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Testing violations of Bose and *CPT* symmetries via Dalitz plots and their prismatic generalizations

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Bose symmetry and *CPT* symmetry are two very fundamental symmetries of nature. However, the validity of these symmetries in diverse phenomena must be verified by experiments. We propose new techniques to probe these two fundamental symmetries in the realm of mesons by using the Dalitz plot of a few three-body meson decays. Since these symmetries are very fundamental in nature, their violations, if any, are expected to be extremely small. Hence, observing their violations requires study of a huge data sample. In this context we introduce a new three-dimensional plot which we refer to as the Dalitz "prism." This provides an innovative means for acquiring the huge statistics required for such studies. Using the Dalitz plots and the Dalitz prisms we chart out the way to probe the violations of Bose and *CPT* symmetries in a significant manner. Since mesons are unstable and composite particles, testing the validity of Bose symmetry and the *CPT* symmetry in these cases is of paramount importance for fundamental physics.

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The statement that a state made up of two identical bosons does not alter under exchange of the two bosons is the dictum of Bose symmetry [1]. This along with the Fermi statistics [2] forms one of the cornerstones of modern physics, the famous spin-statistics theorem. Within the conventional Lorentz invariant and local quantum field theory, even a small violation of Bose symmetry is impossible. There has been therefore a lot of interest in experiments looking for Bose symmetry violation as a means of testing the present theoretical framework. Theoretical ideas and experimental investigations for Bose symmetry violations have looked at the spin-0 nucleus of oxygen ^{16}O [3,4], molecules such as $^{16}O_2$ and CO_2 [5–8], photons [9–14], pions [15] and Bose symmetry violating transitions [16–22]. Theoretically a scenario where Bose symmetry is not exact swings open doors to a plethora of avenues for new physics [23–27]. Like the Bose symmetry, the very nature of Lorentz invariant local quantum field theory encompasses another fundamental symmetry of nature, namely the *CPT* symmetry. This symmetry combines the operations of charge conjugation (C), parity (P) and time reversal (T). In the conventional settings of quantum field theory, the *CPT* symmetry is very closely related to both spin-statistics theorem and Lorentz invariance [28–47]. However, *CPT* invariance and the spin-statistics theorem need not be connected [47–49], and there are examples of quantum field theories in the literature [50–52] that explicitly violate the *CPT* invariance. Under *CPT* transformation, a particle becomes its antiparticle and vice versa with the same three-momentum but with its helicity reversed. The *CPT* invariance also implies that a particle and its antiparticle must have the same mass, decay width and lifetime. It is important to note that if *CPT* invariance holds good but *CP* is violated, then partial rate asymmetries for a particle and its antiparticle can be different while keeping their total decay rates unchanged [53]. Similarly, the *CPT* invariance also implies that the total scattering cross section of two particles would be equal to that of their antiparticles, but the partial scattering cross sections need not be equivalent if *CP* is violated [54]. Though *CPT* invariance is in concord with our present theoretical framework of standard model of particle physics, it needs to be thoroughly tested experimentally. The literature is replete with many tests for *CPT* violation, such as in anomalous magnetic moments [55,56], double beta decay [57], some neutral mesons [58–69], muon [70,71], neutrino [72–77], Higgs boson decay [78], neutron [79], photon [80,81], hydrogen atom [82], cosmic microwave background measurements [83] as well as some space based experiments [84]. A summary of results of many such studies and more references can be found in Ref. [85]. The best test of *CPT* invariance has come from polarization studies of cosmic microwave background radiation [85]. In all these studies there is no concrete indication of any breakdown of the *CPT* invariance. However, if there is even an extremely small violation of *CPT*, it would have very significant theoretical ramifications in various models of new physics. If *CP* violation is present in the decay mode, it might overshadow the signature of *CPT* violation in the Dalitz plots. Therefore, usage of Dalitz plot for observation of *CPT* violation must be dealt with deftly. Nevertheless, probing violations of Bose and *CPT* symmetries by new methods is of paramount importance. As was shown in Refs. [46,86] *CPT* violation invariably leads to an associated violation of Lorentz invariance in an interacting field theory. Though very alluring, we do not dwell upon any signatures of Lorentz violation in the Dalitz plot, as this is outside the scope of this paper.

In this paper we shall point out methods, in search for the violations of Bose symmetry and *CPT* symmetry, in some

three-body meson decays via the Dalitz plot. This is in continuation of our efforts to use the Dalitz plot as an experimental tool to search for violations of some of the fundamental symmetries in nature, such as the CP symmetry [87]. In this paper we shall analyze the observational signatures of violation of Bose symmetry and CPT symmetry in the Dalitz plot. On the way we shall elucidate the techniques by considering a few decay modes in which these searches would be fruitful. Finally, we shall introduce the concept of and explain the utility of the Dalitz "prism" which in its simplest form can be realized as a stacked up pile of numerous Dalitz plots with increasing center-of-momentum energy. We conclude emphasizing the importance of these new methods.

Let us consider a general three-body decay process, say $X \to 1+2+3$, where the 4-momentum of the particle i ($i \in \{X, 1, 2, 3\}$) is denoted by p_i and its corresponding mass is denoted by m_i. Let us also define the following Mandelstam-like variables: $s = (p_2 + p_3)^2 = (p_X - p_1)^2$, $t = (p_1 + p_3)^2 = (p_X - p_2)^2$ and $u = (p_1 + p_2)^2 = (p_X - p_3)^2$. It is well known that $s + t + u = m_X^2 + m_1^2 + m_2^2 + m_3^2 = M^2$ (say). We can always construct a ternary plot (see Fig. 1) of which (t, u, s) form the Cartesian coordinates. The ternary plot can also be described by a barycentric rectangular coordinate system (x, y) or a barycentric polar coordinate system (r, θ). For the polar coordinate system the pole is at the centroid of the equilateral triangle of the ternary plot and the polar axis passes through the vertex for which $(t, u, s) = (0, 0, M^2)$. It is quite straightforward to express the variables s, t, u in terms of r, θ and x, y as follows:

$$s = \frac{M^2}{3}(1 + r\cos\theta) = \frac{M^2}{3}(1 + y), \tag{1}$$

$$t = \frac{M^2}{3}\left(1 + r\cos\left(\frac{2\pi}{3} - \theta\right)\right) = \frac{M^2}{6}(2 - \sqrt{3}x - y), \tag{2}$$

$$u = \frac{M^2}{3}\left(1 + r\cos\left(\frac{2\pi}{3} + \theta\right)\right) = \frac{M^2}{6}(2 + \sqrt{3}x - y). \tag{3}$$

We know that s, t, u take values in the following ranges: $(m_2+m_3)^2 \le s \le (m_X-m_1)^2$, $(m_1+m_3)^2 \le t \le (m_X-m_2)^2$, $(m_1 + m_2)^2 \le u \le (m_X - m_3)^2$. So when the final particles are ultrarelativistic, the Dalitz plot tends to occupy the full region inside the equilateral triangle of Fig. 1. In all cases the Dalitz plot is inscribed inside the equilateral triangle. The density of events inside the Dalitz plot is a consequence of the dynamics driving the decay. Mathematically, if $A(r, \theta)$ is the amplitude of the decay under consideration, the Dalitz plot density $D(r, \theta)$ is directly proportional to $|A(r, \theta)|^2$. If the full Dalitz plot can be constructed (i.e. if $0 \le \theta \le 2\pi$), then the decay amplitude $A(r, \theta)$ can be expanded in terms of a Fourier series as follows:

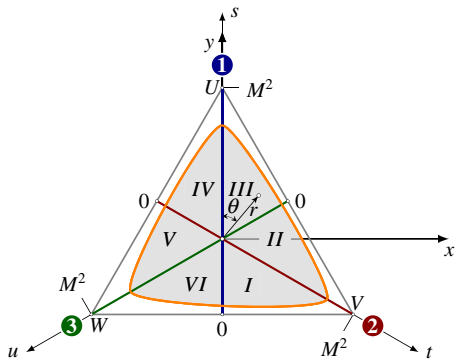

FIG. 1 (color online). A hypothetical Dalitz plot with Mandelstam-like variables s, t, u for the decay $X \to 1+2+3$. The three sides of the equilateral triangle $\triangle UVW$ are given by $s = 0$, $u = 0$ and $t = 0$. At the three vertices we have $s = M^2$, $t = M^2$ and $u = M^2$. The three vertices in terms of the barycentric rectangular coordinate (x, y) are given by $U = (0, 2)$, $V = (\sqrt{3}, -1)$ and $W = (-\sqrt{3}, -1)$. For the barycentric polar coordinates (r, θ), the angle θ is measured from the vertical axis. The blobs with **1**, **2** and **3** serve as a mnemonic to suggest that the exchanges $s \leftrightarrow t \leftrightarrow u$ are equivalent to the particle exchanges $1 \leftrightarrow 2 \leftrightarrow 3$ respectively. The sextants of the Dalitz plot are also shown.

$$A(r, \theta) = \sum_{n=0}^{\infty}(S_n(r)\sin(n\theta) + C_n(r)\cos(n\theta)), \tag{4}$$

where $S_n(r)$ and $C_n(r)$ are the Fourier coefficients. It would be profitable for us to divide the Dalitz plot into six sectors or sextants by the medians of the equilateral triangle as shown in Fig. 1.

It is now easy to explain the idea of observing the violations of Bose symmetry and CPT symmetry in the Dalitz plot. We shall first discuss the Bose symmetry part. If particles 2 and 3 were identical mesons, then the final state must remain symmetric under their exchange as demanded by Bose symmetry. This implies that the Dalitz distribution should remain symmetric under the exchange $t \leftrightarrow u$. The decay amplitude $A(r, \theta)$ can also be written as $A(t, u)$, such that $A(r, -\theta) \equiv A(u, t)$. If the two particles 2 and 3 are not exactly identical, then the Bose symmetry would not be strictly obeyed. In such a case, we can split the decay amplitude into a part which is symmetric under $t \leftrightarrow u$ exchange and another part which is nonsymmetric under the same exchange:

$$A(t, u) = A^S + A^N, \tag{5}$$

where

$$A^S \equiv \frac{1}{2}(A(t, u) + A(u, t)) = \sum_{n=0}^{\infty} C_n(r)\cos(n\theta), \tag{6}$$

$$A^N \equiv \frac{1}{2}(A(t, u) - A(u, t)) = \sum_{n=0}^{\infty} S_n(r)\sin(n\theta). \tag{7}$$

The density of events in the Dalitz plot is proportional to the amplitude mod-square. Only the interference term which is proportional to $\mathrm{Re}(A^S \cdot A^{N*})$ would give rise to an asymmetry in the Dalitz plot under $t \leftrightarrow u$ exchange. Thus, we need the full Dalitz plot in this case. This can be easily obtained if we construct the Dalitz plot from those events in which particles 2 and 3 decay into different and distinct final states. For example, the following decay modes

$$(K^+, D^+, D_s^+) \to \underbrace{\pi^+(p_1)}_{\mu^+\nu_\mu}\underbrace{\pi^0(p_2)}_{e^+e^-\gamma}\underbrace{\pi^0(p_3)}_{\gamma\gamma},$$
$$(K^+, D^+, D_s^+) \to \underbrace{\pi^-(p_1)}_{\mu^-\bar{\nu}_\mu}\underbrace{\pi^+(p_2)}_{e^+\nu_e}\underbrace{\pi^+(p_3)}_{\mu^+\nu_\mu},$$

can be used for such a Bose symmetry violation study, since the particles with 4-momenta p_2 and p_3 are the same but are reconstructed from different final states. The extent of departure from Bose symmetry can be quantified by using the conventional left-right asymmetry of the Dalitz plot.

It is also possible to analyze three-body decays in which all the final states are identical mesons, such that the final state is fully Bose symmetric under the exchange of any two particles in it. Such a situation would demand invariance under the exchange $s \leftrightarrow t \leftrightarrow u$. This would imply that all the sextants of the Dalitz plot would be symmetrical to one another when we go from one to the other. Thus, if all the three final particles are reconstructed from identical final states we would be left with only one of the sextants of the Dalitz plot. For the Bose symmetry test we need to have more than one sextant in our Dalitz plot. For this we reconstruct two particles, say 1 and 2, from identical final states and particle 3 from different final state. In this case we would have sextants *VI*, *I* and *II* (or equivalently *III*, *IV* and *V*) in our Dalitz plot. If particles 2 and 3 are identical bosons then these sextants should map from one to the other. Any asymmetry among these sextants would be a signature of Bose symmetry violation. Decay modes such as $(\eta, K_L^0, D^0) \to \underbrace{\pi^0(p_1)}_{\gamma\gamma}\underbrace{\pi^0(p_2)}_{\gamma\gamma}\underbrace{\pi^0(p_3)}_{e^+e^-\gamma}$, $B^0 \to \underbrace{K_S^0(p_1)}_{\pi^+\pi^-}\underbrace{K_S^0(p_2)}_{\pi^+\pi^-}\underbrace{K_S^0(p_3)}_{\pi^0\pi^0}$, can be profitably used to search for the Bose symmetry violations in their Dalitz plots.

The invariance under *CPT* is a characteristic feature of any Lorentz invariant local quantum field theory. Thus, it applies equally well to both electroweak and strong interactions. In weak interaction, however, *CP* violation is observed, which, as emphasized in the introduction, can make the signature of *CPT* violation unextractable from the Dalitz plot. Keeping this in mind, we consider only those decay modes which can occur via electromagnetic and strong interactions, and thereby have no contribution from *CP* violation in them. *CP* violation might still occur below the current experimental bounds in these modes and mimic the signal for possible *CPT* violation. It is, nevertheless, extremely interesting to look for any unexpected violation of *CP* or *CPT* in strong or electromagnetic interactions. A nice example of such a process, free from *CP* violation, is the decay modes $J/\psi \to N\pi^+\pi^-$, where N can be any of the following: π^0, ω, η, ϕ. The amplitude $A(r,\theta)$ for the process $J/\psi \to N\pi^+\pi^-$ can be expanded in a Fourier series as follows:

$$A(r,\theta) = \sum_{n=0}^{\infty}(s_n(r)\sin(n\theta) + c_n(r)\cos(n\theta)), \quad (8)$$

where $s_n(r)$ and $c_n(r)$ are Fourier coefficients which are in general complex. Under *CPT* the angle θ goes to $-\theta$ and the complex Fourier coefficients [$s_n(r)$ and $c_n(r)$] transform to their respective complex conjugates [$s_n^*(r)$ and $c_n^*(r)$]. Therefore, *CPT* invariance implies that $A(r,\theta) = A^*(r,-\theta)$. Moreover, for a self-conjugate process the initial and final state must have the same *CP* if *CP* is conserved. Hence, conservation of *CP* and *CPT* jointly implies that all the $s_n(r)$ are zero and all the $c_n(r)$ are purely real. This restricts the amplitude in Eq. (8) and the Dalitz plot density to be symmetric under $\theta \leftrightarrow -\theta$. If *CP* is conserved, any asymmetry in the Dalitz plot under $\theta \leftrightarrow -\theta$ would therefore be a signature of *CPT* violation as discussed below.

The amplitude $\bar{A}(r,-\theta)$ for the *CP* conjugate process, assuming *CPT* violation, is given by

$$\bar{A}(r,-\theta) = \sum_{n=0}^{\infty}(-\bar{s}_n(r)\sin(n\theta) + \bar{c}_n(r)\cos(n\theta)), \quad (9)$$

where $\bar{s}_n(r)$ and $\bar{c}_n(r)$ are Fourier coefficients that are complex and are necessarily different from $s_n^*(r)$ and $c_n^*(r)$ respectively unless *CPT* is conserved [88]. The coefficients $s_n(r)$, $c_n(r)$, $\bar{s}_n(r)$ and $\bar{c}_n(r)$ can be written as

$$s_n(r) = (|s_n(r)| + \epsilon_n^s(r))e^{i\delta_n^s}, \quad \bar{s}_n(r) = (|s_n(r)| - \epsilon_n^s(r))e^{i\delta_n^s},$$
$$c_n(r) = (|c_n(r)| + \epsilon_n^c(r))e^{i\delta_n^c}, \quad \bar{c}_n(r) = (|c_n(r)| - \epsilon_n^c(r))e^{i\delta_n^c},$$

where $\delta_n^{s,c}$ are the strong phases and $\epsilon_n^{s,c}(r)$ are *CPT* violating terms, i.e. for the case of *CPT* invariance they vanish identically. Since we have assumed that *CP* is conserved no explicit weak phase dependence is retained in $s_n(r)$, $c_n(r)$, $\bar{s}_n(r)$ and $\bar{c}_n(r)$.

Since in our case the process and its *CP* conjugate process are the same, the amplitude which comes into picture is the average of both $A(r,\theta)$ and $\bar{A}(r,-\theta)$:

$$A \equiv \frac{1}{2}(A(r,\theta) + \bar{A}(r,-\theta))$$
$$= \sum_{n=0}^{\infty}(\epsilon_n^s(r)\sin(n\theta)e^{i\delta_n^s} + |c_n(r)|\cos(n\theta)e^{i\delta_n^c}). \quad (10)$$

The logic for the average is easy to realize by observing that if CPT is conserved, $\epsilon_n^{s,c}(r) = 0$ and the amplitude in Eq. (10) reduces to that in Eq. (8) as expected. The Dalitz distribution is proportional to $|A|^2$ and any asymmetry under $\theta \leftrightarrow -\theta \equiv t \leftrightarrow u$ can arise only from the term odd under θ which is proportional to

$$\sum_{n,m=0}^{\infty} |c_n(r)|\epsilon_m^s(r)\cos(\delta_n^c - \delta_m^s)\cos(n\theta)\sin(m\theta).$$

This interference term survives only if CPT is violated. We have thus demonstrated mathematically how CPT violation leads to asymmetry in the Dalitz plot. It should be noted that the observation of such an asymmetry would be an unambiguous signature of CPT violation (or CP violation in strong and electromagnetic interaction) and would demand the presence of CPT (or CP) violating new physics. The usual left-right Dalitz plot asymmetry can be used to quantify this asymmetry in the Dalitz plot. It is possible to look for CPT violation in any self-conjugate process of the form $X \to NM\bar{M}$ which proceeds via strong or electromagnetic interactions preserving CP in the decay.

The Bose and CPT symmetries are expected to hold firmly. Their violations, if any, would by virtue be extremely small. In order to possibly observe such tiny numbers, one would require as large a sample of events as possible. In purview of this *a new concept* of Dalitz prism is developed here. So far in the discussions on Bose and CPT symmetries, details of the initial particle X played no role. In fact the particle X can be replaced by, say e^+e^-, such that m_X denotes the total energy in the center-of-momentum frame. In such a situation we are dealing with continuum production of particles 1, 2 and 3, e.g. $e^+e^- \to \pi^+\pi^-\pi^0$. Considering such continuum productions in association with the decays of several resonances would provide significantly larger statistics to study the violations of Bose and CPT symmetry. To facilitate such a study we note that the Dalitz plot can be generalized into a three-dimensional plot, which we call the Dalitz prism (Fig. 2). This prism is a regular right triangular prism. For a given value of m_X one can slice this prism to obtain the Dalitz plot. Decay events corresponding to all possible values of m_X fill up only those regions of the prism which are allowed by conservation of energy and momentum. This idea of Dalitz prism can be extended to include cases such as $X(p_X) \to N(p_1)M(p_2)\bar{M}(p_3)$ where N can represent more than one particle and p_1 is, therefore, the total 4-momentum of all those particles denoted by N. One example of such a mode is $X \to N\pi^+\pi^-$ where the initial

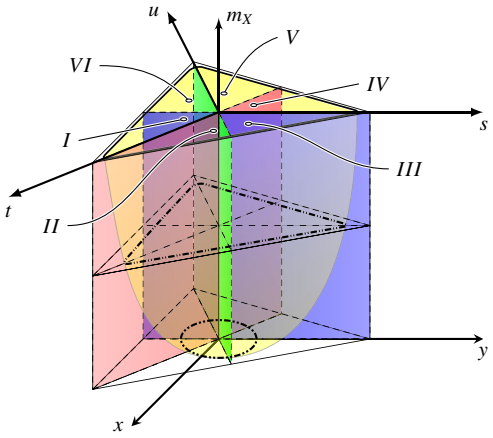

FIG. 2 (color online). The schematic figure of the Dalitz prism. To get the Dalitz plot corresponding to a certain value of m_X one simply slices the Dalitz prism parallel to the xy plane at that particular value of m_X. The sextants of the Dalitz plots are subsumed into the six analogously labeled identical wedges of the Dalitz prism.

state X can be a resonance such as J/ψ or even e^+e^-, the final state N can be K^+K^-, $\pi^0K^+K^-$, $K^+K^-\eta$, $\omega\pi^0$, $p\bar{p}$, $p\bar{p}\pi^0$ and $n\bar{n}$. In such a case, the value of p_1^2 is not fixed even though for a given initial state configuration $p_X^2 = m_X^2$ is fixed at a constant value. One can also vary both $p_1^2 = m_1^2$ as well as m_X^2, such as when $X = e^+e^-$. For such cases we can again construct a prism whose z axis denotes $M^2 = m_X^2 + m_1^2 + m_2^2 + m_3^2$, such that the M^2 value can vary even if either m_X^2 or m_1^2 or both vary. The xy plane of the prism is spanned by the various values of s, t and u as before. When p_X, p_2 and p_3 are precisely measured, p_1 need not be measured, as $p_1 = p_X - p_2 - p_3$ from conservation of 4-momentum. Similarly, one need not measure p_X when p_1, p_2, and p_3 are precisely measured. It is easy to see that our approach is unaffected by any initial and final state radiation effects, since these effects can always be included as part of contributions to N. If we know the total initial 4-momentum of X (such as e^+e^-, say), and the two 4-momenta of π^+ and π^-, the event can be registered in the Dalitz prism. All the events allowed by conservation of energy and 3-momentum populate the interior of this general prism. Even though, slices of this prism do not give any Dalitz plot, because the recorded events are no longer just three-body decays, we shall nevertheless refer to it as Dalitz prism as well. The Dalitz prism can, therefore, subsume all cases where m_X^2 and/or m_1^2 varies. The distribution of events on the z axis is irrelevant for our discussion. One only needs to take a projection of all the events recorded in this unified prism onto its base and look for asymmetry in the resulting triangular plot. Usage of the Dalitz prism as explained above, thus liberates the methods discussed here from the shackles of branching fractions and thereby enhances the sensitivity of the search for violations of Bose and CPT symmetries. It is noteworthy that the use

of the Dalitz prism in the study of *CP* violation [87] can also be advantageous. The Dalitz prism in its generalized form is hence a very significant tool to study the fundamental symmetries of nature using multibody decays.

The *CPT* violation proposed in this paper deals with strong and electromagnetic interactions. This is fundamentally different from the kind of *CPT* violation already studied in K, D, and B meson mixing. Hence, it is not possible to make a meaningful comparison of the *CPT* violation proposed here and the studies in K, D, and B meson mixing. The sensitivity of our method increases with the increase in number of events in the Dalitz prism which can in principle combine many modes and continuum data from various experiments taking systematic errors into account. The sensitivity to the asymmetry that can be probed will naively be inversely proportional to the square root of the number of events that can be combined in the Dalitz prism.

We have thus shown how Bose symmetry and *CPT* symmetry violations can lead to asymmetries in the Dalitz plot. We have also developed the new concept of Dalitz prism which can be used to gather the huge statistics needed for an effective search for the Bose and *CPT* symmetry violations from studies of meson decays. Since both Bose symmetry and *CPT* symmetry are of fundamental importance to foundations of modern field theory, it is worthwhile to check their validity in the realm of unstable and composite particles such as the mesons.

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