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Probing the photon flux in the diffractive quarkonium photoproduction at the LHC

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In this paper we propose the study of the diffractive quarkonium photoproduction in pp collisions at LHC energies to probe the photon flux associated with an ultrarelativistic proton. The total photon distribution is expected to be given in terms of the elastic and inelastic components. Distinctly from the elastic photon component that can be determined from the elastic form factors, the magnitude of the inelastic component still is an open question. We consider the current parametrizations for the photon distribution of a proton and estimate the rapidity distributions and total cross sections for the production of J/Ψ, Ψ', and Υ at LHC energies. We demonstrate that the predictions associated with the inelastic contribution are of the same order as those associated with the elastic one. Our results imply that a dedicated experimental analysis of diffractive quarkonium photoproduction with the tagging of the two protons in the final state can be useful to constrain the magnitude of the inelastic component of the photon distribution.

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I. INTRODUCTION

In recent years a series of experimental results at RHIC [1,2], Tevatron [3], and LHC [4–7] demonstrated that the study of photon-induced interactions in hadronic colliders is feasible and can be used to probe, e.g., the nuclear gluon distribution [8–10], the dynamics of the strong interactions [11–16], and the mechanism of quarkonium production [17–22]. It has stimulated the improvement of the theoretical description of these processes as well as the proposal of new forward detectors to be installed in the LHC. The basic idea in the photon-induced processes is that an ultrarelativistic charged hadron (proton or nuclei) gives rise to strong electromagnetic fields, such that the photon stemming from the electromagnetic field of one of the two colliding hadrons can interact with one photon of the other hadron (photon-photon process) or can interact directly with the other hadron (photon-hadron process) [23]. In these processes the total cross section can be factorized in terms of the equivalent flux of photons into the hadron projectile and the photon-photon or photon-target production cross section. Consequently, a basic ingredient in the analysis of the photon-induced processes is the description of the equivalent photon distribution of the hadron. The equivalent photon approximation of a charged pointlike fermion was formulated many years ago by Fermi [24] and developed by Williams [25] and von Weizsacker [26]. In contrast, the calculation of the photon distribution of the hadrons still is a subject of debate, due to the fact that they are not pointlike particles. In this case it is necessary to distinguish between the elastic and the inelastic components (see Fig. 1). The elastic component [Fig. 1(a)] can be estimated by analyzing the transition $h \rightarrow \gamma h$, taking into account the effects of the hadronic form factors, with the hadron remaining intact in the final state [27]. In contrast, the inelastic contribution [Fig. 1(b)] is associated with the transition $h \rightarrow \gamma X$, with $X \neq h$, and can be estimated taking into account the partonic structure of the hadrons, which can be a source of photons (see, e.g., Refs. [28–35]). Therefore the total photon distribution of a hadron is given by

$$\gamma(x, \mu^2) = \gamma_{\rm el}(x) + \gamma_{\rm inel}(x, \mu^2), \tag{1}$$

where x is the fraction of the hadron energy carried by the photon and μ has to be identified with a momentum scale of the photon-induced process. It is important to emphasize that while $\gamma_{\rm el}$ is proportional to the squared charge of the hadron (Z^2), because of the coherent action of all protons in the nucleus, $\gamma_{\rm inel}$ is proportional to the mass number A. Consequently, for heavy nuclei, the total photon distribution is determined by the elastic component. In contrast, for

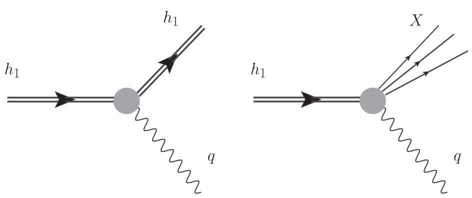

FIG. 1. (a) Elastic and (b) inelastic components of the equivalent photon distributions of a hadron.

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a proton, both components contribute equally and should be taken into account in the study of photon-induced processes. Currently, the description of the inelastic component still is an open question, with the predictions for its x dependence being largely distinct, as we will demonstrate in the next section. Our goal in this paper is twofold: (a) analyze the impact of the inelastic component in the diffractive photoproduction of vector mesons ($V = J/\Psi, \Psi'$, and Υ) in pp collisions at LHC, and (b) verify if a more detailed analysis of this process can be used to constrain the inelastic component of the photon distribution.

This paper is organized as follows. In the Sec. II we briefly review the description of the elastic and inelastic components of the photon distribution of a nucleon. In particular, we present a comparison between the current parametrizations for the photon distribution. In Sec. III we discuss the diffractive photoproduction in pp collisions and the main input used in our calculations. In Sec. IV we present our results for the $J/\Psi, \Psi'$, and Υ production considering the different models for the inelastic component of the photon distribution and compare with those obtained considering the elastic component. Finally, in Sec. V, we summarize our main conclusions.

II. THE EQUIVALENT PHOTON DISTRIBUTION

The concept of the photon content of a charged fermion is based on the equivalent photon approximation [23,28], which implies that the photon distribution of a nucleon consists of two parts: the elastic and the inelastic components. A detailed derivation of the elastic photon distribution of a nucleon was presented in Ref. [27], which can be written as

$$\gamma_{\mathrm{el}}(x) = -\frac{\alpha}{2\pi}\int_{-\infty}^{-\frac{m^2x^2}{1-x}} \frac{dt}{t}\left\{\left[2\left(\frac{1}{x}-1\right)+\frac{2m^2x}{t}\right]H_1(t) + xG_M^2(t)\right\}, \tag{2}$$

where $t = q^2$ is the momentum transfer squared of the photon,

$$H_1(t) \equiv \frac{G_E^2(t)+\tau G_M^2(t)}{1+\tau} \tag{3}$$

with $\tau \equiv -t/m^2$, m being the nucleon mass, and where G_E and G_M are the Sachs elastic form factors. Although an analytical expression for the elastic component is presented in Ref. [27], it is common to find in the literature the study of photon-induced processes considering an approximated expression proposed in Ref. [36], which can be obtained from Eq. (2) by disregarding the contribution of the magnetic dipole moment and the corresponding magnetic form factor. As demonstrated in Ref. [37] the difference between the full and the approximated expressions is smaller than 5% at low x. Consequently, in what follows we will use the expression proposed in Ref. [36], where the elastic photon distribution is given by

$$\gamma_{\mathrm{el}}(x) = \frac{\alpha}{\pi}\left(\frac{1-x+0.5x^2}{x}\right) \times\left[\ln(\Omega)-\frac{11}{6}+\frac{3}{\Omega}-\frac{3}{2\Omega^2}+\frac{1}{3\Omega^3}\right], \tag{4}$$

where $\Omega = 1 + (0.71\ \mathrm{GeV}^2)/Q^2_{\mathrm{min}}$ and $Q^2_{\mathrm{min}} \approx (xm)^2/(1-x)$.

On the other hand, there are different models for the contribution of the inelastic component of the photon distribution of a nucleon. In Ref. [30], a naive approach to the photon flux was proposed, with the photon distribution in the proton being given by a convolution of the distribution of quarks in the proton and the distribution of photons in the quarks as follows:

$$\gamma_{\mathrm{inel}}(x,\mu^2) = \sum_q \int_x^1 \frac{dx_q}{x_q} f_q(x_q,\mu^2)e_q^2 f_{\gamma/q}\left(\frac{x}{x_q}, Q_1^2, Q_2^2\right), \tag{5}$$

where the sum runs over all quark and antiquark flavors and the flux of photons in a quark $f_{\gamma/q}$ is given by

$$f_{\gamma/q}(z) = \frac{\alpha}{2\pi}\frac{1+(1-z)^2}{z}\log\frac{Q_1^2}{Q_2^2}, \tag{6}$$

where Q_1^2 is assumed to be the maximum value of the momentum transfer in the process and Q_2^2 is assumed to be equal to 1 GeV^2 for the parton model to be applicable. Recently, different groups have studied the modification of the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations for the quark and gluon distributions by the inclusion of QED contributions and have performed global parton analysis of deep inelastic and related hard-scattering data [32–35]. Basically, the DGLAP equations and the momentum sum rule are modified considering the presence of the photon as an additional pointlike parton in the nucleon. The parametrizations for the photon distribution currently available in the literature [32,33] differ in the approach for the initial condition for the photon distribution. While the MRST group assumes that $\gamma_{\mathrm{inel}}(x, Q_0^2)$ is given by an expression similar to Eq. (5), the NNPDF group parametrizes the input photon parton distribution function (PDF) and attempts to determine the parameters from the global data. The preliminary CTEQ analysis presented in Ref. [34] assumes a similar theoretical form for $\gamma_{\mathrm{inel}}(x, Q_0^2)$ to that proposed by the MRST group, but with an arbitrary normalization parameter, which is expressed as the momentum fraction carried by the photon. More recently, a distinct approach for the initial condition for the evolution of the photon distribution was proposed in Ref. [35], where the authors have proposed that the starting distribution for the photon PDF should be the total photon distribution, i.e., by the sum of the elastic and inelastic components as given in Eq. (1). The main motivation of this approach is the reduced uncertainty in the input photon PDF, since the major part of the distribution is given by the elastic component, which is well known. As a consequence of this assumption, the elastic component is dominant also at large values of the hard scale μ^2 (see Fig. 5 in [35]). Unfortunately, the current data are not sufficiently accurate to precisely determine the initial condition. Thus the current predictions for the inelastic photon component strongly differ in its x dependence. In what follows we will consider the MRST2004QED and NNPDF parametrizations, since only these two are currently available for public use. In Fig. 2 we present the predictions of the MRST2004QED and NNPDF parametrizations for the inelastic photon distribution considering two different values for the hard scale μ^2. For comparison the predictions of the naive approach [Eq. (5)] and the elastic component [Eq. [36]] are also presented. While the elastic component is independent of the hard scale μ^2, the inelastic component is strongly dependent, increasing at larger values of μ^2. Moreover, all models predict that the inelastic contribution is dominant at very small values of x. However, as demonstrated in Fig. 3, the x dependence of the inelastic parametrizations is very distinct. This result motivates the analysis of observables that are strongly dependent on the photon flux.

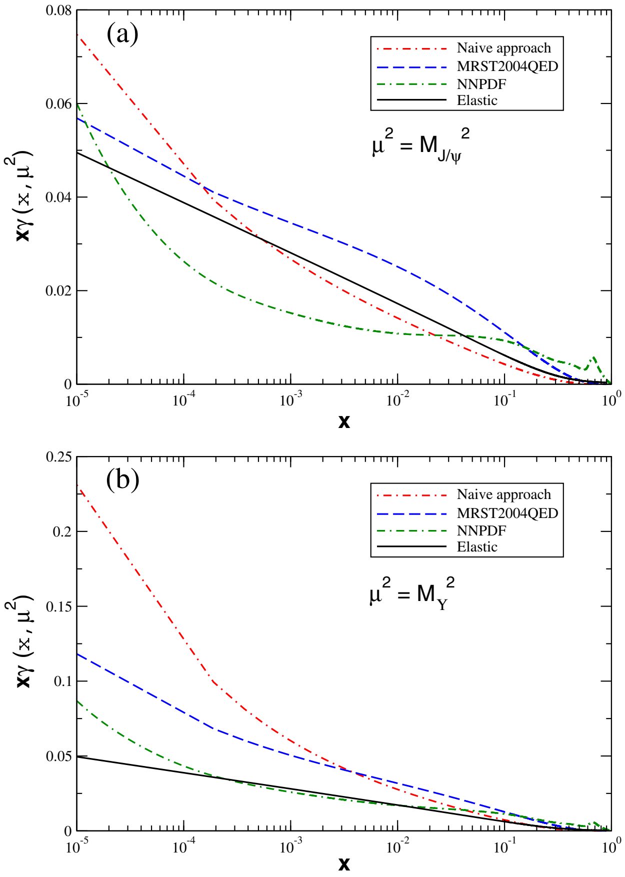

FIG. 2 (color online). Comparison between the different models for the inelastic component of the photon distribution for two different values of the hard scale μ^2: (a) $\mu^2 = M^2_{J/\Psi}$, and (b) $\mu^2 = M^2_{\Upsilon}$. The elastic component is presented for comparison.

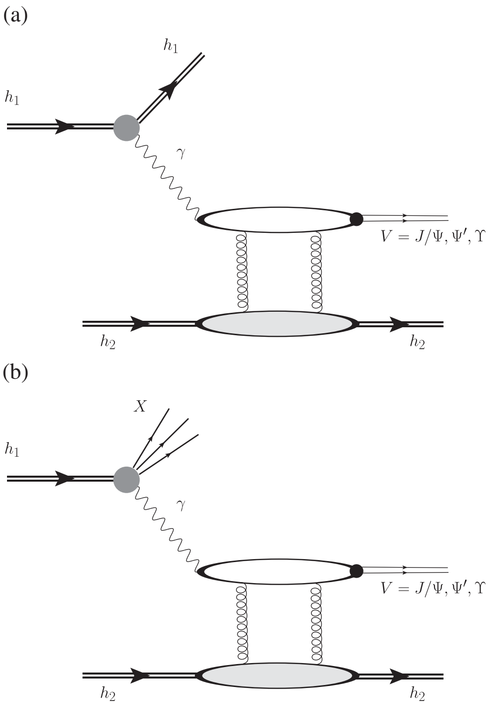

FIG. 3. Diffractive quarkonium photoproduction in hadronic collisions associated with the (a) elastic and (b) inelastic components of the photon distribution of the hadron.

III. THE DIFFRACTIVE QUARKONIUM PHOTOPRODUCTION IN pp COLLISIONS

The basic idea for the description of the diffractive quarkonium photoproduction in pp collisions is that the total cross section can be factorized in terms of the equivalent flux of photons of the hadron projectile and the photon-target production cross section as follows [23]:

$$\sigma(h_1 h_2 \to p \otimes V \otimes h_3) = \sum_{i=1,2} \int dY \frac{d\sigma_i}{dY}, \qquad (7)$$

where $h_1 = h_2 = p$, $\otimes$ represents a rapidity gap in the final state and $h_3 = p$ or X depending on whether the incident proton that emits the photon remains intact or dissociate. Moreover, $d\sigma_i/dY$ is the rapidity distribution for the photon-target interaction induced by the hadron h_i ($i = 1, 2$), which can be expressed as

$$\frac{d\sigma_i}{dY} = x\gamma_i(x,\mu^2)\sigma_{\gamma h_j \to V h_j}(W^2_{\gamma h_j}) \qquad (i \neq j), \tag{8}$$

where γ_i is the equivalent photon flux associated with the hadron i, $W^2_{\gamma h} = 2\omega\sqrt{s_{\rm NN}}$ is the center-of-mass system energy squared of the photon-hadron system, ω is the photon energy, and $s_{\rm NN}$ is the center-of-mass system energy squared of the hadron-hadron system.

Since the pioneering studies [8,38,39] on diffractive vector meson production in ultraperipheral heavy ion collisions (UPHIC) about 14 years ago, a large number of papers on the subject has been published considering several improvements in the theoretical description [9–17,20–22,40–42] and experimental analysis [1–6]. However, such studies have only considered the process in which the hadron that emits the photon remains intact as represented in Fig. 3(a). In other words, these studies have assumed that the total photon distribution is dominated by the elastic component and that the hadron that emits the photon remains intact. As discussed in the Introduction, such an approximation is reasonable for a nuclear projectile. On the other hand, for a proton projectile, the magnitude of the contribution associated with the inelastic component of the photon distribution, where the proton that emits the photon dissociates, represented in Fig. 3(b), remains an open question. This is the main goal of the next section. First, we need to specify the cross section for the diffractive photoproduction of a vector meson ($\sigma_{\gamma p \to Vp}$). In recent years such a process was studied by several theoretical groups considering different formalisms and underlying assumptions, with its predictions in reasonable agreement with the current experimental data. To obtain predictions of the inelastic contribution that are not dependent on the choice of the model used to estimate $\sigma_{\gamma p \to Vp}$, we will assume in what follows the following form obtained in the H1 analyses [43]:

$$\sigma_{\gamma p \to J/\Psi p}(W_{\gamma p}) = N\left(\frac{W_{\gamma p}}{90\ \text{GeV}}\right)^{\lambda},$$
$$\sigma_{\gamma p \to \Psi' p}(W_{\gamma p}) = 0.166N\left(\frac{W_{\gamma p}}{90\ \text{GeV}}\right)^{\lambda}, \tag{9}$$

where $N = 81 \pm 3$ nb and $\lambda = 0.67 \pm 0.03$. Moreover, for the Υ production, we will assume that $\sigma_{\gamma p \to \Upsilon p} = (0.12\ \text{pb})(W_{\gamma p}/W_0)^{1.6}$ with $W_0 = 1$ GeV, as given in Ref. [14].

IV. RESULTS

Let us initially calculate the rapidity distribution and total cross section for the diffractive quarkonium photoproduction in pp collisions at LHC energies. The distribution on rapidity Y of a vector meson $V(= J/\Psi, \Psi', \Upsilon)$ of mass M_V in the final state can be directly computed from Eq. (7), by using its relation with the photon energy ω, i.e., $Y \propto \ln(\omega/M_V)$. Explicitly, the rapidity distribution is written down as

$$\begin{aligned}\frac{d\sigma[h_1 h_2 \to p \otimes V \otimes h_3]}{dY} &= [x\gamma_{h_1}(x,\mu^2)\sigma_{\gamma h_2 \to V\otimes h_2}(W^2_{\gamma h_2})]_{\omega_L} \\ &\quad + [x\gamma_{h_2}(x,\mu^2)\sigma_{\gamma h_1 \to V\otimes h_1}(W^2_{\gamma h_1})]_{\omega_R},\end{aligned} \tag{10}$$

where $\omega_L(\propto e^{-Y})$ and $\omega_R(\propto e^{Y})$ denote photons from the h_1 and h_2 hadrons, respectively. Moreover, $h_3 = p$ or X depending on whether the incident proton that emits the photon remains intact or dissociates, respectively. As the photon fluxes have support at small values of ω (low x), decreasing at large ω (high x), the first term on the right-hand side of Eq. (10) peaks at positive rapidities while the second term peaks at negative rapidities. Consequently, the study of the rapidity distribution can be used to constrain the equivalent photon distribution. Moreover, the total rapidity distributions for pp collisions will be symmetric about midrapidity ($Y = 0$).

To estimate the diffractive quarkonium photoproduction associated with the inelastic component of the photon distribution, we should specify the hard scale μ^2. As can be verified in the literature, the choice of this scale is a bit ambiguous [30,32,33,44–46]. In general it is assumed that it is related to the center-of-mass energy of the photon-induced subprocess or to a hard scale in the final state. Following previous analysis that demonstrates that the mass of the vector meson can be considered a hard scale that justifies a perturbative calculation of its photoproduction (see, e.g., Ref. [47]), in what follows we will assume that $\mu^2 = M_V^2$. It is important to emphasize that larger values of the hard scale increase our predictions, since the magnitude of the inelastic photon distribution is amplified by the DGLAP evolution. Moreover, in order to estimate the inelastic component using the naive approach given by Eq. (5) we will assume that $\mu^2 = Q_1^2 = M_V^2$ and that the parton distributions are given by the MRST 2001 leading order parametrization [48].

In Fig. 4 we present our predictions for the rapidity distribution of the diffractive J/Ψ photoproduction in pp collisions at $\sqrt{s} = 7$ and 14 TeV. We consider three different models for the inelastic component of the photon distribution and also present the predictions associated with the elastic contribution for comparison. We obtain that at midrapidities the NNPDF predictions are a factor of ≈1.5 smaller than the elastic one. On the other hand, the MRST2004QED predictions are larger than the elastic one, with the naive predictions being of the same order of the elastic one. Our results indicate that inelastic predictions obtained using the MRST2004QED parametrization are larger than the elastic one in the full rapidity range. In contrast, the NNPDF parametrization implies that the inelastic contribution is larger than the elastic one at large rapidities. These behaviors are directly related to the x dependence of the inelastic component of the photon

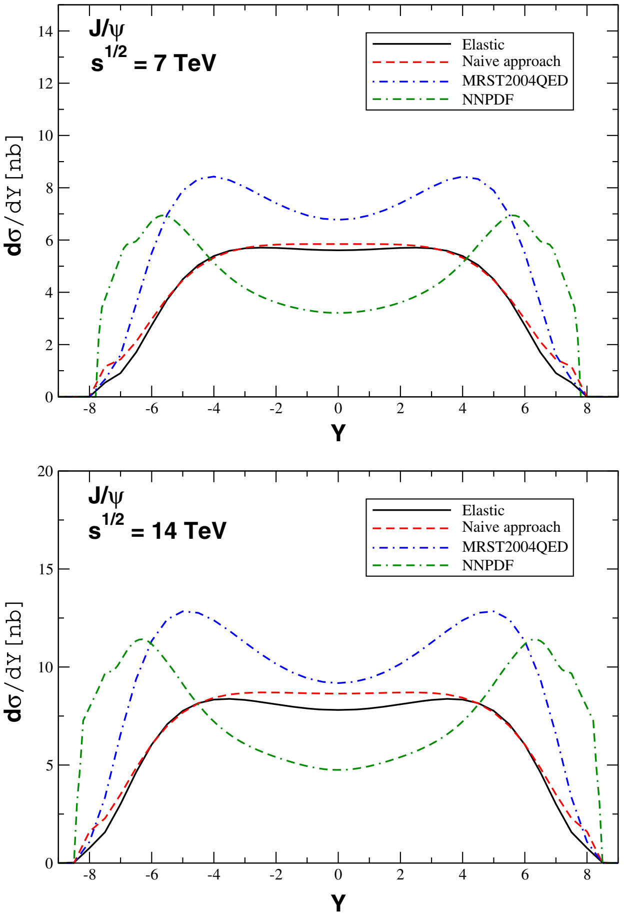

FIG. 4 (color online). Rapidity distribution for the diffractive J/Ψ photoproduction in pp collisions at LHC ($\sqrt{s} = 7$ and 14 TeV) considering different models for the inelastic component of the photon distribution. The predictions associated with the elastic component are also presented for comparison.

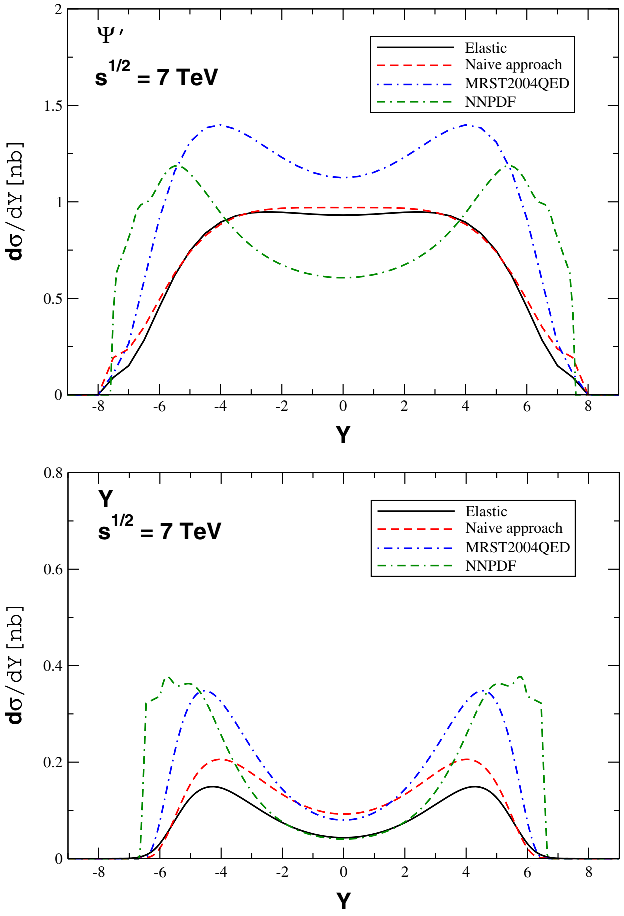

FIG. 5 (color online). Rapidity distribution for the diffractive photoproduction of Ψ' and Υ in pp collisions at LHC ($\sqrt{s} = 7$ TeV) considering different models for the inelastic component of the photon distribution. The predictions associated with the elastic component are also presented for comparison.

distribution for $\mu^2 = M^2_{J/\Psi}$ presented in Fig. 2(a), since at large rapidities we are probing larger values of x.

In Fig. 5 we present our predictions for the Ψ' and Υ photoproduction in pp collisions at $\sqrt{s} = 7$ TeV. For the Ψ' production we obtain that the behavior of the rapidity distributions are very similar to those obtained for the J/Ψ case, which is expected since the energy dependence of the photon-proton cross sections predicted by the H1 parametrizations are the same [See Eq. (9)], differing only in the normalization. In contrast, for the Υ production, we now obtain that at midrapidities the inelastic NNPDF prediction is of the same order as the elastic one. Moreover, we obtain that the inelastic predictions dominate at large rapidities. As in the J/Ψ case, these behaviors are directly related to the x dependence of the inelastic component of the photon distribution $\mu^2 = M^2_{\Upsilon}$ presented in Fig. 2(b).

In Table I we present our predictions for the total cross sections for the different final states discussed above. In particular, we present the predictions associated with the three different models for the inelastic component of the photon distribution. The predictions associated with the elastic component are also presented for comparison. As expected from our predictions for the rapidity distributions, our results indicate that for the three models of the inelastic component considered the associated cross sections are larger than the elastic one. These predictions are very distinct and allow one, in principle, to discriminate between the different models for the inelastic component of the

TABLE I. Total cross sections for the photoproduction of different final states in pp collisions at LHC ($\sqrt{s} = 7$ TeV) considering different models for the inelastic component of the photon distribution. For comparison the predictions considering the elastic component are also presented. Values are in nb.

Vector meson	Naive	MRST2004	NNPDF	Elastic
J/Ψ	69.50	98.70	73.70	66.90
Ψ'	11.50	16.40	12.80	11.10
Υ	1.70	2.40	2.50	1.10

photon distribution. Moreover, if future experimental results indicate a very small fraction of inelastic processes, it can be considered evidence that the more adequate approach for the treatment of the photon distribution of the proton is that proposed in Ref. [35]. To perform such study a dedicated experimental analysis is necessary to separate the inelastic and elastic processes. Basically, it is fundamental to tag the two protons into the final state to separate the elastic contribution. In principle, the products of the proton dissociation in the inelastic processes will travel essentially along the beam pipe. Consequently, both processes will be characterized by two rapidity gaps. Therefore, the presence of forward detectors will be essential to characterize the events.

V. SUMMARY

During the past two decades a rich phenomenology of photon-induced processes in hadronic colliders has emerged. However, several questions still remain to be answered. In particular, the treatment of the photon flux associated with an ultrarelativistic proton still is an open question. In this paper we have proposed, for the first time to our knowledge, the study of the diffractive quarkonium photoproduction in pp collisions at LHC energies as a probe of the photon distribution of a proton. This distribution is expected to be characterized by elastic and inelastic components, which are associated with the coherent or incoherent emission of the photon from the proton, with the proton remaining intact or dissociating, respectively. Currently, several groups have proposed distinct approaches for the treatment of the inelastic contribution and its evolution. In this paper we have considered three different models and estimated the diffractive photoproduction of J/Ψ, Ψ', and Υ in pp collisions. Our results indicated that, for the models considered, the contribution of the inelastic processes is of the same order or larger than the elastic one, with the predictions for the rapidity distributions being largely different, which makes the experimental discrimination feasible, with the detection of the two protons into the final state being indispensable to separate the inelastic and elastic events. Finally, if the contribution of the inelastic events is probed to be very small, it can be interpreted as a signature of a different approach for the treatment of the photon distribution [35].

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[1] C. Adler *et al.* (STAR Collaboration), Phys. Rev. Lett. **89**, 272302 (2002).

[2] S. Afanasiev *et al.* (PHENIX Collaboration), Phys. Lett. B **679**, 321 (2009).

[3] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **102**, 242001 (2009).

[4] B. Abelev *et al.* (ALICE Collaboration), Phys. Lett. B **718**, 1273 (2013).

[5] E. Abbas *et al.* (ALICE Collaboration), Eur. Phys. J. C **73**, 2617 (2013).

[6] R. Aaij *et al.* (LHCb Collaboration), J. Phys. G **40**, 045001 (2013).

[7] R. Aaij *et al.* (LHCb Collaboration), J. Phys. G **41**, 055002 (2014).

[8] V. P. Goncalves and C. A. Bertulani, Phys. Rev. C **65**, 054905 (2002).

[9] A. L. Ayala Filho, V. P. Goncalves, and M. T. Griep, Phys. Rev. C **78**, 044904 (2008); A. Adeluyi and C. Bertulani, Phys. Rev. C **84**, 024916 (2011); **85**, 044904 (2012).

[10] V. Guzey and M. Zhalov, J. High Energy Phys. 10 (2013) 207; 02 (2014) 046.

[11] V. P. Goncalves and M. V. T. Machado, Eur. Phys. J. C **40**, 519 (2005).

[12] V. P. Goncalves and M. V. T. Machado, Phys. Rev. C **73**, 044902 (2006); Phys. Rev. D **77**, 014037 (2008); Phys. Rev. C **80**, 054901 (2009).

[13] V. P. Goncalves and M. V. T. Machado, Phys. Rev. C **84**, 011902 (2011).

[14] L. Motyka and G. Watt, Phys. Rev. D **78**, 014023 (2008).

[15] T. Lappi and H. Mantysaari, Phys. Rev. C **87**, 032201 (2013).

[16] M. B. Gay Ducati, M. T. Griep, and M. V. T. Machado, Phys. Rev. D **88**, 017504 (2013); Phys. Rev. C **88**, 014910 (2013).

[17] W. Schafer and A. Szczurek, Phys. Rev. D **76**, 094014 (2007); A. Rybarska, W. Schafer, and A. Szczurek, Phys. Lett. B **668**, 126 (2008); A. Cisek, W. Schafer, and A. Szczurek, Phys. Rev. C **86**, 014905 (2012).

[18] V. P. Goncalves and M. M. Machado, Eur. Phys. J. C **72**, 2231 (2012).

[19] V. P. Goncalves and M. M. Machado, Eur. Phys. J. A **50**, 72 (2014).

[20] A. Cisek, W. Schafer, and A. Szczurek, arXiv:1405.2253.

[21] V. P. Goncalves, B. D. Moreira, and F. S. Navarra, Phys. Rev. C **90**, 015203 (2014).

[22] V. P. Goncalves, B. D. Moreira, and F. S. Navarra, Phys. Lett. B **742**, 172 (2015).

[23] G. Baur, K. Hencken, D. Trautmann, S. Sadovsky, and Y. Kharlov, Phys. Rep. **364**, 359 (2002); V. P. Goncalves and M. V. T. Machado, Mod. Phys. Lett. A **19**, 2525 (2004); C. A. Bertulani, S. R. Klein, and J. Nystrand, Annu. Rev.

Nucl. Part. Sci. **55**, 271 (2005); K. Hencken *et al.*, Phys. Rep. **458**, 1 (2008).
[24] E. Fermi, Z. Phys. **29**, 315 (1924).
[25] E. J. Williams, Phys. Rev. **45**, 729 (1934).
[26] C. F. von Weizsacker, Z. Phys. **88**, 612 (1934).
[27] B. A. Kniehl, Phys. Lett. B **254**, 267 (1991).
[28] V. M. Budnev, I. F. Ginzburg, G. V. Meledin, and V. G. Serbo, Phys. Rep. **15**, 181 (1975).
[29] A. De Rujula and W. Vogelsang, Phys. Lett. B **451**, 437 (1999).
[30] M. Drees, R. M. Godbole, M. Nowakowski, and S. D. Rindani, Phys. Rev. D **50**, 2335 (1994).
[31] M. Gluck, C. Pisano, and E. Reya, Phys. Lett. B **540**, 75 (2002); C. Pisano, arXiv:hep-ph/0512306.
[32] A. D. Martin, R. G. Roberts, W. J. Stirling, and R. S. Thorne, Eur. Phys. J. C **39**, 155 (2005).
[33] R. D. Ball, V. Bertone, S. Carrazza, L. Del Debbio, S. Forte, A. Guffanti, N. P. Hartland, and J. Rojo (NNPDF Collaboration), Nucl. Phys. **B877**, 290 (2013).
[34] C. Schmidt, J. Pumplin, D. Stump, and C.-P. Yuan, *Proc. Sci.*, DIS2014 (2014) 054.
[35] A. D. Martin and M. G. Ryskin, Eur. Phys. J. C **74**, 3040 (2014).
[36] M. Drees and D. Zeppenfeld, Phys. Rev. D **39**, 2536 (1989).
[37] V. P. Goncalves, D. T. da Silva, and W. K. Sauter, Phys. Rev. C **87**, 028201 (2013).
[38] S. R. Klein and J. Nystrand, Phys. Rev. C **60**, 014903 (1999).
[39] L. Frankfurt, M. Strikman, and M. Zhalov, Phys. Lett. B **540**, 220 (2002).
[40] L. Frankfurt, M. Strikman, and M. Zhalov, Phys. Lett. B **537**, 51 (2002); Phys. Rev. C **67**, 034901 (2003); L. Frankfurt, V. Guzey, M. Strikman, and M. Zhalov, J. High Energy Phys. 08 (2003) 043.
[41] S. R. Klein and J. Nystrand, Phys. Rev. Lett. **92**, 142003 (2004).
[42] S. P. Jones, A. D. Martin, M. G. Ryskin, and T. Teubner, J. High Energy Phys. 11 (2013) 085.
[43] C. Adloff *et al.* (H1 Collaboration), Phys. Lett. B **541**, 251 (2002); C. Alexa *et al.* (H1 Collaboration), Eur. Phys. J. C **73**, 2466 (2013).
[44] J. Ohnemus, T. F. Walsh, and P. M. Zerwas, Phys. Lett. B **328**, 369 (1994).
[45] G. Bhattacharya, P. Kalyniak, and K. A. Peterson, Phys. Rev. D **53**, 2371 (1996).
[46] M. Luszczak and A. Szczurek, arXiv:1309.7201.
[47] P. Newman and M. Wing, Rev. Mod. Phys. **86**, 1037 (2014).
[48] A. D. Martin, R. G. Roberts, W. J. Stirling, and R. S. Thorne, Eur. Phys. J. C **23**, 73 (2002).