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Fluctuation of strongly interacting matter in the Polyakov–Nambu–Jona-Lasinio model in a finite volume

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We estimate the susceptibilities of conserved charges for two-flavor strongly interacting matter with varying system sizes, using the Polyakov loop enhanced Nambu–Jona-Lasinio model. The susceptibilities for vanishing baryon densities are found to show a scaling with the system volume in the hadronic as well as partonic phase. This scaling breaks down for a temperature range of about 30–50 MeV around the crossover region. Simultaneous measurements of the various susceptibilities may, thus, indicate how close to the crossover region the freeze-out occurs for the fireball created in heavy-ion collision experiments.

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Strongly interacting matter under extreme conditions of temperature and density is expected to show a rich phase structure. In the early Universe, a few microseconds after the big bang, when the temperature was extremely high, exotic states, namely, quarks and gluons, may have been prevalent [1]. On the other hand, inside the core of a compact star, where the baryon matter density is extremely high, various exotic phases like color superconductor, color superfluid, etc., may be present [2]. Experiments with heavy ions colliding with each other or with a target at the facilities at CERN (France/Switzerland), BNL (USA), and GSI (Germany) are continuing the search for such exotic states of matter in the laboratory.

The matter formed due to the energy deposition of the colliding particles obviously has a finite volume. It is, therefore, imperative to have a clear understanding of the finite size effects to fully contemplate the thermodynamic phases that may be created in the experiments. These effects depend on the size of the colliding nuclei, the center of mass energy ($\sqrt{s}$), and the centrality of collisions. There have been many efforts to estimate the system size at freeze-out for different $\sqrt{s}$ and different centralities. A study using the measurement of HBT radii [3] indicates that the freeze-out volume increases as the $\sqrt{s}$ increases. In the same work, the freeze-out volume has been estimated to be 2000 to 3000 fm^3. On the other hand, in Ref. [4] the volume of homogeneity has been calculated using the Ultrarelativistic Quantum Molecular Dynamics model [5] and compared with the experimentally available results. The $\sqrt{s}$ considered was in the range of 62.4 to 2760 GeV for lead-lead collisions at different centralities. The system volume has been found to vary from 50 to 250 fm^3. The effect of colliding particles and $\sqrt{s}$ has been further analyzed by the ALICE Collaboration in [6]. Given that these are the volumes at the time of freeze out, one may expect an even smaller system size at the initial equilibration time [7,8].

The importance of finite size effects in the thermodynamics of strong interaction may be brought forward with the help of finite size scaling analysis [9]. In the context of heavy-ion collisions, such a possible analysis has been discussed in the literature (see, e.g., [10–12]).

Other theoretical studies of finite volume effects have been performed in various contexts. In Ref. [13], the effective degrees of freedom have been found to be reduced due to a finite volume using a noninteracting bag model. The effect of a finite volume has been studied also with a two-model equation of state, and it has been found that the critical temperature loses its sharpness [14]. A few first-principles studies of pure gluon theory on space-time lattices were performed, showing the possibility of significant finite size effects [15,16]. The meson properties show a significant volume dependence as found in Refs. [17,18]. In the context of chiral perturbation theory, the implications of finite system size have been discussed [19,20]. There are also studies with four-Fermi-type interactions, like the Nambu–Jona-Lasinio (NJL) [21] models [10,22,23], linear sigma models [11,24,25], and Gross-Neveu models [26]. While in Ref. [24], the scaling behavior of the chiral phase transition for finite and infinite volumes has been studied, the character of phase diagram has been studied in Refs. [10,11,25,26]. In Refs. [22,23], the authors have studied the chiral properties as a function of the radius of a

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finite droplet of quark matter. The stability of such a droplet in the context of strangelet formation within the NJL model has been addressed in Ref. [27]. Size-dependent effects of difermion states within the two-dimensional NJL model has been studied in Ref. [28] and that of magnetic field is discussed in Ref. [29]. Recently, in a $(1+1)$-dimensional NJL model, the induction of a charged pion condensation phenomenon in dense baryonic matter due to finite volume effects has been studied in [30]. Recently, some of us have studied the thermodynamic properties of strongly interacting matter in a finite volume using the Polyakov–Nambu–Jona-Lasinio (PNJL) model [31]. It has been shown there that the critical temperature for the crossover transition at zero baryon density decreases as the volume decreases. Furthermore, at low volume the critical end point is pushed towards the higher μ and lower T domain. At $R = 2$ fm, it was found that the critical end point (CEP) vanishes, and the whole phase diagram becomes a crossover. The possible chiral symmetry restoration in a color confined state has also been discussed.

Though various thermodynamic properties have been studied to some extent in finite size systems, not much has been done to estimate the fluctuations occurring in finite volumes. On the lattice, the Polyakov loop susceptibility has been calculated for a finite volume [32]. A similar work has been done in the PNJL model using Monte Carlo simulation [33] and also in the quark-meson model using a renormalization group approach [34]. Fluctuations of conserved quantum numbers are related to the respective susceptibilities via the fluctuation-dissipation theorem. For a two-flavor strongly interacting system, one has the quark number susceptibility and isospin number susceptibility, etc. These fluctuations are sensitive indicators of the transition from hadronic matter to partonic state. Also, the existence of the CEP may be signaled by the diverging behavior of fluctuations. Here we report our calculations of quark and isospin number susceptibilities of strongly interacting matter using the PNJL model up to sixth order. The report is organized as follows. First, we give a very brief description of the PNJL model and the necessary methodology. Thereafter, we present the results for the various susceptibilities. Finally, we summarize and conclude.

The PNJL model used here is based on a series of works [21,31,35–47]. For some recent progress on this model, see, e.g., [44,48–57]. For a detailed overview, see, e.g., [58] and references therein. The PNJL model for two flavors is described by the Lagrangian,

$$\mathcal{L} = \sum_{f=u,d} \bar{\psi}_f \gamma_\mu i D^\mu \psi_f - \sum_f m_f \bar{\psi}_f \psi_f + \sum_f \mu_f \gamma_0 \bar{\psi}_f \psi_f + \frac{g_S}{2} \sum_{a=1,2,3} [(\bar{\psi}\tau^a\psi)^2 + (\bar{\psi} i\gamma_5 \tau^a \psi)^2] - \mathcal{U}'(\Phi[A], \bar{\Phi}[A], T), \tag{1}$$

where the Polyakov loop potential $\mathcal{U}'(\Phi[A], \bar{\Phi}[A], T)$ can be expressed as

$$\frac{\mathcal{U}'(\Phi[A], \bar{\Phi}[A], T)}{T^4} = \frac{\mathcal{U}(\Phi[A], \bar{\Phi}[A], T)}{T^4} - \kappa \ln(J[\Phi, \bar{\Phi}]). \tag{2}$$

Here, $\mathcal{U}(\Phi, \bar{\Phi}, T)$ is a Landau-Ginsburg-type potential as given in Ref. [38],

$$\frac{\mathcal{U}(\Phi, \bar{\Phi}, T)}{T^4} = -\frac{b_2(T)}{2} \bar{\Phi}\Phi - \frac{b_3}{6}(\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4}(\bar{\Phi}\Phi)^2, \tag{3}$$

where

$$b_2(T) = a_0 + a_1 \left(\frac{T_0}{T}\right) + a_2 \left(\frac{T_0}{T}\right)^2 + a_3 \left(\frac{T_0}{T}\right)^3, \tag{4}$$

b_3 and b_4 being constants. The second term in Eq. (2) is the Vandermonde term,

$$J[\Phi, \bar{\Phi}] = (27/24\pi^2)[1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi\bar{\Phi})^2].$$

The parameters a_i, b_i were fitted from lattice results of pure gauge theory. The set of values chosen here is

$$a_0 = 6.75, \quad a_1 = -1.95, \quad a_2 = 2.625, \quad a_3 = -7.44,$$
$$b_3 = 0.75, \quad b_4 = 7.5, \quad T_0 = 190\,\mathrm{MeV}, \quad \kappa = 0.2.$$

To incorporate the effect of a finite volume, we use a nonzero low momentum cutoff $p_{\min} = \pi/R = \lambda$, where R is the lateral size of a cubic volume $V = R^3$. In principle, one should sum over discrete momentum values, but for simplification, we integrate over continuous values of momentum. Also, we neglect surface and curvature effects. Other parameters of the model were not modified.

We note here that in the NJL model, discussions of using a lower momentum cutoff exists in the literature (see [59,60] and references therein). The motivation for introducing this IR cutoff there has been to mimic confining effects of strong interaction which helps to remove spurious poles in the quark loop diagrams so that unphysical decay of hadrons to quarks does not take place. Since in the PNJL model such unphysical decays are restricted due to the vanishing of the Polyakov loop for low temperatures [38,61], no IR cutoff is necessary in the PNJL model. However, for the two-flavor PNJL model, the unphysical decay does not completely vanish, as pointed out in Ref. [62] where a very small but nonzero sigma meson decay is observed at low temperatures. On the other hand, for $2+1$ flavor, the σ meson becomes a true bound state for small temperatures [63].

Our starting point is the thermodynamic potential given by

$$\Omega = \mathcal{U}'[\Phi,\bar{\Phi},T] + 2g_S\sum_{f=u,d}\sigma_f^2 - 6\sum_f\int_\lambda^\Lambda \frac{d^3p}{(2\pi)^3}E_{p_f}\Theta(\Lambda-|\vec{p}|)$$
$$-2\sum_f T\int_\lambda^\infty \frac{d^3p}{(2\pi)^3}\ln\left[1+3\left(\Phi+\bar{\Phi}\exp\left(\frac{-(E_{p_f}-\mu_f)}{T}\right)\right)\exp\left(\frac{-(E_{p_f}-\mu_f)}{T}\right)+\exp\left(\frac{-3(E_{p_f}-\mu_f)}{T}\right)\right]$$
$$-2\sum_f T\int_\lambda^\infty \frac{d^3p}{(2\pi)^3}\ln\left[1+3\left(\bar{\Phi}+\Phi\exp\left(\frac{-(E_{p_f}+\mu_f)}{T}\right)\right)\exp\left(\frac{-(E_{p_f}+\mu_f)}{T}\right)+\exp\left(\frac{-3(E_{p_f}+\mu_f)}{T}\right)\right], \quad (5)$$

where $E_{p_f} = \sqrt{p^2+M_f^2}$ is the single quasiparticle energy. In the above expression, σ_f is given as

$$\sigma_f = \langle\bar{\psi}_f\psi_f\rangle = -\frac{3M_f}{\pi^2}\int_\lambda^\Lambda \frac{p^2}{\sqrt{p^2+M_f{}^2}}dp. \quad (6)$$

We first obtain the mean fields σ, Φ, and $\bar{\Phi}$ from the extremization conditions: $\frac{\partial\Omega}{\partial\sigma}=0, \frac{\partial\Omega}{\partial\Phi}=0, \frac{\partial\Omega}{\partial\bar{\Phi}}=0$. The field values so obtained are then put back into Ω to obtain the thermodynamic potential, which is then used to obtain various thermodynamic quantities, some of which have been reported by us in Ref. [31]. For example, the pressure in the finite volume system is given by

$$P(T,\mu_q,\mu_I) = -\frac{\partial(\Omega(T,\mu_q,\mu_I)V)}{\partial V}, \quad (7)$$

where T is the temperature, and μ_q and μ_I are the quark and isospin chemical potentials, respectively. The variation of scaled pressure with T/T_c is shown in Fig. 1. The critical temperature T_c is dependent on the system size. We have considered different system sizes corresponding to $R=2$ fm, $R=2.5$ fm, $R=4$ fm, and infinite volume. The corresponding values of T_c are 167, 171, 183, and 186 MeV, respectively.

We now discuss the various susceptibilities of quark number and isospin number. These are defined as

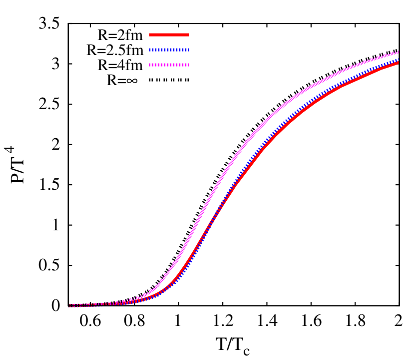

FIG. 1 (color online). Variation of pressure with temperature for different system sizes.

$$c_n(T) = \frac{1}{n!}\frac{\partial^n(\Omega(T,\mu_q,\mu_I)/T^4)}{\partial(\frac{\mu_X}{T})^n}\Bigg|_{\mu_X=0}, \quad (8)$$

where $\mu_X=\mu_q$ or μ_I. For an expansion around $\mu_X=0$, the odd order terms vanish due to CP symmetry. Many of these susceptibilities have been measured for infinite volume systems in first-principles QCD calculations on the lattice [64–72] as well as hard thermal loop calculations [73–82]. At the same time, various QCD-inspired models have also made suitable estimates of these fluctuations for infinite systems (see, e.g., [40,41,58,83–93]). Here we present the first computation of finite size effects on these fluctuations.

For each system volume considered, we have calculated Ω at chemical potentials spaced by 0.1 MeV at a given temperature. These have been fitted it to an eighth order polynomial in μ_X using the GNUPLOT program. We have chosen the maximum range of μ_X to be 200 MeV. From the fit, we have extracted the coefficients c_2, c_4, and c_6 both for quark number and isospin number susceptibilities. This procedure has been repeated for different values of temperature.

The variation of quark number susceptibilities with T/T_c is shown in Fig. 2. The general features for these susceptibilities in finite volumes are quite similar to that for infinite volume. However, quantitatively we observe significant volume dependence. With an increase in system size, there is an enhancement of all the susceptibilities. For the isospin number susceptibilities shown in Fig. 3, we find almost identical behavior. It may be noted that the most significant finite size effects are seen in the higher order susceptibilities close to the crossover region. Given that the detectors in QGP search experiments are expected to observe the system frozen close to the crossover region, one may find an estimate of the system volume from the measurement of various higher order fluctuations.

Alternatively, it is important to realize that for a comparison of fluctuations calculated theoretically with that measured experimentally, one needs to be confident about the measured system volumes since measuring the system size is quite a difficult task in the experiments, and one usually resorts to consider ratios of fluctuations to eliminate the volume factor [66]. However, this assumption is valid when interactions are small and the volume factor

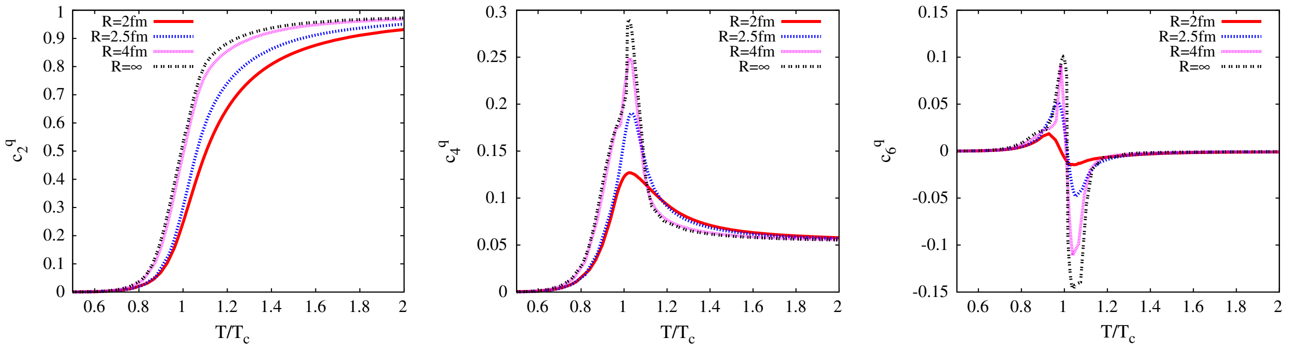

FIG. 2 (color online). Variation of quark number susceptibilities with temperature for different system sizes.

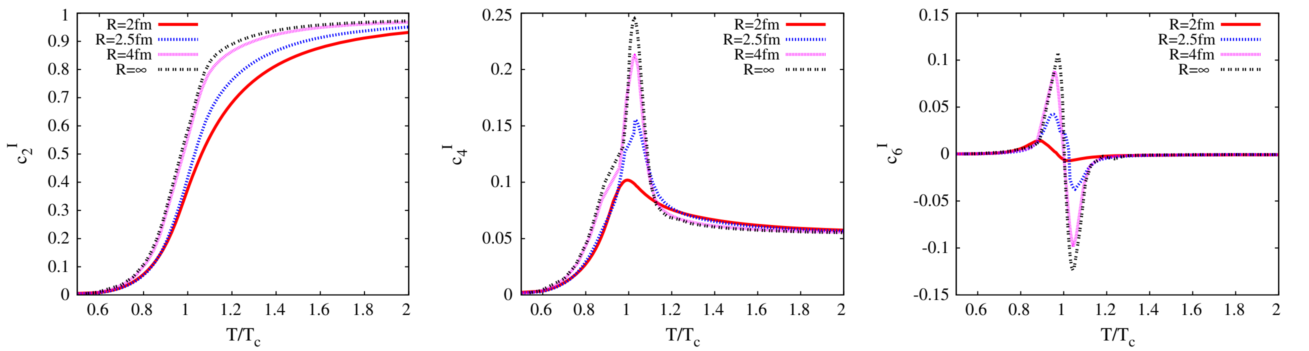

FIG. 3 (color online). Variation of isospin number susceptibilities with temperature for different system sizes.

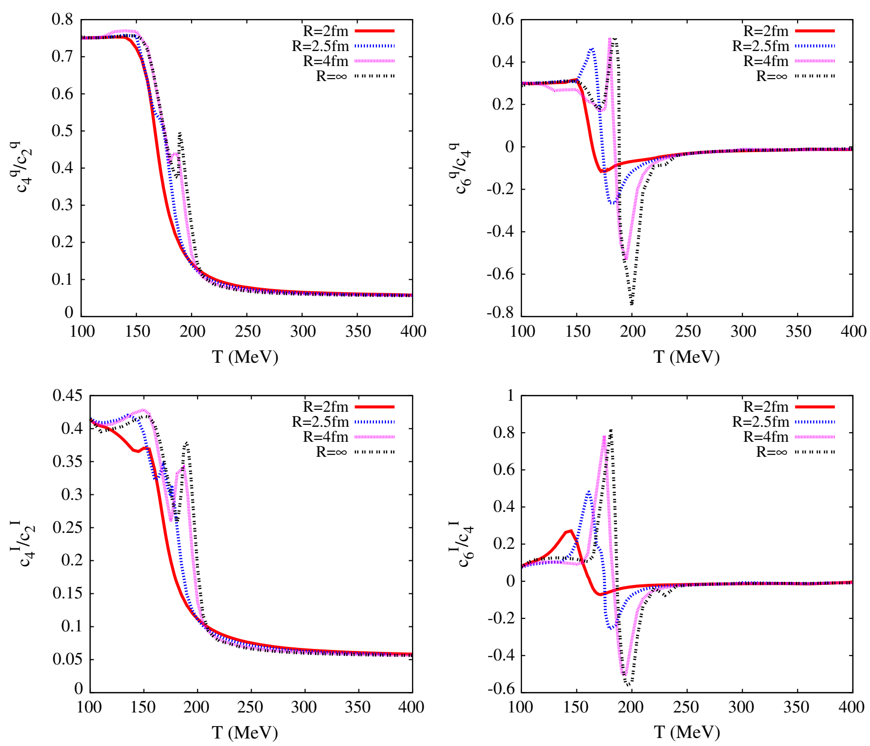

FIG. 4 (color online). Variation of ratios of fluctuations with temperature for different system sizes.

scales out. Therefore, in the purely hadronic or partonic phases, one may observe such a scaling of the fluctuations with system size. However, close to the crossover region such an assumption may not hold as large-scale fluctuations are dominant, and the system deviates from a stable thermodynamic phase. Now that we have the actual calculations of system size effects, we can easily check how the ratio of fluctuations behaves. For this purpose, we present the ratios c_4/c_2 (kurtosis) and c_6/c_4 for both the quark number and isospin number susceptibilities. In this case, we obviously need to plot the variation with temperature rather than with T/T_c. The variations are shown in Fig. 4. We observe that for low and high temperatures, the ratios of fluctuations show the expected scaling with the system volume, while in the crossover region there is significant volume dependence. Thus, if the system created in heavy-ion experiments freeze out much below T_c, the ratios of different susceptibilities would show the corresponding values for the hadronic phase. The amount of deviation of these ratios from the hadronic phase results would indicate the closeness of the system to the crossover region.

To summarize, we have studied the fluctuations of strongly interacting matter in a finite volume using the PNJL model. The susceptibilities in the quark number and isospin number are obtained up to sixth order for different system sizes. We have found a significant volume dependence in these quantities, which may be useful in analyzing the experimental data and obtain the size of the fireball formed in the heavy-ion collision experiments. The volume dependence shows an expected scaling behavior in the hadronic and partonic phases. In the crossover region, the system size scaling breaks down and may be used to estimate the closeness of the created fireball to the crossover region. Given that all our present analysis is at zero density, the results are suitable for analyzing LHC data.

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