Complex linear effective theory and supersymmetry breaking vacua

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We calculate the low-energy effective action of massless and massive complex linear superfields coupled to a massive U(1) vector multiplet. Our calculations include superspace higher-derivative corrections and therefore go beyond previous results. Among the superspace higher derivatives, we find that terms that lead to a deformation of the auxiliary field potential and may break supersymmetry are also generated. We show that the supersymmetry breaking vacua can only be trusted if there exists a hierarchy between the higher-order terms. A renormalization group analysis shows that generically a hierarchy is not generated by the quantum corrections.

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I. INTRODUCTION

Supersymmetric theories (see, for example, Refs. [1,2]) are candidates for describing physics beyond the electroweak scale. Despite their many virtues, the observed elementary particle spectrum signals that we live in a vacuum in which supersymmetry has to be broken, thus leading to a lift of the mass degeneracy of the fermionic and bosonic states. In a quantum field theory, to understand the true vacuum structure, one has to take into account the quantum corrections, which in principle will include all the terms allowed by the symmetries.

For chiral models, the effective Kähler potential and superpotential are well known [3–6]. Effective theories will also include higher-dimension operators that contain superspace higher derivatives. The possible effect of these superspace higher derivatives on the vacuum structure was initially investigated in Ref. [7], but no conventional supersymmetry breaking was found. Indeed, as has been shown in Refs. [8–10], even though new branches exist, they have to be sustained by nontrivial background fluxes. The specific higher-derivative superspace operators used in Refs. [7–10] were shown to be generated by perturbation theory in the work of Refs. [4,5,11], but they were also related to effective actions originating from a more fundamental theory [12–16]. Note that models of this sort have also been used to describe supersymmetric Skyrmions [17–20]. In Ref. [21], linear and complex linear multiplets have also been studied, and the resulting theories have a new broken branch admitting nonlinear supersymmetry. In these theories, there are examples in which no background flux is needed to sustain the broken branch.

Another important aspect of supersymmetric theories is the various existing supermultiplets. The most common in use are the chiral multiplet and the gauge multiplet. The gauge multiplet is somehow unique. The same is not true for the chiral multiplet; there exist other supermultiplets with the same on-shell field content. These superfields are classically equivalent but with different quantum properties. In fact, the role of the *variant* [22,23] scalar multiplets has been little appreciated. It is possible that matter fields are not accommodated into a chiral multiplet, but rather into a complex linear multiplet, for example, and the same goes for the supersymmetry breaking sector.

The quantum properties of complex linear supermultiplets have been studied before [24–27], and it was shown that the general quantization procedure is rather involved. If one solves the constraints by introducing prepotentials, one is faced with a theory with a gauge symmetry that needs to be fixed. In the Batalin-Vilkovisky formalism, this leads to an infinite tower of ghosts. However, as was shown in Ref. [25], if the complex linear superfield prepotentials appear only through their field strengths (i.e., the superfields themselves), the ghosts decouple, and one may ignore many of the complications originating in the infinite ghost sector. In Ref. [25], the effective Kähler potential of the sigma model defined in terms of chiral superfields or its dual defined in terms of complex linear superfields was shown to remain dual at one loop. In this work, we go beyond the known results about the effective Kähler potential, and we calculate all the one-loop corrections that also include superspace higher derivatives.

Superspace higher derivatives of complex linear multiplets have an intriguing effect: they deform the auxiliary field potential in such a way that the auxiliary field equations of motion have more than one solution, and the new solutions lead to vacua in which supersymmetry is spontaneously broken. Recently, the structures of various theories with this possibility were presented [21]. The standard property is the auxiliary structure

$$\mathcal{L}_{\rm aux} = -F\bar{F} + \frac{1}{2f^2}F^2\bar{F}^2, \qquad (1)$$

leading to new nonsupersymmetric vacua when the auxiliary field F is integrated out. Indeed, on top of the standard solution

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$$F = 0 \tag{2}$$

now exists a second solution to the equations of motion of F,

$$F\bar{F} = f^2, \tag{3}$$

which implies supersymmetry breaking. Moreover, in a generic study on supersymmetric effective theories, one should take into account these terms since they deform the auxiliary field potential and thus have an indispensable effect on the scalar potential as well. It is then clear that an effective theory should not be restricted to the evaluation of the form of the effective Kähler potential only but also to the evaluation of the superspace higher-derivative operators, which are always generated.

Our work is organized as follows. In the next section, we review the basic formalism about classical and quantum superfields and present the scalar multiplets variant pictures. In Sec. III, we work with a massless complex linear multiplet coupled to a massive $U(1)$, which we integrate out, and calculate the low-energy effective theory and the deformation in the auxiliary field potential. In Sec. IV, we work with a massive complex linear multiplet coupled to a massive $U(1)$, and by integrating out the massive sector, we calculate the low energy effective theory and study the vacuum structure and the flow of the supersymmetry breaking operators under the renormalization group. We conclude with a short discussion in Sec. V.

II. SHORT REVIEW OF SUPERFIELD METHODS

Here, we review known results in four-dimensional, $\mathcal{N} = 1$ superspace concerning some classical and quantum properties, which we will use throughout our work. For our conventions, one may consult Ref. [1].

A. Scalar multiplets and hypermultiplets

In supersymmetric field theories, there exist different ways of introducing physical scalar fields into supermultiplets. These different ways and their equivalences can be understood in terms of dualities between the various superfields. For example, a chiral superfield is defined by the condition

$$\bar{D}_{\dot{\alpha}}\Phi = 0 \tag{4}$$

and has bosonic components

$$\Phi| = z, \tag{5}$$

$$D^2\Phi| = G. \tag{6}$$

A free chiral superfield is described by the Lagrangian

$$\mathcal{L} = \int d^4\theta\bar{\Phi}\Phi, \tag{7}$$

which leads to the superspace equations of motion

$$\bar{D}^2\bar{\Phi} = 0. \tag{8}$$

On the other hand, a complex linear superfield [22] is defined by the condition

$$\bar{D}^2\Sigma = 0, \tag{9}$$

with superspace Lagrangian

$$\mathcal{L} = -\int d^4\theta\bar{\Sigma}\Sigma, \tag{10}$$

for the free theory. The bosonic components of the complex linear are

$$\Sigma| = A, \tag{11}$$

$$D^2\Sigma| = F, \tag{12}$$

$$\bar{D}_{\dot{\beta}}D_{\alpha}\Sigma| = P_{\alpha\dot{\beta}}. \tag{13}$$

Note that this multiplet can be consistently coupled to supersymmetric Yang–Mills theories [25,26].

The equivalence between the two can be understood by using the Lagrangian

$$\mathcal{L}_{\text{dual}} = -\int d^4\theta\bar{\Sigma}\Sigma + \int d^4\theta\Sigma\Phi + \int d^4\theta\bar{\Sigma}\,\bar{\Phi} \tag{14}$$

for a chiral superfield Φ but an unconstrained Σ. Then, integrating out Φ gives a complex linear condition (9) on Σ, and the dual Lagrangian (14) becomes (10), but integrating out Σ, we find

$$\bar{\Sigma} = \Phi, \tag{15}$$

and after we plug back into Eq. (14), we recover the Lagrangian (7). For this reason, the chiral multiplet is commonly used: it is classically equivalent to the linear super multiplet, and it is simpler to deal with. The propagators of the chiral and the complex linear superfields are given by

$$\bar{\Phi}\Phi\colon\ \frac{D^2\bar{D}^2}{p^2}\delta^4(\theta-\theta'), \tag{16}$$

$$\bar{\Sigma}\Sigma\colon\ \left[1+\frac{\bar{D}^2D^2}{p^2}\right]\delta^4(\theta-\theta'). \tag{17}$$

Because of the structure of the complex linear multiplet, it is only possible that it acquires a mass in tandem with a chiral multiplet [23]. This is done by modifying the complex linear constraint to

$$\bar{D}^2\Sigma = m\Phi. \tag{18}$$

The way to understand the constraint is by dualizing the massive chiral superfield Lagrangian

$$\mathcal{L} = \int d^4\theta\bar{\Phi}\Phi - \frac{m}{2}\int d^2\theta\Phi^2 - \frac{m}{2}\int d^2\bar{\theta}\bar{\Phi}^2. \tag{19}$$

For an unconstrained superfield Σ and a chiral Φ, we have

$$\mathcal{L}_{\rm dual} = -\int d^4\theta\bar{\Sigma}\Sigma + \int d^4\theta\Sigma\Phi + \int d^4\theta\bar{\Sigma}\,\bar{\Phi} - \frac{m}{2}\int d^2\theta\Phi^2 - \frac{m}{2}\int d^2\bar{\theta}\bar{\Phi}^2. \tag{20}$$

Then, by the equations of motion for Σ, we go to Eq. (19). But the equations of motion for Φ give (18)

$$\bar{D}^2\Sigma = m\Phi.$$

If on top of these, we use the Σ equation of motion $\bar{\Sigma} = \Phi$ and find

$$\bar{D}^2\Sigma = m\bar{\Sigma}, \tag{21}$$

which is the equation of motion for a massive chiral superfield; in other words, this is a chiral superfield in the disguise of a complex linear [2]. Thus, it is not possible to have a pure massive complex linear because it will always be, in fact, a relabeling of the massive chiral multiplet. It is then clear that the condition (18) indeed represents a massive complex linear, but it is not possible to solve this condition in terms of Σ alone, as we did before, thus completely disposing of the chiral. To have a truly massive complex linear superfield, the chiral one has to stay in the picture as well. The consistent way to write the massive Lagrangian is

$$\mathcal{L} = -\int d^4\theta\bar{\Sigma}\Sigma + \int d^4\theta\bar{\Phi}\Phi, \tag{22}$$

with the condition (18) for Σ. The mass terms are not manifest in Eq. (22), but they are of course there in the component form [23]. Imposing the condition (18) via a Lagrange multiplier [28] also makes the mass terms appear in superspace, and then one can find the standard propagators for the massive theory,

$$\bar{\Phi}\Phi\colon\ \frac{D^2\bar{D}^2}{p^2+m^2}\delta^4(\theta-\theta'), \tag{23}$$

$$\bar{\Sigma}\Sigma\colon\ \left[1+\frac{\bar{D}^2D^2}{p^2+m^2}\right]\delta^4(\theta-\theta'), \tag{24}$$

and the extra propagators due to the mass terms,

$$\bar{\Sigma}\Phi\colon\ \frac{m\bar{D}^2}{p^2+m^2}\delta^4(\theta-\theta'), \tag{25}$$

$$\bar{\Phi}\Sigma\colon\ \frac{mD^2}{p^2+m^2}\delta^4(\theta-\theta'). \tag{26}$$

Note that this is, in fact, an $\mathcal{N}=2$ hypermultiplet [28,29].

Finally, it is interesting what happens to a supersymmetry breaking theory with a linear superpotential under the duality. We start from

$$\mathcal{L} = \int d^4\theta\bar{\Phi}\Phi - f\int d^2\theta\Phi - f\int d^2\bar{\theta}\,\bar{\Phi}, \tag{27}$$

which leads to the equations of motion for Φ,

$$\bar{D}^2\bar{\Phi} = f, \tag{28}$$

and for the auxiliary field specifically,

$$G = f, \tag{29}$$

signaling supersymmetry breaking. On the other hand, it is again possible to dualize this Lagrangian. For an unconstrained superfield Σ and a chiral Φ, we have

$$\mathcal{L}_{\rm dual} = -\int d^4\theta\bar{\Sigma}\Sigma + \int d^4\theta\Sigma\Phi + \int d^4\theta\bar{\Sigma}\,\bar{\Phi} - f\int d^2\theta\Phi - f\int d^2\bar{\theta}\,\bar{\Phi}\,. \tag{30}$$

Then, by the equations of motion for Σ, we go to Eq. (27). But the equations of motion for Φ give [30]

$$\bar{D}^2\Sigma = f, \tag{31}$$

and the duality Lagrangian (30) becomes

$$\mathcal{L} = -\int d^4\theta\bar{\Sigma}\Sigma. \tag{32}$$

The Lagrangian (32) with the constraint (31) implies supersymmetry breaking. Nevertheless, if one turns to components, all auxiliary fields get a vanishing vacuum expectation value. This peculiarity is resolved by turning back to Eq. (30). Combining the equations of motion for Φ, which are Eq. (31), with the Σ equations of motion

$$\bar{\Sigma} = \Phi, \tag{33}$$

one can see again that Eq. (31) is, in fact, a relabeling of the chiral superfield supersymmetry breaking (28). Thus, the constraint (31) already contains the information of supersymmetry breaking originating by an underlying chiral superfield of which the auxiliary field G indeed gets a vacuum expectation value (vev) from Eq. (29).

Mechanisms to break supersymmetry by a complex linear have recently been found, in which, indeed, the auxiliary field of the complex linear superfield gets a vev and signals supersymmetry breaking. Consider the Lagrangian [21]

$$\mathcal{L} = -\int d^4\theta \Sigma\bar{\Sigma} + \frac{1}{8f^2}\int d^4\theta D^\alpha \Sigma D_\alpha \Sigma \bar{D}^{\dot{\alpha}}\bar{\Sigma}\bar{D}_{\dot{\alpha}}\bar{\Sigma}. \tag{34}$$

The scalar auxiliary sector of this theory reads

$$\mathcal{L}_F = -F\bar{F} + \frac{1}{2f^2}F^2\bar{F}^2, \tag{35}$$

leading to the equations of motion

$$F(F\bar{F} - f^2) = 0. \tag{36}$$

The equation (40) on top of the supersymmetric solution

$$F = 0 \tag{37}$$

has a second solution,

$$F\bar{F} = f^2, \tag{38}$$

which signals supersymmetry breakdown. It has been shown that for models of this sort [21] the goldstino will be one of the previously auxiliary fermion fields of the theory, which propagates only in the broken branch. Its supersymmetry transformation is

$$\delta\lambda \sim \langle F\rangle \epsilon. \tag{39}$$

One may also assume the existence of superspace higher derivatives of higher dimension on top of Eq. (34), for example,

$$\mathcal{L}_F = -F\bar{F} + \frac{1}{2f^2}F^2\bar{F}^2 + \frac{1}{f'^4}F^3\bar{F}^3 + \cdots, \tag{40}$$

and investigate if the solution (38) is still valid even though it has been found by ignoring the $1/f'$ term. There are two limiting cases in which conclusions can be drawn. First, consider

$$\frac{1}{f} \lesssim \frac{1}{f'} \leftrightarrow \frac{f}{f'} \gtrsim 1. \tag{41}$$

In this case, Eq. (40) becomes

$$\mathcal{L}_F = -\frac{1}{2}f^2 + f^2\left(\frac{f}{f'}\right)^4 + \cdots, \tag{42}$$

which means that it is inconsistent to consider the solution (38) since the terms we ignored are larger than the terms we took into account in order to find the solution. The other limiting case is the existence of a large hierarchy between the two scales

$$\frac{1}{f} \gg \frac{1}{f'} \leftrightarrow \frac{f}{f'} \ll 1. \tag{43}$$

Then, Eq. (40) becomes

$$\mathcal{L}_F = -\frac{1}{2}f^2 + f^2\left(\frac{f}{f'}\right)^4 + \cdots, \tag{44}$$

and thus the higher-order terms are highly suppressed and may be safely ignored. In fact, in a generic effective theory, higher-order terms as in Eq. (40) will also be generated, and the verification or not of the hierarchy (43) is a clear signal for the existence or not of the new vacua. Of course, depending on the mechanism responsible for the generation of these superspace higher derivatives, one will in principle find different results.

B. Gauge multiplets and gauging

A massive $U(1)$ vector multiplet is described by the Lagrangian

$$\mathcal{L} = \int d^2\theta W^\alpha W_\alpha + \text{H.c.} + \frac{1}{2}M^2\int d^4\theta V^2, \tag{45}$$

with $V = \bar{V}$ and

$$W_\alpha = i\bar{D}^2 D_\alpha V. \tag{46}$$

The bosonic components of this multiplet are

$$V| = C, \tag{47}$$

$$D^2 V| = N, \tag{48}$$

$$\frac{1}{2}[\bar{D}_{\dot{\alpha}}, D_\alpha]V| = A_{\alpha\dot{\alpha}}, \tag{49}$$

$$\frac{1}{2}D^\alpha \bar{D}^2 D_\alpha V| = \mathrm{D}. \tag{50}$$

One may imagine that the massive $U(1)$ in Eq. (45) has acquired a mass via a gauge-invariant mechanism, for example, in interaction with a real linear multiplet L, as described in Refs. [31,32]. In that case, we have the Lagrangian

$$\mathcal{L} = \int d^2\theta W^\alpha W_\alpha + \text{H.c.} - \frac{1}{2}\int d^4\theta L^2 + M\int d^4\theta LV, \tag{51}$$

which is invariant under

$$V \to V + i\bar{\Lambda} - i\Lambda, \tag{52}$$

due to

$$\int d^4\theta L\Lambda = \int d^4\theta L\bar{\Lambda} = 0, \tag{53}$$

for a chiral superfield Λ and real linear L. Note that by dimensional analysis $[V] = 0$ and $[L] = 1$, and thus, indeed, $[M] = 1$. One can then rewrite Eq. (51) as

$$\mathcal{L} = \int d^2\theta W^\alpha W_\alpha + \text{H.c.} - \frac{1}{2}\int d^4\theta L^2 + M\int d^4\theta LV + \int d^4\theta L(S + \bar{S}), \tag{54}$$

where now L is real but otherwise unconstrained and S is a chiral multiplet that renders L linear on shell. Since now L is unconstrained, we can integrate it out to find

$$\mathcal{L} = \int d^2\theta W^\alpha W_\alpha + \text{H.c.} + \frac{1}{2}\int d^4\theta (MV + S + \bar{S})^2, \tag{55}$$

which, after a redefinition of V of the form (52) with $\Lambda = iS$, becomes Eq. (45). Here, the vector superfield has *eaten* the real linear and became massive.

Since in the Lagrangian (45) we have a massive $U(1)$, it is possible to invert the propagator without gauge fixing. We will nevertheless introduce a gauge-fixing term parametrized by ξ,

$$\mathcal{L}_{\text{GF}} = -\xi\int d^4\theta D^2 V \bar{D}^2 V, \tag{56}$$

which gives the final Lagrangian

$$\mathcal{L} = \int d^2\theta W^\alpha W_\alpha + \text{H.c.} - \xi\int d^4\theta D^2 V\bar{D}^2 V + \frac{1}{2}M^2\int d^4\theta V^2, \tag{57}$$

and the propagator of the massive vector superfield is

$$VV\colon \left(-\frac{1}{p^2+M^2}\frac{D\bar{D}^2 D}{p^2} + \frac{\xi^{-1}}{p^2+\xi^{-1}M^2}\frac{D^2\bar{D}^2 + \bar{D}^2 D^2}{p^2}\right) \times \delta^4(\theta - \theta'). \tag{58}$$

One can now have three different forms for the propagator, depending on the value of ξ:

(i) For no gauge fixing ($\xi = 0$), the propagator becomes

$$VV\colon \left(-\frac{1}{p^2+M^2}\frac{D\bar{D}^2 D}{p^2} + \frac{1}{M^2}\frac{D^2\bar{D}^2 + \bar{D}^2 D^2}{p^2}\right) \times \delta^4(\theta - \theta'). \tag{59}$$

(ii) In the Feynman gauge ($\xi = 1$), it becomes

$$VV\colon -\frac{1}{p^2+M^2}\delta^4(\theta-\theta'). \tag{60}$$

(iii) In the Landau gauge ($\xi = \infty$), it becomes

$$VV\colon -\frac{1}{p^2+M^2}\frac{D\bar{D}^2 D}{p^2}\delta^4(\theta-\theta'). \tag{61}$$

We will perform our calculations in the Feynman gauge. Note that in the Landau gauge (61) and the Feynman gauge (60) it is consistent to set $M = 0$, but for the propagator without gauge fixing (59), taking the limit $M \to 0$ results in a singularity, as expected.

The gauging of a chiral multiplet is well known to be

$$\mathcal{L} = \int d^4\theta \bar{\Phi} e^V \Phi, \tag{62}$$

where the vector transforms as shown in Eq. (52) and the chiral superfield transforms as

$$\Phi \to \Phi' = e^{i\Lambda}\Phi, \tag{63}$$

for a chiral Λ. It is easy to verify that the transformed superfield is still chiral,

$$\bar{D}_{\dot{\alpha}}\Phi' = \bar{D}_{\dot{\alpha}}(e^{i\Lambda}\Phi) = 0. \tag{64}$$

The gauging of the complex linear is

$$\mathcal{L} = -\int d^4\theta \bar{\Sigma} e^V \Sigma, \tag{65}$$

but note that the gauge transformation of Σ is again

$$\Sigma \to \Sigma' = e^{i\Lambda}\Sigma, \tag{66}$$

where Λ is still chiral. It is easy to verify that the transformed superfield remains "complex linear,"

$$\bar{D}^2\Sigma' = \bar{D}^2(e^{i\Lambda}\Sigma) = e^{i\Lambda}\bar{D}^2(\Sigma) = 0. \tag{67}$$

On the other hand, a transformation of the form

$$\Sigma \to \Sigma' = e^{iS}\Sigma \tag{68}$$

for a complex linear S would violate the complex linearity of the Σ superfield due to the fact that

$$\bar{D}^2 S^n \neq 0, \quad \text{for } n > 1. \tag{69}$$

A transformation (68) would also lead to a second complication. The invariance of Eq. (65) would require

$$V \to V + i\bar{S} - iS. \tag{70}$$

But the field strength of the gauge superfield

$$W_\alpha = i\bar{D}^2 D_\alpha V \tag{71}$$

is not invariant under Eq. (70) since

$$i\bar{D}^2 D_\alpha(-iS + i\bar{S}) \neq 0. \tag{72}$$

III. GAUGED MASSLESS COMPLEX LINEAR AND MASSIVE $U(1)$

We start with a model of a massless complex linear coupled to a massive $U(1)$. We calculate the effective low-energy theory for the scalar multiplet by integrating out the massive vector superfield. We find that the effective theory on top of the corrections to the Kähler potential also contains superspace higher derivatives that lead to the deformations of the auxiliary potential.

Our theory has the form in superspace of

$$\mathcal{L} = -\int d^4\theta\bar{\Sigma}e^{gV}\Sigma + \int d^2\theta W^\alpha W_\alpha + \text{H.c.} - \xi\int d^4\theta D^2 V\bar{D}^2 V + \frac{1}{2}M^2\int d^4\theta V^2, \tag{73}$$

which is easily shown to be classically equivalent to a gauged chiral superfield theory coupled to a massive vector superfield by gauging the Lagrangian (14) for the dual superfields having opposite $U(1)$ charge. Here, the gauge transformation for Σ is

$$\Sigma \to e^{ig\Lambda}\Sigma. \tag{74}$$

The model (73) gives rise to the vertices shown in Figs. 1 and 2 [up to $\mathcal{O}(g^2)$]. The renormalizability is understood in terms of gauge invariance, and the no tree-level supersymmetry breaking is understood by a straightforward component expansion. Indeed, the scalar nonderivative sector of the Lagrangian (73) is

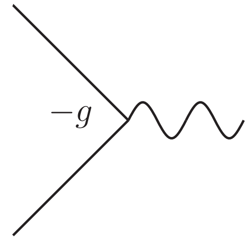

FIG. 1. Vertex due to gauging.

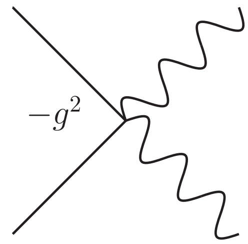

FIG. 2. Vertex due to gauging.

$$\mathcal{L} = -F\bar{F} + N\bar{N} + \text{D}(-gA\bar{A} + M^2C) + (1-\xi)\text{D}^2, \tag{75}$$

which has no other vacuum than

$$\langle F\rangle = \langle N\rangle = \langle \text{D}\rangle = 0. \tag{76}$$

To calculate the vacuum structure of the tree-level theory [Eq. (75)], we have written the gauge-invariant terms in the Wess–Zumino gauge. Of course, when calculating the radiative corrections, one should *not* turn to the W-Z gauge for the quantum fluctuations even though it is possible to do so for the background vector superfield (see, for example, Ref. [3]).

To find the low-energy effective theory for Σ, we work in two steps:

(1) We write down the one-loop corrections to the vertices.

(2) Then, we integrate out the massive vector superfield.

Here, we will keep terms up to

$$\mathcal{O}\left(\frac{1}{M^4}\right), \tag{77}$$

since this is the scale in which the auxiliary field potential deformation comes in. Because of the massless complex linear multiplet, the diagrams under consideration will have infrared divergences, and thus we will calculate the loop momenta integrals with an IR cutoff.

$$\Lambda\text{: IR cutoff}. \tag{78}$$

Moreover, since for the moment we are only interested in finding contributions to the scalar potential, we will set the

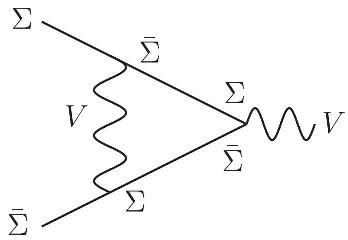

FIG. 3. One-loop correction to the $\Sigma\bar{\Sigma}V$ vertex.

external momenta to zero or, equivalently, impose the condition

$$\partial_a \Sigma = \partial_a \bar{\Sigma} = 0, \tag{79}$$

which is allowed by supersymmetry.

The one-loop corrections to the $V\Sigma\bar{\Sigma}$ vertex are given by the diagrams in Figs. 3, 4, and 5 with corresponding terms in the effective theory:

$$\begin{aligned}
\mathcal{L}_3 &= \frac{g^3}{16\pi^2}\mathcal{A}(\epsilon)\int d^4\theta \Sigma\bar{\Sigma}V \\
&\quad + \frac{g^3}{16\pi^2 M^2}\ln\left(\frac{\Lambda^2+M^2}{\Lambda^2}\right)\int d^4\theta \Sigma\bar{\Sigma}D^2\bar{D}^2 V, \\
\mathcal{L}_4 &= -\frac{g^3}{16\pi^2}\mathcal{A}(\epsilon)\int d^4\theta \Sigma\bar{\Sigma}V, \\
\mathcal{L}_5 &= -\frac{g^3}{16\pi^2}\mathcal{A}(\epsilon)\int d^4\theta \Sigma\bar{\Sigma}V.
\end{aligned} \tag{80}$$

The factor $\mathcal{A}(\epsilon)$ contains the poles due to the UV divergencies and finite terms:

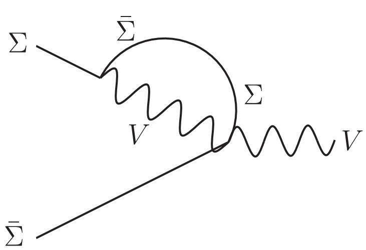

FIG. 4. One-loop correction to the $\Sigma\bar{\Sigma}V$ vertex.

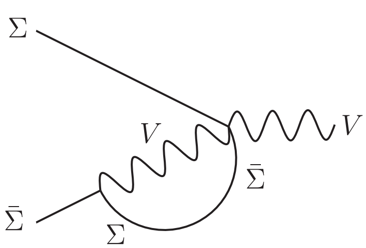

FIG. 5. One-loop correction to the $\Sigma\bar{\Sigma}V$ vertex.

$$\mathcal{A}(\epsilon) = \frac{2}{\epsilon} + 1 - \gamma + \ln(4\pi) - \ln\left(\frac{\Lambda^2+M^2}{\mu^2}\right). \tag{81}$$

We have introduced the renormalization scale μ in order for g to remain dimensionless,

$$g \to \mu^{\epsilon/2} g, \tag{82}$$

during the regularization. Then, we have the diagram shown in Fig. 6, which represents a vertex containing the tree-level vertex (1) along with the loop corrections (3–5) and can be written in the effective theory as

$$\begin{aligned}
\mathcal{L}_3 &= \int d^4\theta(-g\bar{\Sigma}V\Sigma) + \mathcal{L}_{2(a)} + \mathcal{L}_{2(b)} + \mathcal{L}_{2(c)} \\
&= \int d^4\theta\Bigg(-g\bar{\Sigma}V\Sigma \\
&\quad + \frac{g^3}{16\pi^2 M^2}\ln\left(\frac{\Lambda^2+M^2}{\Lambda^2}\right)\Sigma\bar{\Sigma}D^2\bar{D}^2 V\Bigg),
\end{aligned} \tag{83}$$

where we have absorbed the finite parts of the loop corrections to the tree-level vertex by redefining g. Since we want to integrate out the vector superfield, the leading diagrams to this order due to the vertex corrections, relevant to our results, are shown in the Figs. 7 and 8 and lead to the effective interactions

$$\begin{aligned}
\mathcal{L}_7 &= -\frac{g^2}{2M^2}\int d^4\theta \Sigma^2\bar{\Sigma}^2 \\
&\quad + \frac{g^4}{16\pi^2 M^4}\ln\left(\frac{\Lambda^2+M^2}{\Lambda^2}\right)\int d^4\theta D^2(\Sigma\bar{\Sigma})\bar{D}^2(\Sigma\bar{\Sigma})
\end{aligned} \tag{84}$$

and

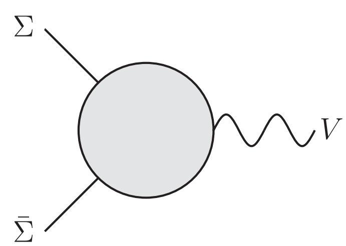

FIG. 6. Tree-level and one-loop corrections to the $\Sigma\bar{\Sigma}V$ vertex.

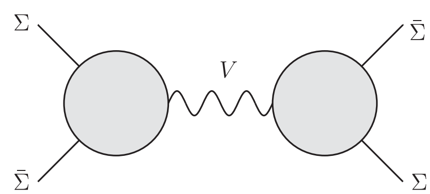

FIG. 7. Effective interactions from integrating out V.

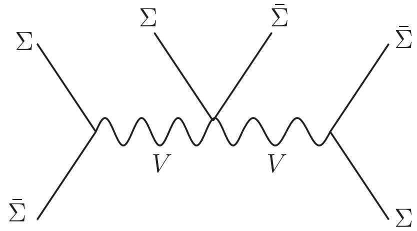

FIG. 8. Effective interactions from integrating out V.

$$\mathcal{L}_8 = -\frac{g^4}{6}\int d^4\theta \frac{1}{M^4}\Sigma^3\bar{\Sigma}^3. \tag{85}$$

The one-loop corrections to the four sigma vertex terms are given by the diagrams in Figs. 9, 10, and 11, with corresponding terms

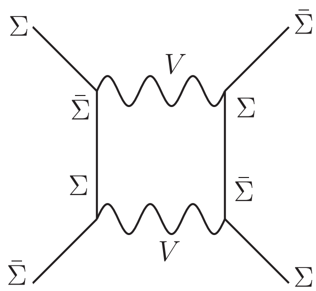

FIG. 9. One-loop correction to the $\Sigma^2\bar{\Sigma}^2$ vertex.

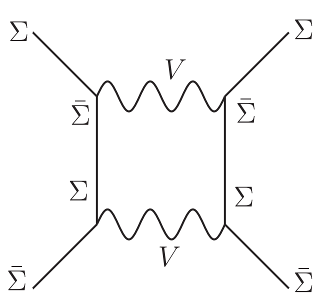

FIG. 10. One-loop correction to the $\Sigma^2\bar{\Sigma}^2$ vertex.

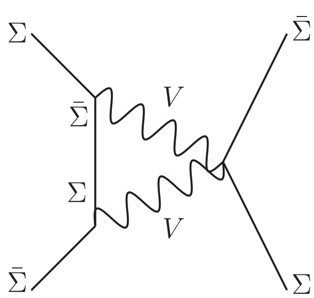

FIG. 11. The one-loop correction to the $\Sigma^2\bar{\Sigma}^2$ vertex.

$$\begin{aligned}\mathcal{L}_9 &= \frac{g^4}{32\pi^2(M^2+\Lambda^2)}\int d^4\theta\Sigma^2\bar{\Sigma}^2\\ &\quad + \frac{g^4}{32\pi^2 M^4}\int d^4\theta\left\{\ln\left(\frac{\Lambda^2+M^2}{\Lambda^2}\right)\right.\\ &\quad \left. - \frac{M^2}{(\Lambda^2+M^2)}\right\}D^2(\Sigma\bar{\Sigma})\bar{D}^2(\Sigma\bar{\Sigma}),\\ \mathcal{L}_{10} &= \frac{g^4}{16\pi^2(M^2+\Lambda^2)}\int d^4\theta\Sigma^2\bar{\Sigma}^2\\ &\quad + \frac{g^4}{32\pi^2 M^4}\int d^4\theta\left\{\ln\left(\frac{\Lambda^2+M^2}{\Lambda^2}\right)\right.\\ &\quad \left. - \frac{M^2}{(\Lambda^2+M^2)}\right\}D^2(\bar{\Sigma}^2)\bar{D}^2(\Sigma^2)\end{aligned} \tag{86}$$

and

$$\mathcal{L}_{11} = -\frac{g^4}{16\pi^2(M^2+\Lambda^2)}\int d^4\theta\Sigma^2\bar{\Sigma}^2. \tag{87}$$

Gathering the quantum corrections up to order $1/M^4$, we have for Σ

$$\begin{aligned}\mathcal{L}_{\Sigma,\mathrm{eff}} &= -\mathcal{T}\int d^4\theta\bar{\Sigma}\Sigma - \mathcal{P}\int d^4\theta\Sigma^2\bar{\Sigma}^2 - \mathcal{Q}\int d^4\theta\Sigma^3\bar{\Sigma}^3\\ &\quad + \mathcal{R}\int d^4\theta D^2(\Sigma\bar{\Sigma})\bar{D}^2(\Sigma\bar{\Sigma})\\ &\quad + \mathcal{S}\int d^4\theta D^2(\bar{\Sigma}^2)\bar{D}^2(\Sigma^2),\end{aligned} \tag{88}$$

with

$$\mathcal{T} = 1 + \mathcal{O}(g^2), \tag{89}$$

$$\mathcal{P} = \frac{g^2}{2M^2} + \mathcal{O}(g^4), \tag{90}$$

$$\mathcal{Q} = \frac{g^4}{6M^4} + \mathcal{O}(g^6) \tag{91}$$

and

$$\begin{aligned}\mathcal{R} &= \frac{g^4}{16\pi^2 M^4}\left\{2\ln\left(\frac{\Lambda^2+M^2}{\Lambda^2}\right) - \frac{M^2}{(\Lambda^2+M^2)}\right\} + \mathcal{O}(g^6),\\ \mathcal{S} &= \frac{g^4}{32\pi^2 M^4}\left\{\ln\left(\frac{\Lambda^2+M^2}{\Lambda^2}\right) - \frac{M^2}{(\Lambda^2+M^2)}\right\} + \mathcal{O}(g^6).\end{aligned} \tag{92}$$

We see that superspace higher derivatives of a similar form as those introduced in Ref. [21] are indeed generated by quantum corrections. The scalar sector of the Lagrangian (88) is

$$\mathcal{L}^{\text{scalar}}_{\Sigma,\text{eff}} = -F\bar{F}[\mathcal{T} + 4\mathcal{P}A\bar{A} + 9\mathcal{Q}A^2\bar{A}^2] + F^2\bar{F}^2\mathcal{R}, \tag{93}$$

where the impact of the superspace higher derivatives on the auxiliary field's effective potential is manifest.

IV. GAUGED MASSIVE HYPERMULTIPLET AND MASSIVE $U(1)$

A massive complex linear Σ is defined by the supersymmetric condition

$$\bar{D}^2\Sigma = m\Phi, \tag{94}$$

where Φ is a chiral superfield. Thus, Σ acquires a mass m in tandem with the chiral multiplet Φ [23]. It is, in fact, equivalent to a massive $N = 2$ hypermultiplet [28]. In a gauged $U(1)$ model, both Σ and Φ transform with the same charge:

$$\Sigma \to e^{ig\Lambda}\Sigma, \qquad \Phi \to e^{ig\Lambda}\Phi. \tag{95}$$

We employ the following superspace Lagrangian:

$$\mathcal{L} = -\int d^4\theta\bar{\Sigma}e^{gV}\Sigma + \int d^4\theta\bar{\Phi}e^{gV}\Phi + \int d^2\theta W^\alpha W_\alpha(V) + \text{H.c.} - \xi\int d^4\theta D^2V\bar{D}^2V + \frac{1}{2}M^2\int d^4\theta V^2. \tag{96}$$

The scalar nonderivative component sector of theory (96) is

$$\mathcal{L} = -F\bar{F} + N\bar{N} + \mathrm{D}(gz\bar{z} - gA\bar{A} + M^2C) + (1-\xi)\mathrm{D}^2 - m^2z\bar{z} - mA\bar{G} - m\bar{A}G + G\bar{G}, \tag{97}$$

with a supersymmetric vacuum

$$\langle F\rangle = \langle G\rangle = \langle N\rangle = \langle \mathrm{D}\rangle = 0. \tag{98}$$

We now turn to the quantum corrections. A way to rewrite the complex linear multiplet constraint is by splitting Σ into a background field Σ_0 and the quantum fluctuations σ as follows:

$$\Sigma = \Sigma_0 + \sigma. \tag{99}$$

For the gauge multiplet, we consider no background field,

$$V = 0 + V, \tag{100}$$

and for the chiral superfield, we have

$$\Phi = 0 + \varphi, \tag{101}$$

where

$$\bar{D}^2\sigma = M\varphi \tag{102}$$

and

$$\bar{D}^2\Sigma_0 = 0. \tag{103}$$

Here, we have set the hypermultiplet mass equal to the vector multiplet mass for simplicity, and we will keep it like this. This is not merely technical, since if these masses originate from the same physics, they will also be of the same order.

A. Propagator renormalization

The corrections to the vector propagator due to σ loops are given by the diagrams in Figs. 12 and 13 and contribute to the effective theory as

$$\mathcal{L}_{12} = \frac{g^2}{32\pi^2}\int d^4\theta\int d^4kV(-k)\left(-M^2 - \frac{1}{6}k^2 + \frac{1}{2}\mathcal{B}(\epsilon)D^\alpha\bar{D}^2D_\alpha\right)V(k),$$
$$\mathcal{L}_{13} = \frac{g^2}{32\pi^2}\int d^4\theta\int d^4kV(-k)M^2(\mathcal{B}(\epsilon)+1)V(k), \tag{104}$$

where

$$\mathcal{B}(\epsilon) = \frac{2}{\epsilon} - \gamma + \ln(4\pi) - \ln\left(\frac{M^2}{\mu^2}\right). \tag{105}$$

Corrections due to φ loops are given by the diagrams shown in Figs. 14 and 15 and contribute to the effective theory as

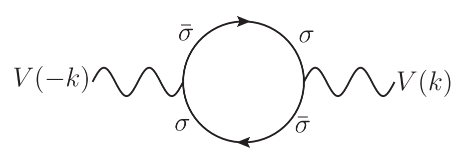

FIG. 12. One-loop correction to the vector propagator from the σ superfield.

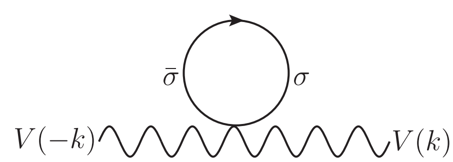

FIG. 13. One-loop correction to the vector propagator from the σ superfield.

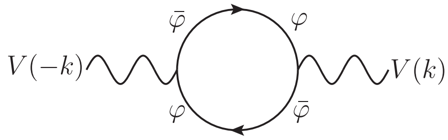

FIG. 14. One-loop correction to the vector propagator from the φ superfield.

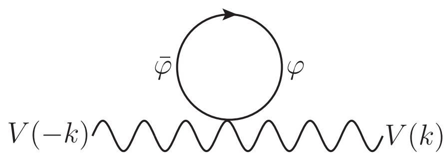

FIG. 15. One-loop correction to the vector propagator from the φ superfield.

$$\mathcal{L}_{14} = \frac{g^2}{32\pi^2}\int d^4\theta \int d^4k V(-k)\left(2\mathcal{B}(\epsilon)M^2 + M^2 - \frac{1}{6}k^2 + \frac{1}{2}\mathcal{B}(\epsilon)D^{\alpha}\bar{D}^2 D_{\alpha}V(k)\right),$$
$$\mathcal{L}_{15} = -\frac{g^2}{32\pi^2}\int d^4\theta \int d^4k V(-k)M^2(\mathcal{B}(\epsilon)+1)V(k). \tag{106}$$

There is finally a mixed-loop diagram shown in Fig. 16, contributing to the effective theory as

$$\mathcal{L}_{16} = -\frac{g^2}{32\pi^2}\int d^4\theta \int d^4k V(-k)\left(2\mathcal{B}(\epsilon)M^2 - \frac{1}{3}k^2\right)V(k). \tag{107}$$

Summing up the different contributions, one gets

$$\mathcal{L} = \mathcal{L}_{12} + \mathcal{L}_{13} + \mathcal{L}_{14} + \mathcal{L}_{15} + \mathcal{L}_{16} = \frac{g^2}{32\pi^2}\int d^4\theta \int d^4k V(-k)\mathcal{B}(\epsilon)D^{\alpha}\bar{D}^2 D_{\alpha}V(k). \tag{108}$$

It is gratifying to see that the quadratic divergencies cancel and that the contribution is purely transversal as it should be for gauge invariance.

For the background complex linear, we have the diagram in Fig. 17,

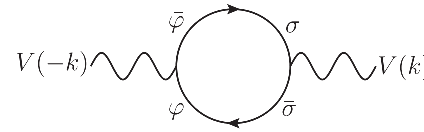

FIG. 16. One-loop correction to the vector propagator from the φ and σ superfields.

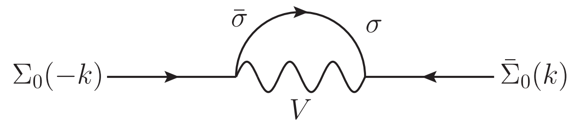

FIG. 17. One-loop correction to the background superfield.

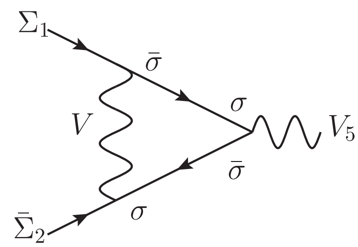

FIG. 18. One-loop correction to the $\Sigma_0\bar{\Sigma}_0 V$ vertex.

$$\mathcal{L}_{17} = -\frac{g^2}{16\pi^2}\int d^4\theta \int d^4k \Sigma(-k)\left(\mathcal{B}(\epsilon)M^2 - \frac{1}{6}k^2\right)\bar{\Sigma}(k), \tag{109}$$

which will give us the Σ_0 wave function renormalization.

B. Vertices and integrating out

We now wish to integrate out all the massive fluctuations and find the low-energy effective theory for the background field Σ_0. Again, we consider the gauge coupling g to be small. Note also that, now the complex linear superfield is massive, there will be no IR divergence.

The one-loop corrections to the $V\Sigma\bar{\Sigma}$ vertex are given by the diagrams in Figs. 18 and 19 with corresponding terms in the effective theory,

$$\begin{aligned}\mathcal{L}_{18} = {}& \frac{g^3}{32\pi^2 M^2}\int d^4\theta \int d^4p_{\text{ext}}\Sigma_1\bar{\Sigma}_2\left(M^2 + 2M^2\mathcal{B}(\epsilon) - \frac{1}{3}p_1^2 - \frac{1}{6}p_1 p_2 - \frac{1}{3}p_2^2\right)V_5 \\ & + \frac{g^3}{32\pi^2 M^2}\int d^4\theta \int d^4p_{\text{ext}}\Sigma_1\bar{\Sigma}_2\left(-\left(\frac{1}{6}p_1 + \frac{1}{3}p_5\right)^{\alpha\dot{\alpha}}\bar{D}_{\dot{\alpha}}D_{\alpha}V_5 - \left(\frac{1}{6}p_2 + \frac{1}{3}p_5\right)^{\alpha\dot{\alpha}}D_{\alpha}\bar{D}_{\dot{\alpha}}V_5\right) \\ & + \frac{g^3}{32\pi^2 M^2}\int d^4\theta \int d^4p_{\text{ext}}\left(\frac{1}{2}\Sigma_1\bar{\Sigma}_2\{D^2, \bar{D}^2\}V_5\right),\end{aligned} \tag{110}$$

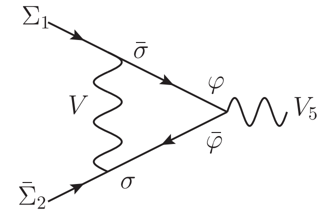

FIG. 19. One-loop correction to the $\Sigma_0\bar{\Sigma}_0 V$ vertex.

$$\mathcal{L}_{19} = -\frac{g^3}{32\pi^2 M^2}\int d^4\theta \int d^4 p_{\rm ext}\Sigma_1\bar{\Sigma}_2 V_5 \times\left(M^2 - \frac{1}{6}p_1^2 - \frac{1}{6}p_1 p_2 - \frac{1}{6}p_2^2\right), \tag{111}$$

where

$$\int d^4 p_{\rm ext} = (2\pi)^4\delta\left(\sum_i p_i\right)\prod_i \int \frac{d^4 p_i}{(2\pi)^4}. \tag{112}$$

Here, p_i are the external momenta, and the subscript i on the superfields implies their momentum, for example,

$$\Sigma_i = \int d^4 x \Sigma_0(x,\theta,\bar{\theta})e^{-ixp_i}. \tag{113}$$

In the above diagrams (and the subsequent), we have calculated the external momenta contribution in the limit

$$\frac{p_{\rm ext}^2}{M^2} \ll 1. \tag{114}$$

We also have the diagrams shown in Figs. 23 and 24, which lead to the terms

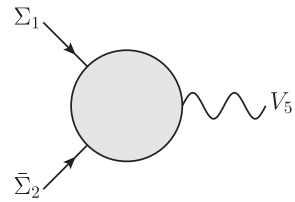

FIG. 20. Tree-level and one-loop corrections to the $\Sigma_0\bar{\Sigma}_0 V$ vertex.

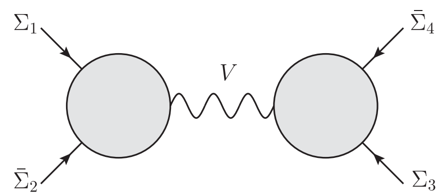

FIG. 21. Effective interactions due to integrating out V.

$$\mathcal{L}_{20} = \frac{g^3}{16\pi^2 M^2}\int d^4\theta \int d^4 p_{\rm ext}\Sigma_1\bar{\Sigma}_2 V_5\left(-M^2\mathcal{B}(\epsilon) + \frac{1}{6}p_1^2\right), \tag{115}$$

$$\mathcal{L}_{21} = \frac{g^3}{16\pi^2 M^2}\int d^4\theta \int d^4 p_{\rm ext}\Sigma_1\bar{\Sigma}_2 V_5\left(-M^2\mathcal{B}(\epsilon) + \frac{1}{6}p_2^2\right). \tag{116}$$

Then, we have a diagram shown in Fig. 20, which represents a vertex containing the tree-level vertex (1) along with the loop corrections (18–24) and can be written in the effective theory as

$$\begin{aligned}\mathcal{L}_{22} &= \int d^4\theta(-g\bar{\Sigma}_0 V\Sigma_0) + \mathcal{L}_{18} + \mathcal{L}_{19} + \mathcal{L}_{20} + \mathcal{L}_{21} \\ &= \int d^4\theta(-g\bar{\Sigma}_0 V\Sigma_0) + \frac{g^3}{32\pi^2 M^2}\int d^4\theta \int d^4 p_{\rm ext}\Sigma_1\bar{\Sigma}_2\left(\frac{1}{6}p_1^2 + \frac{1}{6}p_2^2 + \frac{1}{2}\{D^2,\bar{D}^2\}\right)V_5 \\ &\quad + \frac{g^3}{32\pi^2 M^2}\int d^4\theta \int d^4 p_{\rm ext}\Sigma_1\bar{\Sigma}_2\left(-\left(\frac{1}{6}p_1 + \frac{1}{3}p_5\right)^{\alpha\dot{\alpha}}\bar{D}_{\dot{\alpha}}D_\alpha V_5 - \left(\frac{1}{6}p_2 + \frac{1}{3}p_5\right)^{\alpha\dot{\alpha}}D_\alpha\bar{D}_{\dot{\alpha}}V_5\right) \\ &= \int d^4\theta(-g\bar{\Sigma}_0 V\Sigma_0) + \frac{g^3}{32\pi^2 M^2}\int d^4\theta \int d^4 p_{\rm ext}\Sigma_1\bar{\Sigma}_2\left(\frac{1}{6}p_1^2 + \frac{1}{6}p_2^2 + \frac{1}{2}\bar{D}D^2\bar{D}\right)V_5 \\ &\quad + \frac{g^3}{32\pi^2 M^2}\int d^4\theta \int d^4 p_{\rm ext}\Sigma_1\bar{\Sigma}_2\left(-\frac{1}{12}(p_1 - p_2)^{\alpha\dot{\alpha}}[D_\alpha,\bar{D}_{\dot{\alpha}}]V_5\right),\end{aligned} \tag{117}$$

where we have absorbed the finite parts of the loop corrections by redefining g. Since we want to integrate out the vector superfield, the leading diagrams to this order due to the vertex corrections, relevant to our results, are shown in Figs. 21 and 22 and lead to the effective interactions

$$\begin{aligned}\mathcal{L}_{23} = &-\frac{g^2}{2M^2}\int d^4\theta \Sigma_0^2\bar{\Sigma}_0^2 + \frac{g^4}{64\pi^2 M^4}\int d^4\theta \int d^4 p_{\text{ext}} \Sigma_1 \bar{\Sigma}_2 \bar{D} D^2 \bar{D}(\Sigma_3 \bar{\Sigma}_4) \\ &+ \frac{g^2}{4M^4}\int d^4\theta \int d^4 p_{\text{ext}} \Sigma_1\bar{\Sigma}_2\Sigma_3\bar{\Sigma}_4\{(p_1+p_2)^2 + (p_3+p_4)^2\} \\ &+ \frac{g^4}{32\pi^2 M^4}\int d^4\theta \int d^4 p_{\text{ext}} \Sigma_1\bar{\Sigma}_2\left(\frac{1}{12}p_1^2 + \frac{1}{12}p_2^2 + \frac{1}{12}p_3^2 + \frac{1}{12}p_4^2\right)(\Sigma_3\bar{\Sigma}_4) \\ &+ \frac{g^4}{64\pi^2 M^4}\int d^4\theta \int d^4 p_{\text{ext}} \Sigma_1\bar{\Sigma}_2\left(-\frac{1}{12}(p_1-p_2)^{\alpha\dot{\alpha}}[D_\alpha, \bar{D}_{\dot{\alpha}}](\Sigma_3\bar{\Sigma}_4)\right) \\ &+ \frac{g^4}{64\pi^2 M^4}\int d^4\theta \int d^4 p_{\text{ext}} \Sigma_3\bar{\Sigma}_4\left(-\frac{1}{12}(p_3-p_4)^{\alpha\dot{\alpha}}[D_\alpha, \bar{D}_{\dot{\alpha}}](\Sigma_1\bar{\Sigma}_2)\right) \qquad (118)\end{aligned}$$

and

$$\mathcal{L}_{24} = -\frac{g^4}{6M^4}\int d^4\theta \Sigma_0^3\bar{\Sigma}_0^3. \qquad (119)$$

The one-loop corrections to the $\Sigma_0^2\bar{\Sigma}_0^2$ vertex are given by the diagrams in Figs. 25, 26, and 27, for which we have

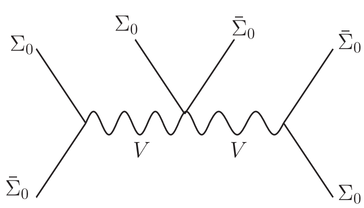

FIG. 22. Effective interactions due to integrating out V.

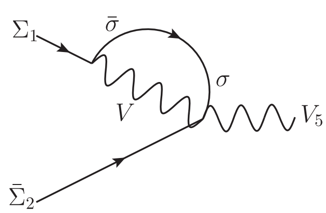

FIG. 23. One-loop correction to the $\Sigma_0\bar{\Sigma}_0 V$ vertex.

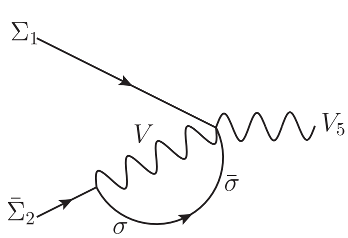

FIG. 24. One-loop correction to the $\Sigma_0\bar{\Sigma}_0 V$ vertex.

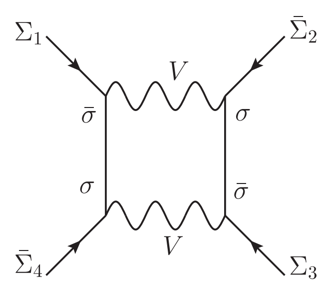

FIG. 25. One-loop correction to the $\Sigma_0^2\bar{\Sigma}_0^2$ vertex.

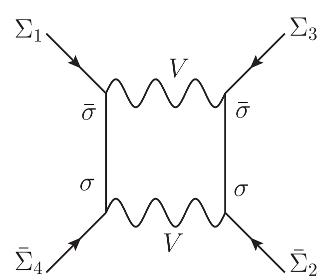

FIG. 26. One-loop correction to the $\Sigma_0^2\bar{\Sigma}_0^2$ vertex.

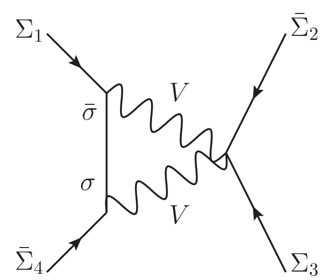

FIG. 27. One-loop correction to the $\Sigma_0^2\bar{\Sigma}_0^2$ vertex.

$$\mathcal{L}_{25} = \frac{g^4}{32\pi^2 M^4}\int d^4\theta \int d^4 p_{\rm ext}\Sigma_1\bar{\Sigma}_2\left(\frac{2}{3}M^2 - \frac{1}{48}(p_1^2+p_2^2+p_3^2+p_4^2) - \frac{23}{120}(p_1+p_2)^2 - \frac{1}{60}(p_1+p_3)^2 + \frac{1}{6}\bar{D}^2D^2 - \frac{1}{48}(p_2+p_4)^{\alpha\dot{\alpha}}[D_\alpha, \bar{D}_{\dot{\alpha}}]\right)\Sigma_3\bar{\Sigma}_4, \tag{120}$$

$$\mathcal{L}_{26} = \frac{g^4}{32\pi^2 M^4}\int d^4\theta \int d^4 p_{\rm ext}\Sigma_1\Sigma_3\left(M^2 - \frac{1}{12}p_1^2 - \frac{1}{12}p_1p_2 - \frac{1}{12}p_2^2 - \frac{1}{12}p_3^2 - \frac{1}{12}p_3p_4 - \frac{1}{12}p_4^2 + \frac{1}{6}\bar{D}^2D^2\right)\bar{\Sigma}_2\bar{\Sigma}_4, \tag{121}$$

$$\mathcal{L}_{27} = -\frac{g^4}{32\pi^2 M^4}\int d^4\theta \int d^4 p_{\rm ext}\Sigma_1\bar{\Sigma}_2\Sigma_3\bar{\Sigma}_4\left(M^2 - \frac{1}{6}p_1^2 - \frac{1}{6}p_1p_4 - \frac{1}{6}p_4^2\right), \tag{122}$$

leading to the effective interactions

$$\mathcal{L}_{25} + \mathcal{L}_{26} + \mathcal{L}_{27} = \frac{g^4}{32\pi^2 M^4}\int d^4\theta \int d^4 p_{\rm ext}\Sigma_1\bar{\Sigma}_2\left(\frac{2}{3}M^2 - \frac{1}{48}(p_1^2+p_2^2+p_3^2+p_4^2) - \frac{23}{120}(p_1+p_2)^2 - \frac{1}{60}(p_1+p_3)^2 + \frac{1}{3}\bar{D}^2D^2 - \frac{1}{48}(p_2+p_4)^{\alpha\dot{\alpha}}[D_\alpha, \bar{D}_{\dot{\alpha}}]\right)\Sigma_3\bar{\Sigma}_4. \tag{123}$$

Now, we can calculate the leading contributions to the off-shell effective potential of the theory. As we see, it receives contributions both from the effective Kähler potential but also from the deformation of the auxiliary field potential. Gathering the relevant quantum corrections, we have for the background superfield Σ_0

$$\mathcal{L}_{\Sigma_0,\rm eff} = -\mathcal{T}'\int d^4\theta\bar{\Sigma}_0\Sigma_0 - \mathcal{P}'\int d^4\theta\Sigma_0^2\bar{\Sigma}_0^2 - \mathcal{Q}'\int d^4\theta\Sigma_0^3\bar{\Sigma}_0^3 + \mathcal{R}'\int d^4\theta D^2(\Sigma_0\bar{\Sigma}_0)\bar{D}^2(\Sigma_0\bar{\Sigma}_0) + \mathcal{S}'\int d^4\theta D^2(\bar{\Sigma}_0^2)\bar{D}^2(\Sigma_0^2), \tag{124}$$

with

$$\mathcal{T}' = 1 + \mathcal{O}(g^2), \tag{125}$$

$$\mathcal{P}' = \frac{g^2}{2M^2} + \mathcal{O}(g^4), \tag{126}$$

$$\mathcal{Q}' = \frac{g^4}{6M^4} + \mathcal{O}(g^6), \tag{127}$$

and for the superspace higher-derivative operators

$$\mathcal{R}' = \frac{7g^4}{192\pi^2 M^4} + \mathcal{O}(g^6), \tag{128}$$

$$\mathcal{S}' = \frac{g^4}{192\pi^2 M^4} + \mathcal{O}(g^6). \tag{129}$$

Note that in the Lagrangian (124) various superspace higher derivatives have been generated radiatively.

C. Vacuum structure

Let us focus on the scalar nonderivative sector of Lagrangian (124), which reads

$$\mathcal{L}^{\rm scalar}_{\Sigma_0,\rm eff} = -F_0\bar{F}_0[\mathcal{T}' + 4\mathcal{P}'A_0\bar{A}_0 + 9\mathcal{Q}'A_0^2\bar{A}_0^2] + F_0^2\bar{F}_0^2\mathcal{R}'. \tag{130}$$

We see that the effective theory has an intriguing similarity to the models of Ref. [21]. Indeed, the equation of motion for F_0 has two solutions:

(i) Standard branch:

$$F_0 = 0. \tag{131}$$

(ii) Broken branch:

$$F_0\bar{F}_0 = \frac{1}{2\mathcal{R}'}[\mathcal{T}' + 4\mathcal{P}'A_0\bar{A}_0 + 9\mathcal{Q}'A_0^2\bar{A}_0^2]. \tag{132}$$

The scalar potential of the broken branch will have the form

$$\mathcal{V} = \frac{1}{4\mathcal{R}'}[\mathcal{T}' + 4\mathcal{P}'A_0\bar{A}_0 + 9\mathcal{Q}'A_0^2\bar{A}_0^2]^2, \tag{133}$$

leading to a positive vacuum energy, and a supersymmetry breaking scale

$$\langle F_0 \bar{F}_0 \rangle = \frac{\mathcal{T}'}{2\mathcal{R}'} = \frac{96\pi^2 M^4}{7g^4}. \quad (134)$$

Note that in this new vacuum $|F_0|$ has a dependence on M^2/g^2, which gives rise to the question of higher-order corrections. In other words, this solution can only be trusted if there exists a hierarchy between the leading superspace higher derivatives and the subsequent ones. If one naively estimates the one-loop contribution to higher point functions such as the six- and eight-point graphs of Figs. 28 and 29, one sees that the minimum of the potential is shifted. Namely, the contribution from the 2n-point diagram to the effective potential of the auxiliary field goes as

$$(-1)^n M^4 \left(\frac{g^2 F\bar{F}}{M^4}\right)^n. \quad (135)$$

The auxiliary field potential will have a generic form,

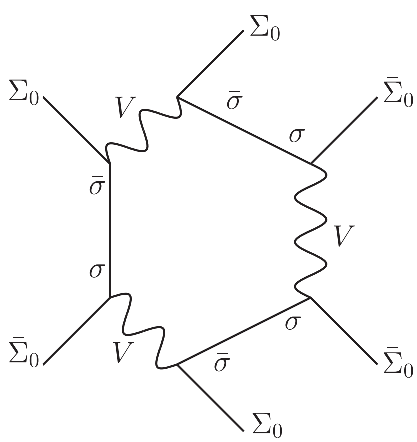

FIG. 28. Higher-order diagram.

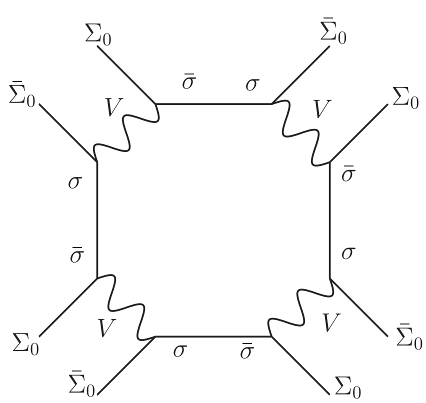

FIG. 29. Higher-order diagram.

$$\mathcal{L}_F = M^4 h\left(\frac{g^2 F\bar{F}}{M^4}\right) - F\bar{F}, \quad (136)$$

but from Eq. (135), we see that the higher-order terms do not satisfy the hierarchy criterion (43) required for reliable results on supersymmetry breaking, and a complete knowledge of the form of Eq. (136) would in principle be required.

Even though we do not have an exact form for the higher-order corrections, we can still ask if generically supersymmetry is broken in the low-energy limit by studying the renormalization group flow. We can draw reliable results if we study a generic term of the form

$$\int d^4\theta \lambda^{(j)} (D\Sigma)^2 (\bar{D}\bar{\Sigma})^2 (D^2\Sigma \bar{D}^2 \bar{\Sigma})^j, \quad (137)$$

which could arise from the quantum corrections, in which the dimension of $\lambda^{(j)}$ is $-4(j+1)$.

Let us define the bare action as

$$\begin{aligned}\mathcal{L}_{\rm bare} &= \int d^4\theta (V_b D^\alpha \bar{D}^2 D_\alpha V_b + M_b^2 V_b^2 - \Sigma_b \bar{\Sigma}_b - g_b V_b \Sigma_b \bar{\Sigma}_b) \\ &= \int d^4\theta (\mathcal{Z}_V V D^\alpha \bar{D}^2 D_\alpha V + M^2 V^2 \\ &\quad - \mathcal{Z}_\Sigma \Sigma\bar{\Sigma} - \mathcal{Z}_\alpha g V \Sigma\bar{\Sigma}), \end{aligned} \quad (138)$$

from which we find

$$\begin{aligned}\mathcal{Z}_\Sigma &= 1 - \frac{4\alpha}{\epsilon}, \\ \mathcal{Z}_V &= 1 - \frac{2\alpha}{\epsilon}, \\ \mathcal{Z}_\alpha &= 1 - \frac{4\alpha}{\epsilon}, \end{aligned} \quad (139)$$

with

$$\alpha = \frac{g^2}{32\pi^2}. \quad (140)$$

Since the bare coupling

$$\alpha_b = \mu^\epsilon \alpha \frac{\mathcal{Z}_\alpha^2}{\mathcal{Z}_\Sigma^2 \mathcal{Z}_V} \quad (141)$$

is independent of the scale μ, we have

$$\frac{d\ln\alpha}{d\ln\mu} = -\epsilon + 2\alpha + \mathcal{O}(\alpha^2), \quad (142)$$

giving $\beta_\alpha = 2\alpha$. Similarly, we find

$$\frac{1}{M}\frac{dM}{d\ln\mu} = \alpha + \mathcal{O}(\alpha^2). \tag{143}$$

The bare action for the higher-dimension operators is defined as

$$\int d^4\theta \lambda_b^{(j)} (D\Sigma_b)^2 (\bar{D}\bar{\Sigma}_b)^2 (D^2\Sigma_b \bar{D}^2\bar{\Sigma}_b)^j. \tag{144}$$

The wave function renormalization gives us a relation between the bare and the renormalized λ,

$$\lambda_b^{(j)} = \lambda^{(j)} (\mathcal{Z}_\Sigma)^{-j-2}. \tag{145}$$

In terms of the dimensionless parameter $\tilde{\lambda}^{(j)} = \lambda^{(j)}\mu^{(j+1)(4-\epsilon)}$, we have

$$\lambda_0^{(j)} = \tilde{\lambda}^{(j)} (\mathcal{Z}_\Sigma)^{-j-2} \mu^{(j+1)(\epsilon-4)}, \tag{146}$$

which gives

$$\frac{d\ln\tilde{\lambda}^{(j)}}{d\ln\mu} = 4(j+1) + 4(j+2)\alpha. \tag{147}$$

We now want to study the emergence of the hierarchy. This translates into comparing $(\lambda^{(0)})^{\frac{1}{4}}$ to $(\lambda^{(j)})^{\frac{1}{4(j+1)}}$ in the low-energy limit. We find

$$\frac{d}{d\ln\mu}\left\{\frac{1}{4}\ln(\lambda^{(0)}) - \frac{1}{4(j+1)}\ln(\lambda^{(j)})\right\} = \frac{j}{j+1}\alpha. \tag{148}$$

From formula (148), we conclude that, even though the quantum corrections generate the superspace higher derivatives responsible for supersymmetry breaking, they do not generate the required hierarchy between the leading and the subsequent terms, and thus the solution leading to the broken branch cannot be trusted. In other words, the quantum corrections alone cannot lead to the supersymmetry breaking. Our results show that in the case in which supersymmetry is broken by the mechanism of Ref. [21], these superspace higher derivatives have to be related to the underlying theory or rely on some other mechanism to be generated.

Note that in a different setup in which $\beta_\alpha < 0$ in formula (142) the model would have the opposite behavior in the low energy, leading to a hierarchy and a reliable supersymmetry breaking branch.

V. DISCUSSION

In this work, we have studied low-energy effective theories for complex linear superfields. We have calculated the quantum corrections to the effective action including also the superspace higher-derivative terms on top of the usual corrections to the Kähler potential. This was done by calculating tree-level and one-loop quantum corrections and then integrating out the massive sector.

Our motivation was related to the properties of such operators concerning supersymmetry breaking. We underlined that a hierarchy between the higher-dimension operators is essential for the supersymmetry breaking vacua to be consistent. Turning to the effective theory, we have verified that indeed these operators are generated by the radiative corrections. On the other hand, the required hierarchy between the leading terms and the subsequent ones sufficient for supersymmetry breaking was not found. This led us to conclude that if supersymmetry is broken by the specific superspace higher derivatives these terms have to originate from the underlying theory or another mechanism with different IR properties for the beta function.

We close with a comment on the case in which one does not integrate out the massive modes. In such a case, the auxiliary field deformation terms for all the multiplets have to be taken into account. The theory, after the radiative corrections are introduced, would be of the form (e.g., for a complex linear and a vector superfield)

$$\mathcal{L} \supset \mathrm{D}^2 - F\bar{F} + \frac{1}{M^4}\mathrm{D}^4 + \frac{1}{M^4}F^2\bar{F}^2 \cdots \tag{149}$$

The study of the vacuum structure of a theory like Eq. (149) is left for future work.

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