\frac{z^2}{a^2} \tag{8}$$

with $6 = 2\Delta(\bar{q}q)$. Here $\alpha = l_s^2$, and the string tension $\sigma_T = 1/2\pi\alpha$. The string coupling is $g_s \equiv 4\pi\lambda/N_c$ with $l_s^4 \equiv \lambda$ in (walled) AdS_5.

IV. QUARK CONDENSATES AROUND A HEAVY COLOR SOURCE

The vacuum solution to the soft-wall EKSS model (1) describes a scalar condensate. For that, we set $A_L = A_R = 0$ and choose $\Delta = 3$ and $p = 0$ to describe the quark condensate $\bar{q}q$, so that $m_5^2 = -3$. The equation of motion is

$$\frac{d}{dz}\left(e^{-B}\frac{d}{dz}X_0\right) + 3\frac{e^{-B}}{z^2}X_0 = 0, \tag{9}$$

subject to the ultraviolet boundary condition

$$X_*(z) = mz + \langle\bar{q}q\rangle z^3 + \mathcal{O}(z^4). \tag{10}$$

Equation (9) admits a unique solution. The actual form of $X_*(z)$ is not needed for the analysis of (11) since the fluctuations around it decouple thanks to the quadratic nature of the action in X in (1).

It is now straightforward to estimate the amount of quark condensate around a heavy color source represented by the small circular Wilson loop described in the preceding section. Specifically, the connected quark condensate is

$$\frac{\langle\bar{q}q(x)W(C)\rangle_c}{\langle W(C)\rangle} = \frac{1}{2\pi\alpha N_c}\int dA\frac{-6z^2}{a^2 z_*^3}G(x,z;x_*,z_*), \tag{11}$$

where $(x_*, z_* \to 0)$ is the position of the quark operator on the boundary as illustrated by the blob in Fig. 1. Equation (11) is readily understood as the scalar quark condensate around the Wilson loop.

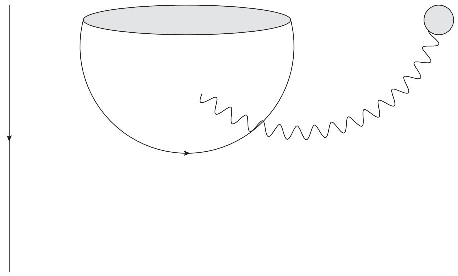

FIG. 1. Circular Wilson loop probed by $X \equiv \langle \bar{q}q \rangle$ at the boundary. See the text.

With this in mind, the Green function for the scalar field X in the bulk admits a Fourier transform,

$$G(x, z; x_*, z_*) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-x_*)} iG(p; z, z_*), \quad (12)$$

owing to space translational invariance, with a mode decomposition,

$$G(p; z, z_*) = \sum_{n=0}^{\infty} \frac{-iX_n(z)X_n(z_*)}{p^2 + m_{Xn}^2}. \quad (13)$$

The modes in (13) are solutions to the equation of motion for the X field in bulk after substituting $X \to e^{-ip\cdot x}X$ for $p^2 = m_X^2$ or

$$\frac{d}{dz}\left(e^{-B}\frac{d}{dz}X\right) + 3\frac{e^{-B}}{z^2}X + m_X^2 e^{-B}X = 0 \quad (14)$$

with $B = z^2 + 3\ln z$. Following Ref. [13], we define $X = e^{B/2}\psi$ and solve for ψ. Thus,

$$X_n = e^{B/2}\psi_n = z^3\sqrt{\frac{2}{1+n}}L_n^1(z^2) \quad (15)$$

with

$$m_{Xn}^2 = 4n + \frac{9}{2}. \quad (16)$$

Using (13)–(15) in (12), one can undo the p-integration (12) in terms of Bessel functions. For equal times or $x = (0, \mathbf{x})$,

$$\int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip\cdot x}}{p^2 + m_{Xn}^2} = \frac{m_{Xn}}{4\pi^2|\mathbf{x}|}\mathbf{K}_1(m_{Xn}|\mathbf{x}|). \quad (17)$$

Thus,

$$\frac{\langle \bar{q}q(x)W(C)\rangle_c}{\langle W(C)\rangle} = -12\frac{\sqrt{\lambda}}{N_c}\sum_{n=0}^{\infty}\int_0^a k^6 z^3 L_n^1(z^2k^2) \times \frac{m_{Xn}\mathbf{K}_1(m_{Xn}k\sqrt{x^2 + a^2 - z^2 - 2x\sqrt{a^2 - z^2}cos\phi})}{ka\sqrt{x^2 + a^2 - z^2 - 2x\sqrt{a^2 - z^2}cos\phi}} dz d\phi. \quad (18)$$

Assuming that the distance $|x - x_*| \approx |0 - \mathbf{x}_*| \equiv L$ between the center of the loop and the probing scalar is large, we may ignore the details of the shape of the loop and set $L_n^1(z^2) \approx L_n^1(0) = n + 1$. The result after the z-integration is $(L/a \gg 1)$

$$\frac{\langle \bar{q}q(x)W(C)\rangle_c}{\langle \bar{q}q\rangle\langle W(C)\rangle} \approx -\frac{6\pi k^6 a^3}{\langle \bar{q}q\rangle}\frac{\sqrt{\lambda}}{N_c}\sum_{n=0}^{\infty}\frac{(n+1)\sqrt{m_{Xn}}}{Lk}\mathbf{K}_1(m_{Xn}kL), \quad (19)$$

where the soft-wall scale $k = m_\rho/2$ was reinstated. The exponential falloff caused by the confining soft wall in (19) is to be contrasted with the power falloff as $1/L^6$ for $\mathcal{N} = 4$ SYM [12] (see also below). The lowest scalar mass in the exponent is $m_{X0}k = 3m_\rho/2\sqrt{2} \approx 817$ MeV. Recall that in AdS/QCD $\langle \bar{q}q \rangle$ is an input through (10).

The pseudoscalar condensate $\bar{q}i\gamma_5 q$ around a Wilson loop in AdS/QCD involves the dual axion field $\xi(x, z)$ in the bulk [17]. This condensate can exist despite the absence of CP violation around flux tubes. It is supported by the dipole content generated by the probe Willson loop. The axion contribution to the bulk action is similar to the action for the X field above but only $1/N_c^2$ suppressed. Therefore, we may just assume that in AdS/QCD the bulk action for the axion field is $1/N_c^2$ that of the scalar in (1). The axion coupling to the string world sheet is also $1/N_c^2$ suppressed with respect to (8) or

$$S_\xi = \frac{1}{2\pi\alpha N_c^3}\int dA(-6\xi)\frac{z^2}{a^2}. \quad (20)$$

A rerun of the preceding arguments for the scalar form factor gives

$$\frac{\langle \bar{q}i\gamma_5 q(x)W\rangle}{\langle W(C)\rangle} \approx -6\pi k^6 a^3\frac{\sqrt{\lambda}}{N_c^3}\sum_{n=0}^{\infty}\frac{(n+1)\sqrt{m_{Xn}}}{Lk}\mathbf{K}_1(m_{Xn}kL), \quad (21)$$

which is $1/N_c^2$ suppressed in comparison to (19).

V. GLUON CONDENSATES AROUND A HEAVY COLOR SOURCE

To probe the scalar gluon condensate around a heavy quark source, we proceed through similar arguments by replacing $\bar{q}q$ with $(gF)^2 \equiv F^2$ (Renormalization Group Invariant). In bulk, the dual of the latter is a dilaton fluctuation φ. Recall that in the bottom-up approach the dilaton background is an input and not a solution to the coupled gravitational equations. This notwithstanding, the form of the action for φ is analogous to (1) for X but with $m_5^2 = 0$ as noted earlier,

$$S_\varphi = \int d^4xdz\sqrt{-g}e^{-\phi}(-|D\varphi|^2). \tag{22}$$

The equation of motion for φ proceeds as before for both the vacuum solution $\varphi_*(z)$ through the ultraviolet-boundary condition $\varphi_*(z) = C + \langle F^2\rangle z^4 + \mathcal{O}(z^5)$ and normal modes in the bulk

$$\varphi_n = z^4\sqrt{\frac{2}{(1+n)(2+n)}}L_n^2(z^2) \tag{23}$$

with

$$m_{\varphi n}^2 = 4n + 13/2. \tag{24}$$

The coupling of the gluonic scalar to the string world sheet in the Einstein frame is analogous to (8),

$$\frac{1}{2\pi\alpha N_c}\int dA\frac{1}{2}\varphi, \tag{25}$$

without the extra minus sign and z-warping. A rerun of the previous arguments yields for the gluonic condensate of the Willson loop

$$\frac{\langle F^2(x)W\rangle_c}{\langle F^2\rangle\langle W(C)\rangle} \approx +\frac{\pi k^8a^4}{3\langle F^2\rangle}\frac{\sqrt{\lambda}}{N_c}\sum_{n=0}^{\infty}\frac{(n+1)(n+2)\sqrt{m_{\varphi n}}}{Lk}\mathbf{K}_1(m_{\varphi n}kL). \tag{26}$$

The leading exponential decay is governed by the lowest mass $m_{\varphi 0}k = \sqrt{13/8}m_\rho \approx 982$ MeV. As indicated earlier, the gluon condensate $\langle F^2\rangle$ is an input in AdS/QCD. We note that when rotated to Euclidean space the left-hand side in (26) does not change sign, while the right-hand side does since $\langle F^2\rangle \to \langle F^2\rangle_E < 0$. This result is in overall agreement with the results obtained through the field strength correlator method [18] and quenched lattice results [19].

The topological gluonic condensate $F\tilde{F}$ of the Wilson loop in AdS/QCD can be sought along the same arguments as the pseudoscalar form factor. The dual field $\chi(x,z)$ is represented by a bulk action that is similar to (22), but with a coupling,

$$S_\chi = \frac{1}{2\pi\alpha N_c^3}\int dA(-8\chi)\frac{z^2}{a^2}, \tag{27}$$

with $8 = 2\Delta(F\bar{F})$, by analogy with the axion coupling (20). A rerun of the preceding arguments for the topological condensate of the circular Wilson loop yields

$$\frac{\langle F\tilde{F}(x)W\rangle}{\langle W(C)\rangle} \approx -\frac{4\pi k^8a^4}{5}\frac{\sqrt{\lambda}}{N_c^3}\sum_{n=0}^{\infty}\frac{(n+1)(n+2)\sqrt{m_{\varphi n}}}{Lk}\mathbf{K}_1(m_{\varphi n}kL), \tag{28}$$

which is opposite to (26) and $1/N_c^2$ suppressed. The depletion of the topological charge near the Wilson loop with a strong flux sheet is plausible, justifying *a posteriori* the negative coupling in (27) by analogy with the axion coupling. This result is also in overall agreement with calculations using the field strength correlation method [18] and quenched lattice results [19].

VI. REMARKS

In deriving (19) using the soft-wall model (1), we have made some assumptions: 1) the dual X of $\bar{q}q$ has no backreaction on the space-time geometry, and 2) its coupling to the NG world sheet is the same as in $\mathcal{N} = 4$ SYM [12]. Some of these issues can be overcome by introducing the backreaction of X on the gravity sector using a more realistic bottom-up approach to the gauge/gravity dual of QCD such as Veneziano-QCD [20]. Specifically, the action is now

$$S = (M^3N_c^2)\int\left[\sqrt{-g}\left(R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2}\right) + V_g(\lambda) - xV_f(\lambda,T)\sqrt{det(g_{ab} + h(\lambda,T)\partial_aT\partial_bT}\right] \tag{29}$$

with R the Ricci curvature for the metric g, λ the dilaton field, and T the tachyon field. $V_{g,f}$ are the gluonic and sermonic potentials respectively with $x = N_f/N_c$ fixed at large N_c. The ensuing equations of motion naturally couple T and g. Recall that T is dual to the quark condensate at the boundary:

$$T_{ij}(x,z) \equiv m_{ij}(x)z + \sigma_{ij}(x)z^3 + \mathcal{O}(z^4). \tag{30}$$

To proceed, we need to solve for T subject to the boundary condition (30) and find the metric g as a function of the mass matrix m_{ij}. The knowledge of $g[m_ij]$ allows the construction of the minimal surface for the circular Wilson loop or $\mathcal{A}[m_ij]$. The scalar condensate in the presence of a Wilson loop follows then through

$$\langle \bar{q}q(x)W(C)\rangle = \frac{\delta\mathcal{A}[m_{ij}]}{\delta m_{ij}(x)} \tag{31}$$

after setting $m_{ij}(x) = m\delta_{ij}$. The local coupling to the string world sheet will follow from

$$\delta\mathcal{A} = \int dA\mathbf{C}(x,z)\delta T(x,z), \tag{32}$$

where $\mathbf{C}(x,z)$ is a local function on the surface. For $L/a \gg 1$, it is sufficient to take $\mathbf{C}(0,z)$ at the center of the world sheet and use the tachyon-tachyon correlator $\langle \delta T(0,z_*)\delta T(x,z')\rangle$ for the Green function $G(0,z_*;x,z')$. This procedure is numerically involved and will be reported elsewhere. Overall, we expect the results to be qualitatively similar to the soft-wall result quoted above.

VII. STATIC DIPOLE-DIPOLE POTENTIAL

The dipole distribution around another dipole of size a is best captured by the dipole-dipole potential. For simplicity, consider static and equal size dipoles of spatial extension a away from each other by a distance L. In QCD this dipole-dipole potential was analyzed in the random-instanton vacuum in Refs. [10,21] and more recently in $\mathcal{N}=4$ SYM in Ref. [12]. We now address it in the context of the soft-wall model. By definition the dipole-dipole potential follows from the long time connected correlator

$$V(L) = -\lim_{T\to+\infty}\frac{1}{T}\ln\left(\frac{\langle W(L)W(0)\rangle_c}{\langle W(L)\rangle\langle W(0)\rangle}\right), \tag{33}$$

with $W(0)$ and $W(L)$ two identical and rectangular Willson loops of width a and infinite time extent T, centered at 0 and L respectively.

In AdS/QCD and for $L/a \gg 1$ and small dipole sizes a, the minimal surface in the x,z plane is unaffected by the soft wall. Thus, its shape is given by

$$\left(\frac{dz}{dx}\right)^2 = \frac{z_h^4}{z^4} - 1, \quad |x-x_o| < a/2, \quad z < z_h, z_h \propto a \tag{34}$$

with x_0 the center of the rectangular loop.

The scalar coupling X to the rectangular Wilson loops now reads

$$\frac{1}{2\pi\alpha N_c}\int dA\frac{z^2}{z_h^2}(-6X) \tag{35}$$

instead of (8). For $L/a \gg 1$, we can use arguments similar to those developed earlier to reexpress the connected correlation function in (33) in terms of the scalar propagator (12) folded with the couplings to the respective couplings (35) on the world sheets. The result for the potential is

$$V(L) \approx -4\pi k^5 a^4 \frac{\lambda}{N_c^2}\sum_{n=1}^{\infty}(n+1)\frac{m_{Xn}}{kL}\mathbf{K}_1(km_{Xn}L) \tag{36}$$

with $k = m_\rho/2$. The overall minus sign follows from (33). The $n=1$ leading contribution asymptotically to (36) is

$$V(L) \approx -\frac{\pi^{\frac{3}{2}}m_{X1}}{2}\frac{\lambda}{N_c^2}\frac{(am_\rho)^4}{L}\frac{e^{-m_{X1}m_\rho L/2}}{\sqrt{m_{X1}m_\rho L}} \tag{37}$$

with $m_{X1} = \sqrt{17/2}$ from (16). The scalar exchange is attractive and exponentially suppressed in the confined phase. The result (36) is to be compared to $-a^4/L^5$ in $\mathcal{N}=4$ SYM [12] (see below).

Although we only considered the exchange of the lowest dimensional scalar X in deriving (36), additional exchanges in the form of gravitons and B-fields are also expected. In the soft-wall model considered here, they are characterized by similar couplings but their boundary squared masses are all larger than the $9k/2$ for the scalar in (16). Indeed, the squared mass of the dilaton is $13k/2$ as in (24), while that of the graviton is $8k$, both of which are heavier.

It is instructive to derive the results in the nonconfining or Coulomb phase using the qualitative arguments developed in Refs. [10,21]. Indeed, in $\mathcal{N}=4$, the composite coupling for the $\Delta=2$ operator made of two adjoint scalars $\Phi\Phi$ each with coupling $g\Phi$ to a heavy quark or Wilson loop follows from second-order perturbation theory or $|<0|g\Phi|1>|^2/\Delta E \approx (a\sqrt{\lambda}/N_c)(\Phi\Phi)$ with the energy splitting $\Delta E \approx \sqrt{\lambda}/a$ at strong coupling [1]. Inserting these operators between a heavy quark-antiquark pair at a distance L as shown in Fig. 2 (left) yields the potential

$$V_{\Phi\Phi}(L) \approx -\left(\frac{a\sqrt{\lambda}}{N_c}\right)^2\int d\tau\left(\frac{1}{L^2+\tau^2}\right)^2 \approx -\frac{\lambda a^2}{N_c^2 L^3}. \tag{38}$$

The composite coupling for the $\Delta=4$ operator made of say two adjoint electric gluons E^2 each with coupling gaE is $a^3\sqrt{\lambda}/N_c E^2$. From Fig. 2 (right), a rerun of the preceding arguments yields the potential

$$V_{FF}(L) \approx -\left(\frac{a^3\sqrt{\lambda}}{N_c}\right)^2\int d\tau\left(\frac{1}{L^2+\tau^2}\right)^4 \approx -\frac{\lambda a^6}{N_c^2 L^7}. \tag{39}$$

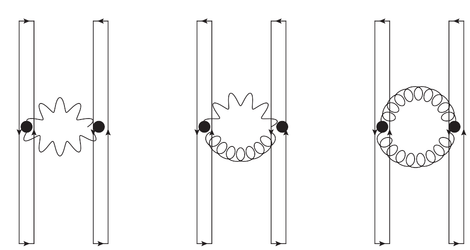

FIG. 2. Static dipole-dipole interactions: $\Phi\Phi$ exchange (left), ΦF exchange (middle), and FF exchange (right). See the text.

The composite coupling for $\Delta = 3$ follows from the cross term arising from the second-order perturbative expression or $|\langle 0|(g\Phi + ga\cdot E)|1\rangle|^2/\Delta E$ leading to $\sqrt{\lambda}a^2/N_c\Phi E$. The dipole-dipole potential is then

$$V_{\Phi F}(L) \approx -\left(\frac{a^2\sqrt{\lambda}}{N_c}\right)^2 \int d\tau \left(\frac{1}{L^2+\tau^2}\right)^3 \approx -\frac{\lambda a^4}{N_c^2 L^5} \quad (40)$$

(38)–(40) summarize to

$$V_\Delta(L) = -\mathcal{C}(\Delta)\frac{\lambda a^{2\Delta-2}}{N_c^2 L^{2\Delta-1}} \quad (41)$$

in agreement with the result in Ref. [12]. The overall constant $\mathcal{C}(\Delta)$ is beyond the qualitative nature of our arguments. Its precise value can be found in Ref. [12] using a more detailed analysis. For completeness, we quote its value

$$\mathcal{C}(\Delta) = \frac{\Gamma(1/4)^{4\Delta-4}}{32^{2\Delta+9}\pi^{3\Delta-7/2}} \times \frac{(\Delta-1)^2(\Delta-2)^2(\Delta-3)\Gamma(\Delta-1/2)\Gamma(\frac{\Delta-1}{4})^4}{\Gamma(\Delta)\Gamma(\frac{\Delta-1}{2})^2}. \quad (42)$$

VIII. CONCLUSIONS

To probe flux tubes in QCD is notoriously hard outside lattice simulations. We have suggested the gauge/gravity correspondence as a simple framework for addressing this issue. We have represented static color charges by circular Wilson loops and shown how to probe the analog of the chiral and gluon condensate around these charges.

In AdS/QCD, the scale is set by the rho mass m_ρ. The quark clouds are dominated by the exchange of a light mass of order $3m_\rho/2\sqrt{2}$, while the gluonic clouds involves a light mass of order $\sqrt{13/8}m_\rho$. The quark and pseudoscalar gluon clouds are depleted by the heavy quark source (negative contribution), while the scalar gluon cloud is enhanced by the heavy quark source (positive contribution). Their strong coupling to the heavy quark world sheet as a loop of radius a is generically $(-am_\rho^2)^\Delta\sqrt{\lambda}/N_c$ with $\Delta = 3, 4$ for the scalar quark and gluon insertions respectively. The pseudoscalar gluon coupling is $1/N_c^2$ the scalar gluon coupling.

The depletion of the scalar quark condensate around a rectangular Wilson loop was noted in the recent lattice simulations [6]. The depletion of the scalar and pseudoscalar gluon condensates around a confining string was observed using the field strength correlator method [18] and also quenched lattice simulations [19]. These is strong support for our AdS/QCD analysis. It would be interesting to assess the pseudoscalar quark condensate in the same simulations for a comparison with our final results.

The interaction between small size and static dipoles in the present holographic construction provides us with some insight into the nature of the static interactions between QCD strings in both the confined and Coulomb phases. In AdS/QCD, the interaction is attractive and dipolelike in the Coulomb phase, of the form $-\lambda a^{2\Delta-2}/N_c^2 L^{2\Delta-1}$. In QCD, the dominance of the $\Delta = 3$ or quark exchange ($\bar{q}q$) and $\Delta = 4$ or glueball exchange (F^2) is likely to saturate the dipole-dipole exchange at large distances with an edge for the light scalar exchange or $\Delta = 3$ [7]. Dynamical strings involve both static and nonstatic or velocity-dependent potentials. The latter are outside the scope of this work.

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