Cosmic neutrino spectrum and the muon anomalous magnetic moment in the gauged $L_{\mu} - L_{\tau}$ model

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The energy spectrum of cosmic neutrinos, which was recently reported by the IceCube Collaboration, shows a gap between 400 TeV and 1 PeV. An unknown neutrino interaction mediated by a field with a mass of the MeV scale is one of the possible solutions to this gap. We examine whether the leptonic gauge interaction $L_{\mu} - L_{\tau}$ can simultaneously explain the two phenomena in the lepton sector: the gap in the cosmic neutrino spectrum and the unsettled disagreement in the muon anomalous magnetic moment. We illustrate that there remain regions in the model parameter space which account for both of the problems. Our results also provide a hint to the distance to the source of the high-energy cosmic neutrinos.

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I. INTRODUCTION

Following the observations of two celebrated events [1,2]. IceCube has accumulated 37 high-energy neutrino events, which is significantly greater than the expected number of background events [3]. These events start showing us an energy spectrum of cosmic neutrinos at the uncharted high-energy regions. The spectrum is consistent with the Waxman-Bahcall bound [4,5] estimated from the high-energy cosmic-ray observations. An interesting feature of the IceCube spectrum is that there is a gap in the energy range between 400 TeV and 1 PeV. Although the gap has not been statistically established yet, some attempts to explain it have been examined [6-9]. An attractive candidate for explanation is an attenuation process driven by an unknown interaction between the high-energy cosmic neutrino and the cosmic neutrino background (C ν B). This type of interaction, the secret neutrino interaction [10], has been discussed particularly in the context of cosmology and astrophysics [11–16]. In this work, we introduce the gauged U(1) leptonic interaction associated with the muon number minus the tau number $L_{\mu} - L_{\tau}$, which is anomaly free within the Standard Model (SM) particle contents [17,18] and can naturally explain the large atmospheric mixing [19–22].

Although the secret neutrino interactions are not tightly constrained from cosmology and laboratory experiments, it is difficult to construct theoretical models with large couplings of the interactions [23]. Because of the SU(2) symmetry in the SM, a secret neutrino interaction, in general, results in providing an interaction with the corresponding charged lepton with the size of the same

order as the neutrino interaction. In Refs. [8,9], an SU(2)violation is introduced to circumvent the bound from the charged lepton sector of their leptonic force. Unlike previous works, the $L_{\mu} - L_{\tau}$ interaction in our scenario does not discriminate between the charged lepton and the corresponding neutrino. We take advantage of the interaction in the charged lepton sector to account for the inconsistency in the muon anomalous magnetic moment $(g_{\mu} - 2)$ [24]. In short, we examine whether this leptonic force simultaneously explains the two phenomena in the lepton sector: the gap in the cosmic neutrino spectrum and the long-standing inconsistency in the $g_{\mu} - 2$. It has been pointed out [25] that one of the attractive scenarios for solving the g_{μ} – 2 problem is a new muonic force mediated by a field with a mass of $\mathcal{O}(1)$ MeV, which is, by accident, within the mass range of the mediation field of the neutrino secret interaction that can attenuate the cosmic neutrinos with energy around $\mathcal{O}(1)$ PeV [6–9]. We will demonstrate that the strength of the leptonic force which can explain the observed value of the $g_{\mu} - 2$ reproduces the gap in the IceCube spectrum. Interestingly, the model parameters in our scenario are manifestly related to the distance to the source of the cosmic neutrinos. We will briefly discuss this point later.

II. MODEL

We consider the following gauge interaction:

$$\mathcal{L}_{Z'} = g_{Z'} Q_{\alpha\beta} (\bar{\nu_{\alpha}} \gamma^{\rho} P_L \nu_{\beta} + \bar{\ell_{\alpha}} \gamma^{\rho} \ell_{\beta}) Z'_{\rho}, \qquad (1)$$

where Z' is the new gauge boson with the gauge coupling $g_{Z'}$, $\alpha, \beta = e, \mu, \tau$, and $Q_{\alpha\beta} = \text{diag}(0, 1, -1)$ represents the charge matrix of $L_{\mu} - L_{\tau}$. After $L_{\mu} - L_{\tau}$ is spontaneously broken, Z' acquires a mass, $m_{Z'}$. In order to keep generality, however, we do not go into the details of the symmetry breaking and simply treat $m_{Z'}$ as a model parameter. Also, the kinetic mixing with the SM $U(1)_Y$ is set to zero. The first term of Eq. (1) is the source of the secret neutrino

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interaction. In the $L_{\mu} - L_{\tau}$ model, as discussed in the next section, a mean free path (MFP) of the cosmic neutrino is calculated to be > $\mathcal{O}(1)$ Mpc, which is many orders of magnitude larger than the coherence length. Traveling such a long distance, neutrino flavor eigenstates are expected to lose their coherence, and thus the scattering process can be described in terms of mass eigenstates with the Lagrangian

$$\mathcal{L}_{Z'\nu\nu} = g'_{ij}\bar{\nu}_i\gamma^{\rho}P_L\nu_j Z'_{\rho},\tag{2}$$

where $g'_{ij} = g_{Z'}(V^{\dagger}QV)$, with i, j = 1...3, and V is the lepton mixing matrix. In order to realize the gap in the cosmic neutrino spectrum, we utilize a resonant interaction and take a Breit-Wigner form. Then the scattering cross section of a $\nu_i \bar{\nu_j} \rightarrow \nu \bar{\nu}$ process is obtained as

$$\sigma_{ij} = \frac{1}{6\pi} |g'_{ij}|^2 g_{Z'}^2 \frac{s}{(s - m_{Z'}^2)^2 + m_{Z'}^2 \Gamma_{Z'}^2},$$
 (3)

where \sqrt{s} is the center-of-mass energy and $\Gamma_{Z'} = g_{Z'}^2 m_{Z'}/(12\pi)$ is the decay width of Z'. As for g'_{ij} , defined in Eq. (2), throughout this study, we use those evaluated with the best fit values of the neutrino mixing parameters [26], yielding

$$\frac{|g'_{ij}|}{g_{Z'}} = \begin{pmatrix} 0.054(0.051) & 0.163(0.158) & 0.555(0.556) \\ 0.163(0.158) & 0.088(0.082) & 0.806(0.808) \\ 0.555(0.556) & 0.806(0.808) & 0.143(0.133) \end{pmatrix}$$
(4)

for the inverted (normal) mass hierarchy, IH (NH). For the mass-squared differences, we also use the best fit values [26].

Analogous to previous works [8,9], we assume that the ratio of initial fluxes in the flavor basis is $\phi_e:\phi_\mu:\phi_\tau = 1:2:0$, which is converted into that in the mass basis via $\phi_i \equiv \sum_{\beta} |V_{\beta i}|^2 \phi_{\beta}$. In view of $\theta_{13} \approx 0$ and $\theta_{23} \approx \pi/4$, we assume $\phi_1:\phi_2:\phi_3 = 1:1:1$ throughout this study. Note that our results are not largely affected by the changes of the initial flux ratio, since all mass eigenstates of the cosmic neutrinos can be attenuated by one C ν B state.

The introduction of the $L_{\mu} - L_{\tau}$ symmetry brings not only the secret neutrino interaction but also the new interaction among the charged leptons. This gives us a chance to solve the inconsistency in the $g_{\mu} - 2$ [27]. In Fig. 1, we show the parameter region favored by the observations of the $g_{\mu} - 2$ within 2σ with the shaded (red) band [24]. The region excluded by the neutrino trident production process [25] from the Columbia-Chicago-Fermilab-Rochester experiment [28] is also indicated by the hatched (grey) region. We will demonstrate that the gap is successfully reproduced with the parameters in the shaded (red) region.



FIG. 1 (color online). The shaded (red) band is the $\pm 2\sigma$ parameter space for the $g_{\mu} - 2$ [24]. The hatched (gray) region is excluded by the constraint from the neutrino trident production process at 95% C.L. [25]. The symbol × indicates the set of parameters used in Figs. 2, 3, 4, and 5 as reference.

III. RESULT

We consider that the cosmic neutrinos, ν_i , are attenuated by the interaction Eq. (2) with the $C\nu$ B, $\bar{\nu}_j$. As reference, in what follows, we will use $(g_{Z'}, m_{Z'}) = (5 \times 10^{-4}, 1.9 \text{ MeV})$, which is represented by × in Fig. 1. Also, z = 0.2, $m_{\nu_3} = 3 \times 10^{-3}$ eV, and IH are assumed. As for the NH case, several comments are given at the end of this section. The MFP λ_i of the cosmic neutrino ν_i with energy E_{ν_i} is described as

$$\lambda_i(E_{\nu_i}, z) = \left[\sum_{j=1}^3 \int \frac{d^3 \mathbf{p}}{(2\pi)^3} f_j(|\mathbf{p}|, z) \sigma_{ij}(\mathbf{p}, E_{\nu_i}^s)\right]^{-1}, \quad (5)$$

where z is the parameter of redshift, **p** is the momentum of the C ν B, and $f_j(|\mathbf{p}|, z) = (e^{|\mathbf{p}|/(T_{\nu 0}(1+z))} + 1)^{-1}$ is the distribution function with the C ν B temperature $T_{\nu 0} \sim 1.95$ K at present. Note that $E_{\nu_i}^s$ is the energy of a cosmic neutrino ν_i , which is measured at the position z where the ν_i is scattered, and E_{ν_i} is the energy measured at IceCube [29,30]. They are related by $E_{\nu_i}^s = (1+z)E_{\nu_i}$. The survival rate R_i of the cosmic neutrino ν_i traveling from the source at z to us (z = 0) is evaluated by

$$R_i = \exp\left[-\int_0^z \frac{1}{\lambda_i(E_{\nu_i}, z')} \frac{dL}{dz'} dz'\right],\tag{6}$$

where $dL/dz = c/(H_0\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda})$, with the present values of the cosmological parameters; the matter energy density $\Omega_m = 0.315$, the dark energy density $\Omega_\Lambda = 0.685$, and the Hubble constant $H_0 = 100h \text{ km/s/Mpc}$, with h = 0.673 [31]. Here we assume that the source of the cosmic neutrinos is located at a particular *z*. We expect that the effect caused by the



FIG. 2 (color online). The MFPs of ν_1 (solid red line), ν_2 (dashed green line), and ν_3 (dash-dotted blue line) for IH. The horizontal (gray) line represents L/4 for reference.

inclusion of the realistic (widespread) distribution of neutrino sources is limited in our case since z is chosen to be small. We present an example of the MFPs of each mass eigenstate of the cosmic neutrinos in Fig. 2. When the resonance condition $s \simeq m_{Z'}^2$ in σ_{ij} is satisfied, the MFP takes its minimum value. With the energy $E_{\nu_i}^{\text{res}}$ at which the resonance takes place, the condition for the cosmic neutrino ν_i is described as

$$m_{Z'}^2 \simeq 2E_{\nu_i}^{\rm res}(1+z) \Big[\sqrt{|\boldsymbol{p}|^2 + m_{\nu_j}^2} - |\boldsymbol{p}|\cos\theta \Big], \quad (7)$$

where θ is the angle between the momentum of the cosmic neutrino and that of the C ν B. Applying the parameters adopted in Fig. 2 to Eq. (7), we find the following two resonant energies:

$$E_{\nu_{i}}^{\text{res}} = \begin{cases} \frac{1}{1+z} \frac{m_{Z'}^{2}}{2m_{\nu_{1}(2)}} \approx 30 \text{ TeV}, \\ \frac{1}{1+z} \frac{m_{Z'}^{2}}{2m_{\nu_{3}}} \approx 500 \text{ TeV}. \end{cases}$$
(8)

Indeed, one can see the two resonance structures in Fig. 2. Note that the dip around 500 TeV on each MFP λ_i is created by the scattering with the lightest C ν B state $\bar{\nu}_3$, and a narrow dip around 30 TeV consists of the contributions from two heavy states, $\bar{\nu}_1$ and $\bar{\nu}_2$. The resonant condition Eq. (7) would help us to reproduce the IceCube gap at the suitable energy range in the calculation of the total flux which will be shown below.

Let us explain four important points to understand the features of the MFPs shown in Fig. 2. First, each mass eigenstate of the cosmic neutrinos is attenuated by all mass eigenstates of the $C\nu B$ since all elements of Eq. (4) are not vanishing. This is one of the distinctive features of our scenario. Second, the difference in the depth of the MFPs at each resonance energy stems from the difference of the

couplings g'_{ii} for each combination of *i* and *j*. For example, at the resonance around 500 TeV in Fig. 2, all mass eigenstates $\nu_{1,2,3}$ of the cosmic neutrinos are attenuated by the lightest C ν B, i.e., $\bar{\nu}_3$. Because $|g'_{33}| < |g'_{23}|, |g'_{13}|$ [see Eq. (4)], the MFP λ_3 for ν_3 becomes longer than the others, $\lambda_{1,2}$. Although the strength of the interactions between ν_3 and $\bar{\nu}_3$ is relatively small, it is still large enough to scatter off ν_3 around the resonance. Third, the thermal distribution effect of the C ν B becomes more important for the C ν B with a smaller mass, which is apparent from Eq. (7). The effect makes the energy range at which the resonance condition is satisfied broader. This feature can be clearly read off from Fig. 2: the resonance around 500 TeV, which is associated with the lightest $C\nu B$ state, is broader than that around 30 TeV, which is related to the heavier $C\nu B$ states. Finally, we pay attention to the fact that the resonance energy measured at IceCube depends on z, as described by Eq. (7). The observed gap in the spectrum results from superposition of the resonant effect in the MFP with a different zalong the path of the cosmic neutrino from the source to IceCube. To investigate the resonance energy range from another point of view, we draw the MFPs λ_2 for ν_2 with various values of z in Fig. 3, where the other parameters are fixed. As is expected from the z dependence of the distribution function f_i , a larger value of z makes the resonance region broader. The position of the resonant energy varies with the change of z along the path of the cosmic neutrino. This behavior can be understood from the redshift of the resonant energy; cf. Eq. (7). From these two effects brought by z, we expect that a choice of a larger value of z makes the gap width broader in the calculation of the total flux. Since the width of the gap is determined mainly by the smallest neutrino mass and the distance z to the neutrino source, there is a strong correlation between them. For example, when z is taken to be small, the lightest neutrino mass must also be small. In terms of the survival rate R, we can also confirm this correlation in Fig. 4, where only the scattering between the cosmic neutrino ν_2 and the $C\nu B \bar{\nu}_3$ is considered. In the upper (lower) panel, R_2 is



FIG. 3 (color online). The MFPs of the cosmic neutrino ν_2 for IH with various values of *z*.



FIG. 4 (color online). The survival rates of the cosmic neutrino ν_2 for IH with various values of z (upper panel) and m_{ν} (lower panel). Only the scattering between the cosmic neutrino ν_2 and the C ν B $\bar{\nu}_3$ is considered.

calculated with various values of z (m_{ν}), while the other parameters are fixed at their reference values. These plots tell us that when we know the distance (z) to the neutrino source, we can predict the neutrino mass associated with the gap. By scanning the values of z and m_{ν_i} , we find that the lightest neutrino mass should be larger than $\mathcal{O}(10^{-3})$ eV in a small z region.

In Fig. 5, we finally calculate the total flux $\varphi(E_{\nu})$ of the cosmic neutrinos $(\nu + \bar{\nu})$ with the set of parameters, which



FIG. 5 (color online). The total flux of the cosmic neutrinos $(\nu + \bar{\nu})$ for IH.

solves the $g_{\mu} - 2$ problem. Although we have an additional resonance caused by the interaction with the other mass eigenstates of $C\nu B$ in the low energy region, it may be smeared by atmospheric neutrino events. We have also checked to see whether a realistic gap is obtained for NH as well by using similar values of the parameters, but we found this difficult. This is because a gap between 400 TeV and 1 PeV caused by the lightest $C\nu B \bar{\nu}_1$ is always accompanied by that caused by $\bar{\nu}_2$; the latter attenuates the flux below 400 TeV. It may be interesting to focus on larger mass regions, where the lighter (or all) neutrino masses are degenerated. Then, the two (or all) gaps are merged into one gap between 400 TeV and 1 PeV. We will study this possibility elsewhere.

IV. SUMMARY

We have discussed the possibility of whether the gap in the cosmic neutrino flux in the 400 TeV-1 PeV range reported by the IceCube experiment can be explained in the gauged $L_{\mu} - L_{\tau}$ model. We have shown that the MFPs of the cosmic neutrinos can be reduced by the resonant scattering with the lightest $C\nu B$ for IH. We have also shown that the MFP has a dip with an appropriate width for the lightest neutrino mass with a value of around several 10^{-3} eV. This is because the thermal distribution of $C\nu B$ makes the resonant energy range satisfying Eq. (7) broader. The dip in the flux becomes broader as the redshift is higher, also because of the effect of superposition of the MFPs with different z's (cf. Fig. 3). Once m_{ν} is fixed, the redshift is determined so as to explain the observed gap (cf. Fig. 4). In Fig. 5, we have shown that the observed gap in the cosmic neutrino spectrum is obtained for the lightest neutrino mass 3×10^{-3} eV and the Z' boson mass 1.9 MeV for the IH case. The gauge coupling constant is taken as 5×10^{-4} , which can settle the $g_{\mu} - 2$ problem. Importantly, in this example, the redshift is determined as 0.2, which corresponds to about 0.845 Gpc to neutrino sources.

Before closing the summary, three comments are in order. (1) With the neutrino masses and the mixing parameters applied in our analysis, the effective neutrino mass of neutrinoless double decay processes is between 4.81×10^{-2} and 1.67×10^{-2} eV in the IH case, which will be examined by the KamLAND-Zen experiment [32]. (2) Also, the sum of the neutrino masses is 0.102 eV, which will be explored in future astrophysical observations [33]. (3) In this study, the effect caused by the neutrinos after the scattering was not taken into account. Inclusion of the secondary neutrinos may explain a small bump at the lower energy bin next to the gap. We also did not consider the distribution of sources of cosmic neutrinos. The impact of the source distribution on our results may be limited because the distance to the source was taken to be small. We leave a detailed study for our next work.

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