Orthogonal bipolar spherical harmonics measures: Scrutinizing sources of isotropy violation

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The two-point correlation function of the cosmic microwave background temperature anisotropies is generally assumed to be statistically isotropic (SI). Deviations from this assumption could be traced to physical or observational artifacts and systematic effects. Measurement of nonvanishing power in the bipolar spherical harmonic spectra is a standard statistical technique to search for isotropy violations. Although this is a neat tool allowing a blind search for SI violations in the cosmic microwave background sky, it is not easy to discern the cause of isotropy violation by using this measure. In this article, we propose a novel technique of constructing orthogonal bipolar spherical harmonic estimators, which can be used to discern between models of isotropy violation.

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I. INTRODUCTION

The cosmic microwave background (CMB) temperature anisotropy measurements are one of the cleanest probes of cosmology. The CMB full sky temperature anisotropy measurements have been used to test the assumption of the isotropy of the Universe. The study of full sky maps from the WMAP 5 year data [1–3], WMAP 7 year data [4], and the recent Planck data [5] has led to some intriguing anomalies that may be interpreted to indicate deviations from statistical isotropy (SI). Deviations from SI can be caused by a number of physical, observational, and systematic effects, such as nontrivial cosmic topology [6], Bianchi models [7–9], primordial magnetic fields [10,11], anisotropic primordial baryon-photon distribution [12], weak gravitational lensing [13], asymmetric power spectrum [14,15], modulation of the CMB sky [5], and measurement of SI CMB skies with a noncircular beam [16–18] being a small subset of the possibilities.

The CMB temperature anisotropies are assumed to be Gaussian, which is in good agreement with current CMB observations. Hence, the two-point correlation function contains complete information about the underlying CMB temperature field. The generic two-point correlation function can be completely encoded in the bipolar spherical harmonic (BipoSH) basis. The coefficients of expansion in this set of basis function are termed as BipoSH spectra $A^{LM}_{l_1 l_2}$. While the BipoSH spectra with $L = 0$ encode information of the SI part of the correlation function, the rest of the BipoSH spectra detail the deviations from isotropy. Measuring nonvanishing BipoSH spectra ($A^{LM}_{l_1 l_2}; L \neq 0$) forms the basic criteria behind searches for deviations from SI in CMB maps.

In the search for deviations from SI, one has to use either one the following two strategies:

(i) Search for deviation from SI by measuring deviations from zero in BipoSH spectra. This method has the advantage of being model independent but nonoptimal as it lacks sensitivity.

(ii) Search for deviation from SI by constructing an optimal estimator by resorting to a chosen model of SI violation. This method has the advantage of being optimal at the cost of being biased, as it requires working with a specific model of isotropy violation.

The study discussed in this article presents a strategy to combine the advantages presented by the above two strategies while minimizing their drawbacks.

II. INTRODUCTION TO THE BIPOSH FORMALISM

The general two-point correlation function can be expressed in terms of the spherical harmonic coefficients of CMB temperature maps:

$$C(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2) = \langle \Delta T(\hat{\mathbf{n}}_1) \Delta T(\hat{\mathbf{n}}_2) \rangle = \sum_{lml'm'} \langle a_{lm} a^*_{l'm'} \rangle Y_{lm}(\hat{\mathbf{n}}_1) Y^*_{l'm'}(\hat{\mathbf{n}}_2). \quad (1)$$

In the SI case, this correlation function depends only on the angular separation between the two directions and not on the directions $\hat{\mathbf{n}}_1$ and $\hat{\mathbf{n}}_2$ explicitly. This property makes it possible to expand the correlation function in the Legendre polynomial (P_l) basis:

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$$C(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2) = C(\hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2) = \sum_l \frac{2l+1}{4\pi} C_l P_l(\hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2), \tag{2}$$

$$\langle a_{lm} a^*_{l'm'} \rangle = C_l \delta_{ll'} \delta_{mm'}, \tag{3}$$

where C_l is the standard angular power spectrum.

In the absence of SI, the correlation function does have an explicit dependence on the two directions $\hat{\mathbf{n}}_1$ and $\hat{\mathbf{n}}_2$. In this case, the BipoSH which forms a complete orthonormal basis for functions defined on $S^2 \times S^2$ forms the ideal basis for studying the direction-dependent two-point correlation function [19,20]. The CMB two-point correlation function is expressed in the BipoSH basis in the following manner:

$$C(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2) = \sum_{LMl_1l_2} A^{LM}_{l_1l_2} \{Y_{l_1}(\hat{\mathbf{n}}_1) \otimes Y_{l_2}(\hat{\mathbf{n}}_2)\}_{LM} = \sum_{LMl_1l_2} A^{LM}_{l_1l_2} \sum_{m_1m_2} \mathcal{C}^{LM}_{l_1m_1l_2m_2} Y_{l_1m_1}(\hat{\mathbf{n}}_1) Y_{l_2m_2}(\hat{\mathbf{n}}_2), \tag{4}$$

where $\mathcal{C}^{LM}_{l_1m_1l_2m_2}$ are the Clebsch-Gordon coefficients, the indices of which satisfy the following triangularity relations: $|l_1 - l_2| \leq L \leq l_1 + l_2$ and $m_1 + m_2 = M$.

These BipoSH coefficients can be expressed in terms of the covariance matrix derived from observed CMB maps,

$$A^{LM}_{l_1l_2} = \sum_{m_1m_2} \langle a_{l_1m_1} a_{l_2m_2} \rangle \mathcal{C}^{LM}_{l_1m_1l_2m_2}. \tag{5}$$

In the case of SI, the only nonvanishing BipoSH coefficients are A^{00}_{ll}, and they can be expressed in terms of the CMB angular power spectrum C_l:

$$A^{00}_{ll} = (-1)^l \Pi_l C_l, \tag{6}$$

where $\Pi_l = \sqrt{2l+1}$.

A. Convenient notation: $A^{LM}_{l_1l_2} \to A^{LM}_{ll+D}$

We introduce the reindexed BipoSH coefficients A^{LM}_{ll+D} which are more convenient to interpret. One can now think of the indices $M \in \{-L, L\}$ and $D \in \{-L, L\}$ as independent parameters with l representing the inverse angular scale of the CMB map.

III. CONSTRUCTING ORTHOGONAL BIPOSH MEASURES

Many isotropy violation mechanisms [13,15,18] can be shown to give rise to BipoSH spectra of the following form:

$$A^{LM}_{ll+D} = (-1)^l \Pi_l C_l \delta_{L0} \delta_{M0} \delta_{D0} + p_{XY} G^L_{ll+D}, \tag{7}$$

where C_l is the angular power spectrum, G^L_{ll+D} denotes the template shape function arising from any model of isotropy violation which may be expressed as a function of C_l, and p_{XY} denotes the parameter characterizing the particular form of isotropy violation. In most of the isotropy-violating models, as will be seen in subsequent sections, the subscripts X and Y map to BipoSH indices L and M except for the noncircular beam.

One can now estimate the parameter p_{LM} by inverting Eq. (7) to yield the following estimator:

$$\hat{p}_{XY} = \frac{A^{LM}_{ll+D}}{G^L_{ll+D}}. \tag{8}$$

Each value of the indices l and D provides an independent estimate of p_{XY}.

It is common practice to make an optimal minimum variance sum to extract the best estimate of p_{XY}. In this article, we draw attention to an alternate construct in which we take an optimal minimum variance difference of the independent estimates of p_{XY}. The basic idea behind this construction is to check for consistency between parameter values as estimated from independent BipoSH spectra. If the model for isotropy violation is appropriate, then it predicts the correct template shape function G^L_{ll+D} encoded in the independent BipoSH spectra, hence yielding the same parameter value irrespective of the BipoSH spectra used to evaluate it. On the contrary, if the model for isotropy violation is invalid, then it predicts a wrong shape function encoded in the different BipoSH spectra, leading to discrepant values of the parameter as evaluated from different BipoSH spectra. Therefore, assessing the statistical significance of the differences between parameter values as extracted from independent BipoSH spectra can be used as a self-consistency test to rule out models of isotropy violation.

We propose the following estimator as a self-consistency test for isotropy violation models:

$$\hat{d}_{XY} = \hat{p}_{XY} - \hat{p}'_{XY} = \sum_{l=l_{\min}}^{l_{\min}+l_{\text{bin}}} w^L_l \left[\frac{A^{LM}_{ll+D}}{G^L_{ll+D}} - \frac{A^{LM}_{ll+D'}}{G^L_{ll+D'}} \right], \tag{9}$$

where the summation is over CMB multipoles and w_l are the weights used to minimize the variance on the estimator. l_{bin} denotes the width of the multipole bins over which the summation is performed. It can be shown that this change in sign makes no difference to the statistics of the estimator, as the independent BipoSH modes are uncorrelated. Generally, for any model of isotropy violation which can be cast in the form of Eq. (7), it can be shown that the weights that minimize the variance are given by the following expression:

$$w^L_l = \frac{\left[\frac{C_l C_{l+D}(1+\delta_{D0})}{(G^L_{ll+D})^2} + \frac{C_l C_{l+D'}(1+\delta_{D'0})}{(G^L_{ll+D'})^2} \right]^{-1}}{\sum_{l=l_{\min}}^{l_{\min}+l_{\text{bin}}} \left[\frac{C_l C_{l+D}(1+\delta_{D0})}{(G^L_{ll+D})^2} + \frac{C_l C_{l+D'}(1+\delta_{D'0})}{(G^L_{ll+D'})^2} \right]^{-1}}. \tag{10}$$

Note that these weights are derived under the assumption of SI and are similar to the minimum variance estimator used in the case of lensing reconstruction [21], except that the plus sign is replaced with a minus sign in Eq. (9). It can be also be shown that the sensitivity of the orthogonal BipoSH estimator is given by the following expression:

$$N_l^L = \frac{1}{\sum_{l=l_{\min}}^{l_{\min}+l_{\text{bin}}} \left(\frac{C_l C_{l+D}(1+\delta_{D0})}{(G^L_{ll+D})^2} + \frac{C_l C_{l+D'}(1+\delta_{D'0})}{(G^L_{ll+D'})^2} \right)^{-1}}, \quad (11)$$

where N_l^L is the variance of the orthogonal BipoSH estimator for an isotropic CMB sky whose statistical properties are described by the angular power spectrum C_l.

IV. TESTING MODELS OF STATISTICAL ISOTROPY VIOLATION

In the following section, we discuss some models of isotropy violation and the template shape functions arising from them. Following that, we demonstrate the ability of the orthogonal BipoSH estimators to discern between models of isotropy violation by evaluating them on ideal full sky, non-SI simulated CMB maps. Non-SI CMB simulations can also be generated by using CoNIGS which can incorporate any kind of SI violation [22]. Finally, we present the results of an identical study carried out on WMAP observed maps.

A. Sources of isotropy violation

1. Weak lensing

Since any given realization of large-scale structure surrounding an observer is anisotropic, it imprints a signature of isotropy violation in the lensed CMB sky. Weak lensing of the CMB photons results in the CMB temperature map getting remapped as described by the following equation:

$$\tilde{T}(\hat{n}) = T(\hat{n} + \vec{\nabla}\psi(\hat{n})), \quad (12)$$

where $\tilde{T}$ denotes the lensed CMB temperature anisotropies, T denotes the unlensed CMB temperature anisotropies, and ψ is the projected lensing potential.

Making no assumptions about isotropy of the lensed CMB sky and evaluating the two-point correlation in the BipoSH basis results in the following expression:

$$\tilde{A}^{LM}_{ll+D} = (-1)^l \Pi_l C_l \delta_{L0}\delta_{M0}\delta_{D0} + \psi_{LM} G^L_{ll+D}, \quad (13)$$

where the lensing shape function G^L_{ll+D} is expressed in terms of the CMB angular power spectrum as follows:

$$G^L_{ll+D} = \frac{C_l F(l+D, L, l) + C_{l+D} F(l, L, l+D)}{\sqrt{4\pi}} \times \frac{\Pi_l \Pi_{l+D}}{\Pi_L} \mathcal{C}^{L0}_{l0(l+D)0}, \quad (14)$$

where

$$F(l_1, L, l_2) = \frac{[l_2(l_2+1) + L(L+1) - l_1(l_1+1)]}{2}. \quad (15)$$

2. Modulation

Modulation of the CMB sky is another model which violates SI. This is being used as a phenomenological model to explain the anomalous, large angular scale dipolar asymmetry seen in the observed CMB maps [4,5]. The modulated CMB temperature anisotropy map is mathematically expressed as follows:

$$\tilde{T}(\hat{n}) = T(\hat{n})[1 + P(\hat{n})], \quad (16)$$

where P denotes the modulating field.

Expressing the two-point correlation function for the modulated temperature field in BipoSH spectra results in an equation having the following form:

$$\tilde{A}^{LM}_{ll+D} = (-1)^l \Pi_l C_l \delta_{L0}\delta_{M0}\delta_{D0} + P_{LM} G^L_{ll+D}, \quad (17)$$

where P_{LM} are the spherical harmonic coefficients of the modulating field, while the modulation shape function G^L_{ll+D} is expressed in terms of the CMB angular power spectrum as follows:

$$G^L_{ll+D} = \frac{C_l + C_{l+D}}{\sqrt{4\pi}} \frac{\Pi_l \Pi_{l+D}}{\Pi_L} \mathcal{C}^{L0}_{l0(l+D)0}. \quad (18)$$

3. Anisotropic primordial power spectrum (PPS)

Another model of isotropy violation is where the primordial fluctuations are described by a direction-dependent power spectrum [13,23,24]:

$$P(\vec{k}) = \mathcal{P}(k)[1 + g(\hat{k})] = \mathcal{P}(k)\left[1 + \sum_{LM} g_{LM} Y_{LM}(\hat{k})\right], \quad (19)$$

where $\mathcal{P}(k)$ describes the standard isotropic power spectrum while $g(\hat{k})$ encodes the directional dependence of the statistical properties of the primordial density fluctuations.

It can be shown that a direction-dependent PPS results in the following set of BipoSH spectra:

$$A^{LM}_{ll+D} = (-1)^l \Pi_l C_l \delta_{L0}\delta_{M0}\delta_{D0} + g_{LM} G^L_{ll+D}, \quad (20)$$

where g_{LM} are the spherical harmonic coefficients of the direction-dependent part of the primordial power spectrum and the shape function introduced due to the anisotropic power spectrum is given by the following expression:

$$G^{L}_{ll+D} = \frac{i^D}{\sqrt{4\pi}}\frac{\Pi_l\Pi_{l+D}}{\Pi_L}\mathcal{C}^{L0}_{l0l+D0} \times \int \frac{2k^2 dk}{\pi}\Delta_l(k)\Delta_{l+D}(k)P(k), \quad (21)$$

where $\Delta_l(k)$ are the CMB transfer functions. Note that the shape function G^L_{ll+D} can be expressed in terms of the angular power spectrum C_l in the case where $D = 0$; however, the rest of the coefficients with $D \neq 0$ require explicit evaluations of the integral in Eq. (21).

4. Noncircular beam

It is known that measurement of CMB temperature anisotropies by noncircular instrument beams results in the observed sky being non-SI [19,25]. This has been realized as one of the most prominent systematics which needs to be accounted for while searching for deviations from SI in the CMB sky.

Most generally, the measured CMB temperature anisotropies are related to the true CMB sky through the following expression:

$$\tilde{T}(\hat{n}) = \int d\hat{n}' B(\hat{n},\hat{n}')T(\hat{n}'), \quad (22)$$

where $\tilde{T}(\hat{n})$ represents the measured CMB sky, $B(\hat{n},\hat{n}')$ denotes the beam sensitivity function, and $T(\hat{n}')$ represents the true CMB sky. The beam function is characterized as being circular if it satisfies $B(\hat{n},\hat{n}') = B(\cos^{-1}(\hat{n}\cdot\hat{n}'))$, while any deviations from this condition render it noncircular.

It can be shown that mildly noncircular beams result in the generation of the BipoSH spectra [18], which can be expressed in the following form:

$$\tilde{A}^{LM}_{ll+D} = (-1)^l \Pi_l C_l B_l^2 \delta_{L0}\delta_{M0}\delta_{D0} + b_{l2} G^L_{ll+D}, \quad (23)$$

where B_l is the beam transfer function which characterizes the circular part of the beam, while $b_{l2}(\hat{z})$ are the beam spherical harmonics [$b_{lm}(\hat{z})$ with $|m| = 2$] when the beam is pointing along the north pole ($\hat{z}$), which characterize the noncircularity of the beam up to the leading order effects, and G^L_{ll+D} is the induced shape function which can be expressed as follows:

$$G^L_{ll+D} = \frac{2\pi\Pi_L}{(\Pi_{ll+D})\mathcal{C}^{L0}_{l0l+D0}}[C_{l+D}B_{l+D}\xi^{L0}_{ll+D} + C_l B_l \xi^{L0}_{l+D,l}], \quad (24)$$

where

$$\xi^{L0}_{l_1 l_2} = \frac{\Pi_{l_1}}{\sqrt{(4\pi)}}\sum_{m_2}(-1)^{m_2}C^{L0}_{l_1 -m_2 l_2 m_2} \times \int_0^\pi d^{l_2}_{m_2 2}(\theta) d^{l_1}_{m_2 0}(\theta)\sin\theta d\theta, \quad (25)$$

where $d^l_{mm'}$ are the Wigner-d functions. In arriving at Eq. (23), we have made a number of assumptions. We assume the beam noncircularity to be small, which allows us to retain terms only up to first order in the beam noncircularity parameter (b_{lm}) and truncate modes above $|m| = 2$ for all the multipoles l. Furthermore, to be able to arrive at an semianalytically evaluable expression for the noncircular-beam-induced shape function, we assume a constant scanning strategy. Specifically in the case of WMAP, it has been shown that [18] the beam noncircularity and the detailed scanning strategy are well approximated by an effective beam and a constant scan strategy (i.e., the orientation or inclination of the beam with the latitude or longitude at each pixel remains fixed).

An interesting point to note is that all the source models of isotropy violation under study (modulation, weak lensing by large-scale structure, anisotropic primordial power spectrum, and non-circular beam effects) induce only even-parity (i.e., $D + L$ is even) BipoSH spectra. Explicitly, this behavior is due to the presence of $\mathcal{C}^{L0}_{l0(l+D)0}$, which vanishes when $D + L$ is odd. Though not considered here, the construction of an orthogonal BipoSH estimator is equally valid even to models which generate odd-parity BipoSH spectra.

B. Testing the orthogonal estimators on simulated SI-violating maps

Given the shape function induced due to any particular source of SI violation, it is possible to arrive at an orthogonal BipoSH estimator, following the construction discussed in Sec. III. To test and demonstrate the effectiveness of these estimators, we evaluate them on a set of simulated SI-violating CMB maps. For this example study, we consider the following two cases:

(i) A modulated CMB map, constructed as in Eq. (16), where we have used a modulation field of the form $P(\hat{n}) = 0.1Y_{20}(\hat{n})$.

(ii) A noncircular-beam-convolved CMB map. These maps are constructed by following the same procedure as described in Ref. [16]. We use side A and side B of the $W1$ beam of WMAP for the convolution process and use a realistic WMAP scanning strategy.

We evaluate the BipoSH estimator and the orthogonal BipoSH estimators for weak lensing, modulation, anisotropic power spectrum, and noncircular beams on these simulated maps.

Before discussing the results of our analysis on these simulated maps, we reiterate that the orthogonal BipoSH

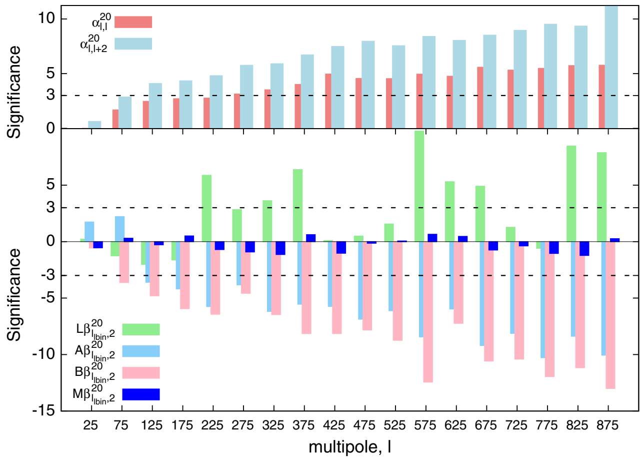

FIG. 1 (color online). Top panel: The red and the blue bars denote the significance of detection of the BipoSH spectra evaluated from simulated modulated maps constructed by using a modulation field given by $P(\hat{n}) = 0.1Y_{20}(\hat{n})$. Bottom panel: The green, light blue, and pink bars representing the sources, lensing (L), anisotropic power spectrum (A), and noncircular beam (B), respectively, show highly discrepant model parameters. Note that the dark blue bars which represent the modulation (M) model are seen to be consistent with zero within 3σ.

estimates yield a difference between model parameter estimates as evaluated from independent BipoSH spectra. The higher the significance of the orthogonal BipoSH estimates, the more likely it is for the source model under consideration to be invalid.

Note that, in the results presented below, $\alpha^{LM}_{l_1 l_2}$ denotes the significance of the BipoSH spectrum defined as $A^{LM}_{l_1 l_2}(\Pi_L/(\Pi_{l_1}\Pi_{l_2}\mathcal{C}^{L0}_{l_1 0 l_2 0}))$ following Ref. [4], and $\beta^{LM}_{l_1 l_2}$ denotes the significance of the difference parameter d_{XY} defined in Eq. (9).

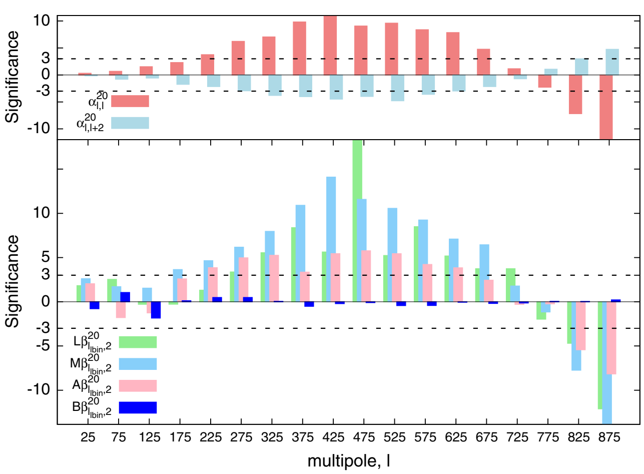

FIG. 2 (color online). As in Fig. 1, but for simulated maps convolved with a noncircular beam. Note that the dark blue bars which represent the anisotropic beam (B) model are seen to be consistent with zero within 3σ.

TABLE I. This table represent the cumulative significance of the difference between the model parameter estimates as derived from independent BipoSH modes. While the rows indicate the nature of the non-SI map used for the study, the columns indicate the source model assumed for the analysis. The source model which yields the least significant orthogonal BipoSH estimate is highlighted in bold.

Source simulation	Weak lensing	Anisotropic PPS	NC beam	Modulation
Modulation	64.1	112.1	141.9	**11.0**
NC beam	92.6	66.3	**7.1**	122.7

On evaluating these orthogonal estimators on modulated CMB maps, it is seen that all other source models, i.e., weak lensing, anisotropic power spectrum, and noncircular beam, show high significance detections in the orthogonal BipoSH estimates, implying that these models yield a statistically significant discrepancy in the same model parameter as estimated from independent BipoSH modes. However, in the case when the source model assumed is modulation, the orthogonal BipoSH estimates are found to have an extremely low significance as compared to other models. This means that the parameters for the assumed source model (modulation field harmonics in this case), as evaluated from independent BipoSH modes, are consistent with each other within error bars, as seen in Fig. 1. Similarly, when these estimators are evaluated on CMB maps convolved with a noncircular beam, the discrepancy between model parameters is found to be least significant only when the source model assumed is that of a noncircular beam, as seen in Fig. 2, while all the other source

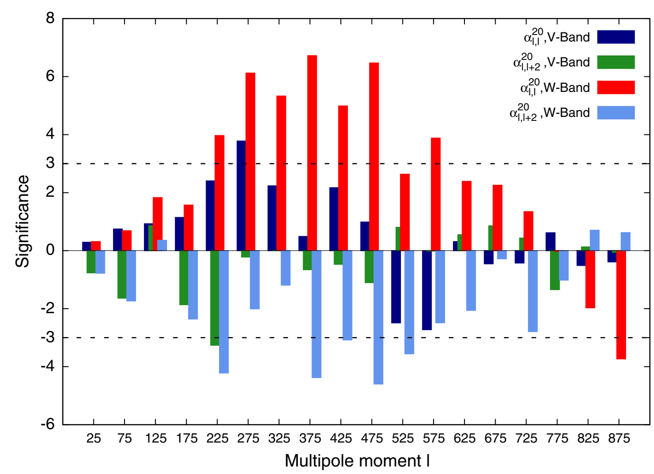

FIG. 3 (color online). This plot depicts the significance of BipoSH spectra from WMAP 9 foreground reduced temperature maps (LAMBDA site). We evaluate the BipoSH spectra in the ecliptic coordinate system and use a KQ75 temperature mask to negotiate with foregrounds. Note that there are highly significant detections in α^{20}_{ll} and α^{20}_{ll+2} modes in both the V band and the W band.

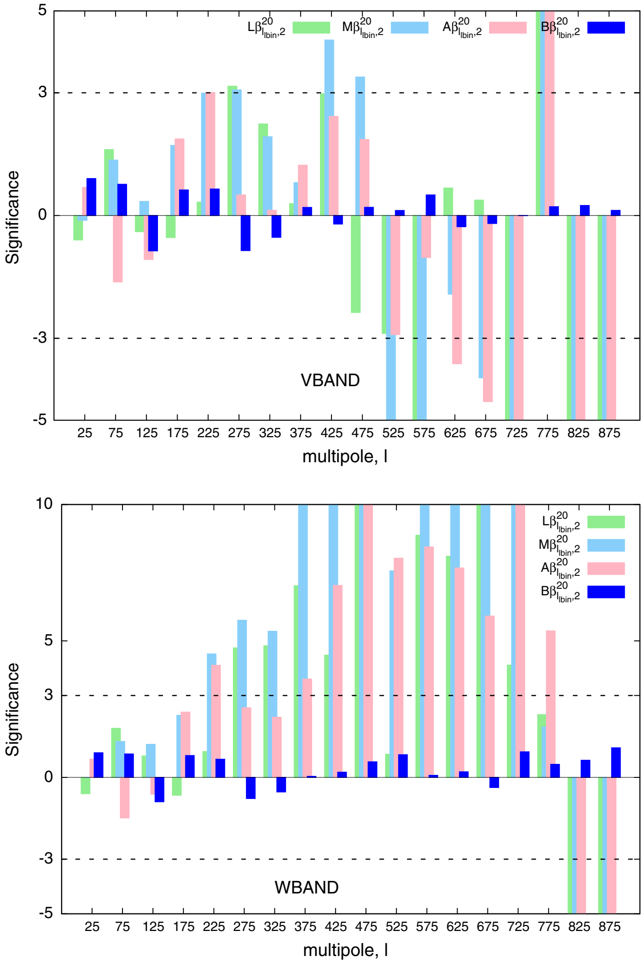

FIG. 4 (color online). The green bars denote the significance of the parameter difference defined via Eq. (9) for the lensing-independent estimator, while the pale-blue and pink bars denote the same for the modulation and anisotropic power spectrum-independent estimators, respectively. To retain spectral information, the orthogonal BipoSH measures were evaluated by computing the minimizing variance sum over multipoles of bin width $\Delta l = 50$.

models under consideration yield high discrepant values for the same model parameter.

The results from this exercise are summarized in Table I, where we quote the cumulative significance (sum of significances in all bins) of the model parameter discrepancy as evaluated from independent BipoSH modes. The most favored model, i.e., the source model showing the least discrepant model parameters, is highlighted.

Through these example case studies, we have demonstrated that the orthogonal BipoSH estimators can be used to quantitatively assess the most favored model of isotropy violation given the data.

C. Revisiting the WMAP quadrupolar anomaly

In this section, we address the following question: Given the source models of isotropy violation (weak lensing,

TABLE II. This table represent the cumulative significance of the difference between the model parameter estimates as derived from independent BipoSH modes. The most preferred model is one for which this difference is least significant. The difference between the noncircular beam parameters is found to be least significant as compared to other source models of SI violation, indicating that the effects of a noncircular beam are the most likely source for the signal seen in WMAP measurements of the CMB sky. The lower cumulative significance seen for the V band is indicative of the lower significance of BipoSH spectra detections themselves.

Source band	Weak lensing	Anisotropic PPS	NC beam	Modulation
V	94.2	71.4	**7.5**	121.9
W	141.3	117.7	**10.8**	188.4

modulation, noncircular beam, and anisotropic power spectrum), can the orthogonal BipoSH estimators be used to assess which of the models provides the most viable explanation for the WMAP quadrupolar anomaly [4]?

Unlike in the case of the example case studies presented in Sec. IV B, where we dealt with a CMB sky with no noise, no foreground contaminations, and no sky cuts, real data are invariably plagued by these systematics. The biases introduced due to these known systematics need to be carefully accounted. While evaluating the orthogonal BipoSH estimator, we follow the analysis procedure described in Ref. [13], which attempted to explain the WMAP quadrupolar anomaly as arising from weak lensing of the CMB photons.

We first evaluate the BipoSH spectra from WMAP maps and evaluate the significance of the detections as depicted in Fig. 3. Recall that detection of nonvanishing BipoSH spectra indicates a violation of SI. Finally, we evaluate the orthogonal BipoSH estimators in order to converge on the most viable explanation for these highly significant detections seen in the WMAP maps. The results of the orthogonal BipoSH estimation analysis on the V band and W band are presented in Fig. 4. It is found that the orthogonal BipoSH estimates for the source models, namely, lensing, modulation, and anisotropic power spectrum, show strong deviations ($>3\sigma$) from nullity, in many CMB multipole bins, clearly indicating that these models provide a poor explanation for the BipoSH spectra detections seen in WMAP maps. On the contrary, for the case of the model of noncircular-beam-induced SI violation, it is seen that the orthogonal BipoSH estimates are consistent with nullity within error bars, for all the CMB multipole bins, unambiguously pointing to the most favored model. We have also summarized this plot in Table II, where we quote the cumulative significance of deviation from nullity for all the orthogonal BipoSH estimators. It is found that the cumulative significance of deviation from nullity is expectedly minimal in the case of noncircular-beam-induced anisotropy as compared to any other model of isotropy violation under consideration.

These findings are in complete synchrony with existing studies which addressed these detections in WMAP data and have thoroughly established that these detections were indeed due to unaccounted noncircular beam effects [5,12,16,17].

V. DISCUSSION

In this article, we have proposed the novel orthogonal BipoSH estimators which can be used to discern between models of isotropy violation. These estimators can be used as a self-consistency test for any model trying to explain non-SI signatures seen in the data. We have given a general prescription for constructing such orthogonal BipoSH estimators for any model of isotropy violation which can be cast in the form of Eq. (7). This happens to be the case in many popular models of SI violation like modulation of the CMB sky [5], anisotropic primordial power spectrum [15], weak lensing, and asymmetric-beam-induced SI violations [18].

We have constructed the orthogonal BipoSH estimators for the above-mentioned models and carried out a systematic study of these estimators on ideal (full sky and no noise) non-SI simulated maps. Through this systematic study, we have established that this method can be used to quantitatively discern between models of isotropy violation and converge to the most preferred model given the data.

Finally, we have carried out a similar analysis on WMAP data. Our study reveals that the most preferred cause for the WMAP quadrupolar anomaly is that of noncircular beam effects. This only reconfirms in an independent fashion this well-known result. This exercise serves to demonstrate the usability of these techniques on real data.

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