CP violation and chiral symmetry breaking in hot and dense quark matter in the presence of a magnetic field

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We investigate chiral symmetry breaking and strong charge parity (*CP*) violation effects on the phase diagram of strongly interacting matter in the presence of a constant magnetic field. The effects of a magnetic field and strong *CP* violating term on the phase structure at finite temperature and density are studied within a three flavor Nambu-Jona-Lasinio model including the Kobayashi-Maskawa-t'Hooft determinant term. This is investigated using an explicit variational ansatz for ground state with quark-antiquark pairs leading to condensates both in scalar and pseudoscalar channels. A magnetic field enhances the condensate in both the channels. Inverse magnetic catalysis for *CP* transition at finite chemical potential is seen for zero temperature and for small magnetic fields. The *CP* transition becomes first order at finite baryon chemical potential and could be relevant for generating *CP*-odd metastable domains in heavy ion collision experiments.

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I. INTRODUCTION

The study of charge parity (*CP*) violation in strong interaction is of immense importance in the context of the early universe scenario [1,2] as well as heavy ion collision experiments [3–5]. It is well known that the existence of instanton configurations of the gauge fields and their intimate connection with axial anomaly allow a *CP* violating topological term in the QCD Lagrangian given as $\mathcal{L}_\theta = \frac{\theta}{64\pi^2} g^2 F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$, θ being the QCD vacuum angle. However, so far experiments indicate that in nature θ parameter is vanishingly small for QCD vacuum [6]. For vanishing θ, the QCD Lagrangian is *CP* invariant and, in such a case, it has been proved that spontaneous parity violation cannot arise [7]. On the other hand, for the case of $\theta = \pi$, although the QCD Lagrangian is *CP* conserving, spontaneous *CP* violation can occur through the Dashen phenomenon [8].

Even if *CP* is not violated for QCD vacuum, it is conceivable that it can be violated for QCD matter at finite temperature and/or density. Indeed, it has been argued that the hot matter produced in heavy ion collision experiments can give rise to domains of metastable states that violate *CP* locally [3] through sphaleron activated processes. These states with an effective nonvanishing value of θ can decay through *CP*-odd processes [9]. Further, apart from producing high temperature, colliding nuclei also produce transient strong magnetic fields for noncentral collisions. A nonzero θ leads to a deviation of left- and right-hand helicity quarks. As a consequence an electromagnetic current is generated along the magnetic field. Such a mechanism, known as the chiral magnetic effect [4], may explain the charge separation in the recent STAR results [5]. On the other hand, in the context of cold and dense matter, compact stars can be strongly magnetized. The magnetars, which are strongly magnetized neutron stars, may have strong magnetic fields of the order of 10^{15}–10^{16} gauss [10–16], which is of relevance for the physics of the dense matter in the core of such compact objects.

The presence of a nonzero θ leads to a very rich vacuum structure with the possibility of having quark condensates in the pseudoscalar channel and generates a more complex phase structure for the phase diagram of strong interaction. Because of the nonperturbative structure of the θ term , it is very difficult to study spontaneous *CP* violation in full QCD for arbitrary values of θ. Therefore, *CP* violation effects in strong interactions have been studied extensively using low energy effective theories like the chiral perturbation theory [17] as well as in specific models like the Nambu-Jona-Lasinio (NJL) model [18] and the linear sigma model coupled to quarks [19].

In the present work, we intend to investigate how chiral transition is affected when the *CP*-odd effects and a strong

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magnetic background are present for hot and dense matter. This is, therefore, of relevance also for the heavy ion collision experiments that explore the nonzero baryon density region of the QCD phase diagram. For this purpose, we adopt the three flavor Nambu-Jona-Lasinio model as an effective theory for chiral transitions. The effect of axial anomaly and the strong CP violation here is included through the Kobayashi-Maskawa-t'Hooft (KMT) determinant term that mimics the effects of nontrivial gauge field configuration. This term is also a function of θ and is responsible for CP violation for nonvanishing values of θ. Such a term has been extensively studied earlier for NJL models with two flavors in Refs. [18,20,21] for studying the effects of nonzero θ on the chiral transition. This has been further extended to the two flavor NJL model including Polyakov loop potential [22,23]. We had earlier considered the effects of strong CP violation on chiral symmetry breaking for the realistic case of $2+1$ flavor using the NJL model [24]. This was further extended in Ref. [23] to include the effects of Polyakov loop potential. In all these investigations the effects of the magnetic field were not included. It is this question that we investigate here.

The modification of the ground state of QCD for $\theta = 0$ in connection with chiral symmetry breaking in the presence of a magnetic field has been investigated in different effective models, e.g., the chiral perturbation theory [25], the NJL model [26–29], as well as different quark models of hadrons. In various models it was seen that while a magnetic field acts as a catalyser of chiral symmetry breaking, it was observed that medium effects can lead to inverse magnetic catalysis for the same, particularly at finite chemical potentials. The effects of a magnetic field as well as nonzero values for θ have been considered in Ref. [19] within the chiral sigma model at finite temperatures. It was shown here that for zero baryon density and at finite temperature, the chiral transition becomes a weak first order one from a crossover in the presence of magnetic fields. In the present work, we also look into the finite density effects on chiral transition in the presence of CP violation and strong constant magnetic fields.

In the present work, we further look into the effects of the mass of the strange quarks on the quark-antiquark condensates both in scalar and pseudoscalar channels. The main property that distinguishes the strange quark from the u and d quarks is its mass, which is more than an order of magnitude larger. The KMT interaction term that mixes the flavors along with the θ dependence is therefore expected to have a significant effect on the condensate structure. Let us note that spontaneous CP violation at $\theta = \pi$ depends both on the strength of the determinant interaction and on the current quark masses. For the three flavors it was shown in Ref. [30] that a region exists in the plane of light quark masses where CP is violated spontaneously depending upon the mass of the strange quark for $\theta = \pi$. In this context, we had earlier observed that spontaneous CP violation occurs at $\theta = \pi$ for the phenomenologically consistent current quark masses as well as the determinant coupling within the framework of the $2+1$ flavor NJL model [24]. In the present work we explore how the condensate structure in various channels depends on the current quark mass of the strange quark.

We organize the paper as follows. In Sec. II, we consider the three flavor NJL model along with the CP violating θ dependent six fermion determinant interaction term that also breaks axial symmetry. Here, we spell out the variational ground state with quark-antiquark pairs that is related to chiral symmetry breaking. The ansatz is taken to be general enough to include both scalar as well as pseudoscalar condensates. These condensate functions are determined through an extremization of thermodynamic potential as in Ref. [24,27]. The pseudoscalar condensate takes nonvanishing values for finite values of θ. In Sec. III, we discuss the resulting phase diagram at finite temperature as well as finite density for different strengths of magnetic field and for various values of θ. Finally, in Sec. IV, we summarize the results and conclusions with a possible outlook.

II. NJL MODEL WITH *CP* VIOLATION AND AN ANSATZ FOR THE GROUND STATE

To describe the chiral phase structure of strong interactions including the CP violating effects and external magnetic field, we use the three flavor NJL model along with the flavor mixing determinant term. The Lagrangian is given by [24]

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi + G_s \sum_{A=0}^{8} [(\bar{\psi}\lambda^A\psi)^2 + (\bar{\psi}i\gamma^5\lambda^A\psi)^2] - K[e^{i\theta}\det\{\bar{\psi}(1+\gamma^5)\psi\} + e^{-i\theta}\det\{\bar{\psi}(1-\gamma^5)\psi\}], \tag{1}$$

where $\psi^{i,a}$ denotes a quark field with color a $(a = r, g, b)$ and flavor i $(i = u, d, s)$. $D_\mu = \partial_\mu - iqA_\mu$ is the covariant derivative in the presence of the external magnetic field B, which we assume to be constant and in the z direction. Further, we choose the gauge such that the corresponding electromagnetic potential is given as $A_\mu = (0, 0, Bx, 0)$. The matrix of current quark masses is given by $\hat{m} = \mathrm{diag}_f(m_u, m_d, m_s)$ in the flavor space. We shall assume isospin symmetry with $m_u = m_d$ in the present investigation. Strictly speaking, when the electromagnetic effects are taken into account, the current quark masses of the up and down quarks should be different. However, due to the smallness of the electromagnetic coupling, we ignore such a tiny difference and continue to take the current quark masses to be the same for up and down quarks. The second term is the four Fermi contact interaction and, for three flavor case, λ^A with $A = 0\cdots 8$ are generators of $U(3)$ with $\lambda^0 = \sqrt{2/3}\times 1$ and λ^A being the standard SU(3) Gell-Mann matrices for $1 \le A \le 8$. The third term is the 't-Hooft determinant interaction that depends upon the QCD vacuum angle θ. By taking divergence of the flavor singlet axial current $J_5^\mu = \bar\psi\gamma^\mu\gamma^5\psi$, one can show that the effect of the topological Chern-Simons term in terms of gluon fields can be simulated by the imaginary part of the determinant term of the quark sector. Such a term can lead to formation of condensates in the pseudoscalar channel for nonvanishing θ as we investigate in the following.

A comment regarding the three flavor NJL Lagrangian given by Eq. (1) may be relevant here. The determinant interaction here leads to a six fermion point interaction. From the point of view of $1/N_c$ expansion, one can also have eight quark interactions having the same N_c order as the KMT determinant term [31,32]. The corresponding coupling, however, is suppressed by $\frac{1}{\Lambda^3}$ compared to the determinant term. The effect of such a term for thermodynamics in the Polyakov loop NJL model has been investigated regarding the critical point in the phase diagram that moves toward a lower chemical potential and higher temperature [32]. We expect that the inclusion of such an eight fermion interaction will not have any qualitative difference regarding the features of *CP* violation that we consider here and continue with the calculations using the Lagrangian given by Eq. (1).

To explore the vacuum structure of the model in the presence of finite θ and $\boldsymbol{B}$, we next consider an ansatz for the ground state with quark-antiquark pairs as

$$|\Omega\rangle = U_{II}U_I|\mathrm{vac}\rangle, \tag{2}$$

where U_I and U_{II} are unitary operators described in terms of the creation and annihilation operators of the quarks and antiquarks. Explicitly, these are given as

$$U_I = \exp\left(\sum_{n=0}^{\infty}\int d\boldsymbol{p}_x q_r^{i\,\dagger}(n,\boldsymbol{p}_x)a^i_{r,s}(n,p_z)f^i(n,\boldsymbol{p}_x)\tilde q^i_s(n,-\boldsymbol{p}_x) - \mathrm{H.c.}\right). \tag{3}$$

In the above, $q_r^{i\,\dagger}(n,\boldsymbol{p}_x)$, $\tilde q^i_r(n,\boldsymbol{p}_x)$, are two component quark creation and antiquark creation operators, respectively, defined through the Fourier expansion of the quark field ψ^i in the presence of a magnetic field. The index n denotes the Landau level and $\boldsymbol{p}_x = (p_y, p_z)$. We have here used the notation of Ref. [27]. Further, in the above equation, the spin dependent structure $a^i_{r,s}$ is given by [27]

$$a^i_{r,s} = \frac{1}{|\boldsymbol{p}^i|}\left[-\sqrt{2n|q^i|B}\,\delta_{r,s} - ip_z\delta_{r,-s}\right] \tag{4}$$

with $|\boldsymbol{p}^i| = \sqrt{p_z^2 + 2n|q^i|B}$ denoting the magnitude of the three momentum of the quark/antiquark of ith flavor (with electric charge q^i) in the presence of a magnetic field. This ansatz for U_I is the same as considered in Ref. [27] in the context of chiral symmetry breaking to include the effects of a magnetic field.

Next, to include the effects of *CP* violation leading to pseudoscalar condensates, the unitary operator U_{II} is given as

$$U_{II} = \exp\left(\sum_{n=0}^{\infty}\int d\boldsymbol{p}_x q_r^{i\,\dagger}(n,\boldsymbol{p}_x)rg^i(n,\boldsymbol{p}_x)\tilde q^i_s(n,-\boldsymbol{p}_x) - \mathrm{H.c.}\right). \tag{5}$$

The above construct of Eq. (2) is a generalization of the ground state structure in the presence of a *CP* violating term as in Ref. [24] to include the effects of a nonvanishing constant magnetic field. Clearly, in Eq. (2) the ansatz for the ground state has two arbitrary functions f^i and g^i that are related to the condensates in the scalar and pseudoscalar channel, respectively, to be determined through a minimization of thermodynamic potential. The above ansatz in Eq. (2) is a variational state for the vacuum case. The effects of temperature and density can be included within the framework of thermofield dynamics again in a variational manner [33,34] by performing another unitary transformation on Eq. (2) to obtain the "thermal vacuum" such that the thermal average of any operator becomes an expectation over the thermal vacuum. The ansatz functions in this thermal state are determined by minimization of the thermodynamic potential and ultimately get related to the thermal distribution functions. The nicety of the approach lies in the fact that the distribution functions here get determined through a minimization principle including the interaction. We here take the ansatz for the thermal vacuum exactly in the same way as in Ref. [27]. Realizing the fact that the unitary transformation of the vacuum as in Eq. (2) corresponds to successive Bogoliubov transformations, one can calculate the Hamiltonian expectation values corresponding to the Lagrangian given in Eq. (1) with respect to the state of Eq. (2) to obtain the energy density as well as the thermodynamic potential.

The procedure is identical to that done in Ref. [27]. The functions in the variational states in Eqs. (3) and (5) can then be determined by minimizing the thermodynamical potential. Since the procedure for determining these ansatz functions is identical as performed in Refs. [24] and [27], we do not repeat the details here and refer to these references for the interested reader.

The resulting thermodynamic potential $\Omega(\beta,\mu,B)$ is then given as

$$\Omega = -\frac{N_c}{4\pi^2}\sum_{n=0}^{\infty}\alpha_n\sum_i|q^iB|\int dp_z\omega_n^i - \frac{N_c}{4\pi^2\beta}\sum_{n=0}^{n_{\max}}\sum_i|q^iB| \times\int dp_z[\ln\{1+e^{-\beta(\omega_n^i-\mu)}\}+\ln\{1+e^{-\beta(\omega_n^i+\mu^i)}\}] + 2G_s\sum_i[I_s^{i\,2}+I_p^{i\,2}]+4K\left[\cos\theta\prod_{i=1}^{3}I_s^i+\sin\theta\prod_{i=1}^{3}I_p^i\right] - 2K|\epsilon_{ijk}|[\cos\theta I_p^iI_p^jI_s^k+\sin\theta I_s^ii_s^jI_p^k]. \tag{6}$$

In the above, $\omega_{i,n}=\sqrt{M^{i2}+p_z^2+2n|q^i|B)}$ is the excitation energy of the nth Landau level with the constituent quark mass $M^i=\sqrt{M_s^{i\,2}+M_p^{i\,2}}$, M_s^i, M_p^i being the dynamical masses arising from scalar and pseudoscalar condensates, respectively, which satisfy the gap equations

$$M_s^i = m^i+4GI_s^i+K|\epsilon_{ijk}|\{\cos\theta(I_s^jI_s^k-I_p^jI_p^k) - \sin\theta(I_s^jI_p^k+I_p^jI_s^k)\}, \tag{7}$$

$$M_p^i = 4GI_p^i-K|\epsilon_{ijk}|\{\cos\theta(I_s^jI_p^k+I_p^jI_s^k) - \sin\theta(I_p^jI_p^k-I_s^jI_s^k)\}. \tag{8}$$

In Eqs. (6)–(8), $I_s^i=-\langle\bar\psi^i\psi^i\rangle$ and $I_p^i=-\langle\bar\psi^ii\gamma^5\psi^i\rangle$ (i not summed) are the condensates in the scalar and pseudoscalar channels, respectively, for the ith flavor and are given as

$$\frac{I_s^i}{M_s^i}=\frac{I_p^i}{M_p^i}\equiv I^i = \sum_{n=0}^{\infty}\frac{N_c|q^i|B\alpha_n}{(2\pi)^2}\int dp_z\left(\frac{1}{\omega_{i,n}}\right)(1-\sin^2\theta_-^i-\sin^2\theta_+^i), \tag{9}$$

where $\sin^2\theta_\mp=1/(\exp(\beta(\omega^{i,n}\mp\mu^i))+1)$ are the particle and antiparticle distribution functions. Thus, the gap equations, Eqs. (7)–(8), are actually self-consistent equations for M_s^i and M_p^i since I_s^i and I_p^i again are given in terms of M_s^i and M_p^i as can be seen from Eqs. (9).

In Eq. (6), the first term is the zero temperature and zero density term in the presence of a constant magnetic field. The second term is the thermodynamic potential of quarks and antiquarks with a medium dependent mass while the last two terms are the remaining interaction terms of the Lagrangian. For the CP violating parameter $\theta\to0$, and the pseudoscalar density $I_p^i\to0$, the thermodynamic potential reduces to the same as in Ref. [27]. The first term in Eq. (6) is ultraviolet divergent, which is also transmitted to the gap equations, Eqs. (7)–(8), through the integrals I_s^i and I_p^i in Eqs. (9) and need to be regularized to get any meaningful result. There have been different regularization schemes to tackle this divergence, e.g., the Schwinger proper time method [35,36] or using a smooth cutoff [37]. We perform the regularization as in Ref. [27] by adding and subtracting a zero field (vacuum) contribution that is also divergent. This puts the first term of Eq. (6) in a rather appealing form of separating the zero field vacuum contribution that is divergent and a field dependent contribution that is finite. The divergent zero field vacuum contribution is then evaluated with a finite cutoff in the three momentum Λ as is usually done in the NJL model without a magnetic field. This way the regularized thermodynamic potential can be written as

$$\Omega=\Omega_{\rm vac}+\Omega_{\rm field}+\Omega_{\rm med} + 2G_s\sum_i[I_s^{i\,2}+I_p^{i\,2}]+4K\left[\cos\theta\prod_{i=1}^{3}I_s^i+\sin\theta\prod_{i=1}^{3}I_p^i\right] - 2K|\epsilon_{ijk}|[\cos\theta I_p^iI_p^jI_s^k+\sin\theta I_s^ii_s^jI_p^k]. \tag{10}$$

In the above, the vacuum ($T=0=\mu$, $B=0$) energy density $\Omega_{\rm vac}$, evaluated with a three momentum cutoff Λ, is given by [27]

$$\Omega_{\rm vac}=-\frac{N_c\Lambda}{8\pi^2}\sum_i\left[(\Lambda^2+M^{i2})^{1/2}(2\Lambda^2+M^{i2}) - M^{i4}\log\frac{\Lambda+\sqrt{\Lambda^2+M^{i2}}}{M^i}\right]. \tag{11}$$

$\Omega_{\rm field}$ is the field contribution to Ω and is given by, with $x^i=\frac{M^{i2}}{2|q^i|B}$,

$$\Omega_{\rm field}=-\frac{N_c}{2\pi^2}\sum_i|q^iB|^2\left[\zeta'(-1,x^i)-\frac{1}{2}(x^{i2}-x^i)\ln x^i+\frac{x^{i2}}{4}\right], \tag{12}$$

where the derivative of the Riemann-Hurwitz zeta function $\zeta(z,x)$ at $z=-1$ is given by [38]

$$\zeta'(-1,x) = -\frac{1}{2}x\log x - \frac{1}{4}x^2 + \frac{1}{2}x^2\log x + \frac{1}{12}\log x + x^2\int_0^\infty \frac{2\tan^{-1}y + y\log(1+y^2)}{\exp(2\pi xy)-1}dy. \tag{13}$$

Finally, the medium contribution $\Omega_{\rm med}$ towards the thermodynamic potential is given by

$$\Omega_{\rm med} = -\sum_{n,i}\frac{N_c\alpha_n|q^iB|}{(2\pi)^2\beta}\int dp_z[\ln\{1+e^{-\beta(\omega_n^i-\mu)}\} + \ln\{1+e^{-\beta(\omega_n^i+\mu)}\}]. \tag{14}$$

Similarly, the condensate $I_s^i/M_s^i = I_p^i/M_p^i \equiv I^i$ of Eq. (9) can be regularized and is given as

$$I^i = I^i{}_{\rm vac} + I^i{}_{\rm field} + I^i{}_{\rm med}, \tag{15}$$

where,

$$I^i_{\rm vac} = \frac{N_c}{2\pi^2}\left[\Lambda\sqrt{\Lambda^2+M^{i2}} - M^{i2}\log\left\{\frac{\Lambda+\sqrt{\Lambda^2+M^{i2}}}{M^i}\right\}\right], \tag{16}$$

$$I^i_{\rm field} = \frac{N_c|q^iB|}{(2\pi)^2}\left[x^i(1-\ln x^i) + \ln\Gamma(x^i) + \frac{1}{2}\ln\frac{x^i}{2\pi}\right], \tag{17}$$

and

$$I^i{}_{\rm med} = \sum_{n=0}^{n_{\rm max}}\frac{N_c|q^i|B\alpha_n}{(2\pi)^2}\int dp_z\frac{M_s^i}{\sqrt{M^{i2}+p_z^2+2n|q^iB|}} \times(\sin^2\theta_-^i + \sin^2\theta_+^i) \tag{18}$$

are, respectively, the contributions from the vacuum, from the field and from the medium, to the condensates [27]. In the presence of a magnetic field, $|\mathbf{p}| = \sqrt{p_z^2+2n|q_iB|}$, a finite three momentum cutoff Λ results in a maximum value of the Landau level $n_{\rm max} = {\rm Int}[\frac{\Lambda^2}{2|q^iB|}]$. Further, in Eq. (18), this condition also leads to a cutoff for $|p_z|$ as $\Lambda' = \sqrt{\Lambda^2 - 2n|q^i|B}$ for a given value of the Landau level n.

The coupled mass gap equations given by Eqs. (7)–(8) and the thermodynamic potential given by Eq. (10) constitute the basis for our numerical results for various physical situations, which we shall discuss in the following section.

III. RESULTS AND DISCUSSIONS

The three flavor NJL model that we investigate here has five parameters in total, namely, the current quark masses for the nonstrange and strange quarks, m_q and m_s, the two couplings G_s, K, $\Lambda = 0.6023$ GeV, $G_s\Lambda^2 = 1.835$, $K\Lambda^5 = 12.36$, $m_q \equiv m_u = m_d = 5.5$ MeV, and $m_s = 140.7$ MeV as has been used in Ref. [39]. After choosing m_q the remaining four parameters are fixed by fitting to the pion decay constant and the masses of pion, kaon, and η'. With this set of parameters the mass of η is underestimated by about six percent and the constituent masses of the light quarks turn out to be $M^q = 0.368$ GeV for u-d quarks, while the same for strange quark turns out as $M^s = 0.549$ GeV, at zero temperature and zero baryon density.

In the numerical calculations that follow, we have taken the quark chemical potential μ to be the same for all the three flavors. For given values of T, μ and strength of a magnetic field B, we first solve the gap equations (7)–(8) self consistently along with the condensates given in Eq. (9) with the parameters of the NJL model as above. Since we have assumed $m_u = m_d$, the equations actually represent four coupled equations for a zero magnetic field: two corresponding to the scalar contributions towards the masses, i.e., $M_s^u = M_s^d$ and M_s^s, and two corresponding to the pseudoscalar contributions towards the masses, i.e., $M_p^u = M_p^d$ and M_p^s, which has been considered earlier in Ref. [24]. However, this degeneracy is lifted in the presence of a nonzero magnetic field. Thus, Eqs. (7)–(8) actually represent six coupled mass gap equations—the contributions to the masses arising from the scalar and pseudoscalar condensates for each flavor. Once the solutions to these coupled equations for the masses and the condensates are found, they are then substituted in Eq. (10) to find the thermodynamic potential Ω. In case of more than one solution to the gap equation, the solution with the minimum Ω is chosen.

In the present analysis, we investigate the behavior of scalar and pseudoscalar contributions to the quark mass with temperature, chemical potential, and magnetic field for different values of θ.

Let us first discuss the effect of a magnetic field on the vacuum properties within the model. In Fig. 1 we plot the constituent quark masses given as $M^i = \sqrt{M_s^{i2}+M_p^{i2}}$ as a function of magnetic field for different representative values of θ. Because of the different charges of the u and d quarks, the isospin symmetry is lost between the light quarks when an external magnetic field is applied to the system. The magnetic catalysis of dynamical generation of mass is seen for all the quarks with the constituent quark masses increasing with magnetic field for all values of θ. The constituent quark masses here, however, are generated from quark-antiquark condensates both in scalar and pseudoscalar channels for nonzero values of θ.

In Fig. 2 the condensates in scalar and pseudoscalar channels for u quarks are plotted as functions of θ for different strengths of magnetic field. The condensates in the scalar and pseudoscalar channel vary in a complimentary

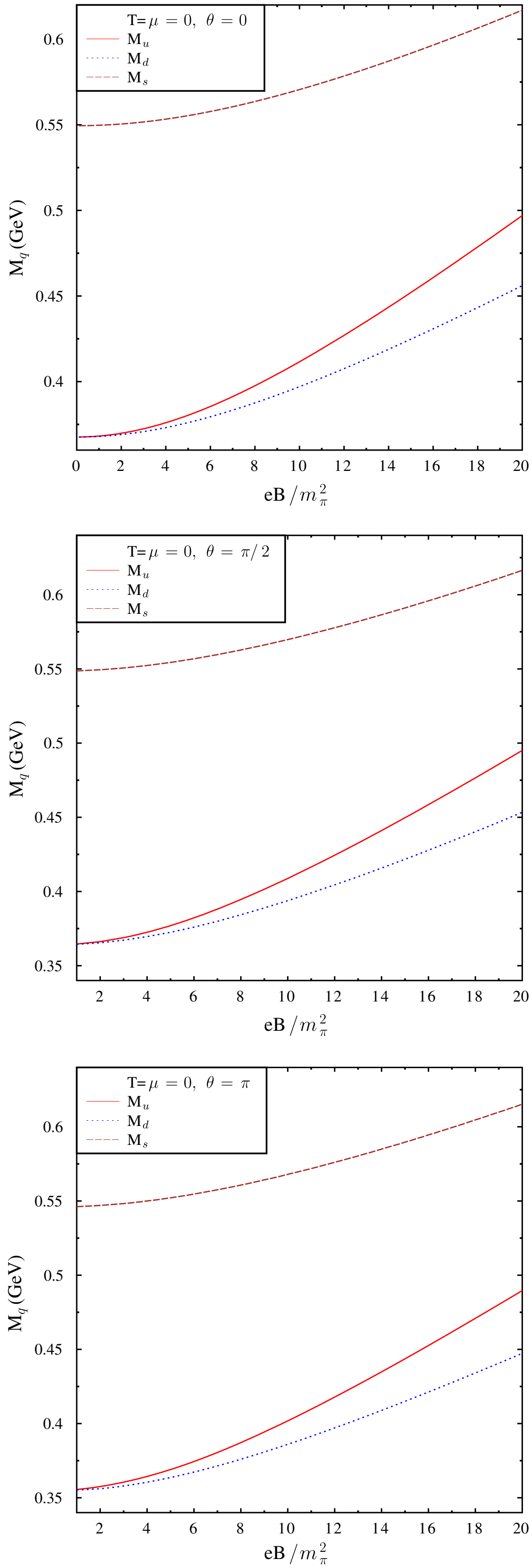

FIG. 1 (color online). Constituent quark mass $M^i = \sqrt{M_s^{i\,2} + M_p^{i\,2}}$ at $T = 0$, $\mu = 0$ as a function of magnetic field for $\theta = 0$ (top), $\theta = \pi/2$ (middle), and $\theta = \pi$ (bottom).

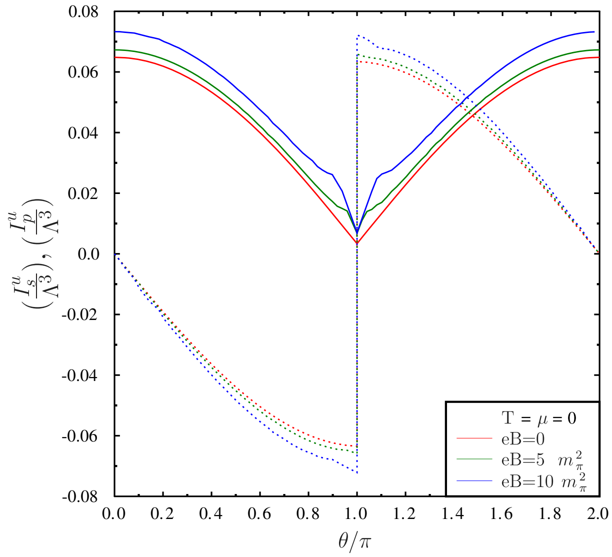

FIG. 2 (color online). θ dependence of the order parameters for u quarks for different strengths of magnetic field. The solid lines correspond to condensates in the scalar channel while the dotted lines correspond to condensates in the pseudoscalar channel. Spontaneous breaking of CP at $\theta = \pi$ with two degenerate vacua for pseudoscalar condensates may be noted.

manner so that the total constituent mass remains almost constant as θ is varied. This behavior is also seen with increasing magnetic field, along with the fact that the condensates in both the channels become larger in magnitude for a larger magnetic field. The spontaneous CP violation is seen for $\theta = \pi$ with two degenerate solutions for the pseudoscalar condensate differing by a sign.

In Fig. 3 we display the variation of the effective potential with θ for different strengths of magnetic fields.

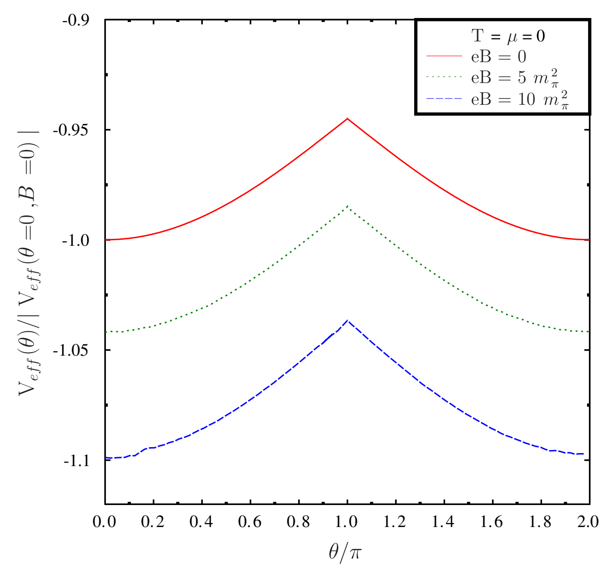

FIG. 3 (color online). Effective potential at $T = \mu = 0$ as a function of θ for different strengths of magnetic field.

The effective potential shown here is normalized with respect to the effective potential at $\theta = 0, |\boldsymbol{B}| = 0$. It is minimum when $\theta = 0$, which is consistent with the Vafa-Witten theorem. The behavior we see here is similar to what we observed without the magnetic field [24]. The magnetic field only reduces the effective potential.

In studies of effective theories of strong interactions, spontaneous *CP* violation at $\theta = \pi$ depends upon the strength of the determinant interaction, the number of flavors, as well as the current quark masses [20,30]. It might be interesting at this point to explore the effect of the current quark mass of the strange quark on the behavior of condensates. For $\theta = 0$, this was investigated in Ref. [40] for the NJL model regarding chiral transition arising from scalar condensates of quark-antiquark pairs. We, on the other hand, look into the effect of current quark masses of strange quarks on the pseudoscalar condensates in the presence of a magnetic field and focus on the case of $\theta = \pi$ where spontaneous *CP* violation can arise.

In Fig. 4, we have plotted the masses arising from condensates in scalar and pseudoscalar condensates as a function of magnetic field using three different values of the s-quark current quark mass: (i) $m_s = 5.5$ MeV, i.e., degenerate strange quark mass with the u, d quarks; (ii) $m_s = 50$ MeV, intermediate between the light quarks and the heavy strange quark; and (iii) $m_s = 140.7$ MeV as is phenomenogically taken to fit the low energy hadronic properties [39]. In all three figures we have taken $\theta = \pi$.

Let us note that the magnetic field effects become noticeable when the strength of the magnetic field is of the order of the square of the quark masses. In the case (i) of the degenerate three flavor case with tiny current quark mass, the condensates in both the scalar and pseudoscalar channels contribute significantly to the effective constituent quark masses for all three flavors as can be seen in the top panel of Fig. 4. For a vanishing magnetic field, the scalar condensate contribution is about 168 MeV while that of the pseudoscalar condensate is about 270 MeV for all three flavors. The dynamically generated masses for the strange quark and the down quark are identical both for the scalar as well as pseudoscalar channels due to their identical electric charges and are different from up quarks in the presence of a magnetic field. Further, this also leads to a significant contribution to the u-quark condensates in the scalar channel. This is because, for $\theta = \pi$, in the right-hand side of Eq. (7), for u quarks, the contribution of the determinant term becomes weaker as $I_s^d I_s^s$ is of the same order as the product $I_p^d I_p^s$ when the strange quark condensate in the pseudoscalar channel is non-negligible. This leads to a significant contribution to the up quark masses arising from scalar condensates.

The degeneracy of the d- and s-quark condensates is not there anymore when $m_{u,d} \neq m_s$ as may be seen from the middle panel ($m_s = 50$ MeV) and the bottom panel

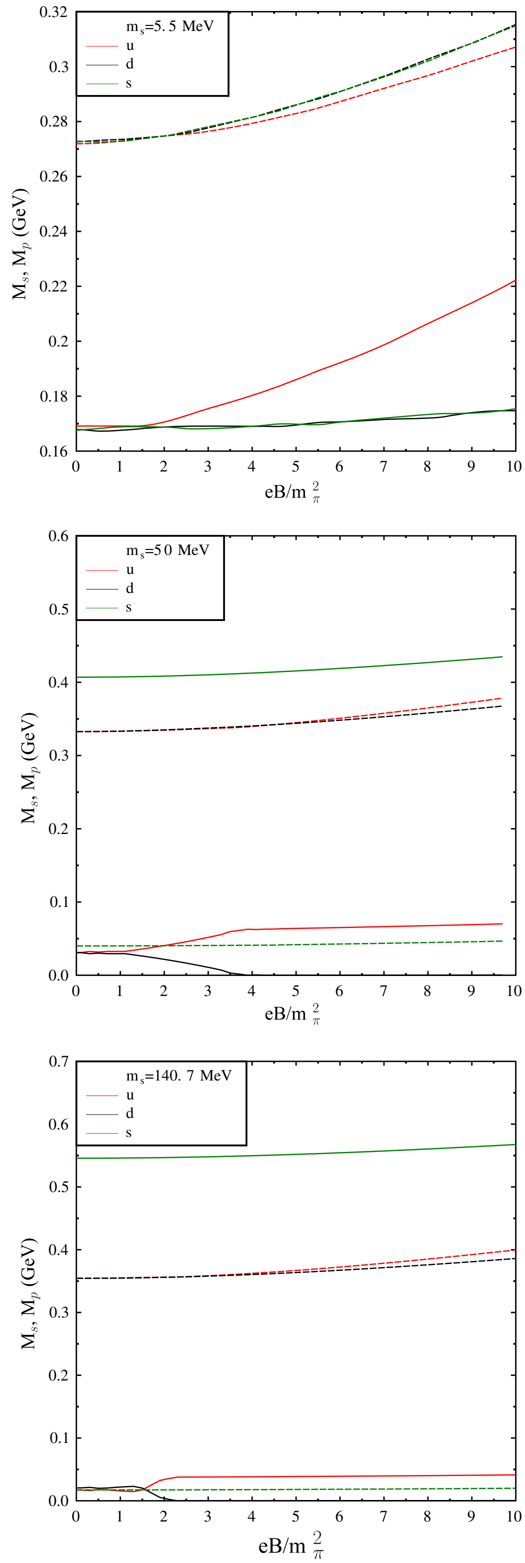

FIG. 4 (color online). Effect of strange quark mass on the constituent quark masses as a function of magnetic field for $\theta = \pi$. The solid lines correspond to masses arising from scalar condensates (M_s) and the dashed lines correspond to the same arising from pseudoscalar condensates (M_p). The various s-quark current quark masses are taken as $m_s = 5.5$ MeV (top), $m_s = 50$ MeV (middle), and $m_s = 140.7$ MeV (bottom).

($m_s = 140$ MeV) in Fig. 4. As the strange quark mass is increased, the magnetic catalysis effects for the strange condensates become weaker. Further, because of the large current quark mass, the strange condensates become dominant in the scalar channel even for $\theta = \pi$. This is in contrast to condensates of light quarks in the scalar channel, which are negligible compared to the light quark condensates in the pseudoscalar channel. This effect is further enhanced when $m_s = 140.7$ MeV. Here, the dynamical mass arising from strange quarks in the pseudoscalar channel is about 18 MeV, which is negligible compared to the same arising from condensates in the scalar channel, which is about 550 MeV. Further, because of the large mass of strange quark, these values are not affected by the magnetic field we have considered here.

In Fig. 5 we show the temperature dependence of the total mass of the u quark for different values of magnetic field at zero chemical potential. For vanishing θ, the total mass gets a contribution only from the condensates in the scalar channel. Within the model, the chiral transition temperature increases with the magnetic field similar to that observed in several effective models as well as some lattice QCD models [41]. The general reason for this behavior is that the magnetic field enhances the condensates and hence requires higher temperatures to melt the condensate. As a result of the charge difference we obtain a higher transition temperature for u quarks than for d quarks with the difference becoming larger with larger values of the magnetic field. The chiral transition is a crossover due to finite current quark mass. However, in some of the recent lattice calculations, inverse magnetic catalysis near the critical temperature is observed leading to a reduction of the crossover transition temperature with a magnetic field [42]. At sufficiently lower temperature, on the other hand, magnetic catalysis is observed in these lattice simulations with the condensates getting enhanced with a magnetic field. Such an effect can be generated in an *ad hoc* manner by reducing the effective four fermion coupling by making it a function of temperature and magnetic field as in Ref. [43]. There have been other attempts to explain this by invoking paramagnetic contributions to the pressure with large magnetization [44], magnetic inhibition due to neutral meson fluctuation [45], as well as a backreaction of the Polyakov loop that could be affected by a magnetic field [46]. In the present work, however, we continue to consider the consequences of the ansatz as in Eq. (2) to discuss the effects of the nonvanishing θ and magnetic field on the phase structure within the premises of the NJL model.

As θ is increased, the contribution to the mass from the pseudoscalar condensates also increases. We also plot the temperature dependence of the pseudoscalar component of the u-quark mass arising from pseudoscalar condensates M_u^p for $\theta = \frac{\pi}{2}$ and $\theta = \pi$ in Fig. 6. As may be observed from Fig. 6(a), for $\theta = \pi/2$ the CP transition is a crossover transition. For $\theta = \pi$, however, the CP transition is a

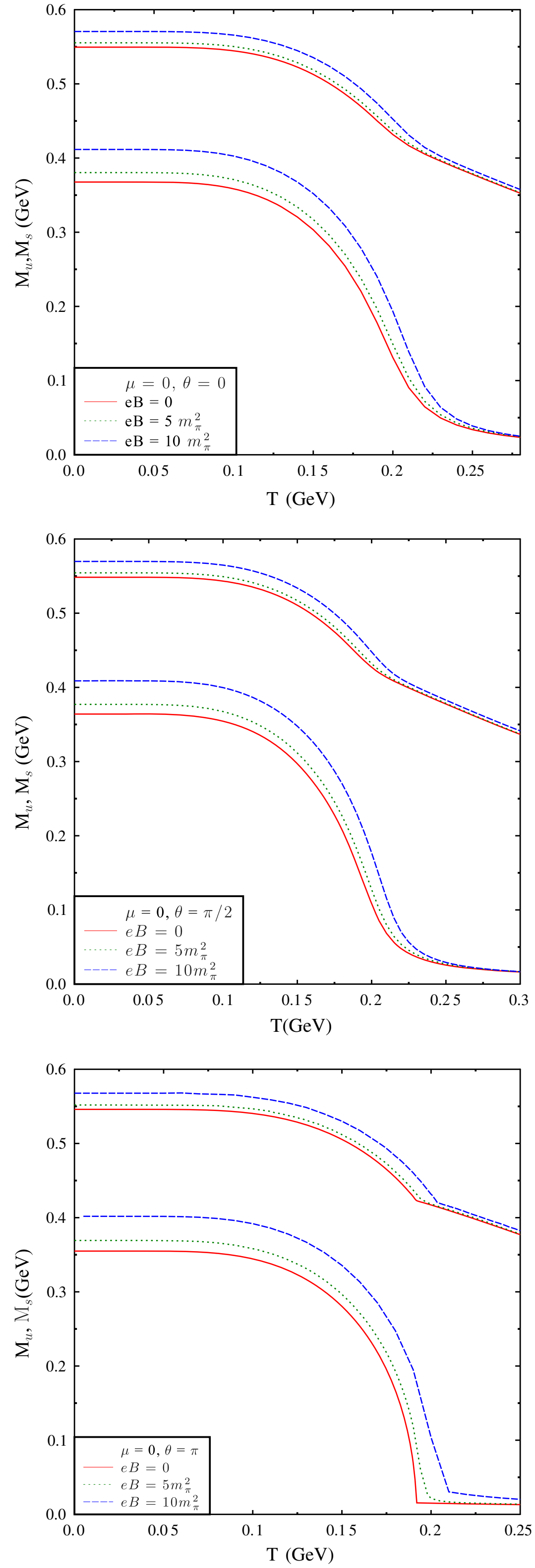

FIG. 5 (color online). Temperature dependence of constituent quark masses for up and strange quarks for $\theta = 0$ (top), $\theta = \pi/2$ (middle), and $\theta = \pi$ (bottom) with different strengths of magnetic field. In each plot the lower curves are for the up quark mass variation while the upper curves show temperature dependence of the strange quark mass.

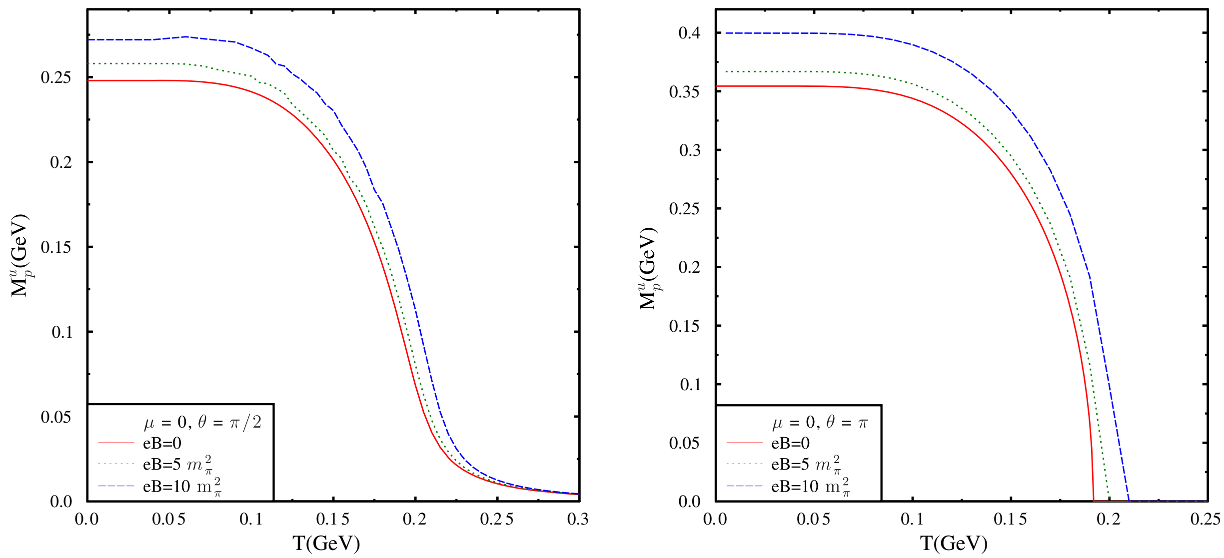

FIG. 6 (color online). Temperature dependence of the pseudoscalar condensates for $\theta = \pi/2$ (a) and $\theta = \pi$ (b) with different strengths of magnetic field.

second order transition with the pseudoscalar condensates smoothly vanishing at the critical temperature. Further, this *CP* restoration transition temperature increases with the magnetic field. We should, however, note that what we considered here is the equilibrium uniform *CP* violating phase structure induced by the determinant term. However, the local parity violating phase can also arise due to fluctuations of topological charges induced through sphaleron configuration that are not exponentially suppressed [3]. On the other hand, such domains can also arise due to nonequilibrium situations depending upon the kinetics of the phase transition. Such *CP*-odd domains can decay via *CP*-odd processes and can have observable effects like a chiral magnetic effect for noncentral heavy ion collisions [4] as well as a possible excess in dilepton production for central collisions [47].

In Fig. 7, we display the dependence of the constituent quark mass for strange and up quarks on quark chemical potential at zero temperature for different values of the strength of magnetic field. As chemical potential is increased, a first order transition is observed for all values of θ and magnetic field, e.g., for $\theta = 0$ (top panel), for a vanishing magnetic field the u-quark mass decreases from about 370 to about 50 MeV for u quarks. Because of the flavor mixing KMT interaction, this is also reflected in a discontinuity for the strange quark mass from its vacuum value of about 550 to 465 MeV. For $\theta = 0$, the critical chemical potential μ_c where the constituent quark mass shows this discontinuity decreases with magnetic field. Let us note that for $\theta = 0$, the entire mass arises from quark condensates in the scalar channel apart from the current quark masses. This behavior of having lower critical chemical potential for a higher magnetic field is the phenomenon of inverse magnetic catalysis of chiral symmetry breaking at finite chemical potential [27,35]. For finite θ, however, the mass is generated by condensates in both scalar and pseudoscalar channels. For $\theta = \pi$ (bottom panel), however, while the dominant contribution for light quark masses arises from the pseudoscalar channel, the same arises from the scalar channel for the strange quarks. It turns out that for $\theta = \pi$, at a zero magnetic field the critical quark chemical potential is $\mu_c \sim 345$ MeV with a first order transition. As the magnetic field is increased, μ_c decreases and is minimum at $eB = 7m_\pi^2$ with $\mu_c \sim 323$ MeV. As the magnetic field is increased further, the critical chemical potential also increases and becomes about $\mu_c \sim 360$ MeV for $eB = 10m_\pi^2$. Such a behavior of decrease of critical chemical potential for intermediate strengths of the magnetic field and then increase for stronger fields is also observed in Ref. [48].

In general, the phase transition line for the light quarks at zero temperature in the plane of the magnetic field and quark chemical potential is shown in Fig. 8. The massive quark phase exists to the left of the critical line while the region to the right of the critical line corresponds to (almost) massless quark phase. For $\theta = 0$, the mass here arises entirely from the scalar condensates while for $\theta = \pi$, the quark mass arises mostly from the pseudoscalar condensates but for the small current quark mass contributions to the scalar condensates. The discontinuous structures in the critical line for lower magnetic fields are due to the effects of discrete Landau levels. There is a region in the phase diagram, where, for fixed μ, with an increase in the magnetic field massless phase is restored. The critical chemical potential decreases with magnetic field up to eB $\simeq 13\mathrm{m}_\pi^2$, eB $\simeq 11\mathrm{m}_\pi^2$, eB $\simeq 7\mathrm{m}_\pi^2$ for $\theta = 0$, $\theta = \pi/2$, and $\theta = \pi$, respectively.

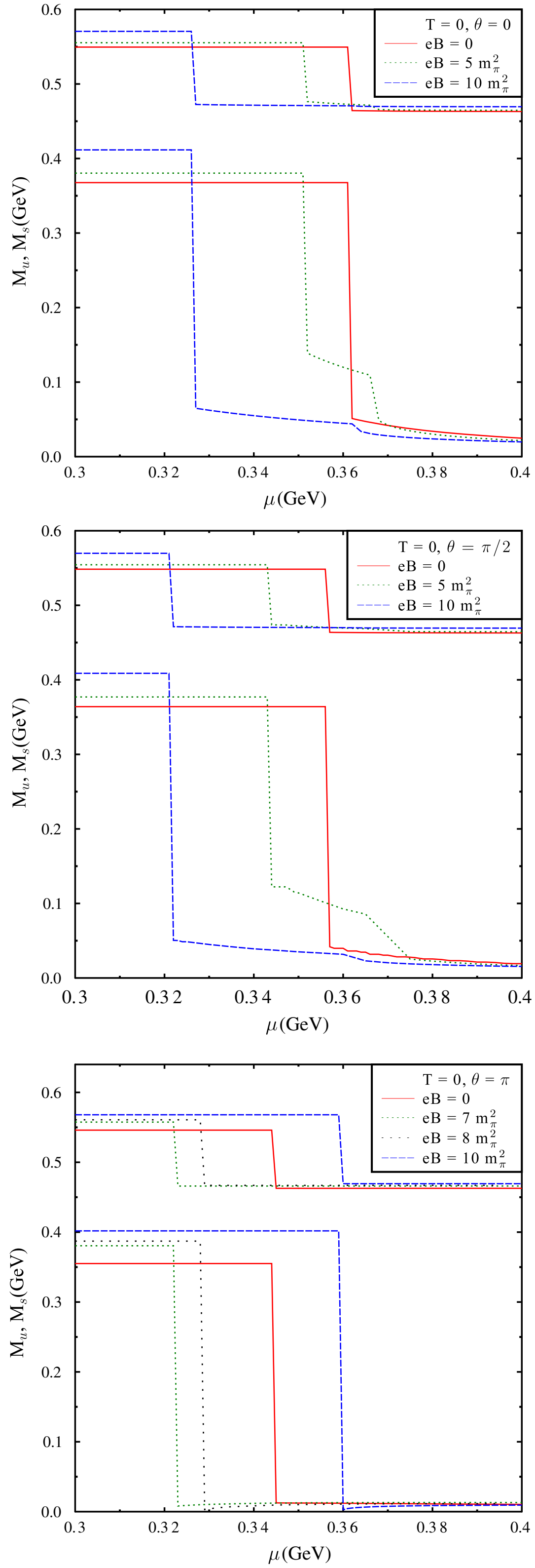

FIG. 7 (color online). Up and strange quark masses as a function of quark chemical potential at zero temperature for $\theta = 0$ (top), $\theta = \pi/2$ (middle), and $\theta = \pi$ (bottom) with different strengths of magnetic field. In each plot the upper curves show μ dependence of strange quarks while the lower ones are for the up quarks.

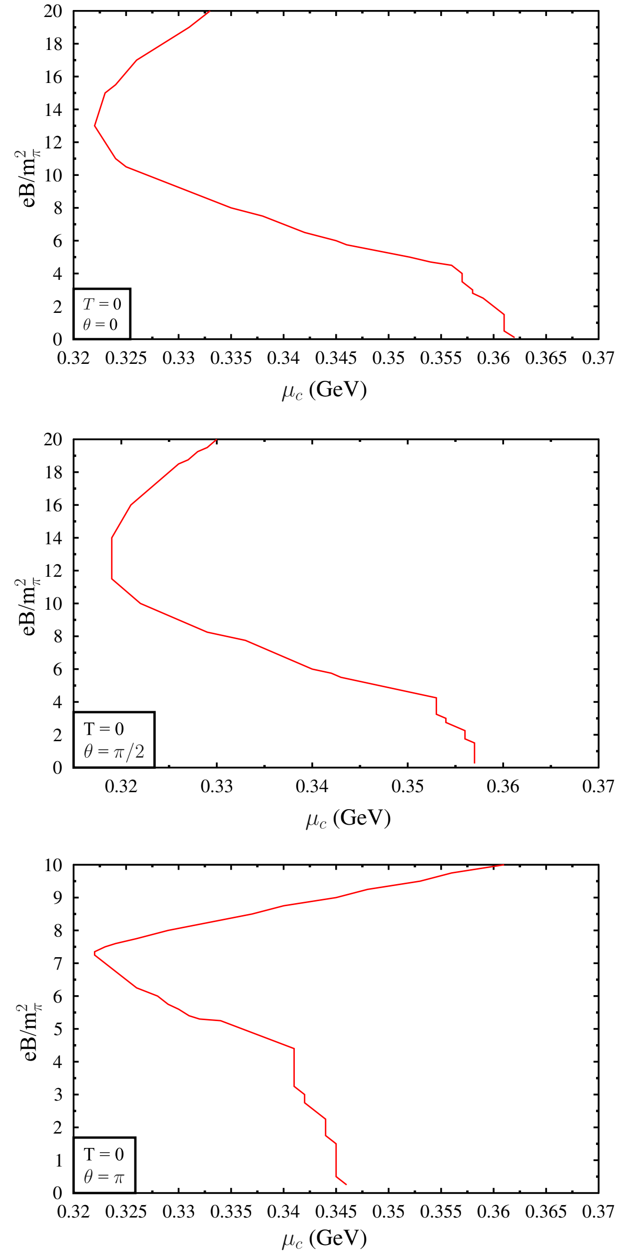

FIG. 8 (color online). μ_c as a function of magnetic field. for $\theta = 0$ (top), $\theta = \pi/2$ (middle), and $\theta = \pi$ (bottom).

This inverse magnetic catalysis of *CP* transition at finite θ can be understood in a manner similar to Ref. [48] discussed for chiral symmetry breaking in the NJL model. Without loss of generality, this analysis can be carried out for $\theta = \pi$ by analyzing the pseudoscalar gap Eq. (8) in this limit. Here, we can approximate the scalar condensates $I_s^i \simeq 0$ as well as the corresponding masses $M_s^i \simeq 0$ for the light quarks. Further, as the coupling is large, we can approximate the solution of the pseudoscalar gap Eq. (8) to be that given by the $\mu = 0$ solution. For nonzero but small magnetic fields $|q^i|B \ll M^2$, we can get the solution up to second order in the magnetic field as

$$M_p^i \simeq M_0^i\left(1+\frac{G_s|q_iB|^2}{M_0^{i2}(1-\frac{6G}{\pi^2}f(M_0^i,\Lambda))}\right), \tag{19}$$

where M_0^i is the solution of the pseudoscalar mass gap Eq. (8) with $\mu=0$, $B=0$, and $f(M_0^i,\Lambda)=\Lambda^2[(1+\hat{M}_0^{i2})-\hat{M}_0^{i2}\log(\frac{1+\hat{M}_0^{i2}/2}{\hat{M}_0^i})]$, where $\hat{M}_0^i=M_0^i/\Lambda$. Inserting this solution in the thermodynamic potential of Eq. (10) and subtracting out $M_p^i=0$ free energy density, one gets the thermodynamic potential difference between the condensed phase and the noncondensed phase, up to second order in B as

$$\Delta\Omega \simeq -\frac{3}{8\pi^2}\sum_i M_0^{i2}\Lambda^2\left(1-\frac{\pi^2}{3G\Lambda^2}\right)+\frac{3}{4\pi^2}\sum_i|q_iB|\mu^2 - \frac{3}{4\pi^2}\sum_i|q_iB|^2\left(1+\log\frac{M_0^{i2}}{2|q_i|B}\right). \tag{20}$$

In the above, the first term is the vacuum ($T=0$, $B=0$, $\mu=0$) contribution to the free energy difference between the condensed and noncondensed phase. The term linear in the magnetic field corresponds to the free energy cost to form a quark-antiquark pair in the pseudoscalar channel at finite μ, which depends upon the magnetic field along with the chemical potential. The last term is the gain in thermodynamic potential due to condensation that is quadratic in magnetic field strength. Therefore, as we turn on the magnetic field, and start from broken phase with $\Delta\Omega<0$, for the small field it can make $\Delta\Omega$ positive with the symmetry restored. However, as the field strength is increased further, the quadratic term starts dominating and the symmetry broken phase is preferred again, leading to the increase of the critical chemical potential with a magnetic field. This behavior of the critical chemical potential with a magnetic field is reflected in Figs. 6–7.

IV. SUMMARY

In the present investigation, we have focused on the effect of the θ vacuum on the chiral transition for hot and dense matter in the presence of a magnetic field. The effect of the *CP* violating θ term in QCD is incorporated through a θ dependent flavor mixing determinant interaction within a three flavor NJL Lagrangian. The methodology uses an explicit variational construct for the ground state in terms of quark-antiquark paring, instead of performing a chiral rotation of quark fields [22,23]. The ansatz functions in the variational construct for the ground state are determined from the minimization of the thermodynamic potential solving self-consistent gap equations for the condensates in the scalar as well as the pseudoscalar channels.

For nonvanishing θ, the constituent quark masses arise from quark-antiquark condensates both in scalar and pseudoscalar channels. With increasing values of *CP* violating parameter θ, and for light quarks, the pseudoscalar condensates increase and become maximum in magnitude at $\theta=\pi$. On the other hand, the condensate in the scalar channel decreases with θ and almost vanishes for $\theta=\pi$ but for the current quark mass contribution. The condensates in the two channels vary in a complimentary way such that the constituent quark mass remains almost constant with θ variation. The magnetic field enhances the condensates in both the channels and breaks the isospin symmetry of the light quarks.

The effective potential as a function of θ shows the minimum at $\theta=0$ with the cusp at $\theta=\pi$ consistent with the Vafa-Witten theorem. Introduction of a magnetic field does not change this behavior. It only increases the magnitude of the effective potential.

We also examined the effects of s-quark mass on the vacuum structure in the presence of finite θ and a magnetic field. It is observed that when the mass of the strange quark is similar to that of u and d quarks, its condensate structure is similar to that of d quarks essentially dictated by its charge. In particular, for $\theta=\pi$, this leads to the condensate structure for u and d quarks in the scalar channel of similar magnitude as that in the pseudoscalar channel. On the other hand, for the phenomenological value of m_s (140.7 MeV) of the model, the s-quark condensate is vanishingly small in the pseudoscalar channel as compared to the scalar channel. This further leads to negligible magnitude of the light quark condensates in the scalar channel for $\theta=\pi$. Although the s-quark condensates are vanishingly small in the pseudoscalar channel, the scalar channel s-quark condensate significantly affects the light quark condensates in the pseudoscalar channel due to the flavor mixing terms in the mass gap equations.

At vanishing chemical potential, with temperature, the condensates in both the channels decrease. The *CP* transition is a second order transition with a critical temperature $T_c\simeq 200$ MeV. With a magnetic field, this *CP* transition still remains second order with the transition temperature increasing as magnetic field strength is increased similar to the magnetic catalysis of chiral symmetry breaking at finite temperature. This high temperature restoration of *CP* is expected as the instanton effects responsible for the *CP* violating phase become suppressed exponentially at high temperatures [49].

At finite chemical potential, however, the *CP* transition is a first order transition. Further, inverse magnetic catalysis for the *CP* transition is observed at finite chemical potential at zero temperature, i.e., the corresponding critical chemical potential decreases with magnetic fields for small magnetic fields. The possibility of a first order phase transition can lead to formation of *CP*-odd metastable domains that could be of relevance for heavy ion collisions at the Facility for Antiproton and Ion Research as well as at the Nucleotron-based Ion Collider facility at Dubna. However, it ought to be mentioned that for the application to heavy ion collision, it is crucial to include the non-equilibrium dynamics of the formation of domains, which will provide the relevant time scales and also provide information on the possibility of measuring the effects arising from the formation of such CP-odd domains [50].

We have considered the quark-antiquark pairing in our ansatz for the ground state that is homogeneous with zero total momentum as in Eq. (2). However, it is possible that the condensate could be spatially nonhomogeneous with a net total momentum [51–53] or for very strong fields could be nonisotropic with vector condensation [54]. Further, one could include the effect of deconfinement transition by generalizing the present model to Polyakov loop NJL models for three flavors to investigate the inter-relationship of deconfinement and the chiral transition [55] as well as CP violation [23] in the presence of strong fields for the three flavor case considered here. This will be particularly important for finite temperature and low baryon densities. On the other hand, at finite density and small temperatures, the ansatz can be generalized to include the diquark condensates in the presence of a magnetic field [56–58]. Some of these calculations are in progress and will be reported elsewhere.

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