D-brane solitons in various dimensions

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We construct a D-brane soliton, a composite topological soliton sharing some properties with a D-brane, in a Skyrme model in $4+1$ dimensions, in which Skyrmions are strings ending on a domain wall. We further generalize this D-brane soliton to diverse dimensions. A string, carrying the π_{N-1} topological charge, ends on a domain wall in an O(N) model with higher-derivative terms in $N+1$ dimensions.

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I. INTRODUCTION

Dirichlet (D-)branes are solitonic objects in string theory, on which fundamental strings can end [1]. Since their discovery, D-branes have provided a fundamental tool to study nonperturbative dynamics of string theory and even quantum field theory such as a brane realization [2] and via the AdS/CFT correspondence [3]. The Dirac-Born-Infeld (DBI) action describes the collective-coordinate motion of D-branes. In the DBI action, endpoints of fundamental strings can be regarded as solitonic excitations called BIons [4]. After some attempts in field theory to mimic D-branes [5], constructing the first exact solution of a composite soliton looking like a D-brane was achieved in Ref. [6], in which a lump string ends on a domain wall [7] in a supersymmetric $\mathbb{C}P^1$ nonlinear sigma model. When one looks at this solution from the domain-wall effective theory, it reproduces a BIon, and therefore this solution was named a D-brane soliton. The D-brane soliton was promoted to that in supersymmetric QED, which is a U(1) gauge theory coupled with two charged scalar fields [8,9]. More general D-brane solitons were constructed in supersymmetric $\mathbb{C}P^n$ and Grassmann sigma models, and corresponding supersymmetric U(N) gauge theories [10], in which exact solutions with an arbitrary number of strings at an arbitrary position stretched between multiple domain walls [11,12] were found (for a review see Ref. [13]). Low-energy dynamics, such as scattering of strings stretched between branes, was studied in the moduli-space approximation [14]. In Ref. [10], a negative monopole charge was found at the endpoint of a string, which was later named a boojum [15]; see also Ref. [16], and the boojum charge was also reproduced in a domain-wall effective action [17]. Strings stretched between a brane and antibrane pair, their approximate solutions, and a fate after a pair annihilation were discussed in Ref. [18]. A wall-vortex junction in the large magnetic flux limit was studied in Ref. [19].

The term boojum was taken from condensed matter physics, and in fact, boojums have been already studied in various condensed matter systems [20] such as nematic liquids [21], superfluids at the edge of a container filled with ^{4}He, at the A-B phase boundary of ^{3}He [22], multi-component Bose-Einstein condensates (BEC) of ultracold atomic gases [23], spinor BECs [24], and in even dense quark matter [25]. Among others, in particular, D-brane solitons accompanied by boojums of the same type were constructed in two-component BECs [26].

Strings in D-brane solitons found thus far are of codimension two. In this paper, we offer a very simple model admitting strings (of higher codimensions) ending on a domain wall in higher dimensions. It is an O(N) nonlinear sigma model with higher-derivative (Skyrme-like) term(s) in $N+1$ dimensions and a quadratic potential term with two vacua and thus admitting a domain wall. The O(3) model is a baby-Skyrme model [27] with a quadratic potential [28] in $3+1$ dimensions, while the O(4) model is the Skyrme model with the quadratic potential [29–33] in $4+1$ dimensions. The model admits a baby-Skyrmion string with π_2 lump charge ($N = 3$), a Skyrmion string with π_3 Skyrmion (baryon) charge ($N = 4$), or higher dimensional analogs with π_{N-1} topological charge. We numerically construct solutions of these D-brane solitons for $N = 3, 4, 5, 6$. For the O(3) model in $3+1$ dimensions, we construct a baby-Skyrmion string ending on a domain wall, which is baby-Skyrmion version of the prototype of a lump-string ending on a wall [6]. For the O(4) model in $4+1$ dimensions, we construct a Skyrmion string with π_3 topological (baryon) charge, ending on a domain wall. For the O(N) model in $N+1$ dimensions, we have a higher dimensional Skyrmion-like string of codimensions $N-1$, supported by the π_{N-1} topological charge, ending on a domain wall. For $N > 3$ higher-derivative terms are needed to prevent the string from collapsing to a singular solution. We study the shapes of domain walls pulled by such (finite) strings. It is known that the shape is logarithmic for the O(3) model without the fourth-order derivative term. We find that once the higher-derivative terms are considered,

the shape is $1/\rho^{\#}$, where the power # is fitted to be about 5 and perhaps is universal.

This paper is organized as follows. Section II presents our model: an O(N) sigma model with higher-derivative terms. In Sec. III, we review the D-brane soliton in the O(3) model without any higher-derivative terms in $d = 3 + 1$. In Sec. IV we construct the D-brane soliton in the Skyrme model for O(3) in $d = 3 + 1$ dimensions and for O(4) in $d = 4 + 1$ dimensions. In Sec. V we generalize the construction to $5 + 1$ and $6 + 1$ dimensions, necessitating an even higher-order derivative term; explicitly we consider a sixth-order term for $N = 5, 6$ in $d = 5 + 1$ and $6 + 1$, respectively. Section VI is devoted to a summary and discussion.

II. THE MODEL

We consider the O(N) sigma model with higher-derivative terms in $N + 1$ dimensions whose Lagrangian density reads

$$\mathcal{L} = -\frac{1}{2}\partial_\mu \mathbf{n} \cdot \partial^\mu \mathbf{n} + c_4 \mathcal{L}_4 + c_6 \mathcal{L}_6 - V, \quad (1)$$

where $\mu = 1, ..., N$; $\mathbf{n} = (n_1, ..., n_N)^{\mathrm{T}}$; $\mathbf{n} \cdot \mathbf{n} = 1$ and

$$\mathcal{L}_4 = -\frac{1}{4}(\partial_\mu \mathbf{n} \cdot \partial^\mu \mathbf{n})^2 + \frac{1}{4}(\partial_\mu \mathbf{n} \cdot \partial_\nu \mathbf{n})^2, \quad (2)$$

$$\mathcal{L}_6 = -(\partial_\mu n^{[a}\partial_\nu n^b \partial_\rho n^{c]})^2$$
$$= -\frac{1}{6}(\partial_\mu \mathbf{n} \cdot \partial^\mu \mathbf{n})^3 + \frac{1}{2}(\partial_\mu \mathbf{n} \cdot \partial^\mu \mathbf{n})(\partial_\nu \mathbf{n} \cdot \partial_\rho \mathbf{n})^2$$
$$- \frac{1}{3}(\partial_\mu \mathbf{n} \cdot \partial^\nu \mathbf{n})(\partial_\nu \mathbf{n} \cdot \partial^\rho \mathbf{n})(\partial_\rho \mathbf{n} \cdot \partial^\mu \mathbf{n}), \quad (3)$$

where $a, b, c = 1, ..., N$ and the antisymmetrization is defined as

$$T^{[abc]} \equiv \frac{1}{3!}(T^{abc} + T^{bca} + T^{cab} - T^{cba} - T^{bac} - T^{acb}). \quad (4)$$

We are using the mostly positive metric. In the absence of the potential, the symmetry is O(N) which is spontaneously broken to O($N - 1$). The target space of the sigma model is

$$\mathrm{O}(N)/\mathrm{O}(N - 1) \simeq S^{N-1}. \quad (5)$$

We consider the potential, given by

$$V = \frac{1}{2}m^2(1 - n_N^2). \quad (6)$$

The vacua are

$$+ : n_N = +1, \quad (7)$$

$$- : n_N = -1. \quad (8)$$

The above potential breaks the O(N) symmetry to O($N - 1$) explicitly.

Note that the Lagrangian density (3) is the baryon density squared when $N = 4$ and is the basis of the BPS Skyrme model [34]. In the present formulation of the term, N can be larger than 4 but then the term no longer represents the π_{N-1} charge.

We will consider a domain wall extended in the $z \equiv x^N$ direction, which interpolates between the $-$ and the $+$ vacua of Eqs. (7) and (8). The domain-wall solution is given by

$$n_N = \tanh(mz), \quad (9)$$

and is an exact solution: it is the sine-Gordon soliton [for the O(3) model [7], the O(4) model [29–32,35], and the O(N) model [33]].

In this paper we are interested in the soliton junction composed by the latter domain wall and a "string" carrying π_{N-1} charge. This is possible because zero modes (moduli) are localized on the domain wall, which originate from the spontaneously broken O($N - 1$) symmetry in the presence of the wall. Those moduli are U(1) for the O(3) model [7], S^2 for the O(4) model [30–32,35], and S^{N-2} for the O(N) model [33]. One could construct textures (Skyrmions) supported by $\pi_{N-2}(S^{N-2}) \simeq \mathbb{Z}$ [30–33,36–39], which are localized on the wall. Instead, here, we discuss defects again supported by $\pi_{N-2}(S^{N-2}) \simeq \mathbb{Z}$. As we will see, these defects actually extend in the direction perpendicular to the wall, and it turns out that they are Skyrmion strings supported by the $\pi_{N-1}(S^{N-1}) \simeq \mathbb{Z}$ in the bulk.

An appropriate ansatz for the configuration is

$$\mathbf{n} = \left(\frac{\tilde{\mathbf{x}}}{\rho}\sin f(\rho, z), \cos f(\rho, z)\right)^{\mathrm{T}}, \quad (10)$$

where $\tilde{\mathbf{x}} = (x^1, ..., x^{N-1})$; $\rho^2 = (x^1)^2 + \cdots + (x^{N-1})^2$ is the $(N - 1)$-dimensional radial coordinate and $z \equiv x^N$. Inserting the above ansatz into the Lagrangian density (1) we obtain the following static Lagrangian density:

$$-\mathcal{L} = \frac{1}{2}f_\rho^2 + \frac{1}{2}f_z^2 + \frac{N-2}{2\rho^2}\sin^2 f + c_4 \left[\frac{N-2}{2\rho^2}\sin^2 f(f_\rho^2 + f_z^2)\right.$$
$$\left. + \frac{(N-3)(N-2)}{4\rho^4}\sin^4 f\right]$$
$$+ c_6 \left[\frac{(N-3)(N-2)}{2\rho^4}\sin^4 f(f_\rho^2 + f_z^2)\right.$$
$$\left. + \frac{(N-4)(N-3)(N-2)}{6\rho^6}\sin^6 f\right]$$
$$+ \frac{1}{2}m^2\sin^2 f, \quad (11)$$

where $f_x \equiv \partial_x f$ and the equation of motion reads

$$f_{\rho\rho} + f_{zz} + \frac{N-2}{\rho} f_\rho + c_4 \frac{N-2}{\rho^2} \sin^2 f \left(f_{\rho\rho} + f_{zz} + \frac{N-4}{\rho} f_\rho \right) + c_4 \frac{N-2}{2\rho^2} \sin(2f)(f_\rho^2 + f_z^2)$$

$$+ c_6 \frac{(N-3)(N-2)}{\rho^4} \sin^4 f \left(f_{\rho\rho} + f_{zz} + \frac{N-6}{\rho} f_\rho \right) + c_6 \frac{(N-3)(N-2)}{\rho^4} \sin^2 f \sin(2f)(f_\rho^2 + f_z^2) - \frac{N-2}{2\rho^2} \sin(2f)$$

$$- c_4 \frac{(N-3)(N-2)}{2\rho^4} \sin^2 f \sin(2f) - c_6 \frac{(N-4)(N-3)(N-2)}{2\rho^6} \sin^4 f \sin(2f) - \frac{1}{2} m^2 \sin 2f = 0. \tag{12}$$

The π_{N-1} charge is given by

$$C = \frac{\Gamma(\frac{N}{2})}{2\pi^{\frac{N}{2}}} \int d^{N-1}x \frac{1}{(N-1)!} \epsilon^{i_1 \cdots i_{N-1}} \epsilon^{a_1 \cdots a_N} \partial_{i_1} n_{a_1} \cdots \partial_{i_{N-1}} n_{a_{N-1}} n_{a_N}$$

$$= \frac{\Gamma(\frac{N}{2})}{\sqrt{\pi}\Gamma(\frac{N-1}{2})} \int d\rho \sin^{N-2}(f) \partial_\rho f$$

$$= -\cos f (\text{sign} \sin f)^{N-1} \frac{\Gamma(\frac{N}{2})}{\sqrt{\pi}\Gamma(\frac{N-1}{2})} {}_2F_1 \left(\frac{1}{2}, \frac{3-N}{2}; \frac{3}{2}; \cos^2 f \right) \Big|_{f(0)}^{f(\infty)} = -1, \tag{13}$$

where $i_\# = 1, \ldots, N-1$; $a_\# = 1, \ldots, N$; Γ is the gamma function and ${}_2F_1$ is the usual hypergeometric function. (Note that the overall sign is chosen for convenience and is opposite to the conventional choice.) With the boundary conditions $f(0) = 0$ and $f(\infty) = \pi$, the above charge integrates to $C = -1$ for all $N \geq 3$, as shown in the last step of the last line. The charge density is given by

$$\mathcal{C} = \frac{1}{\rho^{N-2}} \sin^{N-2}(f) \partial_\rho f. \tag{14}$$

The reason for including higher-derivative terms in the action (1) is to prevent the "string" from collapsing to a singular solution (at a finite distance from the domain wall). Let us consider a scaling argument for just the coordinates transverse to the domain wall, $\tilde{\mathbf{x}} \to \mu\tilde{\mathbf{x}}$. The energy thus scales as

$$E \to -\int d^{N-1}x[\mu^{3-N}\mathcal{L}_2^\rho + \mu^{1-N}\mathcal{L}_2^z + \mu^{5-N}c_4\mathcal{L}_4^\rho$$

$$+ \mu^{3-N}c_4\mathcal{L}_4^z + \mu^{7-N}c_6\mathcal{L}_6^\rho + \mu^{5-N}c_6\mathcal{L}_6^z - \mu^{1-N}V], \tag{15}$$

where $\mathcal{L}_d^{\rho,z}$ denotes the part of the Lagrangian density of dth order in derivatives having two derivatives in ρ and z, respectively [40]. Stability of the solitonic solution requires a positive power of μ for at least one term. If the power is zero (i.e. μ^0), the term is classically conformal and cannot provide stabilization.

For $N = 3$ we can have a finite-size (lump-charged) string with just the Dirichlet term and thus no higher-derivative terms ($c_4 = c_6 = 0$). The apparent instability in the z coordinate does not affect the solution because of factorization (see the next section). Note also that the ρ part of the energy is classically conformal; a characteristic of

lumps. For $N = 4$ and just the Skyrme term turned on ($c_4 = 1$ and $c_6 = 0$) we have a stable finite-size Skyrmion-charged string. At each constant-z slice, a domain wall becomes a spherical domain wall studied in Ref. [31] which is nothing but a Skyrmion. For $N = 5, 6$ we need the sixth-order derivative term for stabilizing the string. We will consider these three cases in turn in the next sections.

III. THE PURE SIGMA MODEL

In this section, we use only the kinetic term and the potential, i.e. the Lagrangian density (11) with $c_4 = c_6 = 0$.

This system is special for $N = 3$ where it corresponds to an integrable sector in the supersymmetric O(3) sigma model [11]. For $N = 2$, the system is again integrable, but is somehow trivial as it describes two domain walls orthogonally assembled.

In order to uncover the domain-wall structure of the system, let us change variables as

$$f = 2 \arctan g, \tag{16}$$

which gives us the nonlinear equation of motion (we keep N explicit for illustrative purposes here)

$$(1 + g^2)g_{\rho\rho} - 2gg_\rho^2 + \frac{N-2}{\rho}(1 + g^2)g_\rho$$

$$+ (1 + g^2)g_{zz} - 2gg_z^2 - \frac{N-2}{2\rho^2}g(1 - g^2)$$

$$- m^2 g(1 - g^2) = 0. \tag{17}$$

Regrouping this, we get

$$g_{\rho\rho} + \frac{N-2}{\rho}g_\rho + g_{zz} - \frac{N-2}{\rho}g - m^2 g = 0, \quad (18)$$

with the nonlinear constraint (assuming the above equation is satisfied)

$$\frac{g_\rho^2}{g} - \frac{N-2}{\rho^2}g + \frac{g_z^2}{g} - m^2 g = 0, \quad (19)$$

and inserting a factorizing ansatz $g = R(\rho)Z(z)$ into the latter constraint, we get

$$R = \rho^{\pm\sqrt{N-2}}, \qquad Z = e^{\pm m(z-z_0)}. \quad (20)$$

Inserting this into Eq. (18) yields ($N \geq 2$)

$$\mp\sqrt{N-2} \pm (N-2)^{\frac{3}{2}} = 0, \quad (21)$$

which determines $N = 3$ (or the trivial solution $N = 2$ which is physically not so interesting). The exact solution in the nonlinear sigma model case for $N = 3$ thus reads

$$g = r^{\pm 1}\exp\{\pm m(z-z_0)\}. \quad (22)$$

The two signs are independent of each other and all four possibilities are solutions to the equation of motion. They are however not all physically different as the Lagrangian is invariant under $f \to \pi - f$ which corresponds to $g \to g^{-1}$. Using this fact, we see that there are two distinct configurations which we can think of as a wall junction and antiwall junction. These two are related by sending the coordinate $z \to -z$.

Factorization is possible when $N = 3$ as we have just shown above, but only possible when $N = 3$. For illustrative purposes, let us implement the domain-wall structure explicitly by setting

$$g = \exp\{mz\}h(\rho, z), \quad (23)$$

and study the string solutions on both sides of the domain wall. Notice that $m \to \pm m$ and $z \to z - z_0$ recovers the domain wall/anti-domain-wall and position modulus, respectively. In order not to clutter the notation too much in the following, we will just use e^{mz}. The equation of motion can now be written as

$$h_{\rho\rho}(1 + e^{2mz}h^2) - 2e^{2mz}hh_\rho^2 + \frac{N-2}{\rho}(1 + e^{2mz}h^2)h_\rho$$
$$+ h_{zz}(1 + e^{2mz}h^2) - 2e^{2mz}hh_z^2 + 2m(1 - e^{2mz}h^2)h_z$$
$$- \frac{N-2}{\rho^2}(1 - e^{2mz}h^2)h = 0. \quad (24)$$

The field h will describe the junction in the (fixed) background of the domain wall which is generally a solution to the above partial differential equation and hence a function of both ρ and z. Taking the limit $z \to \infty$, the equation of motion (24) becomes independent of z:

$$h_{\rho\rho} - \frac{2h_\rho^2}{h} + \frac{N-2}{\rho}h_\rho + \frac{N-2}{\rho^2}h = 0, \quad (25)$$

and a power function ansatz $h = \rho^b$ yields the following two solutions:

$$b_+ = N - 2, \qquad b_- = -1. \quad (26)$$

Taking now the limit $z \to -\infty$, the equation of motion (24) becomes again independent of z:

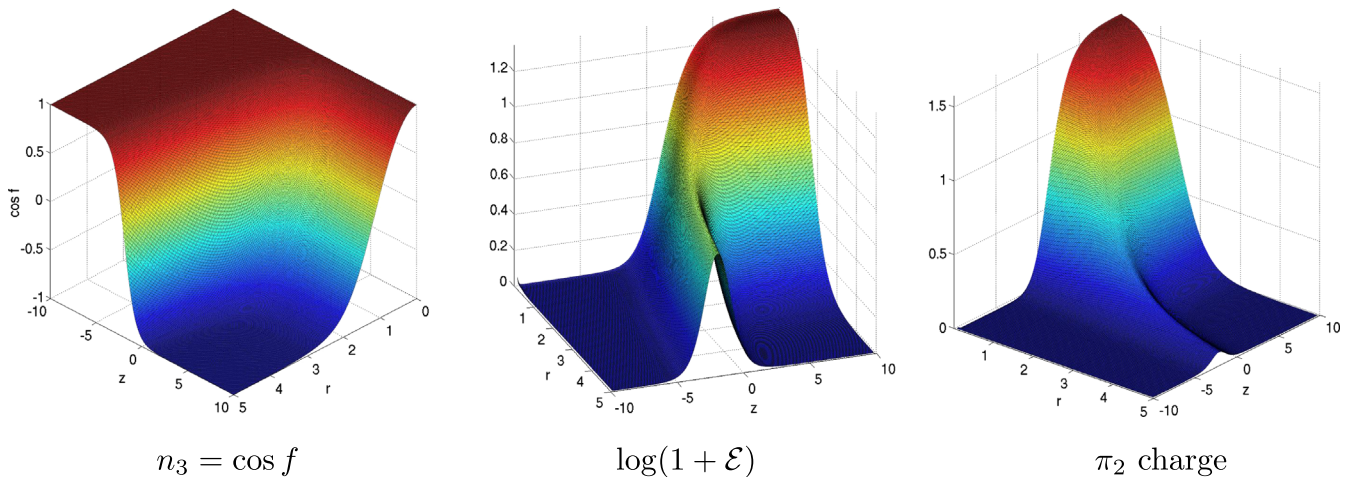$$h_{\rho\rho} + \frac{N-2}{\rho}h_\rho - \frac{N-2}{\rho^2}h = 0, \quad (27)$$

$$n_3 = \cos f \qquad\qquad \log(1 + \mathcal{E}) \qquad\qquad \pi_2 \text{ charge}$$

FIG. 1 (color online). O(3) soliton junction of a domain wall (in the z direction) and a lump string carrying π_2 charge. The three panels show the field n_3, the energy density on a logarithmic scale and the charge density.
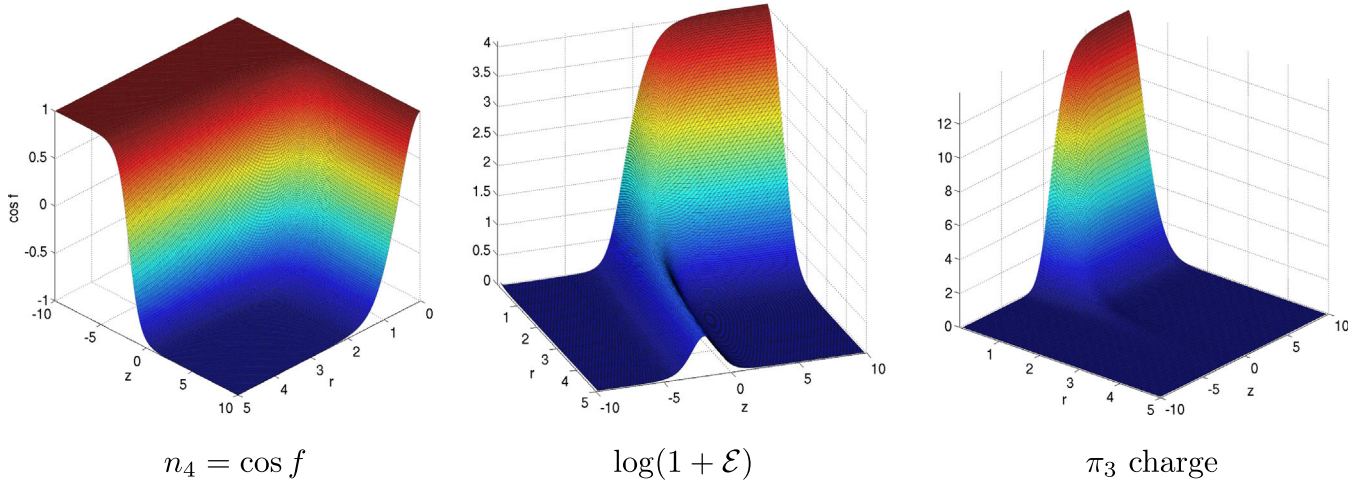
$$n_4 = \cos f \qquad\qquad \log(1 + \mathcal{E}) \qquad\qquad \pi_3 \text{ charge}$$

FIG. 2 (color online). O(4) soliton junction of a domain wall (in the z direction) and a Skyrmion string carrying π_3 charge. The three panels show the field n_4, the energy density on a logarithmic scale and the charge density.
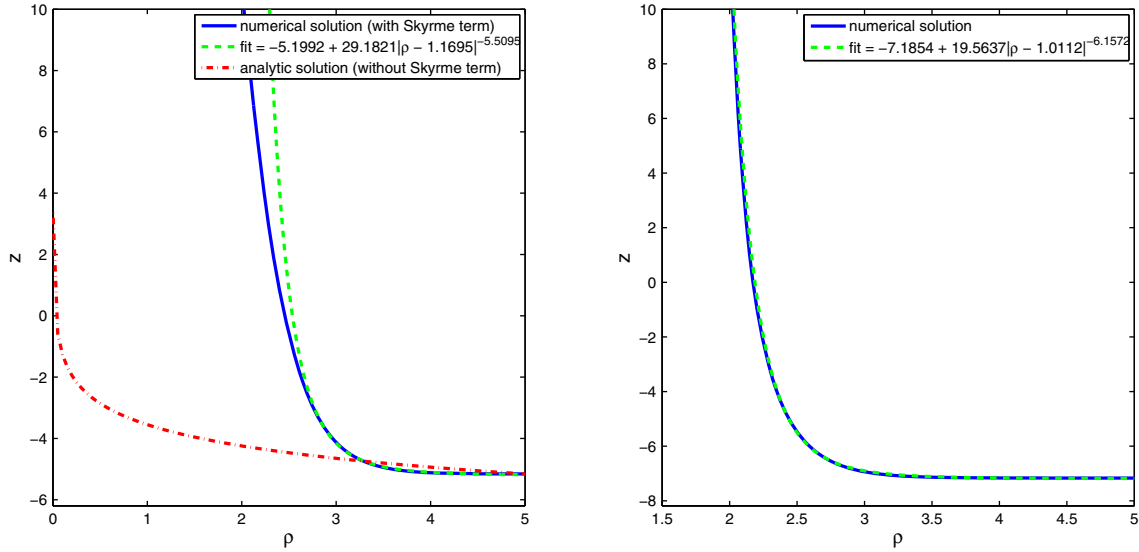

FIG. 3 (color online). Contour line of the soliton junction ($\cos f = 0$), describing the bending of the domain wall due to the attached string. The left-hand (right-hand) side panel is for the O(3)-case [O(4)-case] and the fit is made with the numerical data in the region $z < -3$ ($z < -4$).

and the power function ansatz $h = \rho^b$ now yields the two solutions

$$b_+ = 1, \qquad b_- = -N + 2. \tag{28}$$

Having two different signs on each side of the domain wall corresponds to a composite soliton made of a wall and an antiwall and thus is not a solution on the fixed background. Therefore we need to pick the same sign on each side of the domain wall, which corresponds to choosing a string or an antistring (or alternatively which direction the string is pointed). The factorization is again visible for $N = 3$ because the power function ansatz is the same on both sides of the domain wall (and in fact as we showed earlier, it is a

solution in all space). In principle we could contemplate a solution interpolating the two different power functions when $N \geq 4$, but as shown by a scaling argument in Sec. II [see Eq. (15)], such solution will have a singular (i.e. vanishing thickness) string and the junction will also be pointlike. We can blow up such solutions by adding higher-derivative terms, as shown in Eq. (15). This will be the topic of the next sections.

IV. THE SKYRME MODEL

In this section, we turn on the Skyrme term in the Lagrangian density (11), viz. $c_4 = 1$ and $c_6 = 0$. This will allow for stable finite-size strings for $N = 4$ as shown by
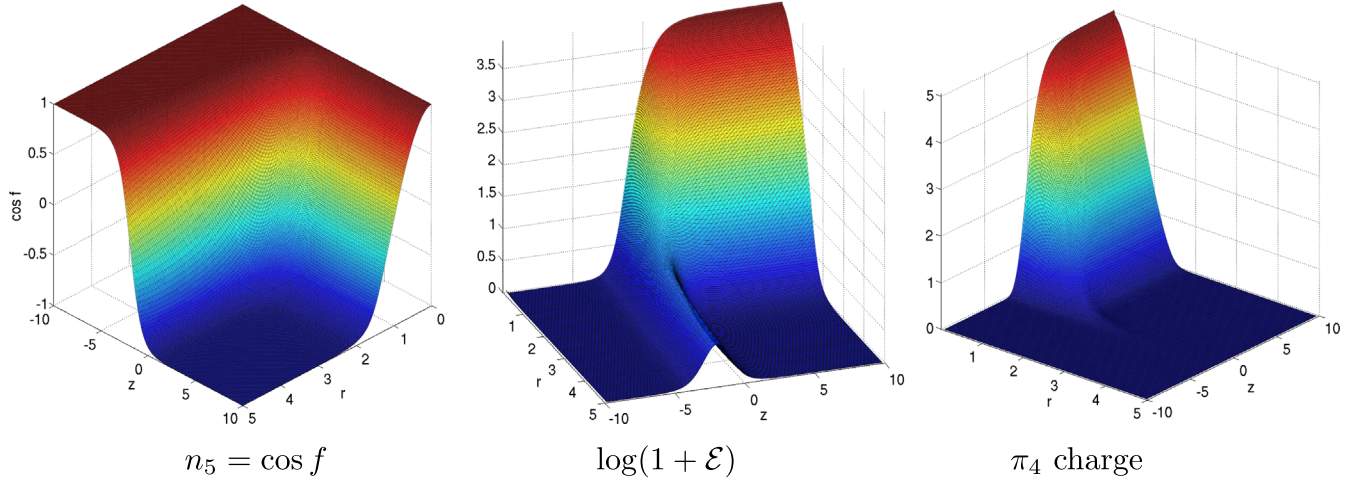
$$n_5 = \cos f \qquad\qquad \log(1+\mathcal{E}) \qquad\qquad \pi_4 \text{ charge}$$

FIG. 4 (color online). O(5) soliton junction of a domain wall (in the z direction) and a string carrying π_4 charge. The three panels show the field n_5, the energy density on a logarithmic scale and the charge density.
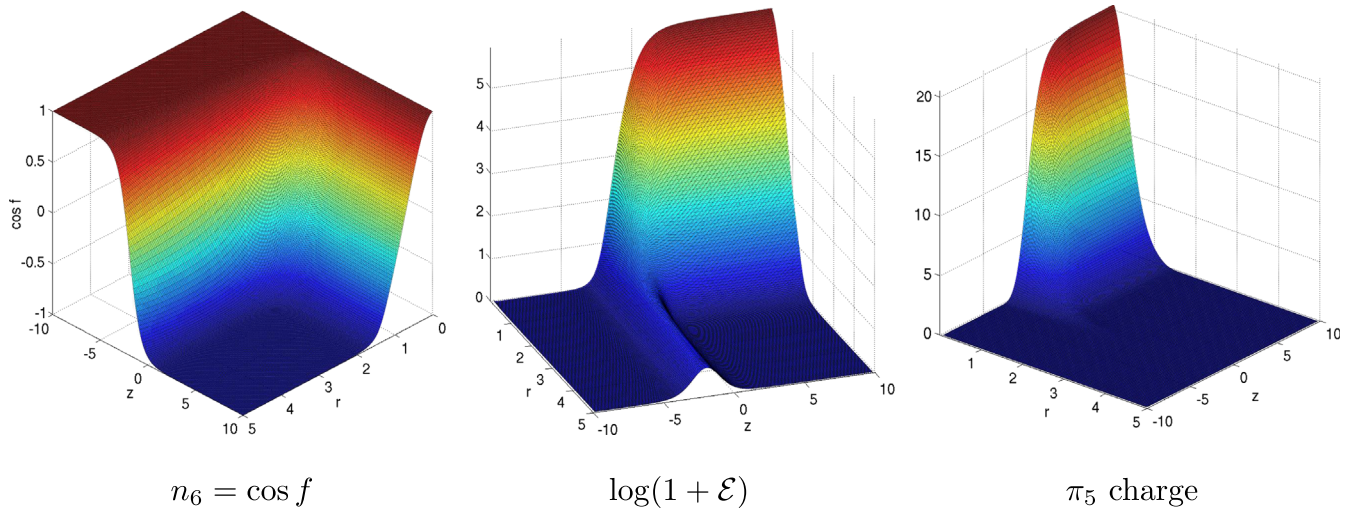

$$n_6 = \cos f \qquad\qquad \log(1+\mathcal{E}) \qquad\qquad \pi_5 \text{ charge}$$

FIG. 5 (color online). O(6) soliton junction of a domain wall (in the z direction) and a string carrying π_5 charge. The three panels show the field n_6, the energy density on a logarithmic scale and the charge density.

the scaling argument in Eq. (15). The equation of motion (12), in this case, is not integrable and we need to turn to numerical methods to obtain solutions.

We will employ a finite-difference scheme on a quadratic square lattice with 256^2 lattice sites and relax initial guesses with the relaxation method.

For completeness, we also calculate the case of $N = 3$ with the Skyrme term, which makes the string thicker than in the sigma model case.

In Figs. 1 and 2 are shown the numerical solutions, the corresponding energy densities and charge densities, for the O(3) and O(4) model, respectively.

Interestingly, the shape of the wall junction is altered somewhat drastically. In Fig. 3 is shown the contour line of the field $n_N = 0$ in the (ρ, z)-plane. In the O(3) case, a comparison with the analytic sigma-model solution is

shown with the red dashed-dotted line. A fit of the asymptotic part of the junction is also shown with a green dashed line. The function is found to be a power function,

$$\text{contour} = z_0 + b|\rho - w|^p, \qquad (29)$$

where z_0 is the position of the domain wall, b is a proportionality constant, w is the width of the string and p is the sought-after power describing the bending of the domain wall. The fits find p to be about 5–6.

V. THE 6TH ORDER MODEL

In this section, we want to consider $N = 5, 6$ which requires at least a sixth-order derivative term, in order for the string to have a finite thickness, see Eq. (15).
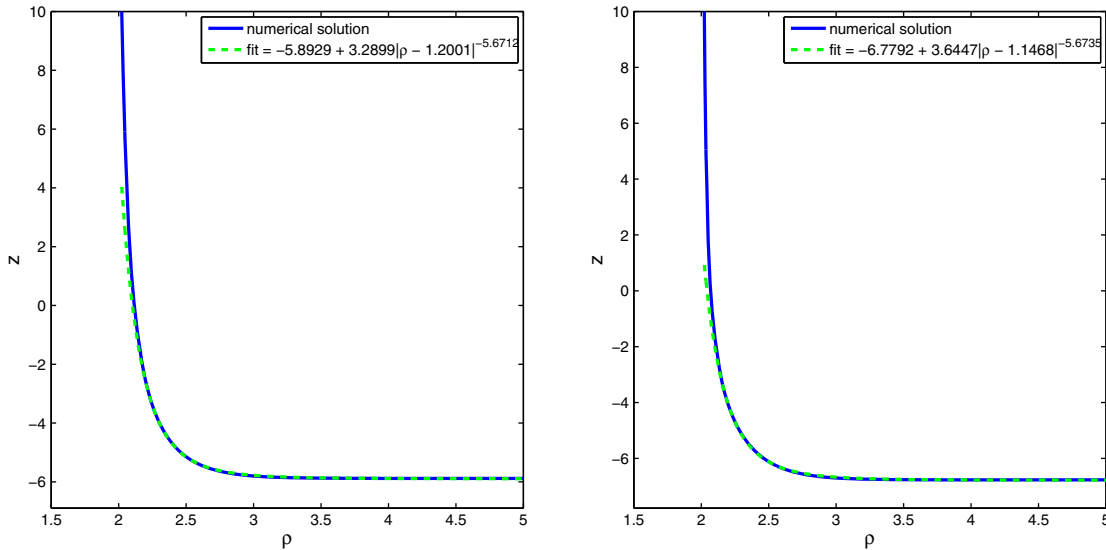
FIG. 6 (color online).   Contour line of soliton junction ($\cos f = 0$), describing the bending of the domain wall due to the attached string. The left-hand (right-hand) side panel is for the O(5)-case [O(6)-case] and the fit is made with the numerical data in the region $z < -3$.

We will again use a finite-difference scheme on a quadratic square lattice with 256^2 lattice sites and relax initial guesses with the relaxation method. In Figs. 4 and 5 are shown the numerical solutions, the corresponding energy densities and charge densities, for the O(5) and O(6) model, respectively.

We consider again the shape of the wall junction and show the contour line of the field $n_N = 0$ in the (ρ, z)-plane as well as a fit of the type (29) in Fig. 6. The powers p are again fitted to be about 5–6.

Finally, we consider the string charge which as function of z has to interpolate from a full charge (1) to zero across the wall junction. Hence we plot Eq. (13) across the domain-wall junction for all the obtained solutions in
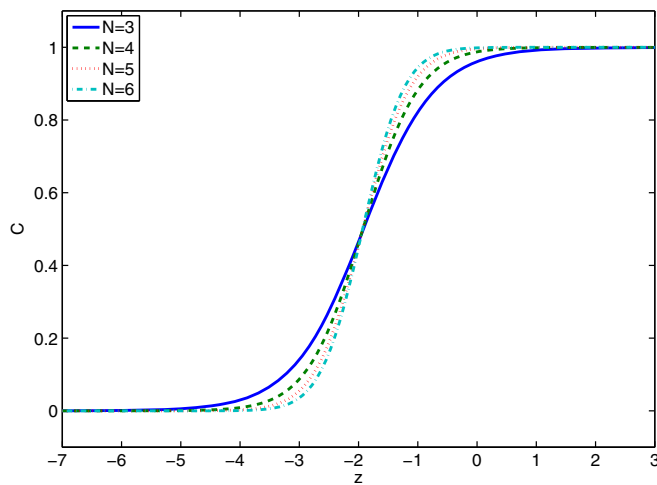

FIG. 7 (color online).   String charge C as function of z. Far away from the domain wall it is 0 and 1, on the left- and right-hand side of the domain wall, respectively.

Fig. 7. It is seen that the transition becomes more steep with increasing N, which may be expected just on dimensional grounds.

VI. SUMMARY AND DISCUSSION

We have constructed D-brane solitons, composite solitons of strings ending on a domain wall in an O(N) model with a higher-derivative term in $d = N + 1$ dimensions. For $N = 3$, it is a baby-Skyrmion string ending on a domain wall in $d = 3 + 1$, while for $N = 4$, it is a Skyrmion string ending on a domain wall in $d = 4 + 1$. In general, a string supported by the π_{N-1} topological charge ends on a domain wall and bends the domain wall like $1/\rho^{\#}$. In this paper, we have considered only $N = 3, 4, 5, 6$ where we have calculated the needed terms explicitly. A generalization to higher N can be carried out by considering the higher-order derivative term

$$\mathcal{L}_{2m} = -(\partial_{\mu_1}^{[a_1} \cdots \partial_{\mu_m}^{a_m]})^2, \qquad (30)$$

where $\mu_i = 1, \ldots, N$; $a_i = 1, \ldots, N$; $i = 1, \ldots, m$ and $m = \lceil N/2 \rceil$ ($\lceil x \rceil$ rounds up a real number to the nearest integer).

In this paper, strings are of various codimensions depending on the dimension, while "D-branes" are all of domain-wall type, that is, of codimension one. For instance, a vortex string (of codimension two) with a confined Skyrmion was constructed in Refs. [32,41]. The generalization to higher codimensions for D-branes remains a future problem. In supersymmetric theories, all possible composite BPS solitons were classified in Ref. [42], which may be useful for this study.

Field theory D-branes beyond the semiclassical approach were studied in Ref. [43], in which the bulk-boundary

correspondence was proposed. Our study could be applied to that direction as well.

ACKNOWLEDGMENTS

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