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Accidental symmetries and massless quarks in the economical 3-3-1 model

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In the framework of a 3-3-1 model with a minimal scalar sector, known as the economical 3-3-1 model, we study its capabilities of generating realistic quark masses. After a detailed study of the symmetries of the model, before and after the spontaneous symmetry breaking, we find a remaining axial symmetry that prevents some quarks from gaining mass at all orders in perturbation theory. Since this accidental symmetry is anomalous, we also consider briefly the possibility of generating their masses for nonperturbative effects. However, we find that nonperturbative effects are not enough to generate the measured masses for the three massless quarks. Hence, these results imply that the economical 3-3-1 model is not a realistic description of the electroweak interaction.

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I. INTRODUCTION

Up to now, from the experimental point of view, neutrino masses and their mixing, and dark matter are the only issues that demand explanations beyond the standard model (SM). On the other hand, from the theoretical point of view, the quest for a deeper understanding leads us to believe that a more fundamental model of the interactions is needed. That model should be able to answer simple but deep questions. Some of these questions are the following. Why are their three families of quarks and leptons? Is there a more fundamental relation (symmetry) between quarks and leptons? Why does the observed pattern for the particle masses have this particular form? Should the parameters involved have any calculability? What is the origin of CP violation? Even in the SM, what is the origin of the CP-violating Cabibbo-Kobayashi-Maskawa phase? Can it be computed? Is there a more efficient mechanism that is able to account for the matter-antimatter asymmetry in the Universe? What is the mechanism that generates masses and mixing angles for neutrinos? Is there CP violation in leptons? What would be its role in the evolution of the Universe? How can dark matter and dark energy be incorporated? Unfortunately, experimental efforts have not been able to indicate exactly what the physics beyond the SM should be.

In the framework of gauge theories, one way of introducing new physics is to consider a gauge symmetry group larger than the SM one. Some years ago models with the $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ gauge symmetry were proposed [1–4], which have experienced considerable developments. The so-called 3-3-1 models present interesting features concerning the questions above. One of them is that, depending on the representation content, the triangle anomalies cancel out, and the number of families has to be a multiple of three. More precisely, the number of families must be three due to the asymptotic freedom. A version of this kind of model (called minimal [1–4]) presents a Landau-like pole when $\sin^2\theta_W = 1/4$ at energies of the order of a few TeV [5]. This particular behavior stabilizes the electroweak scale (avoiding the hierarchy problem) and also explains why it is observed $\sin^2\theta_W < 1/4$. This model also accounts for the electric charge quantization independently of the nature of the massive neutrinos, i.e., whether they are Dirac or Majorana particles [6]. The model also has interesting features concerning the strong CP problem. In the minimal 3-3-1 model there is an almost automatic Peccei-Quinn (PQ) symmetry, and there is an automatic symmetry in the so-called economical version of the model, as we will show below. In both versions there are ways of solving the strong CP problem while keeping the corresponding axion invisible and protected against gravitational effects [7,8]. Due to a larger gauge symmetry group and a rich scalar sector, this kind of model has garnered some attention in many other subjects, such as new sources of CP violation, active neutrino mass generation and mixing, dark matter candidates, and Z'-boson physics.

In this paper we are concerned with the quark mass generation, in the context of the economical 3-3-1 model (E331 model, for short). In particular, we investigate the capabilities of the model in generating realistic quark masses. The quark sector of this model has already been considered in the literature, and conflicting results were found Refs. [8,9]. In this work, in order to clarify this important issue, we perform a detailed study of the symmetries (local and global) of the entire E331-model Lagrangian. Once we have identified all the symmetries,

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and after the spontaneous symmetry breaking of the scalar potential, we investigate are the remaining symmetries (if any) of the vacuum state. In other words, we attempt to find the independent linear combinations (if any) of the group generators that annihilate the vacuum state, in order to know if the corresponding symmetries are realized *á la* Wigner-Weyl (WW) or Nambu-Goldstone (NG). This is of fundamental importance since it will affect the physical particle spectrum. When the total Lagrangian and vacuum state are both invariant under a symmetry transformation, this is called a WW realization of the symmetry. On the other hand, when the vacuum is not invariant, this is called an NG realization, and this implies a massless NG scalar boson. We find that there is a WW realization of a subgroup of the initial symmetry group that protects some quarks from getting mass at all orders in perturbation theory, as expected from quantum field theory.

The paper is organized as follows. In Sec. II we briefly review the economical 3-3-1 model. In Sec. III we make a detailed study of the symmetries of the model—both before and after the spontaneous symmetry breakdown—and its implication for the quark masses. Nonperturbative effects contributing to quark masses are also briefly considered. Our conclusions are presented in Sec. IV.

II. A BRIEF REVIEW OF THE ECONOMICAL 3-3-1 MODEL

The model considered has a matter content given by [10]

$$\begin{aligned}
\Psi_{aL} &= (\nu_a, e_a, (\nu_{aR})^C)_L^T \sim (\mathbf{1}, \mathbf{3}, -1/3),\\
e_{aR} &\sim (\mathbf{1}, \mathbf{1}, -1),\\
Q_{\alpha L} &= (d_\alpha, u_\alpha, d'_\alpha)_L^T \sim (\mathbf{3}, \mathbf{3}^*, 0),\\
Q_{3L} &= (u_3, d_3, u'_3)_L^T \sim (\mathbf{3}, \mathbf{3}, 1/3),\\
u_{aR} &\sim (\mathbf{3}, \mathbf{1}, 2/3),\\
u'_{3R} &\sim (\mathbf{3}, \mathbf{1}, 2/3),\\
d_{aR} &\sim (\mathbf{3}, \mathbf{1}, -1/3),\\
d'_{\alpha R} &\sim (\mathbf{3}, \mathbf{1}, -1/3),\\
\chi &= (\chi^0, \chi^-, \chi_1^0)^T \sim (\mathbf{1}, \mathbf{3}, -1/3),\\
\rho &= (\rho^+, \rho^0, \rho_1^+)^T \sim (\mathbf{1}, \mathbf{3}, 2/3),
\end{aligned} \tag{1}$$

where $a = 1, 2, 3$, $\alpha = 1, 2$, and the values in parentheses denote, respectively, the quantum numbers corresponding to the $(\mathrm{SU}(3)_C, \mathrm{SU}(3)_L, \mathrm{U}(1)_X)$ groups. From now on latin and greek letters always take the values 1, 2, 3 and 1, 2, respectively.

With the quark, lepton, and scalar multiplets in Eq. (1) we have that the most general Yukawa interactions allowed by the gauge symmetries and renormalizability are

$$\begin{aligned}
\mathcal{L}_Y = {}& Y_{ab}\overline{\Psi_{aL}} e_{bR}\rho + Y'_{ab}\epsilon^{ijk}(\overline{\Psi_{aL}})_i (\Psi_{bL})_j^C (\rho^*)_k\\
&+ G^1\overline{Q_{3L}} u'_{3R}\chi + G^2_{\alpha\beta}\overline{Q_{\alpha L}} d'_{\beta R}\chi^* + G^3_a\overline{Q_{3L}} d_{aR}\rho\\
&+ G^4_{\alpha a}\overline{Q_{\alpha L}} u_{aR}\rho^* + G^5_a\overline{Q_{3L}} u_{aR}\chi + G^6_{\alpha a}\overline{Q_{\alpha L}} d_{aR}\chi^*\\
&+ G^7_\alpha\overline{Q_{3L}} d'_{\alpha R}\rho + G^8_\alpha\overline{Q_{\alpha L}} u'_{3R}\rho^* + \mathrm{H.c.},
\end{aligned} \tag{2}$$

where G^i, Y_{ab}, and Y'_{ab} are arbitrary complex matrices and Y'_{ab} is also antisymmetric. We use the convention that addition over repeated indices is implied.

The χ and ρ scalar multiplets break the $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ gauge symmetry spontaneously. The vacuum expectation values (VEVs) in this model satisfy $\langle \mathrm{Re}\rho^0\rangle \equiv v$, $\langle \mathrm{Re}\chi^0\rangle \equiv u \ll \langle \mathrm{Re}\chi_1^0\rangle \equiv w$. The most general scalar potential that is both invariant under the gauge symmetry and renormalizable is

$$\begin{aligned}
V = {}& \mu_\chi^2\chi^\dagger\chi + \mu_\rho^2\rho^\dagger\rho + \lambda_1(\chi^\dagger\chi)^2 + \lambda_2(\rho^\dagger\rho)^2\\
&+ \lambda_3(\chi^\dagger\chi)(\rho^\dagger\rho) + \lambda_4(\chi^\dagger\rho)(\rho^\dagger\chi).
\end{aligned} \tag{3}$$

With only two scalar multiplets the scalar sector is simple, which is (in principle) an appealing feature of this model compared to other 3-3-1 models [1,2,11].

Finally, the electric charge operator is written as

$$Q = T_3 - \frac{1}{\sqrt{3}}T_8 + X, \tag{4}$$

where T_3 and T_8 are the diagonal generators of the $\mathrm{SU}(3)_L$ group and X refers to the quantum number of the $\mathrm{U}(1)_X$ group.

III. SPONTANEOUS SYMMETRY BREAKING AND MASSLESS QUARKS

Before considering which symmetries are broken, we look for all the exact symmetries—local and global—that this model actually has. By doing so we realize that, apart from the local gauge symmetry $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$, this model has two extra global $\mathrm{U}(1)$ symmetries, which we denote generically by $\mathrm{U}(1)_\zeta$. In order to see this, we write down the relations that these symmetries must obey in order to keep the entire Lagrangian invariant. From Eq. (2) we obtain the following relations:

$$\begin{aligned}
-\zeta_{Q_3} + \zeta_{u'_{3R}} + \zeta_\chi &= 0,\\
-\zeta_Q + \zeta_{d'_R} - \zeta_\chi &= 0,\\
-\zeta_{Q_3} + \zeta_{u_R} + \zeta_\chi &= 0,
\end{aligned} \tag{5}$$

$$\begin{aligned}
-\zeta_Q + \zeta_{d_R} - \zeta_\chi &= 0,\\
-\zeta_{Q_3} + \zeta_{d_R} + \zeta_\rho &= 0,\\
-\zeta_Q + \zeta_{u_R} - \zeta_\rho &= 0,
\end{aligned} \tag{6}$$

$$-\zeta_{Q_3} + \zeta_{d'_R} + \zeta_\rho = 0,$$
$$-\zeta_Q + \zeta_{u'_{3R}} - \zeta_\rho = 0,$$
$$-\zeta_\Psi + \zeta_{e_R} + \zeta_\rho = 0, \tag{7}$$

$$2\zeta_\Psi + \zeta_\rho = 0, \tag{8}$$

where the ζ_{ψ_i}'s above denote the $\mathrm{U}(1)_\zeta$ charges of the ψ_i fields. Solving Eqs. (5)–(8), we find that all charges ζ_{ψ_i} can be written in terms of three independent ones. This means that the model has only three independent $\mathrm{U}(1)_\zeta$ symmetries. In principle, we can choose any three independent $\mathrm{U}(1)_\zeta$ symmetries as a basis. However, some physical considerations can help us make more appropriate choices. First, we note that one of these symmetries is the $\mathrm{U}(1)_X$ gauge symmetry, which is anomaly free by construction and has an associated gauge boson. The other two are global symmetries and they can be divided into a vectorial and an axial symmetry acting on the quarks. The vectorial one is the well-known baryon number symmetry, denoted here as $\mathrm{U}(1)_B$, which is an accidental symmetry in this model (as in the SM). The other one is an axial symmetry also acting on the quarks, which we denote as $\mathrm{U}(1)_{\mathrm{PQ}}$. The last symmetry is a PQ one since it is anomalous and A_{PQ}, the coefficient of the $[\mathrm{SU}(3)_C]^2\mathrm{U}(1)_{\mathrm{PQ}}$ anomaly, is $\propto -3$. Also, we notice that the $\mathrm{U}(1)_{\mathrm{PQ}}$ is a natural symmetry in the sense that it is not imposed; rather, it follows from the gauge local symmetry and renormalizability. In other words, the economical model naturally has a PQ symmetry. The assignment of the three independent U(1) quantum charges is shown in Table I. (These quantum charges appeared for the first time in Ref. [8]; we have written them here for the sake of completeness and clarity.) Thus, the model actually has a larger symmetry: $G \equiv \mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X \otimes \mathrm{U}(1)_B \otimes \mathrm{U}(1)_{\mathrm{PQ}}$, where the last two are accidental and global symmetries.

Now, let us search for the remaining symmetries after the χ and ρ scalar triplets obtain their VEVs, $\langle\chi\rangle \equiv V_\chi = \frac{1}{\sqrt{2}}(u, 0, w)^T$ and $\langle\rho\rangle \equiv V_\rho = \frac{1}{\sqrt{2}}(0, v, 0)^T$. To do this, we consider an infinitesimal transformation of the total group G on the vacuum states to find the generators of the unbroken subgroups as a linear combination of the T_i, X, PQ, and B generators. Here, it is important to note that the $\mathrm{SU}(3)_C \otimes \mathrm{U}(1)_B$ subgroups are clearly unbroken and thus we can omit them in the following analysis without affecting our conclusions. Then, under an infinitesimal transformation on the vacuum we have

$$\left(\sum_{i=1}^{8} \alpha_i T_i + \gamma X\chi 1_{3\times3} + \delta \mathrm{PQ}_\chi 1_{3\times3}\right) V_\chi = 0, \tag{9}$$

$$\left(\sum_{i=1}^{8} \alpha_i T_i + \gamma X_\rho 1_{3\times3} + \delta \mathrm{PQ}_\rho 1_{3\times3}\right) V_\rho = 0, \tag{10}$$

where α_i, γ, and δ are independent real constants and $1_{3\times3}$ denotes the 3×3 identity matrix. Also, we have from Table I that $X\chi = -1/3$, $X_\rho = 2/3$, and $\mathrm{PQ}_\chi = \mathrm{PQ}_\rho = 1$. Since the χ and ρ scalar triplets are in the fundamental representation of $\mathrm{SU}(3)_L$, the T_i generators in Eqs. (9) and (10) are given by $\lambda_i/2$, where λ_i are the well known Gell-Mann matrices. From Eqs. (9)–(10) we have

$$v(\alpha_1 - i\alpha_2) = 0, \tag{11}$$

$$v(\alpha_6 + i\alpha_7) = 0, \tag{12}$$

$$u(\alpha_1 + i\alpha_2) + w(\alpha_6 - i\alpha_7) = 0, \tag{13}$$

$$v(-3\alpha_3 + \sqrt{3}\alpha_8 + 4\gamma + 6\delta) = 0, \tag{14}$$

$$3u(\alpha_4 + i\alpha_5) - 2w(\sqrt{3}\alpha_8 + \gamma - 3\delta) = 0, \tag{15}$$

$$u(3\alpha_3 + \sqrt{3}\alpha_8 - 2\gamma + 6\delta) + 3w(\alpha_4 - i\alpha_5) = 0, \tag{16}$$

with $i = \sqrt{-1}$. Solving Eqs. (11)–(16) simultaneously (with $u \neq 0$, $v \neq 0$, and $w \neq 0$), we have that $\alpha_1 = \alpha_2 = \alpha_5 = \alpha_6 = \alpha_7 = 0$ and

$$\alpha_4 = -\frac{6uw}{u^2 + w^2}\delta \equiv -3\sin(2\theta)\delta, \tag{17}$$

$$\alpha_8 = \frac{6}{\sqrt{3}}\left(\frac{2w^2}{u^2 + w^2} - 1\right)\delta - \frac{\alpha_3}{\sqrt{3}} \equiv \frac{6\cos(2\theta)}{\sqrt{3}}\delta - \frac{\alpha_3}{\sqrt{3}}, \tag{18}$$

$$\gamma = -\frac{3w^2}{u^2 + w^2}\delta + \alpha_3 \equiv -\frac{3}{2}(1 + \cos(2\theta))\delta + \alpha_3, \tag{19}$$

where $\tan\theta \equiv u/w$. Since the parameters α_3 and δ are independent, this implies that from the ten generators only two linearly independent combinations, say g_1 and g_2, remain unbroken. At first glance, the choice of these generators is arbitrary. However, we take into consideration that one of them has to be the anomaly-free electric charge generator, which is achieved by taking $\delta = 0$ and $\alpha_3 = 1$. By doing so, $g_1 = Q$. The other generator, g_2, must have $\delta \neq 0$ in order to be linearly independent of g_1 (since all generators with $\delta = 0$ will be proportional to Q). Hence, the unbroken generators are written as

TABLE I. Assignment of the three independent U(1) quantum charges in the economical 3-3-1 model.

	$Q_{\alpha L}$	Q_{3L}	(u_{aR}, u'_{3R})	$(d_{aR}, d'_{\alpha R})$	Ψ_{aL}	e_{aR}	ρ	χ
$\mathrm{U}(1)_X$	0	1/3	2/3	−1/3	−1/3	−1	2/3	−1/3
$\mathrm{U}(1)_B$	1/3	1/3	1/3	1/3	0	0	0	0
$\mathrm{U}(1)_{\mathrm{PQ}}$	−1	1	0	0	−1/2	−3/2	1	1

$$g_1 = T_3 - \frac{1}{\sqrt{3}} T_8 + X, \tag{20}$$

$$g_2 = \Big[3\cos^2(\theta) T_3 - 3\sin(2\theta) T_4 + \frac{1}{2}\sqrt{3}(3\cos(2\theta) - 1) T_8 + \mathrm{PQ}\Big]\delta, \tag{21}$$

with $\delta \neq 0$ in the last equation. The symmetry associated to g_1, $\mathrm{U}(1)_Q$, is anomaly free, as is well known. The g_2 generator, which is independent of g_1, is a linear combination of T_3, T_4, T_8, and PQ generators. We refer to the symmetry associated to g_2 as $\mathrm{U}(1)_H$. The key point here is that a part of the initial axial symmetry, $\mathrm{U}(1)_{\mathrm{PQ}}$, remains unbroken because the coefficient δ in Eq. (21) is always different from zero. In conclusion, the existence of g_1 and g_2 implies that the $\mathrm{U}(1)_Q \otimes \mathrm{U}(1)_H$ subgroup of $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X \otimes \mathrm{U}(1)_{\mathrm{PQ}}$ remains unbroken.

Now, since G is an exact symmetry, i.e., $[G, \mathcal{L}_T] = 0$ (where $\mathcal{L}_T$ is the total Lagrangian of the model) from Goldstone's theorem [12], we have exactly eight NG scalar bosons (an NG scalar boson for each broken generator), which in this model will become the longitudinal components of the eight massive gauge vector bosons via the Higgs mechanism. In the physical scalar spectrum this model has only massive scalar bosons, H_1^0, H_2^0, H^+, H^-, as was shown in Ref. [8]. If the g_2 was broken, an NG scalar boson would appear in the physical scalar spectrum. Because g_2 has a component in the PQ generator, this physical NG scalar boson would be an axion. However, this does not happen and the model has three massless quarks instead (one u-type quark and two d-type quarks). This can be easily seen from the mass matrices, because a pair of rows in the u-quark mass matrix and two pairs of rows in the d-quark mass matrix are proportional to each other [see Eqs. (18) and (19) in Ref. [8]]. The exact form of these massless quarks is neither clarifying nor relevant for our analysis, and thus we do not write them here. These massless quarks are fully expected because an exact axial symmetry that is realized in the WW manner implies massless fermions.

The action of the remaining $\mathrm{U}(1)_H$ symmetry preventing some quarks from gaining mass becomes obvious when we change the basis to work with the mass eigenstates instead of the symmetry eigenstates. The symmetry eigenstates $U_{iL,R}, D_{jL,R}$ (with $i = 1, \ldots, 4$ and $j = 1, \ldots, 5$) and the mass eigenstates $(U_M)_{iL,R}, (D_M)_{jL,R}$ are related by $U_{L,R} = (V^U_{L,R})^\dagger (U_M)_{L,R}$ and $D_{L,R} = (V^D_{L,R})^\dagger (D_M)_{L,R}$, where $V^{U,D}_{L,R}$ are independent unitary matrices such that $V^U_L M^U V^{U\dagger}_R = \hat{M}^U$ and $V^D_L M^D V^{D\dagger}_R = \hat{M}^D$, where $\hat{M}^U = \mathrm{diag}(m_{u_{M1}}, m_{u_{M2}}, m_{u_{M3}}, m_{u_{M4}})$ and $\hat{M}^D = \mathrm{diag}(m_{d_{M1}}, m_{d_{M2}}, m_{d_{M3}}, m_{d_{M4}}, m_{d_{M5}})$. Since the mass matrix of the u- and d-quark types are not Hermitian matrices, in order to obtain the mass eigenstates we have to solve the matrix equations $V^q_L M^q M^{q\dagger} V^{q\dagger}_L = V^q_R M^{q\dagger} M^q V^{q\dagger}_R = (\hat{M}^q)^2$, where $q = U, D$. More specifically, we have to find the base-rotation matrices $V^U_{L,R}$ and $V^D_{L,R}$ to be able to write the Yukawa interactions in terms of the quark-mass eigenstates. This task can be done using standard procedures. Unfortunately, exact analytical expressions for these matrices are enormous, so it is worthless to show them here. The diagonalization study shows that we have one vanishing eigenvalue in the u-quark sector and two in the d-quark sector, as expected. The respective zero-mass eigenstates are clearly identified. Let us call them u_{M1}, d_{M1}, and d_{M2}. This means that there are no mass terms of the form $m_{u_{M1}} \overline{u_{M1L}} u_{M1R} + m_{d_{M1}} \overline{d_{M1L}} d_{M1R} + m_{d_{M2}} \overline{d_{M2L}} d_{M2R} + \mathrm{H.c.}$, i.e., $m_{u_{M1}} = m_{d_{M1}} = m_{d_{M2}} = 0$.

We find more important results by looking at the quark-scalar field interactions coming from the Yukawa interactions in Eq. (2). Here we find that there are no interactions involving the right component of these massless quark states. The right states u_{M1R}, d_{M1R}, and d_{M2R} disappear from the Yukawa interactions. In other words, no left quark states are coupled with these massless states through neutral- or charged-scalar fields. Nonetheless, these zero-mass right components should have some interaction after all. In fact, from the quark kinetic terms we find that they interact only with the neutral vector bosons A_μ, Z_μ, and Z'_μ. These interactions couple only quarks with the same chirality. This means that each of the right components u_{M1R}, d_{M1R}, and d_{M2R} can be transformed by an arbitrary $\mathrm{U}(1)_H$ phase without affecting other terms in the Lagrangian. Hence, looking at the Yukawa and the neutral vector-boson interactions, written in terms of the quark-mass eigenstates, we can identify the $\mathrm{U}(1)_H$ symmetry that prevents these quark states from getting mass: the massless right fields u_{M1R}, d_{M1R}, and d_{M2R} transform as $e^{i\alpha} u_{M1R}, e^{i\alpha} d_{M1R}$, and $e^{i\alpha} d_{M2R}$ and all other fields transform trivially under $\mathrm{U}(1)_H$ (note that this symmetry is anomalous with the $[\mathrm{SU}(3)_C]^2 \mathrm{U}(1)_{\mathrm{H}}$ anomaly $\propto -3$). This is a clear and undoubtable manifestation of the remaining symmetry we have found. If we consider perturbation theory, these massless quarks can not get mass through radiative corrections to their propagators since the right component of these fields disappear from the Yukawa interactions and they only couple to neutral vector bosons, which conserve chirality. Therefore, there is no way to form loop diagrams to give mass for these particular fields, at all orders in perturbation theory.

Now, let us discuss the possibility of generating masses for these massless quarks through nonperturbative corrections and the viability of this model to explain the low-energy hadron phenomenology. Roughly speaking, this can be seen as follows. From both chiral QCD and lattice calculations the ratio μ_u/μ_d is 0.410 ± 0.036 [13–15], where μ_u and μ_d are the "low-energy quark masses." These should be distinguished from the quark-mass parameters m_i of the QCD Lagrangian at high scale [16]. In particular, $\mu_u = \beta_1 m_u + \beta_2 \frac{m_d m_s}{\Lambda_{\chi SB}}$, where $\Lambda_{\chi SB} \sim 1$ TeV (where we have identified naturally the massless quarks as $u = u_{M1}$,

$d = d_{M1}$, and $s = d_{M2}$). This means that μ_μ receives an additive nonperturbative contribution of order $m_d m_s$ in addition to the perturbative one, $\beta_1 m_u$, which is zero because both m_u's are zero. The nonperturbative contribution is also zero because m_d and m_s are both zero and β_2 is estimated to be a number of order one. Thus, $\mu_u = 0$, which is in complete disagreement with the ratio μ_u/μ_d. A similar analysis is also valid for μ_d and μ_s [17].

IV. CONCLUSIONS

The scalar content in the E331 model is not enough to break the initial symmetry, G to $\mathrm{U}(1)_Q \otimes \mathrm{U}(1)_B$. Instead, an extra generator g_2 remains unbroken and thus the model has a $\mathrm{U}(1)_H$ axial symmetry after the spontaneous symmetry breaking. As we have explicitly shown above, g_2 is a linear combination of the T_3, T_4, T_8, and PQ generators and it is linearly independent of the generators of the electric charge and baryonic number, g_1 and B, respectively. Because of the PQ component in the g_2 generator, we have that the initial axial $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry is not completely broken. In other words, the $\mathrm{U}(1)_Q \otimes \mathrm{U}(1)_H$ subgroup of the $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X \otimes \mathrm{U}(1)_{\mathrm{PQ}}$ group remains unbroken. Therefore, the model has three massless quarks. The $\mathrm{U}(1)_H$ symmetry acts on the mass eigenstates as an axial symmetry, $u_{M1R} \to e^{i\alpha} u_{M1R}$, $d_{M1R} \to e^{i\alpha} d_{M1R}$, $d_{M2R} \to e^{i\alpha} d_{M2R}$, and this will protect these massless quarks from acquiring mass at any level of perturbation theory. Furthermore, we recall that the unbroken $\mathrm{U}(1)_H$ subgroup has its origin in an axial symmetry, $\mathrm{U}(1)_{\mathrm{PQ}}$, which—although anomalous—is an accidental symmetry in the sense that it follows from the gauge symmetries and renormalizability. Therefore, the remaining axial symmetry acting on quarks will only be broken by nonperturbative QCD processes [18]. However, these effects are not enough to provide the necessary low-energy quark masses, μ_i, to the three massless quarks to bring the model into agreement with both chiral QCD and lattice calculations, which give the ratio μ_u/μ_d as 0.410 ± 0.036 [13–15]. Hence, the economical version of the 3-3-1 model cannot be considered a realistic description of the electroweak interaction. The original 3-3-1 models, such as the model I presented in Ref. [4], do not suffer from such an illness.

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