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Fermion masses and mixing in $\Delta(27)$ flavor model

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An extension of the Standard Model (SM) based on the non-Abelian discrete group $\Delta(27)$ is considered. The $\Delta(27)$ flavor symmetry is spontaneously broken only by gauge singlet scalar fields, therefore our model is free from any flavor changing neutral current (FCNC). We show that the model accounts simultaneously for the observed quark and lepton masses and their mixing. In the quark sector, we find that the up-quark mass matrix is flavor diagonal and the Cabbibo-Kobayashi-Maskawa (CKM) mixing matrix arises from down quarks. In the lepton sector, we show that the charged lepton mass matrix is almost diagonal. We also adopt type-I seesaw mechanism to generate neutrino masses. A deviated mixing matrix from tri-bimaximal Maki-Nakagawa-Sakata (MNS), with a correlation between $\sin\theta_{13}$ and $\sin^2\theta_{23}$ are illustrated.

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I. INTRODUCTION

The understanding of the origin of quark and lepton families and the observed pattern of their masses and mixing is still one of the major outstanding problems in particle physics. In the SM, these masses and mixing are derived from Yukawa couplings, which are not defined by the gauge symmetry. Therefore, they are arbitrary parameters and another type of symmetry, called flavor symmetry, is required to explain the observed fermion flavor structures. In particular, one aims to interpret the large mass ratios between generations: $m_u \ll m_c \ll m_t$; $m_d \ll m_s \ll m_b$; $m_e \ll m_\mu \ll m_\tau$, and the smallness of the off-diagonal elements of the quark weak coupling matrix, in addition to the tiny neutrino masses and their large mixings as recent data suggests [1].

Two standard approaches for dealing with flavor symmetries in particle physics. The first one is known as "top-down" approach, where one assumes that the SM Lagrangian is invariant under certain flavor group G and a number of Higgs-like scalar bosons, called flavons, are coupled invariantly to SM fermions. The vacuum expectation values (VEVs) of these flavons break the flavor symmetries and generate mass terms for SM fermions. The comparison of the resultant mixing matrices and the mass eigenvalues with the experimental data will confirm or refute if this group represents the correct flavor symmetry. In the second approach, which is known as "bottom-up," one studies the residual symmetry that manifests in the mass matrix and tries to relate it with the flavor symmetry group, for instance, by calculating the matrices S_i that keep the neutrino mass matrix invariant and the matrices T_i that keep the charged leptons mass matrix invariant [2]. The group G generated by these matrices can be considered as the group of the flavor symmetry of the lepton sector. In this regard, it was argued that for Majorana neutrinos, regardless of the form of the mass matrix M_ν, it has $Z_2 \times Z_2$ residual symmetry [3,4], provided that it has three distinct eigenvalues [5].

Many attempts were made to interpret the flavor aspects by using discrete symmetry groups (see [6]). In particular, the non-Abelian groups A_4 and S_4 have been significantly considered and shown to be useful for obtaining tri-bimaximal neutrino mixing matrix [7,8]. Also, $\Delta(27)$ which belongs to $\Delta(3n^2)$ [9] has been considered in Refs. [10–16] and $\Delta(54)$ which belongs to $\Delta(3n^2)$ [17] was considered in [18,19] as examples of discrete symmetries that may deviate MNS mixing matrix from tri-bimaximal. However, in [14,15] the attention has been devoted to the lepton sector only. Also extra Higgs (flavons) doublets have been considered, which could make the model suffer from dangerous flavor changing neutral currents. It is worth noting that within flavor symmetry approaches, the Yukawa couplings are typically generated through nonrenormalizable flavon interactions with the SM fermions, *i.e.*, $Y \sim \langle\phi\rangle^n/\Lambda^n$, $n = 1, 2, \ldots$. In this respect, the hierarchy of fermion masses is related to the order of nonrenormalizable interactions. For instance, third-generation Yukawa couplings can be obtained from $\langle\phi\rangle/\Lambda$, while first- and second-generation Yukawa couplings should correspond to higher-order terms.

In this paper, we explore the possibility that the flavor symmetry based on the group $\Delta(27)$ leads to the correct quarks, charged leptons, and neutrino masses, in addition to the quark and neutrino mixing matrices, consistent with the latest experimental results. We present a new model based on the semi-direct product $\Delta(27) \ltimes S_2$, where S_2 is quite useful symmetry that guarantees the tri-bimaximal mixing as zero approximation in our model. It is worth mentioning that although $\Delta(27) \ltimes S_2$ is isomorphic to $\Delta(54)$, the constructed model based on these two symmetries could be different in the following aspects: (i) $\Delta(54)$ is rather more constrained than $\Delta(27) \ltimes S_2$, since in the latter case we may have a multiplet charged under $\Delta(27)$ and a singlet under S_2 or, conversely, a multiplet under S_2 and a singlet under $\Delta(27)$ where this may not be feasible in $\Delta(54)$. (ii) $\Delta(54)$ contains doublets where $\Delta(27)$ does not. (iii) $\Delta(54)$ contains all permutations of three objects [as S_3 is a subgroup of $\Delta(54)$], while in $\Delta(27) \ltimes S_2$ we chose certain permutations, namely, the permutation of the second and third elements of the triplet. Furthermore, as we will see, other types of symmetry like $U(1)$ should be considered to obtain viable mixings in quark and lepton sectors.

We will show that deviation from the tri-bimaximal neutrino mixing matrix is related to spontaneous breaking of this symmetry. The Higgs sector in our model consists of one Higgs doublet only to break the electroweak symmetry and the SM singlet scalars to break the flavor symmetry. Therefore, our model is free from the famous flavor changing neutral current constraints that most constructed models suffer from due to the existence of more than one $SU(2)$ doublet Higgs.

We will show that the observed hierarchical structure of quark and lepton masses can be accommodated. In addition, the small quark mixing in the $V_{\rm CKM}$ and large neutrino mixing in $U_{\rm MNS}$ can be simultaneously realized. If one assumes that left-handed quarks and right-handed up quarks transform as a triplet under $\Delta(27)$, then one finds that the up-quark mass matrix is flavor diagonal. With right-handed down quarks transform as singlets under $\Delta(27)$, we will show that the $V_{\rm CKM}$ mixing matrix can be obtained from the down quark sector. In the lepton sector, the lepton doublet transforms under $\Delta(27)$ as a triplet, while the right-handed charged lepton transforms as a singlet. In this case, the charged lepton mass matrix is almost diagonal. Finally, we assume right-handed neutrinos as singlets under $\Delta(27)$; thus, with the appropriate singlet scalars [triplet and singlets under $\Delta(27)$], we will show that a generic MNS mixing matrix can be obtained, and different interesting limits will be studied.

The paper is organized as follows. In the next section we briefly introduce $\Delta(27)$ flavor symmetry. In Sec. III we show that the charged lepton mass hierarchy can be naturally accounted for. Section IV is devoted for neutrino masses and mixing, where the observed nearly tri-bimaximal mixing is realized. Quark sector is discussed in Sec. V. In our model the quark mixing matrix, $V_{\rm CKM}$, is obtained from down quarks. Finally, we give our conclusions in Sec. VI.

II. Δ(27) FLAVOR SYMMETRY

The discrete group $\Delta(27)$ is a subgroup of $SU(3)$ and an isomorphic to the semi-direct product group $(Z_3 \times Z_3') \ltimes Z_3''$. It is also one of the groups $\Delta(3n^2)$ with $n = 3$. It has 27 elements and 11 conjugacy classes, so it has 11 irreducible representations, two triplets, **3** and its conjugate $\bar{\mathbf{3}}$, and 9 singlets $\mathbf{1_1}$–$\mathbf{1_9}$. The group multiplication rules for $\Delta(27)$ are

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{\mathbf{3}} \times \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}_{\mathbf{3}} = \begin{pmatrix} x_1 y_1 \\ x_2 y_2 \\ x_3 y_3 \end{pmatrix}_{\bar{\mathbf{3}}} + \begin{pmatrix} x_2 y_3 \\ x_3 y_1 \\ x_1 y_2 \end{pmatrix}_{\bar{\mathbf{3}}} + \begin{pmatrix} x_3 y_2 \\ x_1 y_3 \\ x_2 y_1 \end{pmatrix}_{\bar{\mathbf{3}}}, \tag{1}$$

$$\bar{3} \times \bar{3} = 3 + 3 + 3, \tag{2}$$

with similar multiplication rules as above. $3 \times \bar{3} = \sum_{i=1}^{9} 1_i$, where

$$\begin{aligned} &1_1 = x_1\bar{y}_1 + x_2\bar{y}_2 + x_3\bar{y}_3, \qquad 1_2 = x_1\bar{y}_1 + \omega x_2\bar{y}_2 + \omega^2 x_3\bar{y}_3, \\ &1_3 = x_1\bar{y}_1 + \omega^2 x_2\bar{y}_2 + \omega x_3\bar{y}_3, \quad 1_4 = x_1\bar{y}_2 + x_2\bar{y}_3 + x_3\bar{y}_1, \\ &1_5 = x_1\bar{y}_2 + \omega x_2\bar{y}_3 + \omega^2 x_3\bar{y}_1, \quad 1_6 = x_1\bar{y}_2 + \omega^2 x_2\bar{y}_3 + \omega x_3\bar{y}_1, \\ &1_7 = x_2\bar{y}_1 + x_3\bar{y}_2 + x_1\bar{y}_3, \qquad 1_8 = x_2\bar{y}_1 + \omega^2 x_3\bar{y}_2 + \omega x_1\bar{y}_3, \\ &1_9 = x_2\bar{y}_1 + \omega x_3\bar{y}_2 + \omega^2 x_1\bar{y}_3, \end{aligned} \tag{3}$$

where $\omega = e^{2\pi i/3}$. The singlet multiplications are given in Table I.

Nonvanishing neutrino masses imply the existence of three right-handed neutrinos. Therefore, we consider the matter sector of SM besides three right-handed neutrinos. We assign the lepton doublet to the triplet **3** of $\Delta(27)$, while right-handed components are ascribed to different singlet representations of $\Delta(27)$. As mentioned, we consider only one SM Higgs scalar (H) and the following singlets: χ, ξ, η, σ, and ϕ that break the flavor symmetry.

In order to get the tri-bimaximal mixing as a zero-order approximation in our model, we find that an additional S_2 symmetry should be considered. The S_2, the group of permutation of two objects, has the following generators in the three-dimensional representation:

$$e = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad a = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{4}$$

The particle transformations under S_2 are given by

$$f_1 \leftrightarrow f_1, \qquad f_2 \leftrightarrow f_3, \tag{5}$$

TABLE I. The singlet multiplications of the group $\Delta(27)$.

	$\mathbf{1_2}$	$\mathbf{1_3}$	$\mathbf{1_4}$	$\mathbf{1_5}$	$\mathbf{1_6}$	$\mathbf{1_7}$	$\mathbf{1_8}$	$\mathbf{1_9}$
$\mathbf{1_2}$	$\mathbf{1_3}$							
$\mathbf{1_3}$	$\mathbf{1_1}$	$\mathbf{1_2}$						
$\mathbf{1_4}$	$\mathbf{1_6}$	$\mathbf{1_5}$	$\mathbf{1_7}$					
$\mathbf{1_5}$	$\mathbf{1_4}$	$\mathbf{1_6}$	$\mathbf{1_9}$	$\mathbf{1_8}$				
$\mathbf{1_6}$	$\mathbf{1_5}$	$\mathbf{1_4}$	$\mathbf{1_8}$	$\mathbf{1_7}$	$\mathbf{1_9}$			
$\mathbf{1_7}$	$\mathbf{1_8}$	$\mathbf{1_9}$	$\mathbf{1_1}$	$\mathbf{1_3}$	$\mathbf{1_2}$	$\mathbf{1_4}$		
$\mathbf{1_8}$	$\mathbf{1_9}$	$\mathbf{1_7}$	$\mathbf{1_2}$	$\mathbf{1_1}$	$\mathbf{1_3}$	$\mathbf{1_6}$	$\mathbf{1_5}$	
$\mathbf{1_9}$	$\mathbf{1_7}$	$\mathbf{1_8}$	$\mathbf{1_3}$	$\mathbf{1_2}$	$\mathbf{1_1}$	$\mathbf{1_5}$	$\mathbf{1_4}$	$\mathbf{1_6}$

where f_i stands for ℓ_i, χ_i, η_i, ϕ_i, ξ_i, σ_i, e_{R_i}, and ν_{R_i}. Moreover, we consider an extra $U(1)$ group to get the correct mass hierarchy of the charged leptons. In Table II, we present the field transformations under $\Delta(27)$ and $U(1)$.

Before concluding this section, we comment on possible vacuum alignments for the VEVs of the singlet scalars. The $\Delta(27) \ltimes S_2 \times U(1)$ invariant scalar potential at the renormalizable level is given by

$$\begin{aligned}V = {} & m_1^2\eta_1^\dagger\eta_1 + g_1(\eta_1^\dagger\eta_1)^2 + m_2^2(\eta_2^\dagger\eta_3 + \eta_3^\dagger\eta_2) + g_2\eta_1^\dagger\eta_1\eta_2^\dagger\eta_3 + g_3\eta_2^\dagger\eta_2\eta_2^\dagger\eta_1 + g_4\eta_3^\dagger\eta_3\eta_3^\dagger\eta_1\\ & + g_5\eta_2^\dagger\eta_2\eta_3^\dagger\eta_3 + m_\xi^2\xi^\dagger\xi + h_1(\xi^\dagger\xi)^2 + m_\sigma^2\sigma^\dagger\sigma + h_2(\sigma^\dagger\sigma)^2 + m_\phi^2\phi^\dagger\phi + h_3(\phi^\dagger\phi)^2\\ & + m_\chi^2\chi^\dagger\chi + h_4(\chi^\dagger\chi)^2 + h_5\eta_1^\dagger\eta_1\xi^\dagger\xi + h_6\eta_1^\dagger\eta_1\sigma^\dagger\sigma + h_7\eta_1^\dagger\eta_1\phi^\dagger\phi + h_8\eta_1^\dagger\eta_1\chi^\dagger\chi\\ & + h_9\eta_2^\dagger\eta_3\xi^\dagger\xi + h_{10}\eta_2^\dagger\eta_3\sigma^\dagger\sigma + h_{11}\eta_2^\dagger\eta_3\phi^\dagger\phi + h_{12}\eta_2^\dagger\eta_3\chi^\dagger\chi + h_{13}\xi^\dagger\xi\phi^\dagger\phi + h_{14}\xi^\dagger\xi\chi^\dagger\chi\\ & + h_{15}\phi^\dagger\phi\chi^\dagger\chi + h_{16}\xi^\dagger\xi\sigma^\dagger\sigma + h_{17}\chi^\dagger\chi\sigma^\dagger\sigma + h_{18}\phi^\dagger\phi\sigma^\dagger\sigma + \text{H.c.}\end{aligned} \tag{6}$$

The S_2 symmetry leads to $g_3 = g_4$. From this equation, one can notice that the potential contains 6 free mass parameters and 23 free self-interacting couplings. This large number of free parameters is one of the features of any flavor scalar (nonsupersymmetric) potential. Therefore, the minimization conditions of this potential can imply the following extremum solutions:

$$\begin{aligned}&\langle\xi\rangle = (0,w,0), \quad \langle\phi\rangle = (0,0,w'), \quad \langle\chi\rangle = (v,v,v)\\ &\langle\sigma\rangle = (v',0,0), \quad \langle\eta_1\rangle = u_1, \quad \langle\eta_2\rangle = u_2, \quad \langle\eta_1\rangle = u_3.\end{aligned} \tag{7}$$

We assume that the VEVs w, w', v', and v are of the same order and satisfy the following relation:

$$\frac{w}{\Lambda} \sim \frac{w'}{\Lambda} \sim \frac{v}{\Lambda} \sim \frac{v'}{\Lambda} \sim \mathcal{O}(\lambda_C^2), \tag{8}$$

where λ_C is the Cabibbo angle, *i.e.*, $\lambda_C \sim 0.22$.

III. CHARGED LEPTON MASSES AND Z_4 SYMMETRY

As shown in Table II, the lepton doublet is assigned to the triplet **3** of $\Delta(27)$, while the right-handed leptons l_i^c are ascribed as singlet representations of $\Delta(27)$. We find that the hierarchy between the charged lepton masses may be achieved by imposing an extra $U(1)$ symmetry. A possible set of charge assignments of $U(1)$ that lead to Yukawa interactions compatible with the experimental data is given in Table II. Therefore, the charged lepton Yukawa Lagrangian, invariant under $\Delta(27) \ltimes S_2 \times U(1)$, is given by

TABLE II. Field transformations under $\Delta(27)$ and $U(1)$. Here, α refers to 1, 2, 3.

Fields	ℓ	e_R	μ_R	τ_R	ν_{R_α}	H	χ	η_α	ξ	ϕ	σ
$\Delta(27)$	3	1_1	1_1	1_1	1_α	1_1	3	1_α	$\bar{3}$	3	$\bar{3}$
U(1)	1	−6	2	−2	−1	1	1	2	1	2	0

$$\mathcal{L}_l = \frac{\lambda_e}{\Lambda^4}\bar{\ell}He_R\chi^2\phi^2 + \frac{\lambda_\mu}{\Lambda^2}\bar{\ell}H\mu_R\phi^\dagger\sigma + \frac{\lambda_\tau}{\Lambda}\bar{\ell}H\tau_R\phi + \text{H.c.}, \tag{9}$$

where Λ is nonrenormalization scale, which is $\gg$ TeV. The Yukawa couplings λ_e, λ_μ, λ_τ are of order one. As mentioned in the previous section, the scalar potential $V(\phi,\chi,\xi,\sigma)$ contains several free parameters that can be adjusted to generate the VEVs for the flavons as given in Eq. (7). From Eqs. (1) and (2), one can show that

$$\begin{aligned}\langle\chi^2\rangle \times \langle\phi^2\rangle &= \begin{pmatrix} v^2 \\ v^2 \\ v^2 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ \omega'^2 \end{pmatrix} = \begin{pmatrix} v^2\omega'^2 \\ v^2\omega'^2 \\ v^2\omega'^2 \end{pmatrix},\\ \langle\phi^\dagger\rangle \times \langle\sigma\rangle &= \begin{pmatrix} 0 \\ 0 \\ \omega'^\dagger \end{pmatrix} \times \begin{pmatrix} v' \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \omega'^2 v' \\ 0 \end{pmatrix}.\end{aligned} \tag{10}$$

In this case, one finds that the charged lepton mass matrix m_ℓ is given by

$$m_\ell = \begin{pmatrix} a\lambda_C^6 & 0 & 0 \\ a\lambda_C^6 & b\lambda_C^2 & 0 \\ a\lambda_C^6 & 0 & 1 \end{pmatrix}\lambda_\tau\lambda_C^2\langle H\rangle, \tag{11}$$

where $a = \frac{\lambda_e}{\lambda_\tau}$, $b = \frac{\lambda_\mu}{\lambda_\tau}$. The matrix m_ℓ is not symmetric or Hermitian, so it can be diagonalized by two unitary matrices:

$$\begin{aligned}m_\ell = U_L m_\ell^{\text{diag}} U_R^T \sim \lambda_\tau\lambda_C^2\langle H\rangle & \begin{pmatrix} 1 & \sqrt{2}ab\lambda_C^8 & 0 \\ -\sqrt{2}ab\lambda_C^8 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\\ \times & \begin{pmatrix} a\lambda_C^6 & 0 & 0 \\ 0 & b\lambda_C^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -\frac{a}{b}\lambda_C^4 & -a\lambda_C^6 \\ \frac{a}{b}\lambda_C^4 & 1 & 0 \\ a\lambda_C^6 & 0 & 1 \end{pmatrix}.\end{aligned} \tag{12}$$

Therefore, the charged lepton masses are given by

$$\begin{aligned} m_e &\sim a\lambda_\tau\lambda_C^8\langle H\rangle, \\ m_\mu &\sim b\lambda_\tau\lambda_C^4\langle H\rangle, \\ m_\tau &\sim \lambda_\tau\lambda_C^2\langle H\rangle. \end{aligned} \quad (13)$$

Hence, the following mass relations are satisfied:

$$m_\tau : m_\mu : m_e \approx 1 : b\lambda_C^2 : a\lambda_C^6. \quad (14)$$

We can choose a and b to get the hierarchy between the charged lepton masses. For $a = 2.53$, $b = 1.22$, the masses of the charged particles are

$$\begin{aligned} m_e &= 0.511 \text{ MeV}, \\ m_\mu &= 105.658 \text{ MeV}, \\ m_\tau &= 1.776 \text{ GeV}. \end{aligned}$$

It is worth noting that the left-handed mixing matrix U_L is close to the identity matrix, so the lepton mixing arises mainly from the neutrino sector.

IV. NEUTRINO MASSES AND MIXING

From solar and atmospheric neutrino oscillation data [20], the neutrino mass squared differences are given by

$$\begin{aligned} \Delta m_{21}^2 &= 7.54^{+0.26}_{-0.22} \times 10^{-5} \text{ eV}^2, \\ |\Delta m_{31}^2| &= 2.47^{+0.06}_{-0.22} \times 10^{-3} \text{ eV}^2, \\ |\Delta m_{32}^2| &= 2.46^{+0.07}_{-0.11} \times 10^{-3} \text{ eV}^2. \end{aligned} \quad (15)$$

In addition, the latest best-fit results for the mixing pattern in the lepton sector are given by [1]

$$\begin{aligned} \sin^2\theta_{12} &= 0.308^{+0.017}_{-0.017}, \\ \sin^2\theta_{23} &= 0.437^{+0.033}_{-0.023}, \\ \sin^2\theta_{13} &= 0.0234^{+0.0020}_{-0.0019}. \end{aligned} \quad (16)$$

Having the lepton doublet, ℓ_i, as a $\Delta(27)$ triplet and right-handed neutrinos, ν_{R_j}, as singlets, one can then construct the following invariant interaction terms:

$$\mathcal{L}_D = \frac{1}{\Lambda}\lambda_i \bar{\ell}\nu_{R_i} H\chi, \quad (17)$$

where χ is a $\Delta(27)$ triplet scalar. Therefore, one gets the following terms:

$$\begin{aligned} &\frac{1}{\Lambda}\lambda_1(\bar{\ell}_1\chi_1 + \bar{\ell}_2\chi_2 + \bar{\ell}_3\chi_3)\nu_{R_1}H, \\ &\frac{1}{\Lambda}\lambda_2(\bar{\ell}_1\chi_1 + \omega^2\bar{\ell}_2\chi_2 + \omega\bar{\ell}_3\chi_3)\nu_{R_2}H, \\ &\frac{1}{\Lambda}\lambda_3(\bar{\ell}_1\chi_1 + \omega\bar{\ell}_2\chi_2 + \omega^2\bar{\ell}_3\chi_3)\nu_{R_3}H. \end{aligned}$$

The S_2 flavor symmetry imposes the equality of the second and third couplings: $\lambda_2 = \lambda_3$. After the flavor symmetry breaking through the aligned vacuum, $\langle\chi\rangle = (v, v, v)$, the following Dirac neutrino mass matrix is obtained

$$m_D = \frac{v}{\Lambda}\begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_2 \\ \lambda_1 & \omega^2\lambda_2 & \omega\lambda_2 \\ \lambda_1 & \omega\lambda_2 & \omega^2\lambda_2 \end{pmatrix}\langle H\rangle, \quad (18)$$

which can be expressed as

$$m_D = \frac{v}{\Lambda}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}\begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_2 \end{pmatrix}\langle H\rangle. \quad (19)$$

Note that here all Dirac neutrino masses are generated from the same nonrenormalizable interactions of order v/Λ. Therefore, one would not expect any hierarchy between them.

Furthermore, from the invariant interactions of right-handed neutrinos with $\Delta(27)$ singlets η_i, Majorana mass terms for ν_R can be obtained from the following renormalizable interactions:

$$\mathcal{L}_M = f_{ijk}\bar{\nu}^c_{R_i}\nu_{R_j}\eta_k. \quad (20)$$

According to the $\Delta(27)$ multiplication rules of singlet representations, the invariants that give right-handed neutrino masses are

$$\begin{aligned} &f_1\bar{\nu}^c_{R_1}\nu_{R_1}\eta_1, \quad f_2\bar{\nu}^c_{R_1}\nu_{R_2}\eta_3, \quad f_3\bar{\nu}^c_{R_1}\nu_{R_3}\eta_2, \\ &f_4\bar{\nu}^c_{R_2}\nu_{R_2}\eta_2, \quad f_5\bar{\nu}^c_{R_2}\nu_{R_3}\eta_1, \quad f_6\bar{\nu}^c_{R_3}\nu_{R_3}\eta_3. \end{aligned} \quad (21)$$

The symmetry S_2 imposes the following constraints:

$$f_2 = f_3 \qquad f_4 = f_6.$$

Therefore, after $\Delta(27)$ symmetry breaking through the VEVs of η_k, one obtains the following right-handed Majorana mass matrix:

$$M_R = \begin{pmatrix} f_1u_1 & f_3u_3 & f_3u_2 \\ f_3u_3 & f_4u_2 & f_5u_1 \\ f_3u_2 & f_5u_1 & f_4u_3 \end{pmatrix}. \quad (22)$$

As usual, the light neutrino mass matrix is obtained in terms of the Dirac neutrino mass matrix and right-handed neutrino one through a type I seesaw mechanism as

$$M_\nu = -m_D M_R^{-1} m_D^T. \tag{23}$$

It is noticeable that the mass matrix M_R in Eq. (22) is a generic matrix that can lead to different type of neutrino mixing matrix (tri-bimaximal or nearly tri-bimaximal mixing matrix), depending on the coupling f_3 and the difference between the VEVs u_2 and u_3. In general, the tri-bimaximal mixing matrix, $U_{\rm TBM}$, can be written as [21,22]

$$U_{\rm TBM} = \Gamma_{\rm mag} U', \tag{24}$$

where $\Gamma_{\rm mag}$ is the magic matrix proposed by Cabibbo [23] and Wolfenstein [24] and has the form

$$\Gamma_{\rm mag} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \tag{25}$$

and

$$U' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & i \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} \end{pmatrix}. \tag{26}$$

From Eqs. (19) and (25),

$$M_\nu = -\frac{3v^2}{\Lambda^2} \langle H \rangle^2 \Gamma_{\rm mag} D_\lambda M_R^{-1} D_\lambda \Gamma_{\rm mag}, \tag{27}$$

where $D_\lambda = {\rm diag}(\lambda_1, \lambda_2, \lambda_2)$. If M_ν is diagonalized by a tri-bimaximal mixing matrix, then we can determine the corresponding form of the right-handed neutrino mass matrix, which typically takes the form

$$(M_R)_{\rm TBM} = \begin{pmatrix} x & 0 & 0 \\ 0 & z & y \\ 0 & y & z \end{pmatrix}. \tag{28}$$

Therefore, the exact tri-bimaximal can be naturally obtained within Δ(27) flavor symmetry if the coupling $f_3 = 0$ and the VEVs $u_2 = u_3 = u$, which ensures the S_2 invariance. In this case, one obtains

$$M_\nu^{\rm diag} = -3 \frac{v^2}{\Lambda^2} \langle H \rangle^2 \begin{pmatrix} \frac{\lambda_2^2}{f_5 u_1 + f_4 u} & 0 & 0 \\ 0 & \frac{\lambda_1^2}{f_1 u_1} & 0 \\ 0 & 0 & \frac{\lambda_2^2}{f_5 u_1 - f_4 u} \end{pmatrix}. \tag{29}$$

As expected, unlike the charged lepton masses, here there is no clear argument for neutrino mass hierarchy. Instead, one should assume a hierarchy among the involved couplings of flavon VEVs to achieve the type of desired neutrino mass spectrum. For instance, if one considers $f_4 \sim f_5 \gg f_1$, $u_1 \sim u$, and the couplings λ_s are of the same order, the normal neutrino mass hierarchy is realized, while an inverted neutrino mass hierarchy is obtained if $f_4 \sim f_5 \gg f_1$ and $u_1 \sim -u$. Finally, the degenerate scenario is obtained if $f_1 \sim f_5 \gg f_4$ and $u_1 \gg u$.

Now we consider the case of $f_3 \neq 0$ and $u_2 = u_3 = u$ (*i.e.*, M_ν is still invariant under S_2 symmetry). In this case, the neutrino mass matrix is given by

$$M_\nu = \frac{v^2}{\Lambda^2} \langle H \rangle^2 \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix}, \tag{30}$$

where

$$A = \frac{f_5 u_1 \lambda_1^2 + (f_4 \lambda_1 - 4 f_3 \lambda_2) u \lambda_1 + 2 f_1 \lambda_2^2 u_1}{f_1 u_1 (f_5 u_1 + f_4 u) - 2 f_3^2 u^2},$$

$$B = \frac{f_5 u_1 \lambda_1^2 + (f_4 \lambda_1 - f_3 \lambda_2) u \lambda_1 - f_1 \lambda_2^2 u_1}{f_1 u_1 (f_5 u_1 + f_4 u) - 2 f_3^2 u^2},$$

$$C = \frac{f_5^2 \lambda_1^2 u_1^2 + 2 f_5 \lambda_2 u_1 (f_1 \lambda_2 u_1 + f_3 \lambda_1 u) - u(-f_1 f_4 \lambda_2^2 u_1 + (f_4^2 \lambda_1^2 + 2 f_3 f_4 \lambda_1 \lambda_2 + 3 f_3^2 \lambda_2^2) u)}{(f_5 u_1 - f_4 u)(f_1 u_1 (f_5 u_1 + f_4 u) - 2 f_3^2 u^2)},$$

$$D = \frac{f_5^2 \lambda_1^2 u_1^2 + f_5 \lambda_2 u_1 (-f_1 \lambda_2 u_1 + 2 f_3 \lambda_1 u) - u(2 f_1 f_4 \lambda_2^2 u_1 + (f_4^2 \lambda_1^2 + 2 f_3 f_4 \lambda_1 \lambda_2 - 3 f_3^2 \lambda_2^2) u)}{(f_5 u_1 - f_4 u)(f_1 u_1 (f_5 u_1 + f_4 u) - 2 f_3^2 u^2)}.$$

As emphasized in Ref. [25], the tri-bimaximal mixing matrix corresponds to the neutrino mass matrix that satisfies the following three conditions:

$$(M_\nu)_{12} = (M_\nu)_{13},$$
$$(M_\nu)_{22} = (M_\nu)_{33},$$
$$(M_\nu)_{11} + (M_\nu)_{12} = (M_\nu)_{22} + (M_\nu)_{23}. \quad (31)$$

It is clear that the neutrino mass matrix in our case satisfies the first two conditions only while the third condition is not satisfied. Therefore, this neutrino mass matrix can be diagonalized by a matrix which is very close to tri-bimaximal. However, we find that the resulting mixing matrix still has zero θ_{13} and maximal θ_{23}. It essentially deviates from tri-bimaximal in the first and the second columns. Also, the corresponding eigenvalues of neutrino masses are given by

$$m_1 = -3\frac{v^2}{\Lambda^2}\langle H\rangle^2 \frac{2\lambda_1^2\lambda_2^2}{(x+\sqrt{x^2-y^2})},$$
$$m_2 = -3\frac{v^2}{\Lambda^2}\langle H\rangle^2 \frac{2\lambda_1^2\lambda_2^2}{(x-\sqrt{x^2-y^2})},$$
$$m_3 = -3\frac{v^2}{\Lambda^2}\langle H\rangle^2 \frac{\lambda_2^2}{(f_5u_1 - f_4u)}, \quad (32)$$

where $x = (f_1\lambda_2^2 + f_5\lambda_1^2)u_1 + f_4u\lambda_1^2$ and $y^2 = 4\lambda_1^2\lambda_2^2(-2f_3^2u^2 + f_1u_1(f_5u_1 + f_4u))$. Here the normal hierarchy is achieved if $f_4 \sim f_5 \gg f_1 \gg f_3$, $u_1 \sim u$, and the couplings $\lambda_1 \sim \lambda_2$. The degenerate scenario is obtained if $f_1 \sim f_5 \gg f_4 \gg f_3$ and $u_1 \gg u$. Finally, the inverted hierarchy is obtained if $f_4 \sim f_5 \gg f_1 \gg f_3$ and $u_1 \sim -u$.

Now we turn to the case of spontaneous S_2 symmetry breaking, i.e., $u_2 \neq u_3$, with $f_3 \sim 0$. In this case, all three relations in Eq. (31) are violated. The consequences of the deviation from tri-bimaximal mixing on the symmetry manifesting in the neutrino mass matrix were studied in [25]. Following the notations used in this reference, we define the parameters which characterize the deviation of mixing angles from the tri-bimaximal values as

$$D_{12} \equiv \frac{1}{3} - s_{12}^2, \qquad D_{23} \equiv \frac{1}{2} - s_{23}^2, \qquad D_{13} \equiv s_{13}, \quad (33)$$

where $s_{ij} \equiv \sin\theta_{ij}$. The violation of the tri-bimaximal symmetry of the neutrino mass matrix in Eq. (31) can be written in terms of deviation parameters D_{23} and s_{13} as follows:

$$\Delta_1 = (M_\nu)_{12} - (M_\nu)_{13} = \frac{\sqrt{2}}{3}((2m_1 + m_2)e^{2i\delta} - 3m_3)s_{13}e^{-i\delta} + \frac{2}{3}(m_2 - m_1)D_{23},$$
$$\Delta_2 = (M_\nu)_{22} - (M_\nu)_{33} = \frac{2\sqrt{2}}{3}(m_2 - m_1)s_{13}e^{i\delta} + \frac{1}{3}(m_1 + 2m_2 - 3m_3)D_{23},$$
$$\begin{aligned}\Delta_3 &= (M_\nu)_{11} + (M_\nu)_{12} - ((M_\nu)_{22} + (M_\nu)_{23}) \\ &= \left(\frac{1}{3\sqrt{2}}(3m_3 - (2m_1 + m_2)e^{2i\delta})e^{-i\delta} - \frac{\sqrt{2}}{3}(m_2 - m_1)e^{i\delta}\right)s_{13} \\ &\quad + \left(\frac{2}{3}(3m_3 - (2m_1 + m_2)e^{2i\delta})e^{-2i\delta} - \frac{1}{3}(m_2 - m_1)\right)\frac{s_{13}^2}{2} \\ &\quad - \frac{1}{3}(2m_1 + m_2 - 3m_3)D_{23} - \frac{9}{4}(m_2 - m_1)D_{12}, \end{aligned} \quad (34)$$

where m_i are the masses of the effective neutrinos and δ is the leptonic Dirac phase. In our model the deviations from tri-bimaximal conditions in Eq. (31) can give constrains on our parameters (couplings and VEVs) in order to get the correct mixing angles and desired scenario of mass spectra,

$$\Delta_1 = -\frac{v^2}{\Lambda^2}\langle H\rangle^2 \frac{i\sqrt{3}\lambda_2^2(u_2 - u_3)f_4}{f_5^2u_1^2 - f_4^2u_2u_3}, \qquad \Delta_2 = \frac{v^2}{\Lambda^2}\langle H\rangle^2 \frac{i\sqrt{3}\lambda_2^2(u_2 - u_3)f_4}{f_5^2u_1^2 - f_4^2u_2u_3}, \qquad \Delta_3 = -\frac{v^2}{\Lambda^2}\langle H\rangle^2 \frac{i\sqrt{3}\lambda_2^2(u_2 - u_3)f_4}{f_5^2u_1^2 - f_4^2u_2u_3}. \quad (35)$$

From Eqs. (34) and (35) we can calculate the deviation parameters from tri-bimaximal mixing (33) as follows

$$s_{13} = \frac{v^2}{\Lambda^2}\langle H\rangle^2 \frac{3i\sqrt{\frac{3}{2}}f_4\lambda_2^2(u_2 - u_3)(m_2 - m_3)e^{i\delta}}{((m_1 + 2m_2 - 3m_3)m_3 + e^{2i\delta}(-3m_1m_2 + 2m_1m_3 + m_2m_3))(-f_5^2u_1^2 + f_4^2u_2u_3)},$$
$$D_{23} = -\frac{v^2}{\Lambda^2}\langle H\rangle^2 \frac{i\sqrt{3}f_4\lambda_2^2(u_2 - u_3)(e^{2i\delta}m_2 - m_3)}{2((m_1 + 2m_2 - 3m_3)m_3 + e^{2i\delta}(-3m_1m_2 + 2m_1m_3 + m_2m_3))(-f_5^2u_1^2 + f_4^2u_2u_3)}$$
$$D_{12} = \left(\frac{v^2}{\Lambda^2}\langle H\rangle^2\right)^2 \frac{-3(u_2 - u_3)^2f_4^2\lambda_2^4(e^{2i\delta}(m_1 + m_2) - 2m_3)(m_2 - m_3)^2}{(m_1 - m_2)((m_1 + 2m_2 - 3m_3)m_3 + e^{2i\delta}(-3m_1m_2 + 2m_1m_3 + m_2m_3))^2(f_5^2u_1^2 - f_4^2u_2u_3)^2}. \quad (36)$$

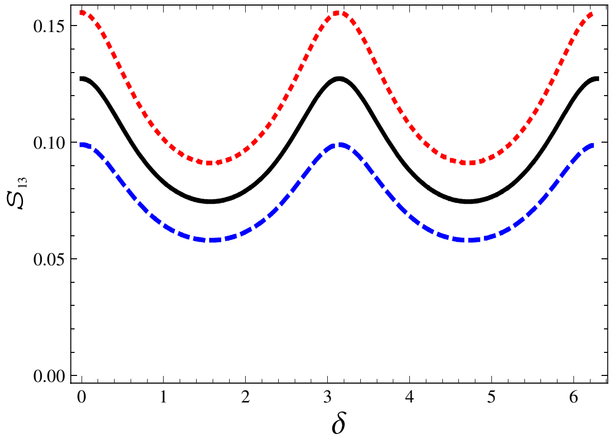

FIG. 1 (color online). s_{13} versus the Dirac phase δ for normal mass hierarchy and different values of D_{23}. The deviation parameter D_{23} is set to its best fit value ~0.066, 1σ limit 0.09 and 0.11 for the dashed, solid, dotted lines, respectively.

Thus, one can write the following relations:

$$s_{13} = \frac{\sqrt{2}e^{i\delta}(-m_2 + m_3)}{(e^{2i\delta}m_2 - m_3)} D_{23}$$
$$D_{12} = \frac{4(e^{2i\delta}(m_1 + m_2) - 2m_3)(m_2 - m_3)^2}{9(m_1 - m_2)(-e^{2i\delta}m_2 + m_3)^2} D_{23}^2. \quad (37)$$

If the Dirac phase $\delta = 0$, from the first relation, one finds that $s_{13} \sim 0.13$ (lower 3σ experimental limit) if $D_{23} \sim 0.09$, which corresponds to 1σ limit of atmospheric neutrino mixing angle [1]. In addition, if $D_{23} \sim 0.11$ (2σ limit), one gets $s_{13} \sim 0.155$ (best-fit value). In Fig. 1, we plot the relation between the Dirac phase δ and s_{13} for different values of D_{23}. As can be seen from this figure, the phase may change the value of s_{13} up to 40% depending on the value of the phase. In Fig. 2 we plot the relation between the lightest neutrino mass m_1 and D_{12} for different values of D_{23}. As can be seen from this figure, for $D_{23} = 0.09$–0.11, which leads to consistent s_{13}, the mass spectrum of the neutrino should be strongly hierarchical, *i.e.*, $m_1 < 0.01$ eV, in order to get D_{12} in the allowed range. We also present the relation between D_{12} and D_{23} for different values of m_1. It confirms the same conclusion that the allowed range for D_{12} can be achieved if $m_1 \lesssim 0.01$ eV for $0.09 \lesssim D_{23} \lesssim 0.11$.

The approximated neutrino mass eigenvalues are given by

$$m_1 \simeq -3\frac{v^2}{\Lambda^2}\langle H\rangle^2 \left(\frac{2\lambda_2^2}{\sqrt{4f_5^2u_1^2 + f_4^2(u_2 - u_3)^2} + f_4(u_2 + u_3)}\right),$$
$$m_2 \simeq -3\frac{v^2}{\Lambda^2}\langle H\rangle^2 \left(\frac{\lambda_1^2}{f_1 u_1}\right),$$
$$m_3 \simeq -3\frac{v^2}{\Lambda^2}\langle H\rangle^2 \left(\frac{2\lambda_2^2}{\sqrt{4f_5^2u_1^2 + f_4^2(u_2 - u_3)^2} - f_4(u_2 + u_3)}\right). \quad (38)$$

We can tune the parameters to obtain the various mass hierarchy spectra as follows: The normal hierarchy is achieved if $f_4 \sim f_5 \gg f_1$, $u_1 \sim (u_2 + u_3)$, and the couplings λ_s are of the same order. The degenerate scenario is obtained if $f_1 \sim f_5 \gg f_4$ and $u_1 \gg u_2, u_3$. The inverted hierarchy is obtained if $f_4 \sim f_5 \gg f_1$ and $u_1 \sim -(u_2 + u_3)$.

To ensure that there are plenty of values for the parameters, flavon VEVs, and coupling, which can account for the recent values of mixing angles and neutrino masses simultaneously, we show in Fig. 3 a correlation between s_{13} and m_3, where all free parameters vary in their allowed ranges.

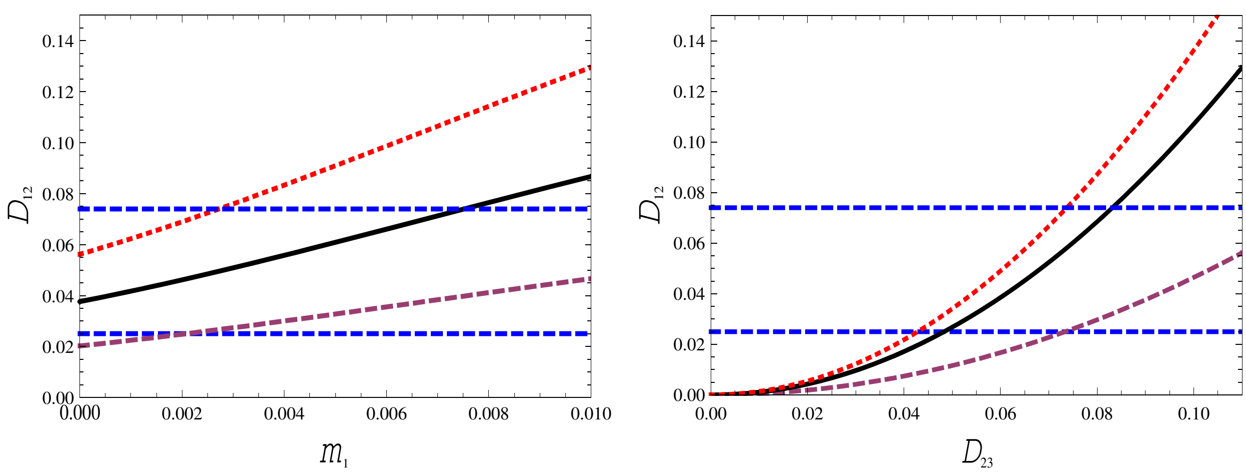

FIG. 2 (color online). (Left) The deviation parameter D_{12} versus the neutrino mass m_1 for different values of D_{23}. The deviation parameter D_{23} is set to its best-fit value ~0.066, 1σ limit 0.09, and 0.11 for the dashed, solid, dotted lines, respectively. (Right) The deviation parameter D_{23} versus D_{12} for different values of m_1. $m_1 = 0$, 0.01, 0.015 eV for the dashed, solid, dotted lines, respectively. The horizontal dashed lines represent the best-fit value and the upper 3σ limit of D_{12}. Here we set the Dirac delta $\delta = 0$:

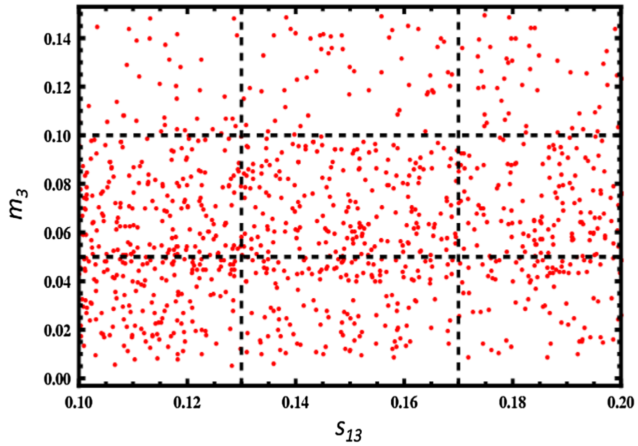

FIG. 3 (color online). $\sin\theta_{13}$ versus the greatest neutrino mass m_3. All free parameters of the model are varying within their allowed regions

It is important to note the essential role of S_2 symmetry, which permutes the second flavor to the third one and leads to the equality of the couplings in the Dirac mass matrix and right-handed mass matrix. In this case, the neutrino mass matrix has the form of Eq. (28) and, hence, tri-bimaximal mixing is realized. When we break the S_2 spontaneously by imposing different VEVs to the second and the third flavor of the flavon η, the deviation from tri-bimaximal is achieved.

V. QUARK MASSES AND CKM MIXING

In this section we analyze the quark masses and mixing in the framework of the symmetry group $\Delta(27)\ltimes S_2\times U(1)$. The quark transformations under $\Delta(27)$ are shown in Table III. We also assume that the left-handed quarks and upright quarks transform under S_2 such that $Q_{L_2}\leftrightarrow Q_{L_3}$ and $U_{R_2}\leftrightarrow U_{R_3}$, while the down right quarks transform trivially under S_2 symmetry. From these charge assignments, one finds that Yukawa interaction terms of the up quarks, invariant under $\Delta(27)\ltimes S_2\times U(1)$, are given by

$$\mathcal{L}_u=\frac{1}{\Lambda}h_i^u\bar{Q}HU_R\eta_i, \tag{39}$$

where $i=1,2,3$. The allowed invariants terms are

TABLE III. Quark assignments under $\Delta(27)$ and Z_4.

Fields	Q	d_R	s_R	b_R	U_R
$\Delta(27)$	3	1_1	1_1	1_1	3
$U(1)$	1	-5	-1	-2	-2

$$\begin{aligned}&\frac{1}{\Lambda}h_1^uH(\bar{Q}_1u_R+\bar{Q}_2c_R+\bar{Q}_3t_R)\eta_1,\\&\frac{1}{\Lambda}h_2^uH(\bar{Q}_1u_R+\omega^2\bar{Q}_2c_R+\omega\bar{Q}_3t_R)\eta_2,\\&\frac{1}{\Lambda}h_3^uH(\bar{Q}_1u_R+\omega\bar{Q}_2c_R+\omega^2\bar{Q}_3t_R)\eta_3.\end{aligned} \tag{40}$$

From the S_2 symmetry, $h_2^u=h_3^u$. The masses of the up quarks are

$$\begin{aligned}m_u&=\frac{1}{\Lambda}\langle H\rangle(h_1^uu_1+h_2^u(2u_2+\Delta)),\\m_c&=\frac{1}{\Lambda}\langle H\rangle(h_1^uu_1+h_2^u(\omega u_2+\omega^2(u_2+\Delta))),\\m_t&=\frac{1}{\Lambda}\langle H\rangle(h_1^uu_1+h_2^u(\omega^2u_2+\omega(u_2+\Delta))),\end{aligned} \tag{41}$$

where $\Delta=u_3-u_2$. In general, the coupling constants h_i^u and VEVs u_i are complex, so the previous three masses are different and can account for the hierarchial mass spectrum of the up-quark sector. As an example for this possibility, if $\frac{u_i}{\Lambda}\sim\mathcal{O}(\lambda_C^2)$ and $h_1^u\simeq 6.85$, $h_2^u\simeq -6.85e^{i\pi/3}$, $\frac{\Delta}{\Lambda}\simeq -0.083e^{i\pi/6}$, one gets the up-quark masses consistent with the following experimental results:

$$\begin{aligned}m_u(1\text{ GeV})&=4.5\pm 1\text{ MeV},\\m_c(m_c)&=1.25\pm 0.15\text{ GeV},\\m_t(m_t)&=166\pm 5\text{ GeV}.\end{aligned} \tag{42}$$

Finally, we consider the down quark mass and mixing. From the charge assignments given in Table III, one can write the following invariants:

$$\begin{aligned}\mathcal{L}_d=&\frac{1}{\Lambda^3}h_d\bar{Q}Hd_R(\chi\eta_1^2+\phi^2\xi)\\&+\bar{Q}Hs_R\left(\frac{1}{\Lambda^3}h_{s_1}\phi\xi^\dagger\sigma+\frac{1}{\Lambda^2}h_{s_2}\xi^\dagger\eta_1+\frac{1}{\Lambda^2}h_{s_3}\xi\sigma\right)\\&+\bar{Q}Hb_R\left(\frac{1}{\Lambda^3}h_{b_1}\xi^\dagger\xi\phi+\frac{1}{\Lambda^2}h_{b_2}\xi\xi+\frac{1}{\Lambda}h_{b_3}\phi\right).\end{aligned} \tag{43}$$

If $h_{s_3}\sim 2h_{s_2}\sim h_{s_1}/2$, then after spontaneous symmetry breaking, the following mass matrix of down quarks is obtained:

$$m_d\simeq\begin{pmatrix}\lambda_C^4 & 2\lambda_C^4 & \lambda_C^4\\ \lambda_C^4 & \frac{\lambda_C^2}{2} & \lambda_C^2\\ \lambda_C^4 & \lambda_C^2 & 1\end{pmatrix}h_{b_1}\lambda_C^2\langle H\rangle. \tag{44}$$

This matrix can be diagonalized by two unitary matrices as

$$m_d=U_Lm_d^{\text{diagonal}}U_R^T, \tag{45}$$

where, for $\lambda_C = 0.22$,

$$U_L = \begin{pmatrix} 0.976 & 0.214 & 0.0026 \\ -0.214 & 0.975 & 0.049 \\ 0.0080 & -0.0488 & 0.9987 \end{pmatrix},$$
$$m_d^{\text{diagonal}} = \begin{pmatrix} 0.0018 & 0 & 0 \\ 0 & 0.0224 & 0 \\ 0 & 0 & 1.0024 \end{pmatrix} h_{b_1}\lambda_C^2\langle H\rangle,$$
$$U_R = \begin{pmatrix} 0.992 & 0.119 & 0.0024 \\ -0.119 & 0.991 & 0.049 \\ 0.0034 & -0.049 & 0.9987 \end{pmatrix}. \tag{46}$$

It is clear that the left-handed rotation matrix is close to the quark V_{CKM} mixing matrix, and the hierarchical spectrum of down quark masses is slightly compatible with measured down quark masses:

$$m_d(1\text{ GeV}) = 8.0 \pm 2\text{ MeV},$$
$$m_s(1\text{ GeV}) = 150 \pm 50\text{ MeV},$$
$$m_b(m_b) = 4.25 \pm 0.15\text{ GeV}. \tag{47}$$

VI. CONCLUSIONS

In this paper we have constructed a model of fermion masses and mixing based on an extension of the SM with a discrete flavor symmetry $\Delta(27)$. Our study is different from the previous $\Delta(27)$ analyses on two main points: (i) One Higgs doublet is used to break the electroweak symmetry, and SM singles only are involved in spontaneous breaking of $\Delta(27)$, so our model is FCNC free. (ii) Both quark and lepton masses and their mixing are simultaneously analyzed under the same flavor symmetry. In fact, most of the work in the literature focuses on the lepton sector only.

By assigning lepton doublets to a $\Delta(27)$ triplet and right-handed leptons to singlets, we show that the charged lepton mass matrix is almost diagonal with the desired hierarchy. Therefore, the neutrino mixing matrix is generated from the neutrino sector. We also argue that deviation from tri-bimaximal is due to spontaneous violation of the imposed S_2 symmetry. Similarly, by assigning quark doublets and right-handed up quarks to a $\Delta(27)$ triplet and right-handed down quarks to singlets, we obtain the diagonal up-quark mass matrix, and the CKM quark mixing matrix arises from the down sector only.

Finally, our model predicts that for $\sin\theta_{13} \simeq 0.13$, the mass of the lightest neutrino is of order $\mathcal{O}(0.1)$ eV and $\sin^2\theta_{23} \simeq 0.41$, which is a remarkable deviation from maximal mixing.

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