

# Open charm production in the parton Reggeization approach: Tevatron and the LHC

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We study the inclusive hadroproduction of  $D^0$ ,  $D^+$ ,  $D^{*+}$ , and  $D_s^+$  mesons at leading order in the parton Reggeization approach endowed with universal fragmentation functions fitted to  $e^+e^-$  annihilation data from CERN LEP1. We have described  $D$ -meson transverse momentum distributions measured in the central region of rapidity by the CDF collaboration at Tevatron ( $|y| < 1$ ) and ALICE collaboration at the LHC ( $|y| < 0.5$ ) within uncertainties and without free parameters, using Kimber-Martin-Ryskin and Ciafaloni-Catani-Fiorani-Marchesini approaches to obtain unintegrated gluon distribution function in a proton.

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## I. INTRODUCTION

The study of the open charm production in the high energy hadronic collisions is considered as a test of general applicability of perturbative quantum chromodynamics (QCD). In the process of charmed meson production one has  $\mu \geq m$ , where  $\mu$  is the typical energy scale of the hard interaction,  $m$  is the charm quark mass, and  $\alpha_S(\mu) \ll 1$ . Nevertheless, this study is also our potential for the observation of a new dynamical regime of perturbative QCD, namely the high-energy *Regge limit*, which is characterized by the following condition  $\sqrt{S} \gg \mu \gg \Lambda_{\text{QCD}}$ , where  $\sqrt{S}$  is the invariant collision energy, and  $\Lambda_{\text{QCD}}$  is the asymptotic scale parameter of QCD. In this limit a new small parameter  $x \sim \mu/\sqrt{S}$  appears.

The small- $x$  effects cause the distinction of the perturbative corrections relative for different processes and different regions of phase space. At first, the higher-order corrections for the production of heavy final states, such as Higgs bosons, top-quark pairs, dijets with large invariant masses, or Drell-Yan pairs, by initial-state partons with relatively large momentum fractions  $x \sim 0.1$  are dominated by soft and collinear gluons and may increase the cross

sections up to a factor 2. By contrast, relatively light final states, such as small-transverse-momentum heavy quarkonia, single jets, prompt photons, or dijets with small invariant masses, are produced by the fusion of partons with small values of  $x$ , typically  $x \sim 10^{-3}$  because of the large values of  $\sqrt{S}$ . Radiative corrections to such processes are dominated by the production of additional hard jets. The only way to treat such processes in the conventional collinear parton model (CPM) is to calculate higher-order corrections in the strong coupling constant  $\alpha_S = g_S^2/4\pi$ , which could be a challenging task for some processes even at the next-to-leading order (NLO) level. To overcome this difficulty and take into account a sizable part of the higher-order corrections in the small- $x$  regime, the  $k_T$ -factorization framework was introduced [1–3].

The theoretical studies of  $D$ -meson hadroproduction were performed both in the NLO of the CPM [4–8] and in the  $k_T$ -factorization with off-shell initial gluons [9], to describe the different experimental data on  $D$ -meson production obtained at Fermilab Tevatron [10] and the CERN LHC, for the central [11–13] and forward [14] regions of rapidity. The aim of the present paper is to study the  $D$ -meson production at Fermilab Tevatron and the CERN LHC at central rapidities in the  $k_T$ -factorization framework [1] endowed with the fully gauge-invariant amplitudes with *Reggeized* gluons in the initial state. We will call this combination the parton Reggeization approach (PRA) everywhere below. We suppose PRA to be

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more theoretically consistent than previous studies in  $k_T$ -factorization, being not a recipe but based on a gauge invariant effective theory for the processes in quasi-multi-Regge kinematics (QMRK) in QCD [15,16]. Therefore it preserves the gauge invariance of high-energy particle production amplitudes and allows a consistent continuation towards the NLO calculations [17].

Recently, PRA was successfully applied for the analysis of inclusive production of single jet [18], pair of jets [19], prompt-photon [20,21], photon plus jet [22], Drell-Yan lepton pairs [23], bottom-flavored jets [24,25], charmonium and bottomonium production [26–30] at the Tevatron and LHC. These studies have demonstrated the advantages of the high-energy factorization scheme based on PRA in the descriptions of data comparing to the collinear parton model calculations.

This paper is organized as follows. In Sec. II we present basic formalism of our calculations, the PRA and the fragmentation model. In Sec. III our results are presented in comparison with the experimental data and discussed. In Sec. IV we summarize our conclusions.

## II. BASIC FORMALISM

The phenomenology of strong interactions at high energies exhibits a dominant role of gluon fusion into heavy quark and antiquark pair in heavy meson production. As it was shown in Ref. [6], a significant part of  $D$ -meson production cross section comes from gluon and  $c$ -quark fragmentation into  $D$ -meson, and the light quark fragmentation turns out to be negligible. Following this, in our study we will consider the  $c$ -quark and gluon fragmentation into different  $D$ -mesons only.

In hadron collisions the cross sections of processes with a hard scale  $\mu$  can be represented as a convolution of scale-dependent parton (quark or gluon) distributions and squared hard parton scattering amplitude. These distributions correspond to the density of partons in the proton with longitudinal momentum fraction  $x$  integrated over transverse momentum up to  $k_T = \mu$ . Their evolution from some scale  $\mu_0$ , which controls a nonperturbative regime, to the typical scale  $\mu$  is described by Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [31] evolution equations which allow to sum large logarithms of type  $\log(\mu^2/\Lambda_{\text{QCD}}^2)$  (collinear logarithms). The typical scale  $\mu$  of the hard-scattering processes is usually of the order of the transverse mass  $m_T = \sqrt{m^2 + |\mathbf{p}_T|^2}$  of the produced particle (or hadron jet) with (invariant) mass  $m$  and transverse two-momentum  $\mathbf{p}_T$ . With increasing energy, when the ratio of  $x \sim \mu/\sqrt{S}$  becomes small, the new large logarithms  $\log(1/x)$ , soft logarithms, are to appear and can become even more important than the collinear ones. These logarithms are present both in parton distributions and in partonic cross sections and can be resummed by the Balitsky-Fadin-Kuraev-Lipatov (BFKL) approach [32].

The approach gives the description of QCD scattering amplitudes in the region of large  $S$  and fixed momentum transfer  $t$ ,  $S \gg |t|$  (Regge region), with various color states in the  $t$ -channel. Entering this region requires us to reduce approximations to keep the true kinematics of the process. It becomes possible introducing the unintegrated over transverse momenta parton distribution functions (UPDFs)  $\Phi(x, t, \mu^2)$ , which depend on parton transverse momentum  $\mathbf{q}_T$  while its virtuality is  $t = -|\mathbf{q}_T|^2$ . The UPDFs are defined to be related with collinear ones through the equation

$$xG(x, \mu^2) = \int^{\mu^2} dt \Phi(x, t, \mu^2). \quad (1)$$

In the case of an inelastic scattering of objects with intrinsic hard scale, such as photons with high center-of-mass energy and virtuality, the UPDFs satisfy the BFKL evolution equation [32,33] which is suited to resum high-energy logarithms and appear in the BFKL approach as a particular result in the study of analytical properties of the forward scattering amplitude. In proton-proton collisions, the initial state does not provide us with intrinsic hard scale, therefore, at small  $x$ , some mixed DGLAP-BFKL evolution is needed. The  $k_T$ -factorization hypothesis is based on the assumption that at small  $x$  the  $k_T$ -ordered DGLAP chain of emissions is followed by at least a few steps of BFKL evolution, with subsequent emissions ordered in rapidity and with broken  $k_T$ -ordering. This assumption allows us to justify the  $k_T$ -factorization ansatz for the cross section, together with a particular procedure to calculate the hard-scattering part of it, based on the Reggeization of partons in the initial state.

The examples of above-mentioned mixed DGLAP-BFKL approaches are Kimber-Martin-Ryskin (KMR) approach [34] and Ciafaloni-Catani-Fiorani-Marchesini (CCFM) approach [35]. The point is that, in the LO of  $k_T$ -factorization, one can use essentially any of these methods to obtain the gluon UPDF, but to calculate the hard-scattering part of the cross section one has to Reggeize the partons in the initial state. This procedure justifies the  $k_T$ -factorization in the leading logarithmic approximation (LLA) and next-to-leading logarithmic approximation (NLLA) [33] and makes the hard-scattering part gauge invariant.

The basis of the BFKL approach is the gluon Reggeization [36], as at small  $x$  the gluons are the dominant partons. The gluon Reggeization appears considering special types of kinematics of processes at high energies. At large  $\sqrt{S}$  the dominant contributions to cross sections of QCD processes gives multi-Regge kinematics (MRK). MRK is the kinematics where all particles have limited (not growing with  $\sqrt{S}$ ) transverse momenta and are combined into jets with limited invariant mass of each jet and large (growing with  $\sqrt{S}$ ) invariant masses of any pair of the jets. At LLA of the BFKL

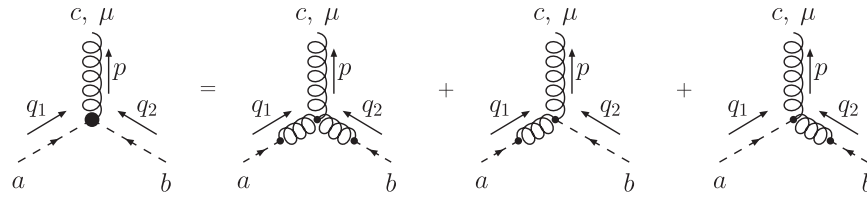


FIG. 1. Feynman diagrams for the subprocess (2).

approach, where the logarithms of type  $(\alpha_s \log(1/x))^n$  are resummed, only gluons can be produced and each jet is actually a gluon. At NLLA the terms of  $\alpha_s (\alpha_s \log(1/x))^n$  are collected and a jet can contain a couple of partons (two gluons or quark-antiquark pair). Such kinematics is called quasi-multi-Regge kinematics. Despite a great number of contributing Feynman diagrams it turns out that at the Born level in the MRK amplitudes acquire a simple factorized form. Moreover, radiative corrections to these amplitudes do not destroy this form, and their energy dependence is given by Regge factors  $s_i^{\omega(q_i)}$ , where  $s_i$  are invariant masses of couples of neighboring jets and  $\omega(q_i)$  can be interpreted as a shift of gluon spin from unity, dependent from momentum transfer  $q$ . This phenomenon is called gluon Reggeization.

Due to the Reggeization of quarks and gluons, an important role is dedicated to the vertices of Reggeon-particle interactions. In particular, these vertices are necessary for the determination of the BFKL kernel. To define them we can notice the two ways: the “classical” BFKL method [37] is based on analyticity and unitarity of particle production amplitudes and the properties of the integrals corresponding to the Feynman diagrams with two particles in the  $t$ -channel has been developed. Alternatively, they can be straightforwardly derived from the non-Abelian gauge-invariant effective action for the interactions of the Reggeized partons with the usual QCD partons, which was first introduced in Ref. [15] for Reggeized gluons only, and then extended by inclusion of Reggeized quark fields in Ref. [16]. The full set of the induced and effective vertices together with Feynman rules one can find in Refs. [16,38].

Recently, an alternative method to obtain the gauge-invariant  $2 \rightarrow n$  amplitudes with off-shell initial-state partons, which is mathematically equivalent to the PRA at the tree level, was proposed in Ref. [39]. These  $2 \rightarrow n$  amplitudes are extracted by using the spinor-helicity representation with complex momenta from the auxiliary  $2 \rightarrow n + 2$  scattering processes which are constructed to include the  $2 \rightarrow n$  scattering processes under consideration. This method is more suitable for the implementation in automatic matrix-element generators, but for our study the use of Reggeized quarks and gluons is found to be simpler.

As we mentioned above, we will consider the  $D$ -meson production by only the  $c$ -quark and gluon fragmentation. The lowest orders in  $\alpha_s$  parton subprocesses of PRA in which gluon or  $c$ -quark are produced are the following: a gluon production via two Reggeized gluons fusion

$$\mathcal{R} + \mathcal{R} \rightarrow g, \tag{2}$$

and the corresponding quark-antiquark pair production

$$\mathcal{R} + \mathcal{R} \rightarrow c + \bar{c}, \tag{3}$$

where  $\mathcal{R}$  are the Reggeized gluons.

According to the prescription of Ref. [38], the amplitudes of relevant processes (2) and (3) can be obtained from the Feynman diagrams depicted in Figs. 1 and 2, where the dashed lines represent the Reggeized gluons. Of course, the last three Feynman diagrams in Fig. 2 can be combined into the effective particle-Reggeon-Reggeon (PRR) vertex [38].

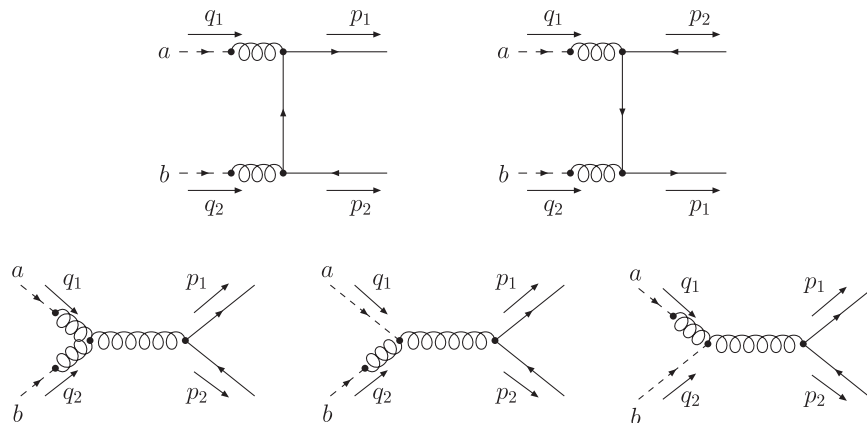


FIG. 2. Feynman diagrams for the subprocess (3).

Let us define four-vectors  $(n^-)^\mu = P_1^\mu/E_1$  and  $(n^+)^\mu = P_2^\mu/E_2$ , where  $P_{1,2}^\mu$  are the four-momenta of the colliding protons, and  $E_{1,2}$  are their energies. We have  $(n^\pm)^2 = 0$ ,  $n^+ \cdot n^- = 2$ , and  $S = (P_1 + P_2)^2 = 4E_1E_2$ . For any four-momentum  $k^\mu$ , we define  $k^\pm = k \cdot n^\pm$ . The four-momenta of the Reggeized gluons can be represented as

$$\begin{aligned} q_1^\mu &= \frac{q_1^+}{2} (n^-)^\mu + q_{1T}^\mu, \\ q_2^\mu &= \frac{q_2^-}{2} (n^+)^\mu + q_{2T}^\mu, \end{aligned} \quad (4)$$

where  $q_T = (0, \mathbf{q}_T, 0)$ . The amplitude of gluon production in fusion of two Reggeized gluons can be presented as a scalar product of Fadin-Kuraev-Lipatov effective PRR vertex  $C_{\mathcal{R}\mathcal{R}}^{g\mu}(q_1, q_2)$  and polarization four-vector of final gluon  $\varepsilon_\mu(p)$ :

$$\mathcal{M}(\mathcal{R} + \mathcal{R} \rightarrow g) = C_{\mathcal{R}\mathcal{R}}^{g\mu}(q_1, q_2) \varepsilon_\mu(p) \quad (5)$$

where

$$C_{\mathcal{R}\mathcal{R}}^{g\mu}(q_1, q_2) = -\sqrt{4\pi\alpha_s} f^{abc} \frac{q_1^+ q_2^-}{2\sqrt{t_1 t_2}} \left[ (q_1 - q_2)^\mu + \frac{(n^+)^\mu}{q_1^+} (q_2^2 + q_1^+ q_2^-) - \frac{(n^-)^\mu}{q_2^-} (q_1^2 + q_1^+ q_2^-) \right], \quad (6)$$

$a$  and  $b$  are the color indices of the Reggeized gluons with incoming four-momenta  $q_1$  and  $q_2$ , and  $f^{abc}$  with  $a = 1, \dots, N_c^2 - 1$  is the antisymmetric structure constants of color gauge group  $SU_C(3)$ . The squared amplitude of the partonic subprocess  $\mathcal{R} + \mathcal{R} \rightarrow g$  is straightforwardly found from Eq. (6) to be

$$|\overline{\mathcal{M}(\mathcal{R} + \mathcal{R} \rightarrow g)}|^2 = \frac{3}{2} \pi \alpha_s \mathbf{p}_T^2. \quad (7)$$

The amplitude of the process (3) can be presented in the same way, as a sum of three terms  $\mathcal{M}(\mathcal{R} + \mathcal{R} \rightarrow c + \bar{c}) = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3$ :

$$\begin{aligned} \mathcal{M}_1 &= -i\pi\alpha_s \frac{q_1^+ q_2^-}{\sqrt{t_1 t_2}} T^a T^b \bar{U}(p_1) \gamma^\alpha \frac{\hat{p}_1 - \hat{q}_1}{(p_1 - q_1)^2} \gamma^\beta V(p_2) (n^+)^\alpha (n^-)^\beta, \\ \mathcal{M}_2 &= -i\pi\alpha_s \frac{q_1^+ q_2^-}{\sqrt{t_1 t_2}} T^b T^a \bar{U}(p_1) \gamma^\beta \frac{\hat{p}_1 - \hat{q}_2}{(p_1 - q_2)^2} \gamma^\alpha V(p_2) (n^+)^\alpha (n^-)^\beta, \\ \mathcal{M}_3 &= 2\pi\alpha_s \frac{q_1^+ q_2^-}{\sqrt{t_1 t_2}} T^c f^{abc} \frac{\bar{U}(p_1) \gamma^\mu V(p_2)}{(p_1 + p_2)^2} \left[ (n^-)^\mu \left( q_2^+ + \frac{q_2^2}{q_1^-} \right) - (n^+)^\mu \left( q_1^- + \frac{q_1^2}{q_2^+} \right) + (q_1 - q_2)^\mu \right], \end{aligned} \quad (8)$$

where  $T^a$  are the generators of the fundamental representation of the color gauge group  $SU_C(3)$ .

The squared amplitudes can be presented as follows:

$$|\overline{\mathcal{M}(\mathcal{R} + \mathcal{R} \rightarrow c + \bar{c})}|^2 = 256\pi^2 \alpha_s^2 \left( \frac{1}{2N_c} \mathcal{A}_{\text{Ab}} + \frac{N_c}{2(N_c^2 - 1)} \mathcal{A}_{\text{NAb}} \right), \quad (9)$$

$$\mathcal{A}_{\text{Ab}} = \frac{t_1 t_2}{\hat{t} \hat{u}} - \left( 1 + \frac{p_2^+}{\hat{u}} (q_1^- - p_2^-) + \frac{p_2^-}{\hat{t}} (q_2^+ - p_2^+) \right)^2, \quad (10)$$

$$\begin{aligned} \mathcal{A}_{\text{NAb}} &= \frac{2}{S^2} \left( \frac{p_2^+ (q_1^- - p_2^-) S}{\hat{u}} + \frac{S}{2} + \frac{\Delta}{\hat{s}} \right) \left( \frac{p_2^- (q_2^+ - p_2^+) S}{\hat{t}} + \frac{S}{2} - \frac{\Delta}{\hat{s}} \right) \\ &\quad - \frac{t_1 t_2}{q_1^- q_2^+ \hat{s}} \left( \left( \frac{1}{\hat{t}} - \frac{1}{\hat{u}} \right) (q_1^- p_2^+ - q_2^+ p_2^-) + \frac{q_1^- q_2^+ \hat{s}}{\hat{t} \hat{u}} - 2 \right), \end{aligned} \quad (11)$$

$$\Delta = \frac{S}{2} \left( \hat{u} - \hat{t} + 2q_1^- p_2^+ - 2q_2^+ p_2^- + t_1 \frac{q_2^+ - 2p_2^+}{q_2^+} - t_2 \frac{q_1^- - 2p_2^-}{q_1^-} \right). \quad (12)$$

Here the bar indicates averaging (summation) over initial-state (final-state) spins and colors,  $t_1 = -q_1^2 = |\mathbf{q}_{1T}|^2$ ,  $t_2 = -q_2^2 = |\mathbf{q}_{2T}|^2$ , and

$$\begin{aligned}\hat{s} &= (q_1 + q_2)^2 = (p_1 + p_2)^2, \\ \hat{t} &= (q_1 - p_1)^2 = (q_2 - p_2)^2, \\ \hat{u} &= (q_2 - p_1)^2 = (q_1 - p_2)^2.\end{aligned}$$

The squared amplitude (9) analytically coincides with that previously obtained in Ref. [1]. We checked that in the

collinear limit, i.e.  $q_{(1,2)T} \rightarrow 0$ , the squared amplitude (9) after averaging over the azimuthal angles transforms to the squared amplitude of the corresponding parton subprocess in collinear model, namely  $g + g \rightarrow c + \bar{c}$ . We perform our analysis in the region of  $\sqrt{S}$ ,  $p_T \gg m_c$ , which allows us to use zero-mass variable-flavor-number-scheme, where the masses of the charm quarks in the hard-scattering amplitude are neglected.

In the  $k_T$ -factorization, the differential cross section for the  $2 \rightarrow 1$  subprocess (2) has the form

$$\frac{d\sigma}{dy dp_T}(p + p \rightarrow g + X) = \frac{1}{p_T^3} \int d\phi_1 \int dt_1 \Phi(x_1, t_1, \mu^2) \Phi(x_2, t_2, \mu^2) \overline{|\mathcal{M}(\mathcal{R} + \mathcal{R}\vec{g})|^2}, \quad (13)$$

where  $\phi_1$  is the azimuthal angle between  $\mathbf{p}_T$  and  $\mathbf{q}_{1T}$ .

The analogous formula for the  $2 \rightarrow 2$  subprocess (3) can be written as

$$\begin{aligned}\frac{d\sigma}{dy_1 dy_2 dp_{1T} dp_{2T}}(p + p \rightarrow c(p_1) + \bar{c}(p_2) + X) &= \frac{p_{1T} p_{2T}}{16\pi^3} \int d\phi_1 \int d\Delta\phi \int dt_1 \\ &\times \Phi(x_1, t_1, \mu^2) \Phi(x_2, t_2, \mu^2) \frac{|\mathcal{M}(\mathcal{R} + \mathcal{R}\vec{c} + \bar{c})|^2}{(x_1 x_2 S)^2},\end{aligned} \quad (14)$$

where  $x_1 = q_1^+/P_1^+$ ,  $x_2 = q_2^-/P_2^-$ ,  $\Delta\phi$  is the azimuthal angle between  $\mathbf{p}_{1T}$  and  $\mathbf{p}_{2T}$ , and the rapidity of the final-state parton with four-momentum  $p$  is  $y = \frac{1}{2} \ln(\frac{p^+}{p^-})$ . Again, we have checked a fact that in the limit of  $t_{1,2} \rightarrow 0$ , we recover the conventional factorization formula of the collinear parton model from (13) and (14).

The unintegrated gluon distribution function  $\Phi(x, t, \mu^2)$  is an important ingredient in our scheme. As default, we obtain it using the prescription of Kimber, Martin and Ryskin [34,40] developed to extract UPDFs from conventional integrated ones and implemented in the C++ code. In the LO KMR scheme, the transverse momentum of a parton in the initial state of the hard scattering comes entirely from the last step of evolution, and the parton radiated at the last step is ordered in rapidity with the particles produced in the hard subprocess. In such a way, KMR distribution corresponds to the DGLAP cascade, followed by the exactly one step of BFKL evolution. This procedure to obtain UPDFs requires less computational efforts than the precise solution of two-scale evolution equations such as, for instance, CCFM [35], nevertheless it is suitable and adequate to the physics of processes under study.

For better control on the uncertainties we reproduce our computations with the UPDF obtained as the solution of the CCFM evolution equation [35], which was constructed for

a smooth matching between DGLAP evolution at small  $k_T$  and BFKL evolution at large  $k_T$ , as implemented in the library of transverse-momentum-dependent distributions TMDlib [41].

The usage of the  $k_T$ -factorization formula and UPDFs with one longitudinal (light-cone) kinematic variable ( $x$ ) requires the Reggeization of the  $t$ -channel partons. Accordingly to Refs. [15,16], Reggeized partons carry only one large light-cone component of the four-momentum and, therefore, its virtuality is dominated by the transverse momentum. Such kinematics of the  $t$ -channel partons corresponds to the MRK of the initial state radiation and particles, produced in the hard process. In our previous analysis [26–30] devoted to the similar processes of heavy quarkonium production we proved that KMR UPDFs give the best description of the charmonium  $p_T$ -spectra measured at the Tevatron [42] and LHC [43]. In our numerical analysis, as input for KMR procedure, we use the LO set of the Martin-Roberts-Stirling-Thorne [44] proton PDFs as our default.

In the fragmentation model the transition from the produced gluon or  $c$ -quark to the  $D$ -meson is described by fragmentation function (FF)  $D_{c,g}(z, \mu^2)$ . According to the corresponding factorization theorem of QCD and the fragmentation model, the basic formula for the  $D$ -meson production cross section reads [45]

$$\frac{d\sigma(p + p \rightarrow D + X)}{dp_D dy} = \sum_i \int_0^1 \frac{dz}{z} D_{i \rightarrow D}(z, \mu^2) \frac{d\sigma(p + p \rightarrow i(p_i = p_D/z) + X)}{dp_{iT} dy_i}, \quad (15)$$

where  $D_{i \rightarrow D}(z, \mu^2)$  is the fragmentation function for the parton  $i$ , produced at the hard scale  $\mu$ , splitting into  $D$ -meson,  $z$  is the longitudinal momentum fraction of a fragmenting particle carried by the  $D$ -meson. In the zero-mass approximation the fragmentation parameter  $z$  can be defined as follows  $p_D^\mu = z p_i^\mu$ ,  $p_D$  and  $p_i$  are the  $D$ -meson and  $i$ -parton four-momenta, and  $y_D = y_i$ . In our calculations we use the LO FFs from Ref. [7], where the fits of nonperturbative  $D^0$ ,  $D^+$ ,  $D^{*+}$ , and  $D_s^+$  FFs, both at LO and NLO in the  $\overline{\text{MS}}$  factorization scheme, to OPAL data from LEP1 [46] were performed. These FFs satisfy two desirable properties: at first, their  $\mu$ -scaling violation is ruled by DGLAP evolution equations; at second, they are universal.

In the fits of Refs. [6–8], the parametrizations at the initial scale  $\mu_0 = m_c$  for the FFs were taken as follows:

$$D_c(z, \mu_0^2) = N_c \frac{z(1-z)^2}{[(1-z) + \epsilon_c]^2}, \quad (16)$$

$$D_{g,q}(z, \mu_0^2) = 0. \quad (17)$$

To illustrate a difference of contributions to the  $D$ -meson production we show in Fig. 3 the  $c$ -quark and gluon FFs into  $D^*$ -meson.

As the contribution of gluon fragmentation at  $\mu > \mu_0$  is initiated by the perturbative transition of gluons to  $c\bar{c}$ -pairs encountered by DGLAP evolution equations, the part of  $c$ -quarks produced in the subprocess (3) with their subsequent transition to  $D$ -mesons are already taken into account considering  $D$ -meson production via gluon

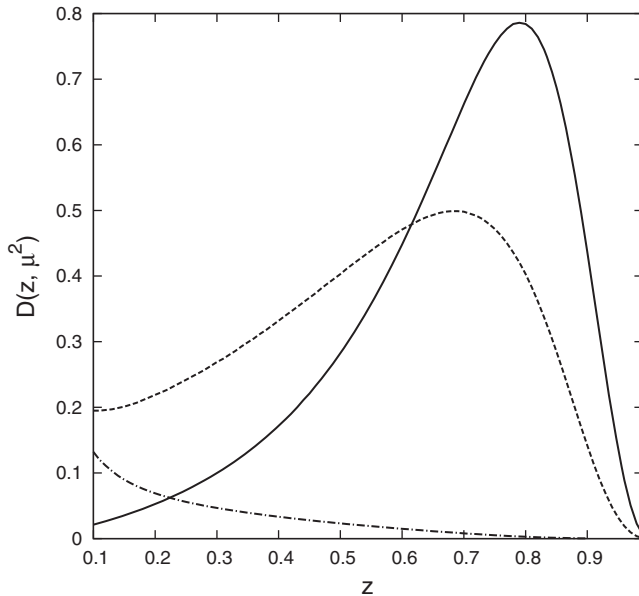


FIG. 3. The fragmentation function  $D(z, \mu^2)$  of  $c$ -quarks and gluons into  $D^*$  mesons from Ref. [7] at the  $\mu^2 = \mu_0^2 = 2.25 \text{ GeV}^2$  (solid curve for  $c$ -quark, the fragmentation function of gluon is negligible) and  $\mu^2 = 100 \text{ GeV}^2$  (dashed line for  $c$ -quark, dash-dotted for gluon).

fragmentation. In such a way, to avoid double counting, we must subtract this contribution, which can be effectively done by the imposing of the lower cut on  $\hat{s}$  at the threshold of the production of the  $c\bar{c}$  pair in (14), i.e.  $\hat{s} > 4m_c^2$ , in accordance with the general scheme of Refs. [6–8]. The precise study of double-counting terms for a zero-mass case needs a separate consideration and can be a subject of our future works.

### III. RESULTS

Recently the ALICE collaboration measured the differential cross sections  $d\sigma/dp_T$  for the inclusive production of  $D^0$ ,  $D^+$ ,  $D^{*+}$ , and  $D_s^+$  mesons [11–13] in proton-proton collisions at the CERN LHC ( $\sqrt{S} = 2.76; 7 \text{ TeV}$ ) as functions of  $D$ -meson transverse momentum ( $p_T$ ) in the central rapidity region,  $|y| < 0.5$ . These measurements extend the CDF collaboration data [10] obtained earlier in proton-antiproton collisions at the Fermilab Tevatron at the  $|y| < 1.0$  and  $\sqrt{S} = 1.96 \text{ TeV}$ . The production of  $D$ -mesons in the forward rapidity region of  $2.0 < y < 4.5$  was investigated at the LHC by the LHCb collaboration and the data in the form of  $d\sigma/dp_T$  were presented for the different intervals of rapidity [14].

These data have been studied in the NLO in the collinear parton model of QCD within the two approaches: the general-mass variable-flavor-number (GM-VFN) scheme [47], and the so-called fixed order scheme improved with next-to-leading logarithms (FONLL scheme) [48]. In the former one, realized in Refs. [6–8], the large fragmentation logarithms dominating at  $p_T \gg m$  are resummed through the evolution of the fragmentation functions, satisfying the DGLAP [31] evolution equations. At the same time, the full dependence on the charm-quark mass in the hard-scattering cross section is retained to describe consistently  $p_T \sim m$  region. The  $D$ -meson FFs were extracted both at leading and next-to-leading order in the GM-VFN scheme from the fit of  $e^+e^-$  data taken by the OPAL collaboration at CERN LEP1 [46]. Opposite, in the FONLL approach, the NLO  $D$ -meson production cross sections are calculated with a nonperturbative  $c$ -quark FF, which is not a subject to DGLAP [31] evolution. The FONLL scheme was implemented in Refs. [4,5] and its main ingredients are the following: the NLO fixed order calculation (FO) with resummation of large transverse momentum logarithms at the next-to-leading level (NLL) for heavy quark production. For consistency of the calculation, the NLL formalism should be used to extract the nonperturbative FFs from  $e^+e^-$  data, and in Refs. [4,5] the scheme of calculation of heavy quark cross section and extraction of the nonperturbative FFs are directly connected and must be used only together.

The overall agreement of data and calculations obtained in Refs. [4–8] is good, the  $D$ -meson spectra measured by the CDF collaboration at Fermilab Tevatron and ALICE

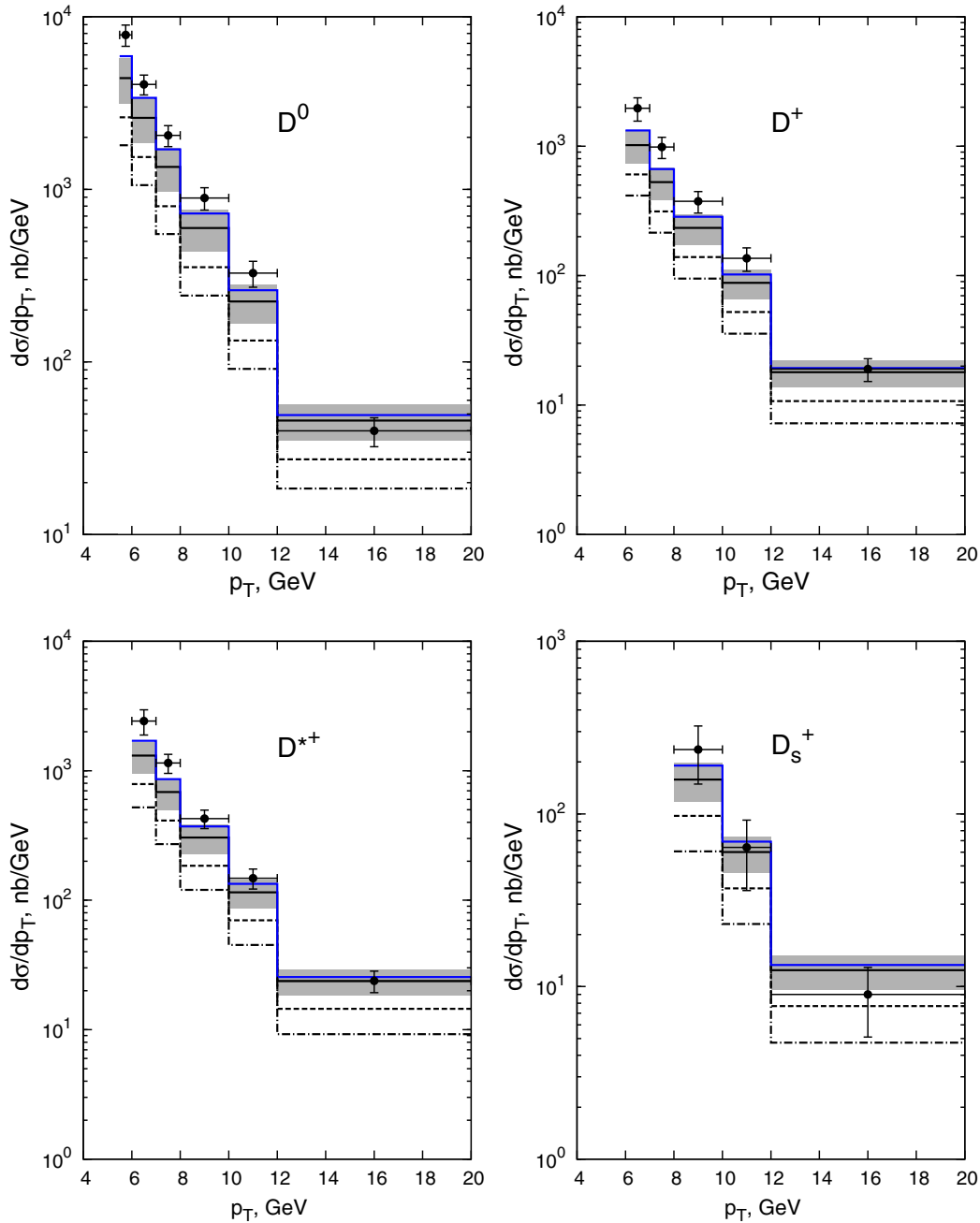


FIG. 4 (color online). Transverse momentum distributions of  $D^0$  (left, top),  $D^+$  (right, top),  $D^{*+}$  (left, bottom), and  $D_s^+$  (right, bottom) mesons in  $p\bar{p}$  scattering with  $\sqrt{S} = 1.96$  TeV and  $|y| < 1.0$ . The dashed line represents the contribution of gluon fragmentation, the dash-dotted line is the  $c$ -quark-fragmentation contribution, and the black solid line is their sum obtained with KMR UPDF. The blue solid line represents the sum of contributions obtained with UPDF ccfm-JH-2013-set1. The CDF data at Tevatron are from Ref. [10].

and LHCb collaborations at the LHC are described within experimental uncertainties.

The study of  $D$ -meson fragmentation production in terms of  $k_T$ -factorization [1–3] was performed also previously in the recent paper [9], with off-shell initial gluons and using the formalism of transverse-momentum dependent parton distributions, whereas the first results in this scheme were obtained for the  $D_0$  production at Tevatron Run I [49]. The resulting curve in Ref. [9] describes the

ALICE experimental data [12] by its upper limit of theoretical uncertainty.

We start the analysis of our results obtained in LO of PRA by their comparison with the data on transverse-momentum distributions of  $D$ -mesons measured by the CDF collaboration at Fermilab Tevatron [10], at the collision energy of  $\sqrt{S} = 1.96$  TeV. The production of the  $D^0$ ,  $D^+$ ,  $D^{*+}$ , and  $D_s^+$  mesons was studied in the central region of rapidity  $|y| < 1.0$  and with transverse momenta

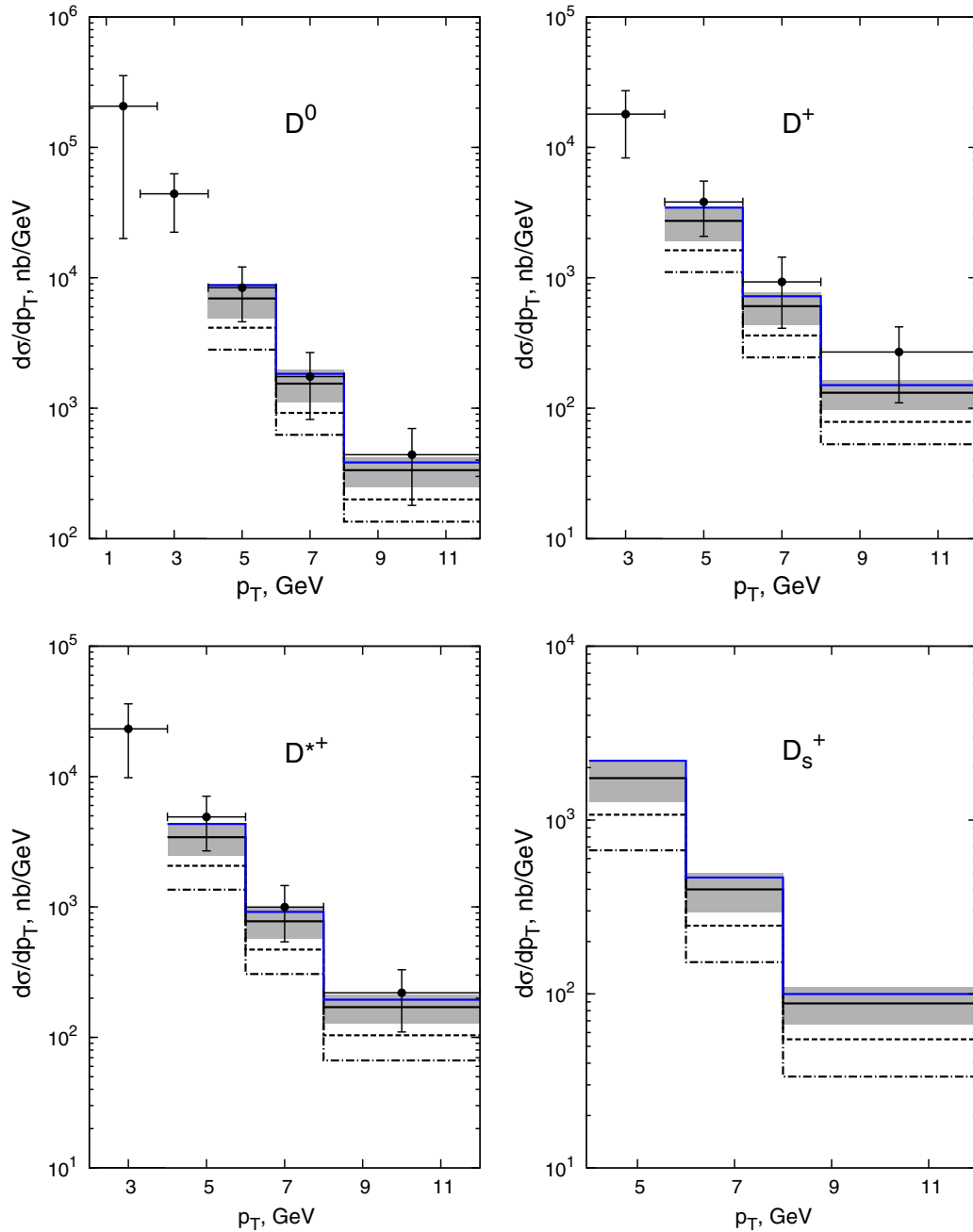


FIG. 5 (color online). Transverse momentum distributions of  $D^0$  (left, top),  $D^+$  (right, top),  $D^{*+}$  (left, bottom), and  $D_s^+$  (right, bottom) mesons in  $pp$  scattering with  $\sqrt{s} = 2.76$  TeV and  $|y| < 0.5$ . The notations are as in Fig. 4. The ALICE data at LHC are from Ref. [11].

up to 20 GeV. In Fig. 4 we introduce these data coming as differential cross sections  $d\sigma/dp_T$ , where the particle and antiparticle contributions are averaged, in comparison with our predictions in the PRA. The dashed lines represent contributions of the process (2) while dash-dotted lines correspond to ones of the process (3). The sum of both contributions is shown as a black solid line. We estimated a theoretical uncertainty arising from uncertainty of definition of factorization and renormalization scales by varying them between  $1/2m_T$  and  $2m_T$  around their central value of

$m_T$ , the transverse mass of fragmenting parton. The resulting uncertainty is depicted in the figures by shaded bands. We find a good agreement between our predictions and experimental data in the large- $p_T$  interval of  $D$ -meson transverse momenta within experimental and theoretical uncertainties. However, our predictions show a tendency to fall below the data in the lower  $p_T$  range. It can point to the significance of  $c$ -quark mass effects in the region, where the hard scale of the process is not much larger than the  $c$ -quark mass. The increasing of the collision energy with



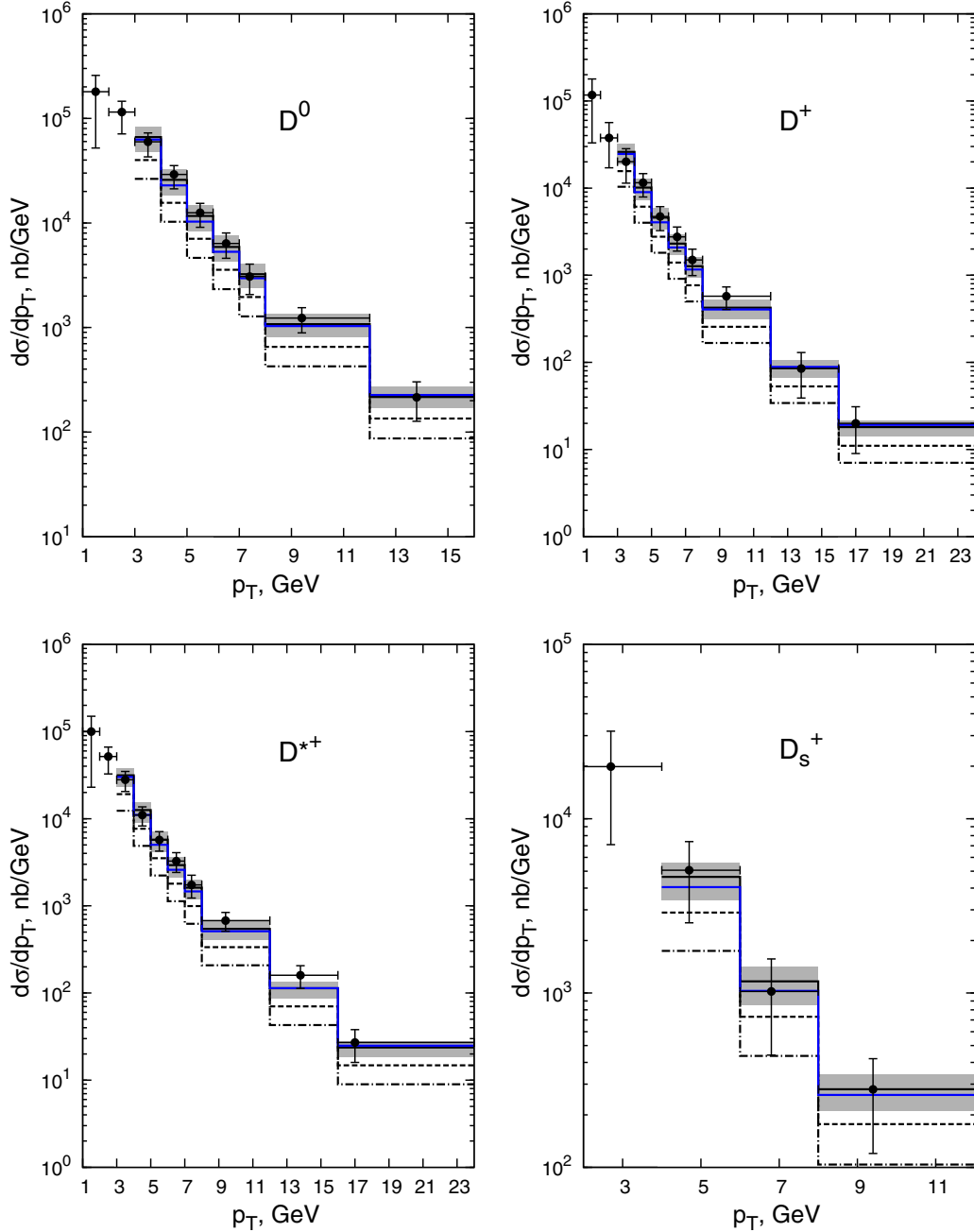


FIG. 6 (color online). Transverse momentum distributions of  $D^0$  (left, top),  $D^+$  (right, top),  $D^{*+}$  (left, bottom), and  $D_s^+$  (right, bottom) mesons in  $pp$  scattering with  $\sqrt{s} = 7$  TeV and  $|y| < 0.5$ . The notations are as in Fig. 4. The ALICE data at LHC are from Ref. [12].

other kinematic conditions preserved is supposed to lead to a better agreement between theory and experiment in our approach as we expect the rise of logarithmic contributions of type  $\log(1/x)$  to be more significant than finite-quark-mass effects.

Our expectations are confirmed when we turn to the description of the recent data from the LHC at its intermediate energy of  $\sqrt{s} = 2.76$  TeV and  $\sqrt{s} = 7$  TeV collected by the ALICE collaboration [11,12]. The previous NLO predictions made in the collinear parton

model in general are in agreement with ALICE data, however, one can find that the FONLL scheme [5] tends to overestimate data and the GM-VFN [8] is to underestimate. In Figs. 5 and 6, we compare our predictions with ALICE data [11,12] keeping the notations of curves the same as in Fig. 4. The current collision energies of LHC is 2–3.5 times larger compared to Tevatron and the interval of  $D$ -meson rapidity is more narrow,  $|y| < 0.5$ . We obtain a good agreement of our predictions with the experiment for all types of  $D$ -mesons at the whole range of their transverse

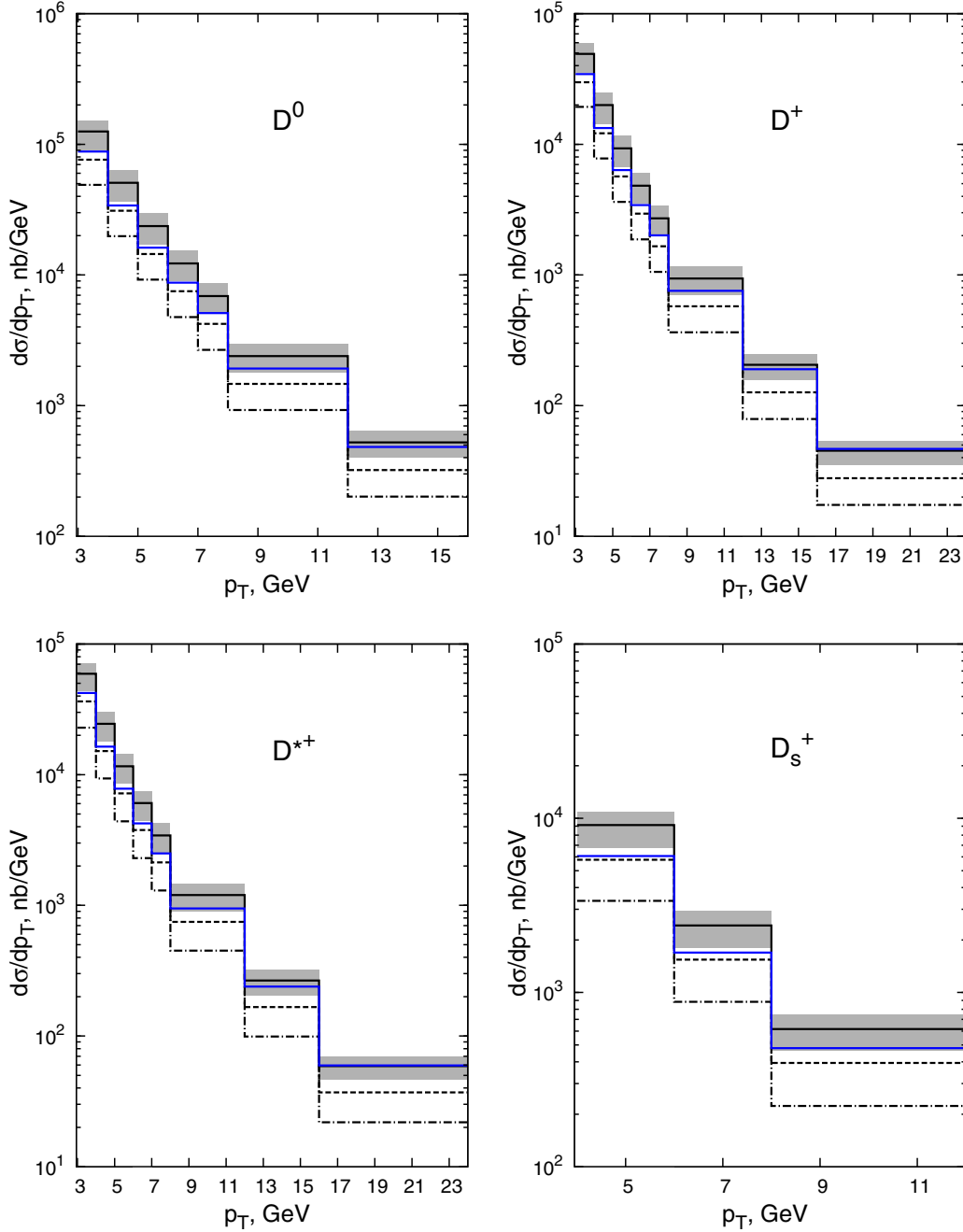


FIG. 7 (color online). Theoretical predictions for the transverse momentum distributions of  $D^0$  (left, top),  $D^+$  (right, top),  $D^{*+}$  (left, bottom), and  $D_s^+$  (right, bottom) mesons in  $pp$  scattering at  $\sqrt{S} = 14$  TeV and  $|y| < 0.5$  obtained in the LO PRA. The notations are as in Fig. 4.

momenta. As there is no experimental data for  $D_s^+$  production at the energy of  $\sqrt{S} = 2.76$  TeV, we introduce the theoretical prediction only. Finally, in Fig. 7 we present our predictions for the planned LHC energy of  $\sqrt{S} = 14$  TeV and the other kinematic conditions as in Ref. [12].

To estimate the theoretical uncertainties originating from the choice of UPDF we performed the same calculations using the UPDF ccfm-JH-2013-set1 from TMDlib [41]. Everywhere in Figs. 4–7 we show the corresponding values

obtained at central  $\mu = m_T$  by blue solid lines. One can find them lying within the range of uncertainties of the predictions made with KMR UPDF, being in a slightly better agreement with experimental data for  $\sqrt{S} = 1.96$  TeV and in the same one for the higher energies.

Considering the  $D$ -meson central rapidity production, we find the MRK subprocess (2) to remain indeed the dominant one for all collision energies. In such a way, we confirm the theoretical suggestion mentioned in Sec. II that

MRK gives the leading logarithmic approximation for the high-energy production processes in the BFKL approach while the QMRK turns out to be subleading. However, in the framework of Ref. [9], which seems to be theoretically close to PRA, this MRK subprocess is absent while the main contribution is coming from the QMRK subprocess.

#### IV. CONCLUSIONS

We introduce a comprehensive study of  $D^0$ ,  $D^+$ ,  $D^{*+}$ , and  $D_s^+$ -meson fragmentation production in proton-(anti)proton collisions with central rapidities at Tevatron Collider and LHC in the framework of the parton Reggeization approach. We use the gauge invariant amplitudes of hard parton subprocesses in the LO level of parton Reggeization theory with Reggeized gluons in the initial state in a self-consistent way together with unintegrated parton distribution functions obtained by the prescription proposed by Kimber, Martin and Ryskin and the ones taken from TMDlib, namely ccfm-JH-2013-set1. The  $2 \rightarrow 1$  hard subprocess of gluon production via a fusion of two Reggeized gluons in the PRA framework is proposed for the first time in the case of  $D$ -meson fragmentation production and proved to be a dominant one. To describe the nonperturbative transition of produced gluons and  $c$ -quarks into the  $D$ -mesons we use the universal fragmentation functions obtained from the fit of  $e^+e^-$  annihilation

data from CERN LEP1. We found our results for  $D$ -meson central-rapidity production to be in excellent coincidence with experimental data from the LHC and good agreement with large-transverse-momenta Tevatron data. The achieved degree of agreement for the LHC exceeds the one obtained by NLO calculations in the conventional collinear parton model and LO calculations in  $k_T$ -factorization with Reggeized gluons. The predictions for the  $D$ -meson production in the central rapidity region for the expected LHC energy of  $\sqrt{s} = 14$  TeV are also presented. We describe  $D$ -meson production without any free parameters or auxiliary approximations.

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