Explaining the proton radius puzzle with disformal scalars

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We analyze the consequences of a disformal interaction between a massless scalar and matter particles in the context of atomic physics. We focus on the displacement of the atomic energy levels that it induces, and in particular the change in the Lamb shift between the 2s and 2p states. We find that the correction to the Lamb shift depends on the mass of the fermion orbiting around the nucleus, implying a larger effect for muonic atoms. Taking the cutoff scale describing the effective scalar field theory close to the QCD scale, we find that the disformal interaction can account for the observed difference in the proton radius of muonic versus electronic hydrogen. Explaining the proton radius puzzle is only possible when the scalar field is embedded in nonlinear theories which alleviate constraints from collider and stellar physics. Short distance properties of the Galileon where nonperturbative effects in vacuum are present ensure that unitarity is preserved in high-energy particle collisions. In matter, the chameleon mechanism alleviates the constraints on disformal interactions coming from the burning rates for stellar objects. We show how to combine these two properties in a single model which renders the proposed explanation of the proton radius puzzle viable.

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I. INTRODUCTION

The Lamb shift is one of the most precisely studied effect in atomic physics. Its relevance has been recently enhanced by the discovery that the Lamb shift behaves differently when muonic atoms are considered, compared to their electronic siblings. The Lamb shift can be used to deduce the value of the proton radius and muonic versus electronic discrepancies imply that the proton radius is lower by four percent in muonic experiments. The combined discrepancy between the proton radius as inferred from muonic hydrogen and that inferred from electronic hydrogen now stands at 7σ. While the muonic results currently only come from one group at Paul Scherrer Institute no systematic uncertainty has been identified that could explain the size of the discrepancy [1].[1] This is the proton radius puzzle which has resisted explanation with standard model physics [3]. Could this be an indication of the need for new physics? Current attempts to explain the proton radius anomaly with new physics have introduced new force carriers with masses in the 1–100 MeV range, which may have non-universal couplings [4–7]; however, these are difficult to reconcile with existing constraints on dark forces. New forces deduced from hidden photons or conformally coupled scalars have also been invoked with limited success [8,9]. In this work we take a different approach, introducing a nearly massless scalar degree of freedom which interacts with matter species through a universal disformal coupling.

The existence of nearly massless scalar fields is strongly suggested by the acceleration of the Universe, as they could act as dark energy or arise in theories of massive gravity as the scalar polarization of a low-mass graviton. This seems far removed from the proton radius puzzle, but in this work we will show that these two motivations for considering new physics can be connected. When conformally coupled to matter, new light scalars are severely constrained by the Cassini probe [10] and tests of the strong equivalence principle such as the lunar ranging experiment [11]. This results in a strong bound on the coupling between the scalar and matter, β, that can, however, be alleviated when screening mechanisms are invoked as nontrivial self-interactions of the field can naturally reduce the strength of the force to observationally undetectable levels in experimental environments. They do this by allowing the properties of the scalar field to vary with the environment. For example; in the chameleon model [12,13] the mass of the scalar field increases in dense environments, in the Galileon model [14], the prefactor of the kinetic term becomes large in the vicinity of dense sources. Even for models with screening mechanisms, however, suitably chosen laboratory tests of theories with screening mechanisms can still be constraining; for models such as chameleons, the conformal coupling to matter could be tested in neutron experiments where the energy levels of the neutron in the terrestrial gravitational field are measured [15,16]. In this article, we will investigate new tests at the atomic level, due to a disformal coupling between matter and scalars, and we will rely on a screening mechanism to alleviate constraints from higher density environments such

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[1]There is an ongoing debate on the proton radius discrepancy inferred from electron-proton scattering experiments [2].

as stellar interiors. Doing so, our theory can be embedded in a dark energy/modified gravity model such as $f(R)$ in the large curvature regime which satisfies all the local gravitational tests thanks to the chameleon mechanism. For these models, gravity is modified on cosmological scales by the scalar field which is nearly massless while being heavy enough in stellar media. In the atomic environment, the scalar field is also nearly massless.

Bekenstein has shown [17] that the most general metric that can be constructed from $g_{\mu\nu}$ and a scalar field that respects causality and the weak equivalence principle is;

$$\tilde{g}_{\mu\nu} = A(\phi, X)g_{\mu\nu} + B(\phi, X)\partial_\mu\phi\partial_\nu\phi, \qquad (1.1)$$

where the first term gives rise to conformal couplings between the scalar field and matter, and the second term is the disformal coupling. Here $X = (1/2)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$. The conformal coupling gives rise to Lagrangian interaction terms of the form

$$\mathcal{L} \supset A(\phi, X)T^\mu_{J\mu} \qquad (1.2)$$

and the disformal interactions give rise to Lagrangian interaction terms of the form

$$\mathcal{L} \supset \frac{B(\phi, X)}{2}\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}_J, \qquad (1.3)$$

where $T^{\mu\nu}_J$ is the energy-momentum tensor of matter fields in the Jordan frame, defined by the metric $g^J_{\mu\nu} = A(\phi, X)g_{\mu\nu}$. The conformal coupling gives rise to Yukawa-type long range forces between matter fields. The disformal coupling has no influence on static configurations of matter as no disformal interaction between static nonrelativistic objects is generated. This follows from the vanishing of the coupling (1.3) when the only nonvanishing component of $T^{\mu\nu}_J$ is T^{00}_J and ϕ is static. This can be extended to all the higher-order terms involving more than two derivatives of ϕ obtained by expanding the matter Lagrangian in perturbations of $B(\phi, X)\partial_\mu\phi\partial_\nu\phi$. This means that constraints on disformal couplings are much weaker than on their conformal counterparts. The leading disformal interaction between two static bodies is a quantum effect at one loop [18–20] which appears at the $1/M^8 = B^2(\phi_0, 0)$ level where the loop has been calculated in a uniform scalar background ϕ_0. This gives rise to a potential of the form $1/M^8r^7$, which will be analyzed and tested here.

We will find that the proton radius puzzle can be explained using the mass dependent disformal potential generated at one loop in $1/M^8r^7$ when M lies close to the QCD scale $M = \Lambda_{QCD} \approx 220$ MeV. This is appropriate for a model which we only require to be valid at low energies, below the QCD phase transition where free quarks and gluons confine into hadrons and which takes place at a

temperature around Λ_{QCD}, and after big bang nucleosynthesis (BBN) in cosmology. This choice of coupling scale would also lead to small anomalous radii for the deuteron [21] and the He nucleus [22]. With new experimental results for deuterons and He nuclei to be soon published, this is a prediction of our model which will be soon tested [23].

In Sec. II we recall salient properties of disformally coupled scalars. In Sec. III we apply these results to the proton radius puzzle, determine the required value of the coupling constant M and predict the ensuing deviations for helium and the deuteron. The disformal coupling, viewed as a higher-order operator, would lead to a violation of unitarity at high energy. It would also increase the burning of stars. In Sec. IV we show that the constraints on disformal couplings coming from the burning of stars can be alleviated in nonlinear models for which the chameleon mechanism, whereby the mass of the scalar becomes large in dense environments, prevents the creation of scalars in stellar media. In Sec. V we turn to the strong constraints which spring from the absence of any violation of unitarity in particle collision at high energy. This is alleviated by embedding the disformal coupling in Galileon models which pass these tests by a novel mechanism whereby classical configurations akin to black holes in trans-Planckian scattering are formed. We also show how the chameleon mechanism, which is effective in dense environments, and the Galileon, which applies in vacuum such as the ones of atomic and particle physics, are compatible. We then conclude in Sec. VI.

II. LIGHT SCALAR FIELDS

We consider a scalar field coupled to matter defined by the action

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N} - \frac{1}{2}(\partial\phi)^2\right) + S_m(\psi_i, \tilde{g}_{\mu\nu}), \tag{2.1}$$

where the metric governing the interactions between the scalar and matter is given by

$$\tilde{g}_{\mu\nu} = A(\phi)g_{\mu\nu} + B(X)\partial_\mu\phi\partial_\nu\phi. \tag{2.2}$$

This is not the most general scalar metric as given by Bekenstein in Eq. (1.1); however, it describes all the leading-order effects of the disformal and conformal couplings.

The metric $\tilde{g}_{\mu\nu}$ is the metric with respect to which matter is conserved. We impose that the conformal coupling function $A(\phi)$ is the only source of (soft) breaking of the shift symmetry $\phi \to \phi + c$, which forces the coupling $B(X)$ to be independent of ϕ. We take a conformal coupling to matter of the form

$$A(\phi) = 1 + \frac{\beta\phi}{m_{\mathrm{Pl}}}, \qquad (2.3)$$

which is the lowest-order breaking of the shift symmetry, and we assume that the disformal coupling function can be expanded as

$$B(X) = \frac{1}{M^4}\left(1 + \sum_{n\geq 1} a_n \frac{X^n}{M^{4n}}\right). \qquad (2.4)$$

As M will be the lowest-energy scale in the disformal sector of our theory, we take the cutoff scale, that defines the model as an effective theory at low energy, to lie just above the scale M. We assume a hierarchy between the scales M and m_{Pl}/β. As we will find that M is of the order the QCD scale, and assuming that $\beta \sim \mathcal{O}(1)$, protecting these scales is similar to the hierarchy problem of the Standard Model. We have nothing to add to the solutions of this problem except to note that hierarchies between disformal and conformal coupling scales arise naturally in theories of massive gravity [24]. When $\beta \sim \mathcal{O}(1)$ in laboratory interactions the conformal coupling is so weak that it can be safely neglected. The disformal coupling scale M appears as a one-loop interaction. For matter sources of masses m_1, m_2 separated by a distance r the potential interaction mediated by the disformal scalars is [19,20]

$$V(r) = -\frac{3m_1 m_2}{32\pi^3 r^7 M^8}, \qquad (2.5)$$

when the scalar is canonically normalized. The coupling scale M is in principle unknown and should be fixed by observations. Here we will focus on theories where this scale is close to the QCD scale

$$M \sim \Lambda_{\mathrm{QCD}}, \qquad (2.6)$$

where $\Lambda_{\mathrm{QCD}} = 217^{+25}_{-23}$ MeV is the strong interaction scale of quantum chromodynamics (QCD). This phenomenological choice is compatible with the desire to view our disformal scalars as a low- energy description of some unknown physics which should appear for scales larger than M. Below the scale M, the physics only involves matter particles which are the electrons, the protons and the neutrons as formed during big bang nucleosynthesis (BBN). We take this scale M to be close to the confinement scale where the QCD phase transition takes places and which is estimated to be close to Λ_{QCD}. At higher energies, the model must either be completed by some new Ultra Violet (UV) physics, or as we shall see with the example of the Galileon, enter a new phase of the model where perturbative calculations fail and nonperturbative phenomena should be taken into account.

Experimental constraints on disformal couplings have been extensively studied in [20] and these are reproduced in Table I. We will discuss in later sections how to make our requirement for $M \sim \Lambda_{\mathrm{QCD}}$ compatible with all current constraints.

In what follows we will restrict ourselves to the leading order effects of the disformal coupling between the scalar field and matter. Therefore we calculate only to leading order in $1/M^4$, implying the action can be expanded as

$$S = \int d^4x \sqrt{-g}\left(\frac{R}{16\pi G_N} - \frac{1}{2}(\partial\phi)^2 + \frac{1}{M^4}\partial_\mu\phi\partial_\nu\phi T_J^{\mu\nu}\right) + S_m(\psi_i, A(\phi)g_{\mu\nu}), \qquad (2.7)$$

where we have introduced the Jordan frame energy-momentum tensor

TABLE I. Summary of the constraints on the disformal coupling scale M derived in [20]. Lab. vac. means the constraint derives from an experiment conducted in a laboratory vacuum on Earth. Horizontal branch means the constraint derives from observations of horizontal branch stars, and similarly for constraints labeled Sun and Supernova SN1987a.

Source of bound	Lower bound on M in GeV	Environment
Unitarity at the LHC	30	Lab. vac.
CMS mono-lepton	120	Lab. vac.
CMS mono-photon	490	Lab. vac.
Torsion Balance	7×10^{-5}	Lab. vac.
Casimir effect	0.1	Lab. vac.
Hydrogen spectroscopy	0.2	Lab. vac.
Neutron scattering	0.03	Lab. vac.
Bremsstrahlung	4×10^{-2}	Sun
	0.18	Horizontal Branch
Compton Scattering	0.24	Sun
	0.81	Horizontal Branch
Primakov	4×10^{-2}	Sun
	0.35	Horizontal Branch
Pion exchange	~ 92	SN1987a

$$T_J^{\mu\nu} = \frac{2}{\sqrt{-g_J}} \frac{\delta S_m}{\delta g_{\mu\nu}^J}. \qquad (2.8)$$

Notice that this last action is written in the Einstein frame and involves the coupling between the Jordan frame energy-momentum tensor T_J and the scalar derivatives.

III. MICROSCOPIC EFFECTS

It has been shown that the disformal coupling to matter induces a one loop potential between matter particles [19,20]. This potential is highly sensitive to short distances as it scales as $1/r^7$. Atomic physics experiments are therefore ideal settings to test the influence of the disformal coupling on the properties of matter. As the conformal coupling scale is $\mathcal{O}(m_{\mathrm{Pl}})$ it can be safely neglected over atomic distance scales. In previous work we have shown that the strongest constraints on M from such experiments comes from precision spectroscopy of hydrogen atoms and constrains the scale $M \gtrsim 200$ MeV. In this section we will determine whether disformal couplings satisfying this bound can explain the proton radius anomaly. This requires determining disformal corrections to the Lamb shift in hydrogen from which the proton radius can be inferred. We will find that the disformal Lamb shift is sensitive to the mass of the particle orbiting around the atomic nucleus, hence inducing different effects in muonic compared to electronic atoms.

A. Lamb shift and proton radius

The scalar interaction due to the one loop effect of the disformal coupling to matter acts as a perturbation of the Coulombic interaction in hydrogenlike atoms

$$V(r) = -\frac{e^2}{r} - \frac{3m_1 m_2}{M^8} \frac{1}{32\pi^3 r^7}. \qquad (3.1)$$

In first order perturbation theory, the atomic levels are perturbed by

$$\delta E = -\frac{3m_f m_N}{32\pi^3 M^8} \left\langle E \left| \frac{1}{r^7} \right| E \right\rangle, \qquad (3.2)$$

where $|E\rangle$ is the unperturbed wave function of a given energy level. Let us focus on hydrogenlike atoms and consider the 2s and 2p levels, as used to calculate the Lamb shift. In each case the perturbed energy levels are sensitive to the small r parts of the wave function, $r \ll a_0$ where a_0 is the Bohr radius and read

$$\psi_{2s}(r) \approx \frac{1}{2\sqrt{2\pi}} \left(\frac{Z}{a_0}\right)^{3/2};,$$

$$\psi_{2p}(r) \approx \frac{1}{\sqrt{\pi}} \left(\frac{Z}{2a_0}\right)^{5/2} r \cos\theta;,$$

resulting in the perturbation of the 2s and 2p levels given by

$$\delta E_{2s} = -\frac{3}{248\pi^3} \left(\frac{Z}{a_0}\right)^3 \frac{m_N m_f}{M^8 r_N^4};, \qquad (3.3)$$

and

$$\delta E_{2p} = -\frac{1}{2^9 \pi^3} \left(\frac{Z}{a_0}\right)^5 \frac{m_N m_f}{M^8 r_N^2}; \qquad (3.4)$$

where the interaction has been cutoff at the nuclear radius r_N as below this scale the internal structure of the nucleus becomes relevant. This leads to a contribution to the Lamb shift $\delta E_{2s-2p} = \delta E_{2p} - \delta E_{2s}$, which is given by

$$\delta E_{2s-2p} = \frac{3}{248\pi^3} \left(\frac{Z}{a_0}\right)^3 \frac{m_N m_f}{M^8 r_N^4} \left[1 - \frac{1}{6}\left(\frac{Z}{a_0}\right)^2 r_N^2\right]. \qquad (3.5)$$

The Lamb shift can be used to infer the proton radius $r_N = r_p$ in atoms where the nucleus reduces to a single proton. The phenomenological parametrization of the Lamb shift in terms of QED and nuclear physics effects and their dependence on the radius r_p is given by [1]

$$\frac{\Delta E_{2s-2p}}{\mathrm{meV}} = 210 - 5.23\left(\frac{r_p}{\mathrm{fm}}\right)^2 + 0.035\left(\frac{r_p}{\mathrm{fm}}\right)^3. \qquad (3.6)$$

A new interaction, such as the disformal one, would lead to a change in the Lamb shift δE_{2s-2p}, which would be read as a corresponding change in the proton radius δr_p:

$$\frac{\delta E_{2s-2p}}{\mathrm{meV}} = -10.31 \frac{\delta r_p}{r_p}. \qquad (3.7)$$

Experimentally, the proton radius deduced from electronic hydrogen measurements is given by $r_p = 0.8758(77)$fm, and this agrees with the charge radius obtained in electron scattering experiments at low energy [3]. The same measurements of the Lamb shift can be conducted for muonic atoms, and surprisingly the proton radius appears to be significantly lower; $r_p = 0.84087(39)$fm, [1], representing a decrease of approximately four percent.

Reinterpreting the disformal contribution to the Lamb shift as a change in the proton radius for muonic hydrogen gives

$$\left(\frac{\delta r_p}{r_p}\right) = -0.90 \left(\frac{217 \text{ MeV}}{M}\right)^8, \qquad (3.8)$$

where we have taken r_N to be the unperturbed proton radius. The electron contribution is suppressed compared to the muonic contribution by the ratio of electron to muon masses. Therefore to account for a four percent shift in the proton radius in muonic hydrogen we must choose

$$M = 320 \text{ MeV}, \tag{3.9}$$

which lies close to the QCD scale. This is compatible with constraints from measurements of hydrogen spectroscopy which require $M \gtrsim 200$ MeV. It is possible to explain the proton radius puzzle because muons orbit closer to the nucleus of an atom than electrons and the disformal force strengthens rapidly with decreases in distance.

One could ask why should the value of M lie close to the QCD scale? We can only give a plausibility argument: we want to describe low-energy physics in the late universe. As we are not sensitive to internal nuclear structure it makes sense to cut off the physical description around Λ_{QCD}, the scale below which confinement of the quarks and gluons takes place to form hadronic matter. We also should not be sensitive to physics in the early universe. The averaged density of the earth, around 5 gcm^{-3}, corresponds to the densities during BBN. In this environment where the density is similar to the density of the Universe during BBN, it is likely that the cutoff scale M should be close to the cutoff scale of the particle physics model during BBN, i.e. Λ_{QCD}.

B. The helium radius

We can extend our study to He ions carrying one muon compared to those with one electron. In this case the Lamb shift is related to the Helium radius as [22]

$$\frac{\Delta E_{2s-2p}}{\text{meV}} = 1670.37 - 105.322 \left(\frac{r_{\text{He}}}{\text{fm}}\right)^2 + 1.529 \left(\frac{r_{\text{He}}}{\text{fm}}\right)^3. \tag{3.10}$$

The disformal interaction would induce a change in the Lamb shift, which is connected to a change in the He radius in the following way:

$$\frac{\delta E_{2s-2p}}{\text{meV}} = -573.5 \frac{\delta r_{\text{He}}}{r_{\text{He}}}. \tag{3.11}$$

Using a value of M determined in Eq. (3.9), $r_{\text{He}} = 1.681$ fm with $m_{\text{He}} = 3.728$ GeV and $Z = 2$ for the two protons in the He nucleus, we find that the disformal coupling would induce a change in the Helium radius

$$\frac{\delta r_{\text{He}}}{r_{\text{He}}} = 0.2\%. \tag{3.12}$$

This is smaller than the uncertainty in the Helium radius coming from e scattering experiments, which is of order 0.3%, and therefore is not currently a detectable effect.

C. The deuterium

New experiments will also give their results on the deuteron's radius as inferred from the Lamb shift in muonic deuterium. The deuteron has a radius of $r_d = 2.1402$ fm and a mass $m_d = 1.875$ GeV. This leads to a shift in the muonic case of the energy levels

$$\delta E_{2s-2p} = 0.023 \text{ meV}. \tag{3.13}$$

This is a prediction of our model which should be compared with future experimental results.

IV. STELLAR BURNING CONSTRAINTS

A. Constraints from stars

We have just seen that the proton radius puzzle seems to indicate that the scale $M \sim \Lambda_{\text{QCD}}$. This is a low-energy scale and one may wonder if the disformal interaction may not have an influence on the burning rate of stars, as is the case for axions and axion-like particles. These constraints are summarized in Table I. For disformally coupled scalar fields the light particles could be emitted by processes such as Compton scattering, bremsstrahlung or Primakov processes in stars of the main sequence and on the horizontal branch of the Hertzsprung-Russell diagram. Two scalars would also be emitted by nuclear processes involving the pion exchange in supernovae. The latter process gives the most severe constraints but suffers from theoretical uncertainties due to the fact that the pion exchange diagram between two nuclei is a strongly coupled effect treated at tree level, although higher-order effects could alter the result drastically. In addition the maximal emissivity of supernovae $\epsilon_{\text{SN}} \lesssim 10^{19}$ erg/g·s as deduced from the SN1987A explosion is only a rough estimate [25]. The constraints from these processes have been presented in [20] and it was shown that for the sun we must impose that $M \gtrsim 240$ MeV, for horizontal branch stars $M \gtrsim 810$ MeV and for supernovae $M \gtrsim 92$ GeV. The solar constraint is always satisfied if we take $M \sim 320$ MeV. The horizontal branch and supernovae constraints need to be analyzed carefully as they rule out, at face value, a disformal explanation of the proton radius puzzle.

B. Alleviating the burning constraints with chameleons

We now embed the disformally coupled scalar field in a chameleon model [26]. The chameleon effect applies to many more models than the original chameleon model. In particular, we shall focus on $f(R)$ theories in the large curvature regime below. These models are scalar field theories where a potential term $V(\phi)$ is added to the Lagrangian,

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G_N} - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) + S_m(\psi_i, \tilde{g}_{\mu\nu}). \tag{4.1}$$

Working to leading order in the disformal coupling, this becomes the effective action

$$S = \int d^4x \sqrt{-g}\left(\frac{R}{16\pi G_N} - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right.$$
$$\left. + \frac{1}{M^4}\partial_\mu\phi\partial_\nu\phi T_J^{\mu\nu}\right) + S_m(\psi_i, A(\phi)g_{\mu\nu}). \qquad (4.2)$$

One of the salient features of these scalar-tensor models is that, in the presence of nonrelativistic matter with a density ρ, the dynamics of the scalar field are governed by the effective potential

$$V_{\text{eff}}(\phi) = V(\phi) + (A(\phi) - 1)\rho, \qquad (4.3)$$

which appears in the Klein-Gordon equation,

$$\Box\phi - \frac{2}{M^4}D_\mu(\partial_\nu T_J^{\mu\nu}) = \frac{\partial_\phi V_{\text{eff}}(\phi)}{\partial\phi}. \qquad (4.4)$$

When the matter density is constant inside a dense region of the Universe, the field settles at the minimum of the effective potential $\phi(\rho)$ satisfying

$$\left.\frac{\partial_\phi V_{\text{eff}}(\phi)}{\partial\phi}\right|_{\phi(\rho)} = 0, \qquad (4.5)$$

when it exists, e.g. for decreasing potentials $V(\phi)$ and increasing coupling functions $A(\phi)$. When the mass of the scalar field, defined as

$$m^2(\rho) \equiv \left.\frac{\partial_\phi^2 V_{\text{eff}}(\phi)}{\partial\phi^2}\right|_{\phi(\rho)}, \qquad (4.6)$$

increases with the matter density ρ, the model is a chameleon theory which evades gravitational tests on scalar fifth forces when the scalar interaction range $\lambda(\rho) = m^{-1}(\rho)$ becomes small enough in dense environments.

Chameleon models are fully characterized by their mass as a function of the density ρ and the coupling β. One prime family of examples is provided by the large curvature $f(R)$ models where

$$f(R) = C + R - \frac{f_{R_0}}{n}\frac{R_0^{n+1}}{R^n}, \qquad (4.7)$$

where n is a positive index and f_{R_0} a parameter constrained to be less than 10^{-6} by local tests of gravity. The scalar field ϕ can be identified via

$$\frac{df}{dR} = e^{-2\beta\phi/m_{\text{Pl}}}, \qquad (4.8)$$

where $\beta = \frac{1}{\sqrt{6}}$ is the coupling to matter and

$$V(\phi) = m_{\text{Pl}}^2 \frac{R\frac{df}{dR} - f}{2(\frac{df}{dR})^2} \qquad (4.9)$$

is implicitly defined. The expression for the mass $m(\rho)$ is known [26,27] and reads

$$m(\rho) \sim m_0 \left(\frac{\rho}{\rho_0}\right)^{(n+2)/2}. \qquad (4.10)$$

The density ρ_0 is the matter density in the Universe now. Local tests of gravity require that

$$m_0 \gtrsim 10^3 H_0 \sim 10^{-30} \text{ eV}, \qquad (4.11)$$

which is a very low mass; i.e., the scalar is nearly massless in vacuum. The derivation of this result is given in Sec. IVC of [26] in terms of the scale factor a related to the matter density by $\rho = \rho_0/a^3$, (see Eqs. (168) and (169) of [26]). For these models, all the tests of gravity are satisfied thanks to the chameleon mechanism. We can also verify that our original assumption in calculating the one loop potential induced by the disformal coupling is not upset by the chameleon mechanism. Indeed, inside a muonic atom, the muon is a pointlike particle with no internal structure while the nucleus can also be seen as a point-like particle when one considers the long range interaction between the nucleus and the muon. In this case, the vacuum between the two particles is the cosmological one filled by the residual dark matter fluid. In this vacuum, the mass of the scalar field is m_0 which is of cosmological size and therefore irrelevant for atomic interactions. In stars such as the ones on the horizontal branch or supernovae, the scalar field settles at its minimum due to the matter density ρ where $\rho/\rho_0 \sim 10^{33}$ and $\rho/\rho_0 \sim 10^{43}$ respectively. Taking $n = 1$ for instance, we find that the mass of the scalar field in these environments far exceed the temperature and therefore scalars are not created inside stars simply for kinematical reason [28]. Hence disformal scalar fields that possess a chameleon mechanism evade the constraints on the disformal coupling coming from the burning rate of stars. Moreover they can be identified with the scalar field responsible for the acceleration of the Universe in models such as $f(R)$ in the large curvature regime.

V. COLLIDER PHYSICS CONSTRAINTS

A. Unitarity constraint

Strong constraints on the scale M can also be obtained from particle physics. Indeed the disformal coupling is nothing but an irrelevant operator of higher order whose presence jeopardizes the UV behavior of the model. We can evaluate when this breakdown occurs and above which scale the effective field description must be altered, by analyzing the unitarity of scattering processes. We focus on high-energy physics experiments and consider that

$M \approx 320$ MeV, as suggested by proton radius measurements. A typical scattering experiment will involve the creation of two scalars from the annihilation of two fermions $f\bar{f} \to \phi\phi$. The disformal matrix element for this process becomes

$$\mathcal{M} = \frac{2\sqrt{2}m_f E^3}{M^4} \qquad (5.1)$$

in terms of the energy of the incoming particles in the center of mass frame. Perturbative unitarity is preserved when $\mathcal{M} \le 16\pi$ implying an energy bound

$$E \le E_{\max} = \left(\frac{8\pi M^4}{\sqrt{2}m_f}\right)^{1/3}. \qquad (5.2)$$

Unitarity has been precisely tested in the standard model with LEP where $m_f = m_e$ and we find

$$E_{\max} \sim 7 \text{ GeV}. \qquad (5.3)$$

Hence unitarity would have been violated at LEP, which reached beam energies of 200 GeV, if we extrapolate the disformal coupling between scalars and matter to such high energies. However, if we expect the cutoff to lie just above the scale M then we are not able to naively extrapolate the theory to such high energies. There are two possible ways to proceed. The first is that, for energies larger than $E_{\max}$, the model must be UV completed in the Wilsonian sense, and the disformal interaction replaced by another interaction between new degrees of freedom, replacing the low-energy field ϕ, and unitarity is restored. However, in this case we can make no statement about whether our model is compatible with collider measurements. The second, more predictive, alternative is to use the classicalization property of Galileon models [29], which we will describe in the following section.

B. Galileons

We embed the disformally coupled scalar field in a Galileon model [14]. These are scalar field theories which have equations of motion that are at most second order in derivatives, despite the presence of nontrivial derivative self-interactions. In flat space the theory also respects the symmetry $\phi \to \phi + b_\mu x^\mu + c$ for constant b_μ and c. We work with this simplest form of the theory, the cubic Galileon, which has the following Lagrangian:

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{\Lambda^3}\Box\phi(\partial\phi)^2 + \frac{\beta\phi}{m_{\text{Pl}}}T + \frac{1}{M^4}\partial_\mu\partial_\nu T^{\mu\nu}. \qquad (5.4)$$

In addition to the coupling scales m_{Pl}/β and M, the theory is determined by Λ, a suppression scale which controls the

derivative self-interactions that define the Galileon. Around a spherically symmetric source of mass m the scalar field profile is

$$\frac{d\phi}{dr} = -\frac{\Lambda^3 r}{4}\left(1 - \sqrt{1 + \left(\frac{R_*}{r}\right)^3}\right), \qquad (5.5)$$

and the nonlinearities dominate the evolution of the scalar within the Vainshtein radius

$$R_* = \frac{1}{\Lambda}\left(\frac{\beta m}{2\pi m_{\text{Pl}}}\right)^{1/3}. \qquad (5.6)$$

Within this radius the nonlinearities act to suppress the scalar force, F_ϕ, compared to that of Newtonian gravity, F_N, so that

$$\frac{F_\phi}{F_N} = \beta^2\left(\frac{r}{R_*}\right)^{3/2}. \qquad (5.7)$$

Outside the Vainshtein radius, the nonlinearities in the kinetic terms become irrelevant and the dominant kinetic term reduces to $-(\partial\phi)^2/2$. Inside the Vainshtein radius, any perturbations around the background of Eq. (5.5) inherit a wave function renormalization such that the kinetic terms of the perturbations read $Z\frac{(\partial\delta\phi)^2}{2}$ where we have

$$|Z| \sim 1 + \frac{\phi'}{r\Lambda^3}, \qquad (5.8)$$

and a prime denotes a derivative with respect to radius. Therefore inside the Vainshtein radius Z can be large.

The Galileon models rely on high-mass dimension operators and therefore are sensitive to quantum corrections at short distance. At the quantum level, the Galileon models receive corrections which preserve the Galilean symmetry. Many operators which are not present at tree level appear, and the effective action calculated using the one-particle irreducible diagrams for the Galileon models can be organized in an infinite series which depends only on the effective cutoff scale

$$\Lambda_Z = \sqrt{Z}\Lambda, \qquad (5.9)$$

and its derivative. This follows from the fact that expanding $\phi = \phi_0 + \delta\phi$ and canonically normalizing the field $\delta\phi$, the only dimensionful quantities controlling self-interactions of the field are Λ_Z and the effective mass of $\delta\phi$ which depends only on $\partial\Lambda_Z$ and its derivatives. As a result, the quantum corrections are given by

$$\delta\mathcal{L} = \Lambda_Z^4 F\left(\frac{\partial^i \Lambda_Z}{\Lambda_Z^{i+1}}\right), \qquad (5.10)$$

where the function F depends on the multiple derivatives of Λ_Z. The overall factor Λ_Z^4 appears for dimensional reason. For instance, at one loop, the mass term behaves like $m_\phi \sim \frac{\partial \Lambda_Z}{\Lambda_Z}$ and the Coleman-Weinberg potential reads $\delta V \sim m_\phi^4$ which corresponds to $F(x) = x^4$ where $x = \frac{\partial \Lambda_Z}{\Lambda_Z^2}$.

This result has important consequences. Firstly, the conformal coupling determined by β and the disformal coupling scale M are not renormalized by Galileon fluctuations [30]. Secondly, the structure of $\delta\mathcal{L}$ implies that the Galileon interactions are not renormalized, the corrections appearing with at least one more derivative at the same order in the ϕ expansion. The higher-order terms from $\delta\mathcal{L}$ are therefore small compared to the Galileon terms provided that $\frac{\partial^i \Lambda_Z}{\Lambda_Z^{i+1}} \lesssim 1$. Inside the Vainshtein radius the dependence of Λ_Z on r is a power law and therefore this condition is met provided

$$r\Lambda_Z \gtrsim 1. \tag{5.11}$$

For the cubic Galileon, we have inside an object and up to its surface

$$\Lambda_Z \sim \left(\frac{\beta \bar\rho \Lambda}{m_{\rm Pl}}\right)^{1/4}, \tag{5.12}$$

for an object of averaged density $\bar\rho$. This scale is independent of r and guarantees that one can trust the Vainshtein solution as long as

$$r \gtrsim r_E \sim \left(\frac{m_{\rm Pl}}{\beta \bar\rho \Lambda}\right)^{1/4}, \tag{5.13}$$

We have used a free scalar field model down to the nuclear scale in the disformal calculation of the Lamb shift, so we must impose that $r_N\Lambda_Z \gtrsim 1$. Taking $\bar\rho \sim r_N^{-4}$ for nuclear matter, we find that $r_E \gg r_N$ for $\Lambda \ll m_{\rm Pl}$ and $\beta = \mathcal{O}(1)$. This would prevent us from trusting our loop calculation of the Lamb shift. The only possibility is to impose that the Vainshtein radius of nucleons is smaller than their size, implying that the Galileon theory is weakly coupled down to nuclear scales. This requires that

$$\Lambda \gtrsim \frac{1}{r_N}\left(\frac{\beta m_N}{m_{\rm Pl}}\right)^{1/3} \sim 0.2 \text{ keV}. \tag{5.14}$$

C. Nonperturbative effects

In some models, the perturbative assumption that single particle states are created from annihilation processes is no longer valid in high-energy collisions. This happens when nonperturbative effects occur at high energy due to the nonlinearities of the theory. In particular, classical lumps can be created in some scalar models that would be akin to the creation of black holes at Planck scale energies. This phenomenon alleviates the perturbative unitarity bound as the growth of the scattering cross section with the energy is modified. Let us illustrate this with the Galileon models which offer the possibility of annulling the constraint from perturbative unitarity.

The Vainshtein radius plays an important role in particle scattering experiments. Take, for instance, the case of a two fermion annihilation process previously discussed. On shell, the trace of the energy-momentum tensor of massive fermions is

$$T = -m_F \bar\psi\psi, \tag{5.15}$$

corresponding to a nonvanishing energy density which is highly concentrated and can be modeled as

$$T^{00} = \sqrt{s}\delta^{(3)}, \tag{5.16}$$

in the center of mass frame, with $s = -p^2$ where p is the total incoming 4-momentum. This is a peaked energy density ρ of order $\sqrt{s}$ over a region of size $R \sim \frac{1}{\sqrt{s}}$ during a time interval of order $\Delta t \sim \frac{1}{\sqrt{s}}$. During the time interval of the collision when the energy density ρ is nonvanishing, the scalar field can be modeled as a solution of the time independent Klein-Gordon equation sourced by T^{00}. In this frame the tree level contribution from the disformal term vanishes as the source is static and the scalar field profile is determined by the same equation as the one used to analyze the Vainshtein screening in Galileon models. The scalar profile becomes a scalar lump with its energy concentrated inside the Vainshtein radius. The nonlinearities dominate inside the Vainshtein radius and are important for scattering experiments provided the source term lies inside its own Vainshtein radius $R \lesssim R_\star$. This is the analogue of the criterion for the formations of black holes, i.e. the requirement that the size of the interaction region must be within its Schwarzschild radius.

Being dominated by the creation of a classical lump, the scattering process has a total cross section which is of the order of the Vainshtein radius squared,

$$\sigma_T \sim R_\star^2. \tag{5.17}$$

After the time interval Δt, the energy density drops to zero and the scalar lump cannot be maintained any more. This triggers the classical decay of the scalar configuration, and the solution becomes time dependent. We do not study the details of this decay here; however, one expects the energy of the initial lump to spread out in space and decay classically. During this process, one also expect that quantum phenomena take place with the emission of on shell particle states.

For the Galileons, a source of mass $\sqrt{s}$ and size $R = 1/\sqrt{s}$ has a Vainshtein radius

$$R_\star(s) = \frac{1}{\Lambda} \left(\frac{\beta\sqrt{s}}{2\pi m_{\text{Pl}}} \right)^{1/3}, \qquad (5.18)$$

and the Vainshtein criterion $R \lesssim R_\star(s)$ is satisfied provided

$$s \gtrsim s_\star \equiv \left(\frac{2\pi\Lambda^3 m_{\text{Pl}}}{\beta} \right)^{1/2}, \qquad (5.19)$$

where $\beta = \mathcal{O}(1)$. The high-energy behavior above $s_\star$ determined here must resolve the breaking of perturbative unitarity implying that $s_\star \leq E_{\text{max}}^2 \sim (7 \text{ GeV})^2$ and, therefore,

$$\Lambda^3 \leq \frac{\beta E_{\text{max}}^4}{2\pi m_{\text{Pl}}}, \qquad (5.20)$$

which corresponds to $\Lambda \leq 6$ keV. Unitarity must also be respected in the scalar sector of the theory where the $\phi + \phi \to \phi + \phi$ process leads to a scattering amplitude at tree level $\mathcal{M} \sim \frac{s^3}{\Lambda^6}$ in a terrestrial environment. At higher energy, one must resum all the ladder diagrams involving the three-point vertex of Galileon models with an amplitude of the form $\mathcal{M} \sim \frac{s^3}{\Lambda^6} (1 + \alpha \frac{s^3}{\Lambda^6})^{-1}$ where α is a numerical factor. This resummation renders the amplitude bounded at large s guaranteeing that unitarity is not violated.

While this classicalization process alleviates the unitarity bounds on the disformal coupling, the decay of the scalar lumps must still be compatible with collider observations. At energies higher than $\sqrt{s_\star}$, fermion annihilation including those at LEP and the LHC would not create two particle scalar states but classical Galileon configurations. These Galileons would then decay classically and quantum mechanically into a few multiparticle states coming from the coupling of the scalar field to matter fields. A full calculation of this process is beyond the scope of this work; however, as the coupling between the scalar and matter particles is suppressed by β/m_{Pl} for the conformal coupling and by loop suppression factors for the disformal coupling, we expect observable signatures of this process to be very difficult to detect.

D. Combining the chameleon and Galileon effects

We have seen that the unitarity bound from particle physics can be alleviated provided $\Lambda \lesssim 6$ keV. We have also found that the calculation of the disformal Lamb shift can be trusted provided that $\Lambda \gtrsim 0.2$ keV. This gives a narrow band of values for $\Lambda \sim 1$ keV. For these values of Λ, the Vainshtein radius of stars of the main sequence and on the horizontal branch are extremely small compared to their sizes. For supernovae, the Vainshtein radius is of the order of the radius of the core. We have used the chameleon mechanism to tackle the burning rate problem for stars. How can we make the chameleon and Galileon compatible? We consider the full model defined by the action

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G_N} - \frac{1}{2}(\partial\phi)^2 - \frac{1}{\Lambda^3}\Box\phi(\partial\phi)^2 \right.$$

$$\left. - V(\phi) + \frac{1}{M^4}\partial_\mu\phi\partial_\nu\phi T_J^{\mu\nu} \right) + S_m(\psi_i, A(\phi)g_{\mu\nu}),$$

$$(5.21)$$

where $V(\phi)$ and $A(\phi)$ depend on the chameleon model. Inside a dense body of almost uniform density, the field settles at the minimum of the effective potential $\phi(\rho)$. This is even true inside the would-be Vainshtein radius $R_\star$ of the object as the source term in the Klein-Gordon for the spatial variation of the field $\frac{\partial V_{\text{eff}}}{\partial\phi}|_{\phi(\rho)}$ vanishes altogether. This implies that the model behaves like a chameleon model inside matter, and the burning rate bounds are evaded provided the mass of the scalar field in dense media is large enough. On the contrary, in the sparse environments of atomic or particle physics experiments where the ambient density is low, the scalar behaves like a nearly massless field with a background value ϕ_0 which minimizes the effective potential. During the course of a high-energy collision, the extreme densities reached in the center of mass frame during a time $\Delta t \sim 1/\sqrt{s}$ far exceed the background density and act as a peaked source localized in space. Expanding $\phi = \phi_0 + \delta\phi$, the action reduces to the one of a cubic Galileon model for a nearly massless field sourced by the matter density coming from the particle collision. As a result, the Galileon lumps can be created at high-enough energy and unitarity is restored.

In conclusion, we have introduced a class of models where the chameleon mechanism dominates in dense environments where the scalar field is extremely massive and cannot be produced by the particle reactions involving the disformal coupling. This prevents the dramatic increase of the burning rate of star that a disformal model with $M \sim \Lambda_{\text{QCD}}$ would entail. On the other hand, in (near) vacuum situations where particle experiments take place, the Galileon interactions become relevant at short distance when the Galileon coupling is approximately $\Lambda \sim 1$ keV.

VI. CONCLUSION

We have considered the effects of a disformal coupling between a massless scalar field and matter in the context of atomic physics. We have shown that the proton radius puzzle, i.e. a difference of four percent between the Lamb shifts of electronic and muonic atoms, can be explained by such a disformal coupling when the cutoff scale of the model for experiments carried out in the terrestrial environment is close to the QCD scale. This allows us to predict that the disformal effect on the He radius should be below the percent level. These results are only valid when the scalar model is embedded in nonlinear models with the chameleon screening mechanism in dense environments.

This helps alleviate the constraints coming from stellar burning rates as scalars are too heavy to be created in such environments. At higher energy and in near vacuum, the Vainshtein mechanisms of cubic Galileon models would prevent the violation of unitarity by disformal interactions. These models would be characterized by the production and decay of classical lumps. The determination of signatures for these events would certainly lead to promising tests of the models presented here at the LHC. This is left for future work.

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