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Does unitarity imply finiteness of electroweak oblique corrections at one loop? Constraining extra neutral Higgs bosons

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Introducing an arbitrary number of neutral Higgs bosons in the electroweak symmetry breaking sector, we derive a set of conditions among Higgs couplings which need to be satisfied to maintain the unitarity of the high energy scattering amplitudes of weak gauge bosons at the tree level (unitarity sum rules). It is shown that the unitarity sum rules require the tree level ρ parameter to be 1, without explicitly invoking the custodial symmetry arguments. The one loop finiteness of the electroweak oblique corrections is automatically guaranteed once these unitarity sum rules are imposed among Higgs couplings. Severe constraints on the lightest Higgs coupling (125 GeV Higgs coupling) and the mass of the second lightest Higgs boson are obtained from the unitarity and the results of the electroweak precision tests (oblique parameter measurements). These results are compared with the effective theory of the light Higgs boson, and we find simple relationships between the mass of the second lightest Higgs boson in our framework and the ultraviolet cutoff in the effective theory framework.

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I. INTRODUCTION

The year 2012 discovery of a Higgs boson at 125 GeV at the Large Hadron Collider (LHC) experiments [1,2] completed the set of all particles predicted in the standard model (SM). We now have a key particle to solve the mystery of the origin of particle masses (electroweak symmetry breaking). Due to the lack of mechanism to stabilize the electroweak scale against the radiative corrections, however, the SM electroweak symmetry breaking (EWSB) sector is believed to be incomplete. Varieties of extended EWSB models have therefore been proposed. These extended models typically contain more particles other than the observed Higgs boson in their EWSB sector.

One of the key roles of the SM Higgs boson is to unitarize the high energy longitudinal weak gauge bosons' scattering amplitudes [3–6]. The Higgs boson also makes the SM renormalizable, i.e., it cancels nonrenormalizable ultraviolet (UV) divergences appearing at the loop level. The Higgs coupling strengths with the weak gauge bosons are precisely adjusted in order to make the SM unitary and renormalizable. Although experimental data accumulated so far on the 125 GeV boson are consistent with the SM Higgs particle [7–9] (see also Refs. [10–17]), in the extended EWSB scenarios, the coupling strengths of the 125 GeV boson still have a chance to deviate largely from the predicted values in the SM. Actually, it has been pointed out that the 125 GeV particle is, within the accuracy of the present data, also consistent [18,19] with a techni-dilaton (light composite scalar particle) composed through hypothetical walking technicolor dynamics. This situation will change drastically in the future. Future LHC experiments with high luminosity will be able to measure $hVV(V=W,Z)$ coupling more accurately [20,21], where h is the observed Higgs boson. Various Higgs coupling strengths will be measured very precisely at the International Linear Collider (ILC) experiment [22,23].

How can we utilize such high precision Higgs measurements? If the measured value of Higgs coupling strengths turn out to deviate from the SM values, in order to make the theory unitary and to keep consistency with the electroweak precision tests (EWPTs), new particles other than the 125 GeV Higgs boson need to exist. Can we make definite predictions for the properties of this required new particle in this case? In this paper, we try to answer this question from the viewpoint of the unitarity and the EWPTs. We assume the EWSB sector contains a richer spectrum of particles, i.e., a zoo of "Higgs" bosons,[1] in addition to the discovered 125 GeV Higgs boson in order to make the deviation of Higgs couplings possible without conflicting with the unitarity and the EWPTs. We do not assume particular Higgs potential models, however, since we seek for clues of physics beyond the SM as model independent as possible. Conditions to keep the scattering amplitudes (perturbatively) unitary at high energy have been derived in

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[1] We call all scalar particles participating the unitarization of the longitudinal gauge boson scattering as Higgs bosons.

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Ref. [24] and are well known as the "unitarity sum rules."[2] However, implications of such unitarity sum rules to the EWPTs at loop level have not been clarified until now. In the former half of this paper, we study the implications of the unitarity sum rules to the finiteness of the electroweak precision parameters (oblique correction parameters S, T, U) at the loop level. For such a purpose, we reanalyze the unitarity sum rules by using the electroweak chiral Lagrangian technique and the equivalence theorem to keep manifest $SU(2) \times U(1)$ gauge invariance, which allows us to use the model not only for the tree level unitarity analysis but also for the loop level oblique correction analysis. We simplify our analysis assuming only neutral Higgs bosons in the EWSB sector. We find the one loop finiteness of the electroweak oblique correction parameters is automatically guaranteed by the unitarity sum rules within this setup. Extensions including charged Higgs bosons and fermions will be discussed elsewhere.

In the latter half of the paper, we study phenomenological implications of the unitarity and the electroweak oblique parameter constraints. We use these constraints to impose upper bounds on the second lightest Higgs boson mass as a function of the deviation of the 125 GeV Higgs coupling $\Delta\kappa_V (\equiv \kappa_V - 1)$. Here $\Delta\kappa_V$ denotes the deviation of the 125 GeV Higgs coupling with weak gauge bosons from its SM value. Once the absence of the second lightest Higgs boson is confirmed below 1 TeV, the electroweak precision constraint will rule out $\Delta\kappa_V \lesssim -0.02$ at 95% C.L.

We keep the tree-level ρ parameter arbitrary in the unitarity analysis, which enables us to investigate theoretical structures which determine the value of the ρ parameter. Especially, we are able to show, without explicitly invoking the custodial symmetry arguments, the unitarity of the scattering amplitudes requires the tree-level ρ parameter to be unity in any EWSB model if it only possesses neutral Higgs bosons. Custodial symmetry is not a required symmetry. Instead, $\rho = 1$ is considered as a result of the unitarity in this class of models. This is consistent with the fact that $\rho = 1$ is predicted in all the known renormalizable EWSB models which do not contain charged Higgs boson couplings with the electroweak gauge bosons. Our finding will be helpful to understand the reason of $\rho = 1$ in the septet Higgs extension model [30–32] which does not enjoy explicit custodial symmetry. We will discuss the septet issue in our separate publication.

Our strategy described in this paper should not be confused with the usual light Higgs effective field theory approaches [33–63]. In the effective field theory approach based on the linear sigma model [33–49], the discovered 125 GeV Higgs boson field is assumed to be a component of a doublet Higgs field just like in the SM. The deviations of Higgs couplings are encoded in the higher dimensional effective Lagrangian coefficients including their renormalization group flow at the loop level [35–40,55–60].

Due to the presence of such higher dimensional operators, perturbative unitarity of the scattering amplitudes is violated at a certain high energy scale (cutoff scale of the effective theory) in the effective field theory [64,65]. Yet unknown UV completion theory therefore needs to replace the effective field theory above the cutoff scale. In this sense, in addition to the studies of the effective field theory, we need to study UV completions model dependently. Actually, many model dependent studies have been performed [27,30,31,66–84]. In this paper, we try to establish a systematic classification of possibilities of perturbative UV completions appearing at the cutoff scale.[3] Especially, we find simple relationships between bounds on the second lightest Higgs boson mass in our framework and the UV cutoff in the effective field theory framework.

This paper is organized as follows: In Sec. II, we describe the model we use in this paper. For simplicity, we restrict ourselves only to the neutral Higgs extension models. We next take the unitary gauge in Sec. III, and compare our model with the gauge noninvariant model used in Ref. [24]. Section IV is devoted to the unitarity sum rules and their possible applications to physics. We then evaluate the one loop radiative corrections to the $f\bar{f} \to f'\bar{f}'$ amplitudes in Sec. V. We explicitly show that the amplitudes automatically remain finite at one loop level if we impose the unitarity sum rules among various Higgs couplings. The explicit formulas of the electroweak oblique parameters [88] (Peskin-Takeuchi parameters) are presented in Sec. VI, and we obtain bounds on the second lightest Higgs boson mass from the unitarity and the EWPTs in Sec. VII. Section VIII discusses extra conditions other than the unitarity sum rules we need to impose to make the theory fully UV complete. The relationship between our approach and the effective field theory will be discussed in Sec. IX. Conclusions and outlook are given in Sec. X.

II. THE MODEL

We use the electroweak chiral Lagrangian [89,90] technique to describe the arbitrary interactions among weak gauge bosons and neutral Higgs bosons in an $SU(2) \times U(1)$ gauge invariant manner. The Lagrangian $\mathcal{L}$ of this model can be decomposed as

$$\mathcal{L} = \mathcal{L}_\chi + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}}, \tag{2.1}$$

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[2] Unitarity sum rules in the Higgsless theories [25], in which a tower of spin-1 resonances exists instead of the spin-0 Higgs boson in the Higgs sector, have been fully investigated in Ref. [26]. Assuming simultaneous existence of both spin-0 and spin-1 particles, Ref. [27] gave model independent sum rules. See also Refs. [28] and [29] for related topics.

[3] Reference [85] found theoretical constraints on effective Lagrangian parameters assuming unitary UV completion behind the effective theory. See also Refs. [86] and [87].

with $\mathcal{L}_\chi$, $\mathcal{L}_{\rm gauge}$, and $\mathcal{L}_{\rm Higgs}$ being the $SU(2)\times U(1)/U(1)$ nonlinear sigma model Lagrangian, the $SU(2)\times U(1)$ gauge Lagrangian, and the Higgs Lagrangian, respectively. Hereafter, we restrict our model Lagrangian to contain only terms of mass dimension four or less and up to (at most) two derivatives [$\mathcal{O}(\partial^2)$ terms] since we are interested in models in which scattering amplitudes remain unitary at high energy.

The spontaneous EWSB sector is described by using the electroweak chiral Lagrangian

$$\mathcal{L}_\chi = \frac{v^2}{4}{\rm tr}[(D_\mu U)^\dagger(D^\mu U)] + \beta\frac{v^2}{4}{\rm tr}[U^\dagger(D_\mu U)\tau_3]{\rm tr}[U^\dagger(D^\mu U)\tau_3]. \tag{2.2}$$

We denote $v\simeq 246$ GeV the decay constant of the charged would-be Nambu-Goldstone boson (NGB). The nonlinear sigma model field U,

$$U = \exp(i\tilde{w}^a\tau_a), \tag{2.3}$$

is introduced in Eq. (2.2), so as to describe the NGB field arising from the spontaneous EWSB. Here τ_a $(a=1,2,3)$ are the Pauli matrices, and $\tilde{w}^a$ are the NGB fields. Note that, under the $SU(2)\times U(1)$ gauge transformation, the NGB field $\tilde{w}^a\tau_a$ transforms nonlinearly,

$$U\to G_L U G_Y^\dagger, \tag{2.4}$$

with

$$G_L\equiv\exp\left(i\frac{\tau_a}{2}\theta_L^a\right),\qquad G_Y\equiv\exp\left(i\frac{\tau_3}{2}\theta_Y\right). \tag{2.5}$$

The covariant derivative $D_\mu U$ is defined as

$$D_\mu U = \partial_\mu U + ig\mathbf{W}_\mu U - ig_Y U\mathbf{B}_\mu, \tag{2.6}$$

with $SU(2)\times U(1)$ gauge fields $\mathbf{W}_\mu$ and $\mathbf{B}_\mu$ being defined by

$$\mathbf{W}_\mu = W_\mu^a\frac{\tau_a}{2},\qquad \mathbf{B}_\mu = B_\mu\frac{\tau_3}{2}. \tag{2.7}$$

The gauge transformation of Eq. (2.6) is

$$D_\mu U\to G_L(D_\mu U)G_Y^\dagger, \tag{2.8}$$

where the gauge fields transform as

$$\mathbf{W}_\mu\to G_L\mathbf{W}_\mu G_L^\dagger + \frac{i}{g}(\partial_\mu G_L)G_L^\dagger, \tag{2.9}$$

$$\mathbf{B}_\mu\to G_Y\mathbf{B}_\mu G_Y^\dagger + \frac{i}{g_Y}(\partial_\mu G_Y)G_Y^\dagger. \tag{2.10}$$

The gauge invariance of the electroweak chiral Lagrangian (2.2) is manifest.

The vacuum expectation value (VEV) of U,

$$\langle U\rangle = 1, \tag{2.11}$$

breaks the electroweak symmetry spontaneously:

$$\langle U\rangle\to\langle G_L U G_Y^\dagger\rangle = G_L G_Y^\dagger \neq 1 = \langle U\rangle. \tag{2.12}$$

The spectrum of physical particles can be obtained by taking the unitary gauge $U=1$, with which the electroweak chiral Lagrangian (2.2) leads to the mass terms of W and Z bosons,

$$M_W^2 = \frac{g^2}{4}v^2,\qquad M_Z^2 = \frac{g_Z^2}{4}v_Z^2, \tag{2.13}$$

with

$$v_Z^2\equiv v^2(1-2\beta), \tag{2.14}$$

and

$$g_Z^2\equiv g^2+g_Y^2. \tag{2.15}$$

Here the charged W boson field (W_μ), the neutral Z boson field (Z_μ) and the photon field A_μ are given by

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1\mp iW_\mu^2), \tag{2.16}$$

and

$$\begin{pmatrix} Z_\mu \\ A_\mu\end{pmatrix} = \begin{pmatrix} c & -s \\ s & c\end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu\end{pmatrix}, \tag{2.17}$$

with

$$s\equiv\frac{g_Y}{\sqrt{g^2+g_Y^2}},\qquad c\equiv\frac{g}{\sqrt{g^2+g_Y^2}}. \tag{2.18}$$

The QED coupling strength e is given by

$$e\equiv gs. \tag{2.19}$$

The coefficient β in the electroweak chiral Lagrangian (2.2) can be related with the tree-level ρ parameter, which is defined as

$$\rho_0\equiv\frac{g_Z^2/M_Z^2}{g^2/M_W^2} = \frac{v^2}{v_Z^2} = \frac{1}{1-2\beta}. \tag{2.20}$$

We keep ρ_0 arbitrary in our analysis of longitudinal gauge boson scattering amplitudes, which makes it possible to investigate the effects of $\rho_0 \neq 1$ in the longitudinal gauge boson scattering amplitudes. This is in contrast to the analysis of Ref. [24] in which $\rho_0 = 1$ is assumed in their practical applications of the unitarity sum rules to the EWSB models.

We investigate the longitudinal gauge boson scattering amplitudes using their equivalence with the NGB scattering amplitudes [4,6,91]. We define the NGB fields $w^{\pm}$ (charged NGB) and z (neutral NGB)

$$\begin{aligned} w^{\pm} &= \frac{v}{\sqrt{2}}(\tilde{w}_1 \mp i\tilde{w}_2), \\ z &= v_Z \tilde{w}_3, \end{aligned} \tag{2.21}$$

to make the kinetic terms of $w^{\pm}$ and z normalized canonically. We then obtain

$$\tilde{w}^a \tau_a = \frac{\sqrt{2}}{v}(w^+\tau_+ + w^-\tau_-) + \frac{1}{v_Z} z\tau_3, \tag{2.22}$$

with

$$\tau_{\pm} \equiv \frac{1}{2}(\tau_1 \pm i\tau_2). \tag{2.23}$$

The $SU(2) \times U(1)$ gauge Lagrangian $\mathcal{L}_{\text{gauge}}$ is given by

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{2}\text{tr}[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}] - \frac{1}{2}\text{tr}[\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}]. \tag{2.24}$$

Here $SU(2) \times U(1)$ field strengths $\mathbf{W}_{\mu\nu}$, $\mathbf{B}_{\mu\nu}$ are

$$\mathbf{W}_{\mu\nu} \equiv \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu + ig[\mathbf{W}_\mu, \mathbf{W}_\nu], \tag{2.25}$$

$$\mathbf{B}_{\mu\nu} \equiv \partial_\mu \mathbf{B}_\nu - \partial_\nu \mathbf{B}_\mu. \tag{2.26}$$

Note the gauge field strengths behave

$$\begin{aligned} \mathbf{W}_{\mu\nu} &\to G_L \mathbf{W}_{\mu\nu} G_L^{\dagger}, \\ \mathbf{B}_{\mu\nu} &\to \mathbf{B}_{\mu\nu}, \end{aligned} \tag{2.27}$$

under the gauge transformation given in Eqs. (2.9) and (2.10). The Lagrangian (2.24) is therefore invariant under the gauge transformation.

We next incorporate neutral spin-0 Higgs bosons (ϕ_n^0, $n = 1, 2, \ldots N_0$) as "matter" particles in the chiral Lagrangian, which keep the model unitary at high energy,

$$\mathcal{L}_{\text{Higgs}} = -V + \frac{1}{2}\sum_{n_1=1}^{N_0}\sum_{n_2=1}^{N_0} K_{n_1 n_2}(\partial_\mu \phi_{n_1}^0)(\partial^\mu \phi_{n_2}^0) + \mathcal{L}_{\text{int}}, \tag{2.28}$$

with V, K being functions of ϕ_n^0.

The masses of these Higgs particles and their self-interactions are described by $V(\phi^0)$.[4] We assume

$$\langle \phi_n^0 \rangle = 0, \tag{2.29}$$

for $n = 1, 2, \ldots N_0$. $V(\phi^0)$ is therefore

$$V(\phi^0) = \frac{1}{2}\sum_{n=1}^{N_0} M_{\phi_n^0}^2 \phi_n^0 \phi_n^0 + \cdots, \tag{2.30}$$

with "$\cdots$" being terms of self-interactions among these Higgs particles. We take $K_{n_1 n_2}$ so as to make the Higgs kinetic term canonically normalized[5]

$$K_{n_1 n_2}(\phi^0) = \delta_{n_1 n_2}. \tag{2.31}$$

Interactions of these Higgs particles with the electroweak gauge bosons are described by $\mathcal{L}_{\text{int}}$,

$$\mathcal{L}_{\text{int}} = \mathcal{L}_{\phi} + \mathcal{L}_{\phi\overset{\leftrightarrow}{\partial}\phi} + \mathcal{L}_{\phi\phi}, \tag{2.32}$$

where

$$\begin{aligned} \mathcal{L}_{\phi} = &-v\sum_{n=1}^{N_0} \kappa_{WW}^{\phi_n^0} \phi_n^0 \text{tr}[U^\dagger (D_\mu U)\tau_+]\text{tr}[U^\dagger (D^\mu U)\tau_-] \\ &- \frac{v}{4}\sum_{n=1}^{N_0} \kappa_{ZZ}^{\phi_n^0} \phi_n^0 \text{tr}[U^\dagger (D_\mu U)\tau_3]\text{tr}[U^\dagger (D^\mu U)\tau_3], \end{aligned} \tag{2.33}$$

$$\mathcal{L}_{\phi\overset{\leftrightarrow}{\partial}\phi} = -\frac{i}{4}\sum_{n=1}^{N_0}\sum_{m=1}^{N_0} \kappa_Z^{\phi_n^0\phi_m^0} (\phi_n^0 \overset{\leftrightarrow}{\partial}_\mu \phi_m^0)\text{tr}[U^\dagger (D^\mu U)\tau_3], \tag{2.34}$$

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[4] Note that ϕ_n^0 in Eq. (2.29) is classified by the unbroken $U(1)_{\text{em}}$ symmetry. Equation (2.29) does not imply the absence of the vacuum expectation values of the linearly realized Higgs multiplet field to which ϕ_n^0 is considered to belong.

[5] In general, $K_{n1,n2}$ induces higher dimensional operators which are functions of Higgs fields. We ignore these operators, however, since they violate the perturbative unitarity of $\phi + \phi \to \phi + \phi$ scattering amplitudes explicitly at high energy.

$$\begin{aligned}\mathcal{L}_{\phi\phi} = &-\frac{1}{2}\sum_{n=1}^{N_0}\sum_{m=1}^{N_0}\kappa_{WW}^{\phi_n^0\phi_m^0}\phi_n^0\phi_m^0\\ &\times \mathrm{tr}[U^\dagger(D_\mu U)\tau_+]\mathrm{tr}[U^\dagger(D^\mu U)\tau_-]\\ &-\frac{1}{8}\sum_{n=1}^{N_0}\sum_{m=1}^{N_0}\kappa_{ZZ}^{\phi_n^0\phi_m^0}\phi_n^0\phi_m^0\\ &\times \mathrm{tr}[U^\dagger(D_\mu U)\tau_3]\mathrm{tr}[U^\dagger(D^\mu U)\tau_3],\end{aligned} \tag{2.35}$$

with

$$\phi_n^0\overleftrightarrow{\partial}_\mu\phi_m^0 \equiv \phi_n^0(\partial_\mu\phi_m^0) - (\partial_\mu\phi_n^0)\phi_m^0. \tag{2.36}$$

Note that our Higgs ϕ_n^0 are all *real* scalar fields. The Higgs coupling parameters $\kappa_{WW}^{\phi_n^0}$, $\kappa_{ZZ}^{\phi_n^0}$, $\kappa_Z^{\phi_n^0\phi_m^0}$, $\kappa_{WW}^{\phi_n^0\phi_m^0}$ and $\kappa_{ZZ}^{\phi_n^0\phi_m^0}$ are therefore required to be real. We also note the $n\leftrightarrow m$ antisymmetry of $\kappa_Z^{\phi_n^0\phi_m^0}$, i.e.,

$$\kappa_Z^{\phi_n^0\phi_m^0} = -\kappa_Z^{\phi_m^0\phi_n^0}, \tag{2.37}$$

and the $n\leftrightarrow m$ symmetry of $\kappa_{VV}^{\phi_n^0\phi_m^0}$, i.e.,

$$\kappa_{WW}^{\phi_n^0\phi_m^0} = \kappa_{WW}^{\phi_m^0\phi_n^0}, \qquad \kappa_{ZZ}^{\phi_n^0\phi_m^0} = \kappa_{ZZ}^{\phi_m^0\phi_n^0}. \tag{2.38}$$

Although the interaction Lagrangian (2.32) has some similarity with the light Higgs effective theory realized in the nonlinear sigma model [51–54,63], our approach differs from the low energy effective theory, since we explicitly introduce heavy Higgs bosons in order to keep the model unitary at high energy as we stressed before.

We here make a couple of comments on the *CP* transformation properties of the model. We know

$$\begin{aligned}(CP)w^+(x^\mu)(CP)^{-1} &= -w^-(x_\mu),\\ (CP)w^-(x^\mu)(CP)^{-1} &= -w^+(x_\mu),\\ (CP)z(x^\mu)(CP)^{-1} &= -z(x_\mu),\end{aligned}$$

and thus[6]

$$(CP)\tilde{w}^a(x^\mu)\tau_a(CP)^{-1} = \tau_2(\tilde{w}^a(x_\mu)\tau_a)\tau_2. \tag{2.39}$$

The *CP* transformation of the nonlinear sigma model field is therefore given by

$$(CP)U(x^\mu)(CP)^{-1} = \tau_2 U(x_\mu)\tau_2. \tag{2.40}$$

In order to keep the electroweak chiral Lagrangian (2.2) invariant under the *CP* transformation, $\mathbf{W}^\mu$ and $\mathbf{B}_\mu$ need to transform as

$$(CP)\mathbf{W}^\mu(x^\mu)(CP)^{-1} = \tau_2\mathbf{W}_\mu(x_\mu)\tau_2, \tag{2.41}$$

and

$$(CP)\mathbf{B}^\mu(x^\mu)(CP)^{-1} = \tau_2\mathbf{B}_\mu(x_\mu)\tau_2. \tag{2.42}$$

It is easy to check that Eqs. (2.41) and (2.42) are consistent with conventional *CP* quantum number assignments of the electroweak gauge bosons. We also find

$$(CP)\mathrm{tr}[U^\dagger(D_\mu U)\tau_\pm](CP)^{-1} = -\mathrm{tr}[U^\dagger(D^\mu U)\tau_\mp], \tag{2.43}$$

$$(CP)\mathrm{tr}[U^\dagger(D_\mu U)\tau_3](CP)^{-1} = -\mathrm{tr}[U^\dagger(D^\mu U)\tau_3]. \tag{2.44}$$

We are now ready to discuss the *CP* transformation properties of neutral Higgs bosons in our model. We assign

$$(CP)\phi_n^0(x^\mu)(CP)^{-1} = \eta_n\phi_n^0(x_\mu), \tag{2.45}$$

with

$$\eta_n = \begin{cases} +1 & \text{for } CP \text{ even}\\ -1 & \text{for } CP \text{ odd.}\end{cases} \tag{2.46}$$

Requiring the Lagrangians (2.33), (2.34) and (2.35) invariant under the *CP* transformation, we obtain

$$\kappa_{WW}^{\phi_n^0}\eta_n = \kappa_{WW}^{\phi_n^0}, \qquad \kappa_{ZZ}^{\phi_n^0}\eta_n = \kappa_{ZZ}^{\phi_n^0}, \tag{2.47}$$

$$-\kappa_Z^{\phi_n^0\phi_m^0}\eta_n\eta_m = \kappa_Z^{\phi_n^0\phi_m^0}, \tag{2.48}$$

and

$$\kappa_{WW}^{\phi_n^0\phi_m^0}\eta_n\eta_m = \kappa_{WW}^{\phi_n^0\phi_m^0}, \qquad \kappa_{ZZ}^{\phi_n^0\phi_m^0}\eta_n\eta_m = \kappa_{ZZ}^{\phi_n^0\phi_m^0}. \tag{2.49}$$

From Eq. (2.47), it is easy to see

$$\kappa_{WW}^{\phi_n^0} = \kappa_{ZZ}^{\phi_n^0} = 0, \quad \text{for } \eta_n = -1. \tag{2.50}$$

Also, combining Eqs. (2.38) and (2.49), we obtain

$$\kappa_{WW}^{\phi_n^0\phi_m^0} = \kappa_{ZZ}^{\phi_n^0\phi_m^0} = 0, \quad \text{for } \eta_n\eta_m = -1, \tag{2.51}$$

if the Higgs sector preserves the *CP* invariance.

III. LAGRANGIAN IN THE UNITARY GAUGE

Unitarity sum rules of longitudinal weak boson scattering amplitudes [3–5] were thoroughly investigated by Ref. [24] in the context of the $SU(2)\times U(1)$ gauge theory with arbitrary Higgs multiplets. Reference [24] performed their analysis without introducing unphysical would-be NGBs, however, in contrast to our chiral Lagrangian

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[6] Precisely speaking, we choose the convention for charged NGBs under the *CP* transformation by Eq. (2.39).

analysis in which $SU(2)\times U(1)$ gauge invariance is kept manifest. In order to make direct comparisons between the results of Ref. [24] and the results presented in this paper, it is convenient to rewrite our model in the unitary gauge

$$U = 1, \tag{3.1}$$

in which unphysical would-be NGBs are absent. We then find

$$\mathcal{L}_\chi = M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu, \tag{3.2}$$

$$\mathcal{L}_\phi = g M_W \sum_{n=1}^{N_0} \kappa_{WW}^{\phi_n^0} \phi_n^0 W_\mu^+ W^{-\mu} + \frac{g_Z}{2}\frac{v}{v_Z} M_Z \sum_{n=1}^{N_0} \kappa_{ZZ}^{\phi_n^0} \phi_n^0 Z_\mu Z^\mu, \tag{3.3}$$

$$\mathcal{L}_{\phi\overleftrightarrow{\partial}\phi} = \frac{g_Z}{4} \sum_{n=1}^{N_0}\sum_{m=1}^{N_0} \kappa_Z^{\phi_n^0\phi_m^0} (\phi_n^0 \overleftrightarrow{\partial}_\mu \phi_m^0) Z^\mu, \tag{3.4}$$

$$\mathcal{L}_{\phi\phi} = \frac{g^2}{4} \sum_{n=1}^{N_0}\sum_{m=1}^{N_0} \kappa_{WW}^{\phi_n^0\phi_m^0} \phi_n^0 \phi_m^0 W_\mu^+ W^{-\mu} + \frac{g_Z^2}{8} \sum_{n=1}^{N_0}\sum_{m=1}^{N_0} \kappa_{ZZ}^{\phi_n^0\phi_m^0} \phi_n^0 \phi_m^0 Z_\mu Z^\mu, \tag{3.5}$$

which correspond to the masses of vector bosons (V), the Higgs-V-V vertices, the Higgs-Higgs-V vertex, and the Higgs-Higgs-V-V vertices of Ref. [24], respectively. It is easy to see that the CP properties [Eqs. (2.47)–(2.49)] are identical to the CP properties of $WW\phi$, $ZZ\phi$, $Z\phi\phi$, $WW\phi\phi$, $ZZ\phi\phi$ couplings obtained in Ref. [24].

IV. UNITARITY SUM RULES

The cancellation of the unitarity violating high energy scattering amplitudes of longitudinally polarized gauge bosons requires a set of conditions among Higgs couplings (“unitarity sum rules”) [3–5]. The unitarity sum rules in the $SU(2)\times U(1)$ gauge theory were studied a couple of decades ago by Ref. [24] and recently by Ref. [92]. In this section, using the equivalence theorem of the amplitudes of longitudinal gauge bosons and the would-be NGBs, we rederive the sum rules [24] in our gauge invariant Lagrangian through the NGB scattering amplitudes. We will then check explicitly the equivalence of our results with the sum rules derived in Ref. [24], which supports the consistency of our method using the gauge invariant Lagrangian.

A. NGB + NGB → NGB + NGB

The NGB scattering amplitudes are calculated in Appendix A in the case of $g = g_Y = 0$ (gaugeless limit). Mandelstam variables s, t, and u are also defined in Appendix A. Requiring the cancellation of the $\mathcal{O}(u)$ divergence in the high energy $w^+w^- \to w^+w^-$ scattering amplitude (A2), we obtain

$$-4 + 3\frac{v_Z^2}{v^2} + \sum_{n=1}^{N_0} \kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0} = 0, \tag{4.1}$$

which agrees with Eq. (4.1) of Ref. [24] in the absence of doubly charged Higgs bosons. Although we here impose the cancellation of scattering amplitude up to the ultimately high energy scale, the energy (cutoff) dependent modifications of $\mathcal{O}(M_V^2/s)$ to the sum rules may be allowed. On the other hand, as we will see later, exact sum rules are required to maintain the finiteness of the oblique corrections. We see, from Eq. (4.1), an inequality

$$v_Z^2 \le \frac{4}{3} v^2, \tag{4.2}$$

which is satisfied in the SM $v_Z^2 = v^2$. However, the inequality (4.2) is not satisfied in the triplet Higgs model ($I = 1$, $Y = 1$), in which $v_Z^2 = 2v^2$ is predicted.[7] Actually, the triplet Higgs model contains (doubly) charged Higgs bosons coupled with electroweak gauge bosons in its spectrum, and thus cannot be covered by the analysis presented in this manuscript.

In a similar manner, using the $w^+w^- \to zz$ amplitude (A4), we find a sum rule,

$$\frac{v_Z^2}{v^2} - \frac{v^2}{v_Z^2}\sum_{n=1}^{N_0} \kappa_{ZZ}^{\phi_n^0}\kappa_{WW}^{\phi_n^0} = 0. \tag{4.3}$$

Again, it is straightforward to see the equivalence of Eq. (4.3) with Eq. (4.2) of Ref. [24] in the absence of charged Higgs bosons.

We note that the $zz \to zz$ amplitude (A6) does not produce extra conditions because of $s + t + u = 0$. Note NGBs are massless in the gaugeless limit.

B. NGB + NGB → ϕ + ϕ

We next consider the $w^+w^- \to \phi_{n_1}^0 \phi_{n_2}^0$ amplitude (A8). The amplitude can be decomposed into two pieces, depending on the relative angular momentum between two scalar bosons in the final state. Requiring the cancellation of the $\mathcal{O}(s)$ enhanced term in the S-wave amplitude, we obtain a relation between $WW\phi\phi$ and $WW\phi$ interaction terms,

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[7] The (pure) triplet Higgs model does not accommodate mass generation mechanisms for SM fermions and cannot be accepted as a phenomenologically viable EWSB model.

$$\kappa_{WW}^{\phi^0_{n_1}\phi^0_{n_2}} - \kappa_{WW}^{\phi^0_{n_1}}\kappa_{WW}^{\phi^0_{n_2}} = 0. \tag{4.4}$$

On the other hand, requiring the cancellation of the $\mathcal{O}(t-u)$ term in the P-wave amplitude, we obtain

$$\kappa_{Z}^{\phi^0_{n_1}\phi^0_{n_2}} = 0. \tag{4.5}$$

Presence of $\kappa_{Z}^{\phi^0_{n_1}\phi^0_{n_2}}$ without introducing extra particles other than the neutral Higgs bosons would therefore cause a violation of unitarity in the $WW \to \phi\phi$ scattering amplitude.

These relations (4.4) and (4.5) correspond to a single equation Eq. (A3) of Ref. [24], which reads

$$\kappa_{WW}^{\phi^0_{n_1}\phi^0_{n_2}} - \kappa_{WW}^{\phi^0_{n_1}}\kappa_{WW}^{\phi^0_{n_2}} + i\kappa_{Z}^{\phi^0_{n_1}\phi^0_{n_2}} = 0, \tag{4.6}$$

in the notation of the present manuscript. Using Eqs. (2.37) and (2.38), however, Eq. (4.6) can be decomposed into $n_1 \leftrightarrow n_2$ symmetric and antisymmetric parts, which can be shown to be identical to our Eqs. (4.4) and (4.5), respectively.

We next move to the $zz \to \phi^0_{n_1}\phi^0_{n_2}$ amplitude (A10). We find a sum rule

$$\kappa_{ZZ}^{\phi^0_{n_1}\phi^0_{n_2}} - \sum_{m=1}^{N_0} \kappa_{Z}^{\phi^0_{n_1}\phi^0_{m}}\kappa_{Z}^{\phi^0_{n_2}\phi^0_{m}} - \frac{v^2}{v_Z^2}\kappa_{ZZ}^{\phi^0_{n_1}}\kappa_{ZZ}^{\phi^0_{n_2}} = 0, \tag{4.7}$$

which is required to cancel the $\mathcal{O}(s)$ divergence of the amplitude. Equation (4.7) is identical to Eq. (A18) of Ref. [24].

C. NGB + NGB → ϕ + NGB

The $w^+w^- \to \phi^0_n z$ amplitude also possesses S-wave and P-wave contributions in Eq. (A12). The cancellation of the high energy P-wave amplitude requires

$$\kappa_{WW}^{\phi^0_n} - \frac{v^2}{v_Z^2}\kappa_{ZZ}^{\phi^0_n} = 0, \tag{4.8}$$

while the S-wave amplitude requires

$$\sum_{m=1}^{N_0} \kappa_{Z}^{\phi^0_n\phi^0_m}\kappa_{WW}^{\phi^0_m} = 0. \tag{4.9}$$

Again, we note that the $zz \to \phi^0_n z$ amplitude (A14) does not produce extra conditions. It is also easy to check the equivalence of Eqs. (4.8) and (4.9) with Eq. (4.5) of Ref. [24].

D. Applications

As emphasized in Ref. [24], the unitarity sum rules can be applied to constrain various extended Higgs models. For an example, as Ref. [24] argued, assuming $v = v_Z$, that the future observation of the Higgs-W-W coupling larger than the SM value would suggest the existence of charged Higgs particles. This fact can be seen from Eq. (4.1), which leads to an upper bound of Higgs-W-W coupling $\kappa_{WW}^{\phi^0_n} \leq 1$ for $v = v_Z$ in any model only having extra neutral Higgs particles.

In this subsection, we list a couple of observations in the unitarity sum rules which have not been stressed in earlier literature.

Let us start with an implication of the unitarity sum rules to the ρ parameter $\rho_0 = v^2/v_Z^2$. Combining Eqs. (4.1) and (4.3), we find

$$\sum_{n=1}^{N_0} \kappa_{WW}^{\phi^0_n}(\kappa_{WW}^{\phi^0_n} - \rho_0\kappa_{ZZ}^{\phi^0_n}) = \frac{4}{\rho_0}(\rho_0 - 1). \tag{4.10}$$

On the other hand, the unitarity sum rules for $w^+w^- \to \phi z$ Eq. (4.8) reads

$$\kappa_{WW}^{\phi^0_n} = \rho_0\kappa_{ZZ}^{\phi^0_n}. \tag{4.11}$$

Plugging Eq. (4.11) into Eq. (4.10), we obtain a condition on the ρ_0 parameter,

$$\frac{1}{\rho_0}(\rho_0 - 1) = 0, \tag{4.12}$$

solely from the unitarity requirements. The ρ_0 parameter needs to be 1 in order to unitarize the $w^+w^- \to w^+w^-$, $w^+w^- \to zz$ and $w^+w^- \to \phi z$ scattering amplitudes in any EWSB model with $v \neq 0$, $v_Z \neq 0$ that only has neutral Higgs particles. Note that this argument cannot be applied to the triplet Higgs mixing model (a doublet and a triplet Higgs fields) [93–97], since we restrict ourselves within the neutral Higgs extension cases only. However, the unitarity argument will be useful when we understand $\rho_0 = 1$ in the septet Higgs case [30–32], in which we do not have manifest custodial symmetry. We will discuss the issue in our subsequent paper, in which we extend our analysis including the charged Higgs bosons.

It is also intriguing that the unitarity sum rule for the $w^+w^- \to zz$ amplitude (4.3) is sensitive to the sign of $\kappa_{ZZ}^{\phi^0_n}\kappa_{WW}^{\phi^0_n}$. Note that the current experimental results on the 125 GeV Higgs boson (h) are sensitive only to the absolute values of hZZ and hWW couplings ($|\kappa_{ZZ}^h|$ and $|\kappa_{WW}^h|$), not to their relative sign.[8] As shown in Eq. (4.3), a wrong sign $\kappa_{ZZ}^h\kappa_{WW}^h$ would cause a violation of unitarity in the $WW \to ZZ$ amplitude. Future measurements on the $WW \to ZZ$ (or $ZZ \to WW$ or $WZ \to WZ$) cross section

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[8] This fact is in contrast to the case of the relative sign between κ_W and κ_t (top-Higgs coupling), which can be determined using the $h \to \gamma\gamma$ channel in the SM.

can thus be used to check whether the $\kappa_{ZZ}^{\phi_n^0}\kappa_{WW}^{\phi_n^0}$ sign is like the SM or not.

The condition (4.5) gives us an insight on the hypothetical CP-odd neutral Higgs boson properties in a model independent manner. Existence of such a CP-odd Higgs boson a, having nonvanishing haZ coupling without introducing extra charged Higgs boson, would contradict with the unitarity relation (4.5) and would therefore cause an enhancement of the $WW \to ha$ cross section.

We finally make an important comment on the implications of the unitarity sum rules to the electroweak radiative corrections. As we will see in the sections below, a violation of the unitarity sum rules often causes a UV divergence in the electroweak radiative corrections. It is therefore severely constrained by the existing precision measurements on the electroweak interactions. The issue is studied extensively in this manuscript in Secs. V and VI.

V. FINITENESS OF $f\bar{f} \to f'\bar{f}'$ AMPLITUDES INCLUDING OBLIQUE CORRECTIONS AT ONE LOOP

Thanks to the gauge invariance of the nonlinear sigma model Lagrangian we use, in the present framework, effects of radiative corrections can be studied without causing unphysical negative metric particle problems even in the R_ξ gauge fixing method. Lack of the renormalizability of the nonlinear sigma model, however, causes UV divergences in the amplitudes, which cannot be renormalized by the redefinitions of the Lagrangian parameters. As we show in this section, one loop UV divergences in the massless fermion scattering amplitudes disappear after appropriate redefinitions of gauge coupling strengths and the VEVs, only when a set of sum rules is satisfied among the Higgs coupling strengths. In this section, we write down such a set of sum rules explicitly. We find these sum rules are automatically satisfied once the Higgs coupling strengths satisfy the unitarity sum rules we found in the previous section.

Before going into detail in the loop analysis, we briefly summarize the relationships between the vacuum polarization functions Π_{33}, Π_{3Q}, Π_{QQ} and Π_{11} and the $f\bar{f} \to f'\bar{f}'$ scattering amplitudes. We assume here the vacuum polarization functions evaluated in the background gauge fixing method, with which the cancellation of the divergences between the one loop vertex corrections and the fermion wave function renormalizations is guaranteed, thanks to the naive Ward-Takahashi identities.

We first discuss the relationship between the vacuum polarization functions Π_{33}, Π_{3Q}, Π_{QQ} and Π_{11},

$$\Pi_{33}(p^2) = \Pi_{33}(0) + p^2\Pi'_{33}(p^2), \tag{5.1}$$

$$\Pi_{11}(p^2) = \Pi_{11}(0) + p^2\Pi'_{11}(p^2), \tag{5.2}$$

$$\Pi_{3Q}(p^2) = p^2\Pi'_{3Q}(p^2), \tag{5.3}$$

$$\Pi_{QQ}(p^2) = p^2\Pi'_{QQ}(p^2), \tag{5.4}$$

and the $f\bar{f} \to f'\bar{f}'$ scattering amplitudes. Here $\Pi_{33}(p^2)$, Π_{11}, and Π_{QQ} are neutral and charged weak $SU(2)$ current correlators, and the electromagnetic current correlator, respectively. Π_{3Q} is the correlator between the neutral weak $SU(2)$ current and the electromagnetic current. These current correlators can be related with the vacuum polarization functions of the electroweak gauge bosons,

$$\Pi_{11} = \frac{1}{g^2}\Pi_{WW}, \tag{5.5}$$

$$\Pi_{33} = \frac{1}{g_Z^2}\left[\Pi_{ZZ} + \frac{g_Y^2}{g^2}\Pi_{AA} + 2\frac{g_Y}{g}\Pi_{ZA}\right], \tag{5.6}$$

$$\Pi_{3Q} = \frac{1}{g^2}\Pi_{AA} + \frac{1}{gg_Y}\Pi_{ZA}, \tag{5.7}$$

$$\Pi_{QQ} = \frac{g_Z^2}{g^2 g_Y^2}\Pi_{AA}. \tag{5.8}$$

The naive Ward-Takahashi identities arising from the conservation of the electromagnetic current give

$$\Pi_{3Q}(p^2 = 0) = \Pi_{QQ}(p^2 = 0) = 0. \tag{5.9}$$

By using these vacuum polarization functions, the neutral and charged current $f\bar{f} \to f'\bar{f}'$ scattering amplitudes ($f \neq f'$) including these oblique corrections can be expressed as

$$-\mathcal{M}_{\rm NC} = e_*^2\frac{QQ'}{-p^2} + \frac{(I_3 - s_*^2 Q)(I'_3 - s_*^2 Q')}{-(\frac{s_*^2 c_*^2}{e_*^2} - \Pi'_{33} + \Pi'_{3Q})p^2 + \frac{v_{Zr}^2}{4}}, \tag{5.10}$$

$$-\mathcal{M}_{\rm CC} = \frac{(I_+ I'_- + I_- I'_+)/2}{-(\frac{s_*^2}{e_*^2} - \Pi'_{11} + \Pi'_{3Q})p^2 + \frac{v_r^2}{4}}, \tag{5.11}$$

with renormalized parameters v_{Zr}^2, v_r^2, e_*^2, s_*^2 and c_*^2 defined by

$$v_{Zr}^2 = v_Z^2 + \delta v_Z^2 + 4\Pi_{33}(0), \tag{5.12}$$

$$v_r^2 = v^2 + \delta v^2 + 4\Pi_{11}(0), \tag{5.13}$$

$$\frac{1}{e_*^2} = \frac{1}{g^2} + \frac{1}{g_Y^2} + \delta\left(\frac{1}{g^2}\right) + \delta\left(\frac{1}{g_Y^2}\right) - \Pi'_{QQ}, \tag{5.14}$$

$$\frac{s_*^2}{e_*^2} = \frac{1}{g^2} + \delta\left(\frac{1}{g^2}\right) - \Pi'_{3Q}, \tag{5.15}$$

$$c_*^2 = 1 - s_*^2, \tag{5.16}$$

with δv_Z^2, δv^2, $\delta(1/g^2)$, $\delta(1/g_Y^2)$ being counterterms to renormalize the divergences in $\Pi_{33}(0)$, $\Pi_{11}(0)$, Π'_{QQ} and Π'_{3Q}. Here the amplitudes are described by using a simplified version of notations of Ref. [98]. The definitions of I_3, $I_\pm$, and $\mathcal{Q}$ are given in Ref. [99]. Finiteness of the scattering amplitudes thus requires

$$\frac{\delta v_Z^2}{4} + \Pi_{33}(0), \tag{5.17}$$

$$\frac{\delta v^2}{4} + \Pi_{11}(0), \tag{5.18}$$

$$\Pi'_{33} - \Pi'_{3Q}, \tag{5.19}$$

$$\Pi'_{11} - \Pi'_{3Q}, \tag{5.20}$$

are all finite. We study these conditions in the subsections below.

A. $\Pi_{33}(0)$ and $\Pi_{11}(0)$

We investigate the conditions of finiteness of Eqs. (5.17) and (5.18). The UV divergences in $\Pi_{11}(0)$ and $\Pi_{33}(0)$ can be absorbed into the renormalizations of v_Z and v if these two parameters are independently adjustable parameters. Triplet Higgs mixing models [93–97] including Georgi-Machacek scenario [100–103] fall into this category. In multi-Higgs doublet models [104] including the SM, and the doublet-septet mixing model [30–32], on the other hand, v_Z and v are linearly related parameters,

$$v_Z^2 = \frac{1}{\rho_0} v^2, \tag{5.21}$$

with ρ_0 being a positive constant. Although the parameter ρ_0 is phenomenologically required to be[9]

$$\rho_0 = 1, \tag{5.22}$$

in this manuscript, we keep this parameter arbitrary for a while in order to clarify the theoretical structure of Eqs. (5.17) and (5.18).

In models satisfying the requirement (5.21), the counterterms we can introduce should satisfy

$$\delta v_Z^2 = \frac{1}{\rho_0} \delta v^2. \tag{5.23}$$

In this class of models, we therefore find

$$v_Z^2 \Pi_{11}(0) - v^2 \Pi_{33}(0) \tag{5.24}$$

needs to be finite in order to keep the $f\bar{f} \to f'\bar{f}'$ amplitude finite at the loop level. In this subsection, we focus on the conditions which guarantee the finiteness of Eq. (5.24) at the one loop level.

We evaluate the vacuum polarization functions $\Pi_{11}(p^2)$ and $\Pi_{33}(p^2)$ at the one loop level. It is convenient to decompose these functions into two pieces,

$$\Pi_{11}(p^2) = \tilde{\Pi}_{11}(p^2) + \Pi_{11}^{\text{Higgs}}(p^2; \kappa), \tag{5.25}$$

$$\Pi_{33}(p^2) = \tilde{\Pi}_{33}(p^2) + \Pi_{33}^{\text{Higgs}}(p^2; \kappa), \tag{5.26}$$

where $\tilde{\Pi}_{11}(p^2)$ and $\tilde{\Pi}_{33}(p^2)$ are contributions arising from loops containing solely the gauge bosons and NGBs, and are independent of the Higgs coupling strengths κ. These contributions are evaluated by using the background gauge fixing method with 't Hooft-Feynman gauge $\xi = 1$. See Appendix B for details. Using the dimensional regularization, we obtain

$$\begin{aligned}\tilde{\Pi}_{11}(p^2 = 0) = {} & (D-2)\left[A(M_W) + \frac{g^2}{g_Z^2} A(M_Z) + \frac{g_Y^2}{g_Z^2} A(0) + \frac{g^2}{g_Z^2} B_0(M_W, M_Z; 0) + \frac{g_Y^2}{g_Z^2} B_0(M_W, 0; 0)\right] \\ & + \frac{v_Z^2}{4v^2}[A(M_W) + A(M_Z) + B_0(M_W, M_Z; 0)] - \frac{1}{4}\left(4 - 3\frac{v_Z^2}{v^2}\right) A(M_W) - \frac{1}{4}\frac{v_Z^2}{v^2} A(M_Z) \\ & - \frac{1}{g_Z^2}\left[g^2 \frac{v}{2}\left(2 - \frac{v_Z^2}{v^2}\right) - g_Y^2 \frac{v_Z^2}{2v}\right]^2 B(M_W, M_Z; 0) - \frac{g^2 g_Y^2}{g_Z^2}\left[\frac{v}{2}\left(2 - \frac{v_Z^2}{v^2}\right) + \frac{v_Z^2}{2v}\right]^2 B(M_W, 0; 0) \\ & - \frac{g^2 v_Z^2}{4} B(M_W, M_Z; 0),\end{aligned} \tag{5.27}$$

$$\tilde{\Pi}_{33}(p^2 = 0) = -\frac{1}{2}\frac{v_Z^4}{v^4} A(M_W) - \frac{1}{2} g^2 \frac{v_Z^4}{v^2} B(M_W, M_W; 0), \tag{5.28}$$

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[9] Strictly speaking, what we need to require is $\rho \simeq 1$ after taking account of the quantum corrections allowing experimental uncertainty of the 10^{-3} level.

with D being the number of space-time dimensions. Here UV divergent loop functions A, B and B_0 are defined by Eqs. (C1), (C2) and (C3).

We next evaluate the Higgs loop contributions to $\Pi_{11}^{\mathrm{Higgs}}$. The corresponding Feynman diagrams are given in Fig. 1. In the 't Hooft-Feynman gauge, we find

$$\begin{aligned}
&\Pi_{11}^{\mathrm{Higgs}}(p^2=0;\kappa)\\
&=\frac{1}{g^2}\Pi_{WW}^{\mathrm{Higgs}}(0)=\frac{1}{4}\sum_{n=1}^{N_0}\kappa_{WW}^{\phi_n^0\phi_n^0}A(M_{\phi_n^0})\\
&\quad+\frac{1}{4}\sum_{n=1}^{N_0}\kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0}\{B_0(M_{\phi_n^0},M_W;0)\\
&\quad-4M_W^2B(M_{\phi_n^0},M_W;0)\}, \qquad (5.29)
\end{aligned}$$

where the first, the second and the third terms are from Figs. 1(a), 1(b), and 1(c), respectively.

In a similar manner, evaluating the Feynman diagrams Fig. 2, we obtain

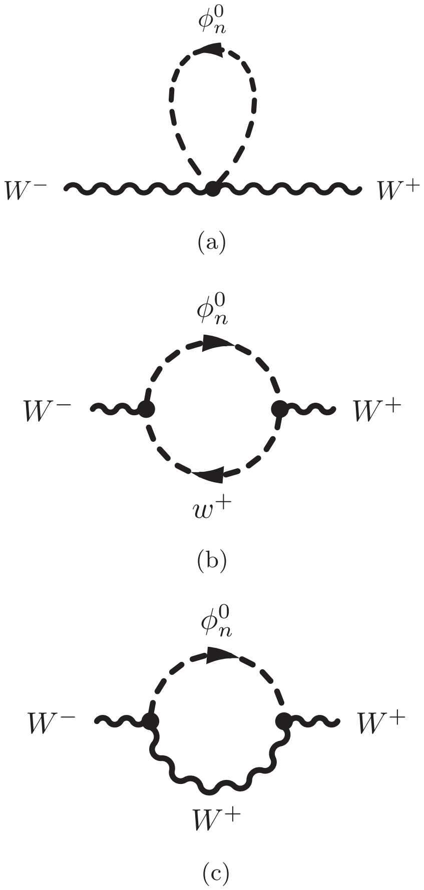

FIG. 1. One loop diagrams for the W boson self-energies $\Pi_{WW}^{\mathrm{Higgs}}$ in our model.

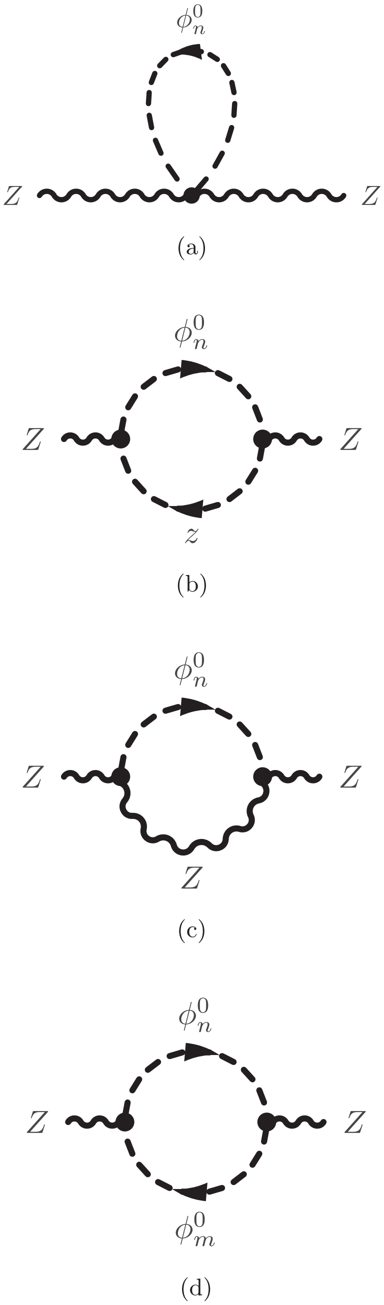

FIG. 2. One loop diagrams for the Z boson self-energies $\Pi_{ZZ}^{\mathrm{Higgs}}$ in our model.

$$\begin{aligned}
&\Pi_{33}^{\mathrm{Higgs}}(p^2=0;\kappa)\\
&=\frac{1}{g_Z^2}\Pi_{ZZ}^{\mathrm{Higgs}}(0)=\frac{1}{8}\sum_{n=1}^{N_0}\sum_{m=1}^{N_0}\kappa_Z^{\phi_n^0\phi_m^0}\kappa_Z^{\phi_n^0\phi_m^0}B_0(M_{\phi_n^0},M_{\phi_m^0};0)\\
&\quad+\frac{1}{4}\sum_{n=1}^{N_0}\kappa_{ZZ}^{\phi_n^0\phi_n^0}A(M_{\phi_n^0})+\frac{v^2}{4v_Z^2}\sum_{n=1}^{N_0}\kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_n^0}\{B_0(M_{\phi_n^0},M_Z;0)\\
&\quad-4M_Z^2B(M_{\phi_n^0},M_Z;0)\}. \qquad (5.30)
\end{aligned}$$

There may also exist tadpole graphs if ϕ_n^0 fields acquire their VEVs at one loop. We assume these one loop VEVs of

ϕ_n^0 are eliminated by introducing appropriate linear potential counterterms in the Higgs potential (2.30).

Our results of the vacuum polarization functions, Eqs. (5.27), (5.28), (5.29), and (5.30), can be compared with the SM, taking $N_0 = 1$, $v = v_Z$, $\kappa_{WW}^{\phi_1^0} = \kappa_{ZZ}^{\phi_1^0} = \kappa_{WW}^{\phi_1^0\phi_1^0} = \kappa_{ZZ}^{\phi_1^0\phi_1^0} = 1$.[10] Comparing them with the SM results of Hagiwara-Matsumoto-Haidt-Kim (HMHK) [105] which employs the pinch technique in their evaluation of the vacuum polarization functions, we find

$$\Pi_{11}^{\rm NTT}(0) - \Pi_{11}^{\rm HMHK}(0) = -\frac{1}{2}A(M_W) - \frac{1}{4}A(M_Z), \tag{5.31}$$

$$\Pi_{33}^{\rm NTT}(0) - \Pi_{33}^{\rm HMHK}(0) = -\frac{1}{2}A(M_W) - \frac{1}{4}A(M_Z), \tag{5.32}$$

where $\Pi_{11}^{\rm NTT}(0)$ and $\Pi_{33}^{\rm NTT}(0)$ denote the results presented in this section with the assumptions above, while $\Pi_{11}^{\rm HMHK}(0)$ and $\Pi_{33}^{\rm HMHK}(0)$ are the SM pinch technique results of Ref. [105]. These differences do not affect physical consequences, however. They actually can be considered to arise from the difference of conventions for the choice of normal ordering in the WW-NGB-NGB and the ZZ-NGB-NGB vertices in the linear sigma model Lagrangian (HMHK) and in the nonlinear sigma model Lagrangian (NTT).

Let us go back to our nonlinear sigma model Lagrangian with arbitrary Higgs coupling strengths κ. Note that the loop functions A, B, and B_0 diverge in the ultraviolet. Introducing the UV cutoff momentum Λ, they can be expressed by using Eqs. (C8), (C9) and (C10). It is now straightforward to obtain the UV divergences in Π_{11} and Π_{33}.

We find

$$\begin{aligned}\Pi_{11}(0)|_{\rm div} = {}& \frac{1}{4}\left(\sum_{n=1}^{N_0}\kappa_{WW}^{\phi_n^0\phi_n^0} - 2\sum_{n=1}^{N_0}\kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0} - 4 + 2\frac{v_Z^2}{v^2}\right)\\ &\times\frac{\Lambda^2}{(4\pi)^2} + \left\{\frac{1}{4}\sum_{n=1}^{N_0}(-\kappa_{WW}^{\phi_n^0\phi_n^0} + \kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0})M_{\phi_n^0}^2\right.\\ &- \frac{3}{16}\sum_{n=1}^{N_0}\kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0}g^2v^2 - \frac{3}{16}g_Z^2\frac{v_Z^4}{v^2}\\ &\left.- \frac{3}{4}g^2v^2 + \frac{9}{16}g^2v_Z^2\right\}\frac{1}{(4\pi)^2}\ln\frac{\Lambda^2}{\mu^2},\end{aligned} \tag{5.33}$$

and

$$\begin{aligned}\Pi_{33}(0)|_{\rm div} = {}& \frac{1}{4}\left(-\sum_{n=1}^{N_0}\sum_{m=1}^{N_0}\kappa_Z^{\phi_n^0\phi_m^0}\kappa_Z^{\phi_n^0\phi_m^0} + \sum_{n=1}^{N_0}\kappa_{ZZ}^{\phi_n^0\phi_n^0}\right.\\ &\left.- 2\frac{v^2}{v_Z^2}\sum_{n=1}^{N_0}\kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_n^0} - 2\frac{v_Z^4}{v^4}\right)\frac{\Lambda^2}{(4\pi)^2}\\ &+ \left\{\frac{1}{4}\sum_{n=1}^{N_0}\left(\sum_{m=1}^{N_0}\kappa_Z^{\phi_n^0\phi_m^0}\kappa_Z^{\phi_n^0\phi_m^0} - \kappa_{ZZ}^{\phi_n^0\phi_n^0} + \frac{v^2}{v_Z^2}\kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_n^0}\right)M_{\phi_n^0}^2\right.\\ &\left.- \frac{3}{16}g_Z^2v^2\sum_{n=1}^{N_0}\kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_n^0} - \frac{3}{8}g^2\frac{v_Z^4}{v^2}\right\}\frac{1}{(4\pi)^2}\ln\frac{\Lambda^2}{\mu^2}.\end{aligned} \tag{5.34}$$

We are now ready to derive conditions to guarantee the finiteness of Eq. (5.24). We obtain a condition,

$$\begin{aligned}0 = {}& \frac{v_Z^4}{v^2} - v_Z^2 + \frac{v_Z^2}{4}\sum_{n=1}^{N_0}(\kappa_{WW}^{\phi_n^0\phi_n^0} - 2\kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0})\\ &- \frac{v^2}{4}\sum_{n=1}^{N_0}\left(\kappa_{ZZ}^{\phi_n^0\phi_n^0} - 2\frac{v^2}{v_Z^2}\kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_n^0} - \sum_{m=1}^{N_0}\kappa_Z^{\phi_n^0\phi_m^0}\kappa_Z^{\phi_n^0\phi_m^0}\right),\end{aligned} \tag{5.35}$$

which guarantees the cancellation of the Λ^2 divergence, and

$$\begin{aligned}0 = {}& -\frac{3}{16}g_Z^2\frac{v_Z^6}{v^2} + \frac{3}{16}g^2v_Z^2(5v_Z^2 - 4v^2)\\ &- \frac{3}{16}v^2\sum_{n=1}^{N_0}(g^2v_Z^2\kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0} - g_Z^2v^2\kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_n^0})\\ &+ \left[-\frac{v_Z^2}{4}\sum_{n=1}^{N_0}(\kappa_{WW}^{\phi_n^0\phi_n^0} - \kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0})M_{\phi_n^0}^2\right.\\ &\left.+ \frac{v^2}{4}\sum_{n=1}^{N_0}\left(\kappa_{ZZ}^{\phi_n^0\phi_n^0} - \frac{v^2}{v_Z^2}\kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_n^0} - \sum_{m=1}^{N_0}\kappa_Z^{\phi_n^0\phi_m^0}\kappa_Z^{\phi_n^0\phi_m^0}\right)M_{\phi_n^0}^2\right],\end{aligned} \tag{5.36}$$

for the cancellation of the $\ln\Lambda^2$ divergence. If we impose conditions that terms proportional to g_Z^2, g^2 and $M_{\phi_n^0}^2$ should vanish separately in Eq. (5.36), we obtain

$$0 = -\frac{v_Z^6}{v^6} + \sum_{n=1}^{N_0}\kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_n^0}, \tag{5.37}$$

$$0 = 5\frac{v_Z^2}{v^2} - 4 - \sum_{n=1}^{N_0}\kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0}, \tag{5.38}$$

$$\begin{aligned}0 = {}& -(\kappa_{WW}^{\phi_n^0\phi_n^0} - \kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0}) + \frac{v^2}{v_Z^2}\left(\kappa_{ZZ}^{\phi_n^0\phi_n^0} - \frac{v^2}{v_Z^2}\kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_n^0}\right)\\ &- \frac{v^2}{v_Z^2}\sum_{m=1}^{N_0}\kappa_Z^{\phi_n^0\phi_m^0}\kappa_Z^{\phi_n^0\phi_m^0}.\end{aligned} \tag{5.39}$$

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[10] $\kappa_Z^{\phi_1^0\phi_1^0} = 0$ is automatic because of the antisymmetry $n_1 \leftrightarrow n_2$ in $\kappa_Z^{\phi_{n_1}^0\phi_{n_2}^0} = 0$.

B. $\Pi'_{33}(0) - \Pi'_{3Q}(0)$

We next turn to the finiteness of Eq. (5.19). In a similar manner to the previous subsection, we decompose

$$\Pi_{3Q}(p^2) = \tilde{\Pi}_{3Q}(p^2) + \Pi_{3Q}^{\text{Higgs}}(p^2;\kappa). \tag{5.40}$$

It is evident

$$\Pi_{AA}^{\text{Higgs}} = \Pi_{ZA}^{\text{Higgs}} = 0, \tag{5.41}$$

since the neutral Higgs bosons do not couple with the photon. Using Eqs. (5.6), (5.7) and (5.8), we therefore obtain

$$\Pi_{33}^{\text{Higgs}} = \frac{1}{g_Z^2}\Pi_{ZZ}^{\text{Higgs}}, \qquad \Pi_{3Q}^{\text{Higgs}} = \Pi_{QQ}^{\text{Higgs}} = 0. \tag{5.42}$$

Analysis similar to Eq. (5.30) then gives the divergent part of $\Pi_{33}'^{\text{Higgs}}(0;\kappa)$ as

$$\begin{aligned}
&\Pi_{33}'^{\text{Higgs}}(0;\kappa)|_{\text{div}} \\
&= -\left[\frac{1}{24}\sum_{n=1}^{N_0}\sum_{m=1}^{N_0}\kappa_Z^{\phi_n^0\phi_m^0}\kappa_Z^{\phi_n^0\phi_m^0} + \frac{1}{12}\frac{v^2}{v_Z^2}\sum_{n=1}^{N_0}\kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_n^0}\right] \\
&\quad \times \frac{1}{(4\pi)^2}\ln\frac{\Lambda^2}{\mu^2}.
\end{aligned} \tag{5.43}$$

Note that the vacuum polarization function $\Pi_{3Q}'^{\text{Higgs}}$ is also trivial

$$\Pi_{3Q}'^{\text{Higgs}}(0;\kappa)|_{\text{div}} = 0. \tag{5.44}$$

The κ independent contributions to the divergent coefficients to Π'_{33} and Π'_{3Q} have been evaluated in the Appendix of Ref. [106]. They are

$$\begin{aligned}
\tilde{\Pi}'_{33}(0)|_{\text{div}} = &\left[\left(\frac{22}{3} - \frac{1}{12}\frac{v_Z^2}{v^2}\right) - \frac{1}{12}\left(1 - \frac{v_Z^2}{v^2}\right)\left(4 - \frac{v_Z^2}{v^2}\right)\right] \\
&\times \frac{1}{(4\pi)^2}\ln\frac{\Lambda^2}{\mu^2},
\end{aligned} \tag{5.45}$$

and

$$\tilde{\Pi}'_{3Q}(0)|_{\text{div}} = \left[\left(\frac{22}{3} - \frac{1}{12}\frac{v_Z^2}{v^2}\right) - \frac{1}{3} + \frac{1}{4}\frac{v_Z^2}{v^2}\right]\frac{1}{(4\pi)^2}\ln\frac{\Lambda^2}{\mu^2}. \tag{5.46}$$

It is now straightforward to obtain a condition guaranteeing the cancellation of the $\ln\Lambda^2$ divergence in Eq. (5.19),

$$\begin{aligned}
0 = &\frac{1}{12}\frac{v_Z^2}{v^2}\left(2 - \frac{v_Z^2}{v^2}\right) - \frac{1}{24}\sum_{n=1}^{N_0}\sum_{m=1}^{N_0}\kappa_Z^{\phi_n^0\phi_m^0}\kappa_Z^{\phi_n^0\phi_m^0} \\
&- \frac{1}{12}\frac{v^2}{v_Z^2}\sum_{n=1}^{N_0}\kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_n^0}.
\end{aligned} \tag{5.47}$$

C. $\Pi'_{11}(0) - \Pi'_{3Q}(0)$

The finiteness condition of $\Pi'_{11}(0) - \Pi'_{3Q}(0)$ can be studied in a similar manner. We find

$$\Pi_{11}'^{\text{Higgs}}(0;\kappa)|_{\text{div}} = -\frac{1}{12}\sum_{n=1}^{N_0}\kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0}\frac{1}{(4\pi)^2}\ln\frac{\Lambda^2}{\mu^2}, \tag{5.48}$$

and

$$\tilde{\Pi}'_{11}(0)|_{\text{div}} = \left(\frac{22}{3} - \frac{1}{12}\frac{v_Z^2}{v^2}\right)\frac{1}{(4\pi)^2}\ln\frac{\Lambda^2}{\mu^2}. \tag{5.49}$$

Using Eqs. (5.44), (5.46), (5.48) and (5.49), we find a condition guaranteeing the finiteness of $\Pi'_{11}(0) - \Pi'_{3Q}(0)$:

$$0 = \frac{1}{3} - \frac{1}{4}\frac{v_Z^2}{v^2} - \frac{1}{12}\sum_{n=1}^{N_0}\kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0}. \tag{5.50}$$

D. Unitarity sum rules vs finiteness of $f\bar{f} \to f'\bar{f}'$

It is easy to show that the conditions of the finiteness of the $f\bar{f} \to f'\bar{f}'$ amplitudes, i.e., Eqs. (5.35), (5.37), (5.38), (5.39), (5.47), and (5.50), are automatically satisfied if the Higgs coupling parameters satisfy the unitarity sum rules [Eqs. (4.1), (4.3), (4.4), (4.5), (4.7) and (4.8)] in the present framework.

Even though we do not require the renormalizability of the model in its construction, any unitary EWSB model with neutral Higgs extension only thus leads to finite $f\bar{f} \to f'\bar{f}'$ amplitude at the one loop level. This fact enables us to perform the EWPTs for any unitary model using $f\bar{f} \to f'\bar{f}'$ amplitudes at the one loop level.

Let us next consider the converse of the problem: Does a model satisfying the finiteness constraints [Eqs. (5.35), (5.37), (5.38), (5.39), (5.47), and (5.50)], automatically satisfy the unitarity sum rules? Evidently, the answer is negative. There is a large class of models which satisfy the finiteness constraints [Eqs. (5.35), (5.37), (5.38), (5.39), (5.47), and (5.50)], but do not satisfy the unitarity sum rules [Eqs. (4.1), (4.3), (4.4), (4.5), (4.7) and (4.8)]. To give an example, the $\kappa_{WW}^{\phi_{n_1}^0\phi_{n_2}^0}$ coupling cannot be constrained by the finiteness conditions [Eqs. (5.35), (5.37), (5.38), (5.39), (5.47), and (5.50)] for $n_1 \neq n_2$. On the other hand, the $\kappa_{WW}^{\phi_{n_1}^0\phi_{n_2}^0}$ coupling not satisfying Eq. (4.4) violates the perturbative unitarity in the $WW \to \phi_{n_1}^0\phi_{n_2}^0$ amplitude. Although the great success of the EWPTs, which use the

$f\bar{f} \to f'\bar{f}'$ processes, suggests the validity of the finiteness conditions, Eqs. (5.35), (5.37), (5.38), (5.39), (5.47), and (5.50), with very high accuracy, it does not imply the perturbative unitarity in the $WW \to \phi^0_{n_1}\phi^0_{n_2}$ process.

It should also be noted that the finiteness conditions are only sensitive to the absolute values of the Higgs-V-V couplings ($\kappa^{\phi^0_n}_{ZZ}$ and $\kappa^{\phi^0_n}_{WW}$) and insensitive to their relative sign $\kappa^{\phi^0_n}_{ZZ}\kappa^{\phi^0_n}_{WW}$. If we adequately choose the other parameters, the finiteness conditions can be satisfied even with a wrong signed $\kappa^{\phi^0_n}_{ZZ}\kappa^{\phi^0_n}_{WW} < 0$. On the other hand, the wrong signed $\kappa^{\phi^0_n}_{ZZ}\kappa^{\phi^0_n}_{WW} < 0$ clearly contradicts with the unitarity sum rule in the $WW \to ZZ$ process [Eq. (4.3)] as we stressed in Sec. IV D.

The numerical comparison between the unitarity sum rules and the finiteness conditions will be performed in Secs. VII and IX in this manuscript.

VI. OBLIQUE CORRECTION PARAMETERS

In order to compare our models with the electroweak precision measurements of the $f\bar{f} \to f'\bar{f}'$ processes, it is most convenient to introduce the electroweak precision parameters such as the oblique correction parameters of Ref. [88] (S, T and U). Hereafter we assume

$$v_Z = v, \tag{6.1}$$

and the bare parameters v and v_Z cannot be adjusted independently to renormalize the UV divergences of $\Pi_{33}(0)$ and $\Pi_{11}(0)$. The electroweak oblique correction parameters are defined by

$$\frac{1}{16\pi}S = (\Pi'_{33}(0) - \Pi'_{3Q}(0)) - (\Pi'_{33}(0) - \Pi'_{3Q}(0))|_{\mathrm{SM}}, \tag{6.2}$$

$$\alpha T = \frac{4}{v^2}(\Pi_{11}(0) - \Pi_{33}(0)) - \frac{4}{v^2}(\Pi_{11}(0) - \Pi_{33}(0))|_{\mathrm{SM}}, \tag{6.3}$$

$$\frac{1}{16\pi}U = (\Pi'_{11}(0) - \Pi'_{33}(0)) - (\Pi'_{11}(0) - \Pi'_{33}(0))|_{\mathrm{SM}}, \tag{6.4}$$

where $\Pi|_{\mathrm{SM}}$ denotes the vacuum polarization function in the SM.

As we did in the previous section, we decompose

$$\Pi(p^2) = \tilde{\Pi}(p^2) + \Pi^{\mathrm{Higgs}}(p^2; M_{\phi^0}, \kappa). \tag{6.5}$$

Under the assumption of Eq. (6.1), $\tilde{\Pi}$ in our generalized model is identical to that of the SM. Also, since the neutral Higgs bosons have no coupling with the photon, we can easily show

$$\Pi^{\mathrm{Higgs}}_{3Q} = \Pi^{\mathrm{Higgs}}_{QQ} = 0. \tag{6.6}$$

Equation (6.2) can therefore be rewritten as

$$\frac{1}{16\pi}S = \Pi'^{\mathrm{Higgs}}_{33}(0; M_{\phi^0}, \kappa) - \Pi'^{\mathrm{Higgs}}_{33}(0; M_h, \kappa_{\mathrm{SM}}), \tag{6.7}$$

with κ_{SM} denoting the SM values of the Higgs coupling strengths. In a similar manner, we obtain

$$\begin{aligned}\alpha T = {}& \frac{4}{v^2}(\Pi^{\mathrm{Higgs}}_{11}(0; M_{\phi^0}, \kappa) - \Pi^{\mathrm{Higgs}}_{11}(0; M_h, \kappa_{\mathrm{SM}})) \\ & - \frac{4}{v^2}(\Pi^{\mathrm{Higgs}}_{33}(0; M_{\phi^0}, \kappa) - \Pi^{\mathrm{Higgs}}_{33}(0; M_h, \kappa_{\mathrm{SM}})),\end{aligned} \tag{6.8}$$

and

$$\begin{aligned}\frac{1}{16\pi}U = {}& (\Pi'^{\mathrm{Higgs}}_{11}(0; M_{\phi^0}, \kappa) - \Pi'^{\mathrm{Higgs}}_{11}(0; M_h, \kappa_{\mathrm{SM}})) \\ & - (\Pi'^{\mathrm{Higgs}}_{33}(0; M_{\phi^0}, \kappa) - \Pi'^{\mathrm{Higgs}}_{33}(0; M_h, \kappa_{\mathrm{SM}})).\end{aligned} \tag{6.9}$$

We are now ready to write down the one loop formulas for the oblique correction parameters,

$$S = S_{\mathrm{log}} + S_f, \tag{6.10}$$

$$T = T_{\mathrm{quad}} + T_{\mathrm{log}} + T_f, \tag{6.11}$$

$$U = U_{\mathrm{log}} + U_f. \tag{6.12}$$

Here T_{quad} denotes the Λ^2 divergent term. S_{log}, T_{log} and U_{log} are the $\ln \Lambda^2$ terms. S_f, T_f and U_f are the finite terms.

We find

$$S_{\mathrm{log}} = \frac{1}{12\pi}\left[1 - \sum_{n=1}^{N_0}\kappa^{\phi^0_n}_{ZZ}\kappa^{\phi^0_n}_{ZZ} - \frac{1}{2}\sum_{n=1}^{N_0}\sum_{m=1}^{N_0}\kappa_Z^{\phi^0_n\phi^0_m}\kappa_Z^{\phi^0_n\phi^0_m}\right]\ln\frac{\Lambda^2}{\mu^2}, \tag{6.13}$$

$$S_f = \frac{1}{4\pi}\sum_{n=1}^{N_0}\kappa^{\phi^0_n}_{ZZ}\kappa^{\phi^0_n}_{ZZ}G^{Z\phi^0_n\prime} - \frac{1}{4\pi}G^{Zh\prime} + \frac{1}{8\pi}\sum_{n=1}^{N_0}\sum_{m=1}^{N_0}\kappa_Z^{\phi^0_n\phi^0_m}\kappa_Z^{\phi^0_n\phi^0_m}F^{\phi^0_n\phi^0_m\prime}, \tag{6.14}$$

$$\alpha T_{\mathrm{quad}} = \sum_{n=1}^{N_0}\left[\kappa_{WW}^{\phi_n^0\phi_n^0} - 2\kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0} - \kappa_{ZZ}^{\phi_n^0\phi_n^0} + 2\kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_n^0} + \sum_{m=1}^{N_0}\kappa_Z^{\phi_n^0\phi_m^0}\kappa_Z^{\phi_n^0\phi_m^0}\right]\frac{\Lambda^2}{(4\pi)^2 v^2}, \tag{6.15}$$

$$\begin{aligned}\alpha T_{\log} = \Bigg\{\sum_{n=1}^{N_0}\Bigg[&\left(-\kappa_{WW}^{\phi_n^0\phi_n^0} + \kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0} + \kappa_{ZZ}^{\phi_n^0\phi_n^0} - \kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_n^0} - \sum_{m=1}^{N_0}\kappa_Z^{\phi_n^0\phi_m^0}\kappa_Z^{\phi_n^0\phi_m^0}\right)\frac{M_{\phi_n^0}^2}{v^2} \\ &- \frac{3}{4}\left(g^2\kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0} - g_Z^2\kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_n^0}\right)\Bigg] - \frac{3}{4}g_Y^2\Bigg\}\frac{1}{(4\pi)^2}\ln\frac{\Lambda^2}{\mu^2},\end{aligned} \tag{6.16}$$

$$\begin{aligned}\alpha T_f = &\frac{1}{(4\pi)^2 v^2}\sum_{n=1}^{N_0}\sum_{m=1}^{N_0}\kappa_Z^{\phi_n^0\phi_m^0}\kappa_Z^{\phi_n^0\phi_m^0}\left(-\frac{1}{2}F^{\phi_n^0\phi_m^0} + M_{\phi_n^0}^2\left(\ln\frac{M_{\phi_n^0}^2}{\mu^2} - 1\right)\right) \\ &+ \frac{1}{(4\pi)^2 v^2}\sum_{n=1}^{N_0}\left(\kappa_{WW}^{\phi_n^0\phi_n^0} - \kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0} - \kappa_{ZZ}^{\phi_n^0\phi_n^0} + \kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_n^0}\right)M_{\phi_n^0}^2\left(\ln\frac{M_{\phi_n^0}^2}{\mu^2} - 1\right) \\ &+ \frac{1}{2(4\pi)^2 v^2}\sum_{n=1}^{N_0}\left(\kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0} - \kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_n^0}\right)M_{\phi_n^0}^2 \\ &+ \frac{1}{(4\pi)^2 v^2}\sum_{n=1}^{N_0}\kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0}\left[G^{W\phi_n^0} - \frac{1}{2}M_{\phi_n^0}^2 - M_W^2\left(\ln\frac{M_W^2}{\mu^2} - 1\right)\right] \\ &- \frac{1}{(4\pi)^2 v^2}\sum_{n=1}^{N_0}\kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_n^0}\left[G^{Z\phi_n^0} - \frac{1}{2}M_{\phi_n^0}^2 - M_Z^2\left(\ln\frac{M_Z^2}{\mu^2} - 1\right)\right] \\ &+ \frac{1}{(4\pi)^2 v^2}\left[-G^{Wh} + M_W^2\left(\ln\frac{M_W^2}{\mu^2} - 1\right) + G^{Zh} - M_Z^2\left(\ln\frac{M_Z^2}{\mu^2} - 1\right)\right],\end{aligned} \tag{6.17}$$

and

$$U_{\log} = \frac{1}{12\pi}\left[\sum_{n=1}^{N_0}\left(-\kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0} + \kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_n^0}\right) + \frac{1}{2}\sum_{n=1}^{N_0}\sum_{m=1}^{N_0}\kappa_Z^{\phi_n^0\phi_m^0}\kappa_Z^{\phi_n^0\phi_m^0}\right]\ln\frac{\Lambda^2}{\mu^2}, \tag{6.18}$$

$$U_f = \frac{1}{4\pi}\sum_{n=1}^{N_0}\kappa_{WW}^{\phi_n^0}\kappa_{WW}^{\phi_n^0}G^{W\phi_n^0\prime} - \frac{1}{4\pi}G^{Wh\prime} - \frac{1}{4\pi}\sum_{n=1}^{N_0}\kappa_{ZZ}^{\phi_n^0}\kappa_{ZZ}^{\phi_n^0}G^{Z\phi_n^0\prime} + \frac{1}{4\pi}G^{Zh\prime} - \frac{1}{8\pi}\sum_{n=1}^{N_0}\sum_{m=1}^{N_0}\kappa_Z^{\phi_n^0\phi_m^0}\kappa_Z^{\phi_n^0\phi_m^0}F^{\phi_n^0\phi_m^0\prime}. \tag{6.19}$$

It is obvious $T_{\mathrm{quad}} = S_{\log} = T_{\log} = U_{\log} = 0$ in models satisfying the conditions Eqs. (5.35), (5.36), (5.47) and (5.50).

VII. CONSTRAINTS ON A HEAVY HIGGS BOSON

If the masses of the extra Higgs bosons become extremely heavy keeping their nonvanishing κs, the longitudinal electroweak gauge boson scattering amplitude is enhanced and the perturbative unitarity can be violated even in the models which satisfy the unitarity sum rules. In a similar manner, the heavy extra Higgs boson mass induces large finite correction to the electroweak precision parameters (S and T) even in the model which satisfies the finiteness conditions. The mass of the extra Higgs boson can therefore be constrained by the perturbative unitarity and the EWPTs.

In this section, we assume models in which the unitarity sum rules [Eqs. (4.1), (4.3), (4.4), (4.5), (4.7) and (4.8)] are satisfied. We also identify the 125 GeV Higgs boson (h) discovered by the LHC experiments as the lightest Higgs boson in our framework (ϕ_1^0), i.e., $M_{\phi_1^0} = M_h = 125$ GeV. The second lightest Higgs boson ϕ_2^0 is denoted by H. In the following subsections, constraints on the second lightest Higgs boson mass $M_H = M_{\phi_2^0}$ are investigated by using the perturbative unitarity argument and the results of EWPTs.

A. Unitarity constraints

Thanks to the equivalence theorem between the high energy longitudinal gauge boson scattering amplitudes and the NGB scattering amplitudes, S-wave amplitude of the $W_L W_L \to W_L W_L$ processes is evaluated as an integral over the scattering angle θ of the corresponding NGB amplitude,

$$t_0^{W_L^+W_L^-\to W_L^+W_L^-} = \frac{1}{32\pi}\int_{-1}^{1} d\cos\theta\,\mathcal{A}_{w^+w^-\to w^+w^-}, \tag{7.1}$$

where the validity of the equivalence is of $\mathcal{O}(M_V^2/s)$. The scattering angle θ is related with the Mandelstam variable s, t as

$$t = -\frac{s}{2}(1-\cos\theta). \tag{7.2}$$

Similarly, the S-wave $Z_LZ_L \to W_LW_L$ and $Z_LZ_L \to Z_LZ_L$ amplitudes are

$$t_0^{Z_LZ_L\to W_L^+W_L^-} = \frac{1}{32\pi}\frac{1}{\sqrt{2}}\int_{-1}^{1} d\cos\theta\,\mathcal{A}_{w^+w^-\to zz}, \tag{7.3}$$

$$t_0^{Z_LZ_L\to Z_LZ_L} = \frac{1}{32\pi}\frac{1}{2}\int_{-1}^{1} d\cos\theta\,\mathcal{A}_{zz\to zz}. \tag{7.4}$$

Factors $1/\sqrt{2}$ in Eq. (7.3) and $1/2$ in Eq. (7.4) arise from the Bose statistics of identical particles in the initial and final states.

We assume the unitarity sum rules [Eqs. (4.1), (4.3), (4.4), (4.5), (4.7) and (4.8)]. The Higgs coupling constants therefore satisfy

$$\kappa_{WW}^{\phi_n^0} = \kappa_{ZZ}^{\phi_n^0}, \qquad \kappa_Z^{\phi_{n_1}^0\phi_{n_2}^0} = 0, \tag{7.5}$$

$$\kappa_{WW}^{\phi_{n_1}^0\phi_{n_2}^0} = \kappa_{WW}^{\phi_{n_1}^0}\kappa_{WW}^{\phi_{n_2}^0}, \qquad \kappa_{ZZ}^{\phi_{n_1}^0\phi_{n_2}^0} = \kappa_{ZZ}^{\phi_{n_1}^0}\kappa_{ZZ}^{\phi_{n_2}^0}, \tag{7.6}$$

and the tree-level ρ parameter restricted to be unity. Plugging these relations into the NGB scattering amplitudes [Eqs. (A2), (A4), and (A6)] and computing the integrals of Eqs. (7.1), (7.3), and (7.4), for sufficiently high energy scale $s \gg M_{\phi_n^0}^2$, we obtain

$$t_0^{W_L^+W_L^-\to W_L^+W_L^-} = \mathcal{T}, \tag{7.7}$$

$$t_0^{Z_LZ_L\to W_L^+W_L^-} = \frac{1}{2\sqrt{2}}\mathcal{T}, \tag{7.8}$$

$$t_0^{Z_LZ_L\to Z_LZ_L} = \frac{3}{4}\mathcal{T}, \tag{7.9}$$

with

$$\mathcal{T} \equiv -\frac{G_F}{4\sqrt{2}\pi}\sum_{n=1}^{N_0}(\kappa_V^{\phi_n^0})^2 M_{\phi_n^0}^2, \qquad G_F = \frac{1}{\sqrt{2}v^2}. \tag{7.10}$$

Here the Higgs-V-V coupling is denoted by $\kappa_V^{\phi_n^0}$,

$$\kappa_V^{\phi_n^0} \equiv \kappa_{WW}^{\phi_n^0} = \kappa_{ZZ}^{\phi_n^0}. \tag{7.11}$$

Using the unitarity sum rule

$$\sum_{n=1}^{N_0}(\kappa_V^{\phi_n^0})^2 = 1, \tag{7.12}$$

and our ordering of neutral Higgs bosons

$$M_h = M_{\phi_1^0} < M_H = M_{\phi_2^0} \le M_{\phi_3^0} \le \cdots, \tag{7.13}$$

we see

$$|\mathcal{T}| \ge \frac{G_F}{4\sqrt{2}\pi}[\kappa_V^2 M_h^2 + (1-\kappa_V^2)M_H^2], \tag{7.14}$$

with κ_V being defined as

$$\kappa_V \equiv \kappa_V^h = \kappa_V^{\phi_1^0} = \kappa_{WW}^{\phi_1^0} = \kappa_{ZZ}^{\phi_1^0}. \tag{7.15}$$

We next deduce the bound on M_H from the perturbative unitarity in the S-wave transition matrix among $W_L^+W_L^-$ and Z_LZ_L states,

$$T = \begin{pmatrix} t_0^{W_L^+W_L^-\to W_L^+W_L^-} & t_0^{W_L^+W_L^-\to Z_LZ_L} \\ t_0^{Z_LZ_L\to W_L^+W_L^-} & t_0^{Z_LZ_L\to Z_LZ_L} \end{pmatrix} = \begin{pmatrix} \mathcal{T} & \frac{1}{2\sqrt{2}}\mathcal{T} \\ \frac{1}{2\sqrt{2}}\mathcal{T} & \frac{3}{4}\mathcal{T} \end{pmatrix}. \tag{7.16}$$

It is easy to calculate the maximum eigenvalue of the transition matrix T,

$$t_0^{\max} = \frac{5}{4}\mathcal{T}. \tag{7.17}$$

Perturbative unitarity requires $|t_0^{\max}|$ should satisfy

$$|t_0^{\max}| < \frac{1}{2}, \tag{7.18}$$

in the off-resonant energy region, which immediately leads to a mass constraint on the second lightest Higgs boson,

$$M_H^2(1-\kappa_V^2) + M_h^2\kappa_V^2 < \frac{16\pi}{5}v^2. \tag{7.19}$$

Once the deviation of the 125 GeV Higgs boson coupling κ_V from its SM value $\kappa_V = 1$ is experimentally confirmed in a future experiment, the inequality (7.19) provides a mass upper bound on the extra Higgs boson.

We here make a comment comparing Eq. (7.19) with the famous Lee-Quigg-Thacker bound [6] on the Higgs boson mass in the SM

$$M_h^2 < \frac{8\pi}{3}v^2. \tag{7.20}$$

The difference of a factor $5/6$ between the right-hand side of Eqs. (7.19) and (7.20) arises from our neglect of the hh,

hH and HH channels in the $\mathcal{T}$-matrix. The amplitudes including these channels depend on the triple-Higgs and quartic-Higgs coupling strengths, which we did not incorporate in our theory, however. We will discuss the issue in our forthcoming publications.

B. Electroweak precision tests

We next study the constraints on the heavier Higgs boson mass M_H given by the EWPTs. In a model with $v = v_Z$ and satisfying the unitarity sum rules, as we found in Sec. VI, the cancellation of UV divergences in the oblique correction parameters,

$$T_{\rm quad} = S_{\log} = T_{\log} = U_{\log} = 0, \tag{7.21}$$

takes place at the one loop level. Moreover, the expressions of finite corrections to the oblique parameters are greatly simplified thanks to the unitarity sum rules. We find

$$S = -\frac{1}{4\pi}(1-\kappa_V^2)G^{Zh\prime} + \frac{1}{4\pi}\sum_{n=2}^{N_0}(\kappa_V^{\phi_n^0})^2 G^{Z\phi_n^0\prime}, \tag{7.22}$$

$$\begin{aligned} T = {} & \frac{1-\kappa_V^2}{16\pi^2 v^2\alpha}[G^{Zh} - G^{Wh}] \\ & - \frac{1}{16\pi^2 v^2 \alpha}\sum_{n=2}^{N_0}(\kappa_V^{\phi_n^0})^2[G^{Z\phi_n^0} - G^{W\phi_n^0}], \end{aligned} \tag{7.23}$$

$$\begin{aligned} U = {} & \frac{1-\kappa_V^2}{4\pi}[G^{Zh\prime} - G^{Wh\prime}] \\ & - \frac{1}{4\pi}\sum_{n=2}^{N_0}(\kappa_V^{\phi_n^0})^2[G^{Z\phi_n^0\prime} - G^{W\phi_n^0\prime}]. \end{aligned} \tag{7.24}$$

Here we used the notations Eqs. (7.11) and (7.15). The loop functions $G^{V\phi}$ and $G^{V\phi\prime}$ are defined in Appendix C.

For sufficiently heavy ϕ_n^0 $(n \geq 2)$, Eqs. (7.22), (7.23) and (7.24) can be approximated by

$$S \simeq \frac{1}{12\pi}\sum_{n=2}^{N_0}(\kappa_V^{\phi_n^0})^2\left[\ln\frac{M_{\phi_n^0}^2}{M_h^2} + 0.86\right], \tag{7.25}$$

$$T \simeq -\frac{3}{16\pi^2 v^2\alpha}(M_Z^2 - M_W^2) \times \sum_{n=2}^{N_0}(\kappa_V^{\phi_n^0})^2\left[\ln\frac{M_{\phi_n^0}^2}{M_h^2} - 1.05\right], \tag{7.26}$$

$$U \simeq \frac{1-\kappa_V^2}{3\pi} \times (-0.028) + \frac{1}{3\pi}\sum_{n=2}^{N_0}(\kappa_V^{\phi_n^0})^2 \frac{M_Z^2 - M_W^2}{M_{\phi_n^0}^2}, \tag{7.27}$$

where we used $M_Z = 91.2$ GeV, $M_W = 80.4$ GeV in the estimates of the numerical coefficients. As we see from Eq. (7.27), the typical value of U parameter prediction is $|U| \lesssim 3 \times 10^{-3}$, which is well below the present value of the measured value of U parameter uncertainty 10^{-2}. We are thus allowed to perform a two-dimensional fit in the S-T plane neglecting the U parameter constraint.

Using the unitarity sum rule (7.12) and the ordering of the Higgs mass (7.13), S and T parameters given in Eqs. (7.25) and (7.26) can be shown to satisfy

$$S \geq S_H \simeq \frac{1-\kappa_V^2}{12\pi}\left[\ln\frac{M_H^2}{M_h^2} + 0.86\right] > 0, \tag{7.28}$$

$$T \leq T_H \simeq -\frac{3(1-\kappa_V^2)}{16\pi^2 v^2\alpha}(M_Z^2 - M_W^2)\left[\ln\frac{M_H^2}{M_h^2} - 1.05\right] < 0, \tag{7.29}$$

with H being the second lightest neutral Higgs boson in the model. Here S_H and T_H denote S and T parameters, respectively, in a model with two neutral Higgs bosons ($N_0 = 2$ model). The inequalities in Eqs. (7.28) and (7.29) guarantee that the limits on M_H deduced from the EWPTs can be regarded as conservative bounds.

Figure 3 shows contours of the likelihood function of S and T corresponding to 95% and 99% confidence level (C.L.) probability, derived from the present limit [107]

$$S = 0.06 \pm 0.09, \qquad T = 0.10 \pm 0.07, \tag{7.30}$$

with

$$\rho_{ST} = 0.91. \tag{7.31}$$

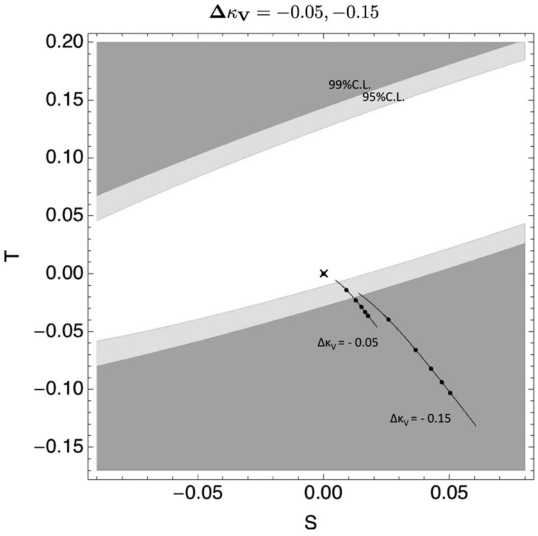

FIG. 3. The behaviors of (S_H, T_H). Contours of likelihood in the S-T plane, corresponding to 95% (gray) and 99% (dark gray) C.L., assuming $M_h = 125$ GeV and $m_{\rm top} = 173$ GeV, are also shown.

Two lines in Fig. 3 show behaviors of (S_H, T_H). The shorter line is for $\Delta\kappa_V = -0.05$, and the longer one is for $\Delta\kappa_V = -0.15$, varying the second lightest Higgs boson mass M_H from 250 GeV to 5 TeV. Five dots on each line starting from the origin of this figure toward the right-bottom direction correspond to the points $M_H = 500$ GeV, 1.0 TeV, 1.5 TeV, 2.0 TeV and 2.5 TeV, respectively. Note that these lines are not straight, since we do not use the large M_H approximations (7.28) and (7.29) in this figure. Also, we obtain $(S_H, T_H) = (0, 0)$ as we expect when we take $M_H = M_h$. If the 125 GeV Higgs boson coupling κ_V turns out to deviate sizably from the SM prediction $\kappa_V = 1$, then we will obtain an upper bound on the extra Higgs boson mass from the EWPTs. Actually, as we see from Fig. 3, $M_H = 283$ GeV (836 GeV) with $\Delta\kappa_V = -0.05$, and $M_H = 171$ GeV (265 GeV) with $\Delta\kappa_V = -0.15$ are ruled out in the present model at 95% C.L. (99% C.L.).

C. Unitarity vs EWPTs

We are now ready to compare the unitarity limit on M_H [Eq. (7.19)] and the EWPT limit shown in Fig. 3. These limits on M_H are depicted in Fig. 4 as functions of $\Delta\kappa_V$. We note, for $-0.008 \lesssim \Delta\kappa_V < 0$ ($-0.03 \lesssim \Delta\kappa_V < 0$), the unitarity limit gives a constraint stronger than that of EWPTs at 95% C.L. (99% C.L.). Note here that, for M_H heavier than the unitarity bound, the theory becomes highly nonperturbative. We cannot make reliable perturbative calculations of S and T parameters in this case.

On the other hand, if the deviation of the Higgs-V-V coupling from its SM value is relatively large, e.g., $\Delta\kappa_V \lesssim -0.03$, then Fig. 4 shows EWPTs give a limit, $M_H \lesssim 450$ GeV at 95% C.L. ($M_H \lesssim 2.4$ TeV at 99% C.L.), which is stronger than the unitarity limit. In this case, the theory remains perturbative and the bounds from EWPTs are considered to be trustable.

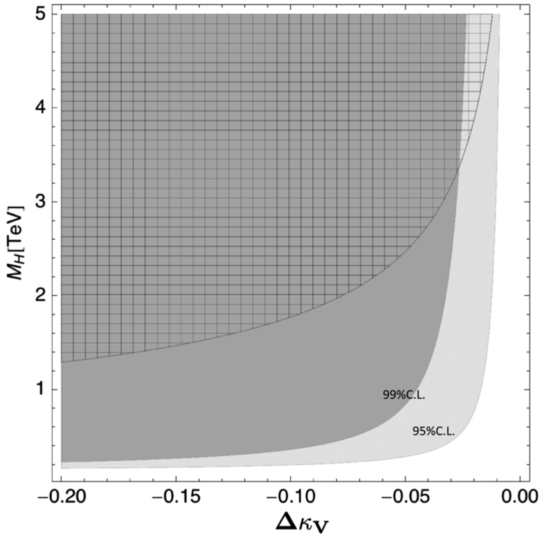

FIG. 4. Limits on the second lightest Higgs boson mass as a function of $\Delta\kappa_V \equiv \kappa_V - 1$. The hatched area is disfavored from the perturbative unitarity. The 95% and 99% C.L. excluded areas from EWPTs are shown by gray and dark gray, respectively.

It is also interesting to compare Fig. 4 with the present experimental value of κ_V measured for the 125 GeV Higgs boson. The ATLAS collaboration reported

$$\kappa_V = 1.15 \pm 0.08, \tag{7.32}$$

in Ref. [7], while the CMS collaboration [8] gave a bound

$$\kappa_V = 1.01 \pm 0.07. \tag{7.33}$$

Results of ATLAS and CMS are both consistent with the SM value $\Delta\kappa_V = 0$, though positive $\Delta\kappa_V = \kappa_V - 1$ is slightly favored by ATLAS, while the CMS experiment prefers the SM prediction.

If the positive $\Delta\kappa_V$ (as favored by the present ATLAS result) would be established by the upgraded LHC in the future, since our model is constrained to be $\Delta\kappa_V < 0$, then we could claim we need a framework of models to include new particles other than the neutral Higgs bosons. On the other hand, in the case of negative $\Delta\kappa_V$, if the observed discrepancy were of order $|\Delta\kappa_V| \simeq 0.02$ or below, it would be difficult to identify the origin of the difference. In this case, as shown in Fig. 4, even a very heavy extra Higgs boson ($M_H \gtrsim 1$ TeV) can explain the EWPT result if we allow 95% C.L. uncertainty. We are able to predict a new neutral Higgs particle below 1 TeV or less only in the case of negative $\Delta\kappa_V$ with $|\Delta\kappa_V| \gtrsim 0.02$.

D. Comparison with the CMS direct search

The LHC experiments continue to search for an extra heavy Higgs boson in various channels [108–115], after the discovery of the 125 GeV Higgs particle. Among them, Ref. [113] searched for the hypothetical heavy extra Higgs boson which arises in a singlet extension of the SM in the $H \to ZZ \to 2\ell 2\nu$ channel, and gave nontrivial constraints in its mass-coupling plane, especially in its high mass region. Note that the heavy Higgs coupling is related with the couplings of the 125 GeV Higgs boson through the unitarity argument,

$$(\kappa_V^h)^2 + (\kappa_V^H)^2 = 1. \tag{7.34}$$

The constraint of Ref. [113] can therefore be superimposed on our Fig. 4, as shown in Fig. 5. Here we assumed that, in addition to the bosonic amplitudes we discussed in this paper, $Zh \to t\bar{t}$ and $ZH \to t\bar{t}$ amplitudes are unitarized solely by two Higgs bosons (125 GeV Higgs boson h and an additional heavy Higgs boson H). This assumption makes it possible to relate the $Ht\bar{t}$ coupling, which affects the $gg \to H$ production cross section, with the value of $\Delta\kappa_V$. See the fermionic unitarity sum rules of Ref. [24]. It is

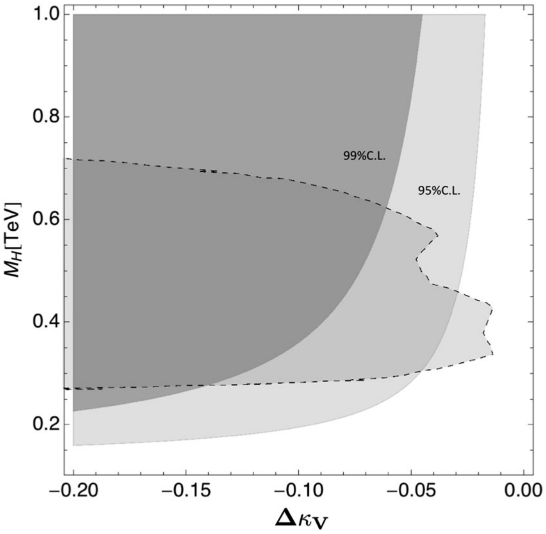

FIG. 5. Limits on the extra Higgs boson of the singlet extension of the SM, as function of $\Delta\kappa_V \equiv \kappa_V - 1$. The 95% and 99% C.L. excluded areas from EWPTs are shown by gray and dark gray, respectively. The region surrounded by the dashed contour is excluded by the CMS direct search [113] at the 95% C.L.

quite interesting that, assuming the extra Higgs boson mass $M_H \simeq 400$ GeV, Fig. 5 excludes $|\Delta\kappa_V| \gtrsim 0.016$ at the 95% C.L., which is stronger than the present signal strength constraints on the 125 GeV Higgs boson coupling $\Delta\kappa_V$ (7.32) and (7.33). On the other hand, for $M_H \lesssim 300$ GeV and $M_H \gtrsim 460$ GeV, the strongest constraint comes from EWPTs. EWPTs have good sensitivity for constraining the Higgs coupling deviations for a wider range of the extra Higgs boson mass.

VIII. A UV COMPLETION AND SELF-INTERACTIONS AMONG HIGGS BOSONS

Although the model we analyze in this paper is based on the nonlinear sigma model, once the unitarity sum rules [Eqs. (4.1), (4.3), (4.4), (4.5), (4.7) and (4.8)] are imposed among its Higgs coupling strengths κs, the longitudinal gauge boson scattering amplitudes can be perturbative enough to satisfy the unitarity constraints. Moreover, the electroweak oblique correction parameters S, T and U are shown to be finite at the one loop level thanks to these unitarity sum rules.

Can the model we analyze in this paper be regarded as a renormalizable model, which does not need further UV completion, then? The answer depends on the assumptions on the Higgs self-interactions. In this section, we take an example of $N_0 = 2$ to study what kind of constraints we need to impose among the self-interactions of the Higgs particles, so as to make the model completely renormalizable.

In the case of $N_0 = 2$, the unitarity sum rules severely constrain the Higgs-gauge boson interaction Lagrangian,

$$\mathcal{L}_{\rm int} = \frac{v}{2}\sum_{n=1,2}\kappa_V^{\phi_n^0}\phi_n^0\,{\rm tr}[(D_\mu U)^\dagger(D^\mu U)] + \frac{1}{4}\sum_{n=1,2}\sum_{m=1,2}\kappa_V^{\phi_n^0}\kappa_V^{\phi_m^0}\phi_n^0\phi_m^0\,{\rm tr}[(D_\mu U)^\dagger(D^\mu U)], \quad (8.1)$$

with

$$\sum_{n=1,2}(\kappa_V^{\phi_n^0})^2 = 1. \quad (8.2)$$

On the other hand, the Higgs self-interaction Lagrangian is left arbitrary from the unitarity arguments:

$$V = \frac{1}{2}\sum_{n=1,2}M_{\phi_n^0}^2\phi_n^0\phi_n^0 + \frac{1}{3!}\sum_{n_1,n_2,n_3}\lambda_{n_1n_2n_3}\phi_{n_1}^0\phi_{n_2}^0\phi_{n_3}^0 + \frac{1}{4!}\sum_{n_1,n_2,n_3,n_4}\lambda_{n_1n_2n_3n_4}\phi_{n_1}^0\phi_{n_2}^0\phi_{n_3}^0\phi_{n_4}^0, \quad (8.3)$$

in which we have 12 free parameters in total (one free parameter in κ_V; two free parameters in $M_{\phi_n^0}^2$; four in triple Higgs couplings $\lambda_{n_1n_2n_3}$; and five in quartic couplings $\lambda_{n_1n_2n_3n_4}$.)

In the absence of heavier particles other than these two neutral Higgs bosons, the model above should be described by the doublet-singlet mixing scenario,[11] which possesses an $SU(2)$ doublet Higgs field (ϕ) and a real singlet Higgs field (σ_2) with $Y = 0$. The Lagrangian of the doublet-singlet mixing scenario is given by

$$\mathcal{L} = (D_\mu\phi)^\dagger(D^\mu\phi) + \frac{1}{2}(\partial_\mu\sigma_2)^2 - V. \quad (8.4)$$

Requiring the renormalizability, the Higgs potential V should be given by

$$V = \frac{\lambda}{2}\left(\phi^\dagger\phi - \frac{1}{2}v^2\right)^2 + \frac{M_{\sigma_2}^2}{2}\sigma_2^2 + \frac{\lambda_{\sigma\sigma\sigma}}{3!}\sigma_2^3 + \frac{\lambda_{\sigma\sigma\sigma\sigma}}{4!}\sigma_2^4 + \lambda_{\phi^\dagger\phi\sigma}\left(\phi^\dagger\phi - \frac{1}{2}v^2\right)\sigma_2 + \frac{1}{2}\lambda_{\phi^\dagger\phi\sigma\sigma}\left(\phi^\dagger\phi - \frac{1}{2}v^2\right)\sigma_2^2. \quad (8.5)$$

Minimizing the Higgs potential V, the doublet Higgs field acquires its VEV

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[11] Reference [116] studied the oblique electroweak corrections in the doublet-singlet mixing scenario by using the effective theory framework. Unitarity constraints of this model are discussed in Ref. [117]. See also Ref. [118] for the studies of radiative corrections in the doublet-singlet mixing model with an extra $U(1)$.

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} v \end{pmatrix}. \tag{8.6}$$

Note that this model is described only by six free parameters. In order for Eq. (8.3) to be regarded as a renormalizable theory, the free parameters in Eq. (8.3) should satisfy $12 - 6 = 6$ constraints.

Hereafter we investigate such constraints. For such a purpose, we introduce the $SU(2)$ matrix field U,

$$\phi = \frac{1}{\sqrt{2}}(v + \sigma_1) U \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{8.7}$$

with v being the VEV of the doublet Higgs field. Using the chiral field U, the Lagrangian (8.4) can be rewritten as

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}(\partial_\mu \sigma_1)^2 + \frac{1}{2}(\partial_\mu \sigma_2)^2 + \frac{v}{2}\sigma_1 \mathrm{tr}[(D_\mu U)^\dagger (D^\mu U)] \\ &+ \frac{1}{4}\sigma_1 \sigma_1 \mathrm{tr}[(D_\mu U)^\dagger (D^\mu U)] - V,\end{aligned} \tag{8.8}$$

with

$$\begin{aligned}V = &\frac{1}{2}(\sigma_1, \sigma_2) M^2 \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} + \frac{\lambda}{2} v \sigma_1^3 + \frac{1}{2}\lambda_{\phi^\dagger\phi\sigma}\sigma_1^2\sigma_2 \\ &+ \frac{1}{2}\lambda_{\phi^\dagger\phi\sigma\sigma} v \sigma_1 \sigma_2^2 + \frac{1}{3!}\lambda_{\sigma\sigma\sigma}\sigma_2^3 + \frac{\lambda}{8}\sigma_1^4 + \frac{1}{4}\lambda_{\phi^\dagger\phi\sigma\sigma}\sigma_1^2\sigma_2^2 \\ &+ \frac{1}{4!}\lambda_{\sigma\sigma\sigma\sigma}\sigma_2^4.\end{aligned} \tag{8.9}$$

Here the 2×2 mass matrix M^2 is given by

$$M^2 \equiv \begin{pmatrix} \lambda v^2 & \lambda_{\phi^\dagger\phi\sigma} v \\ \lambda_{\phi^\dagger\phi\sigma} v & M_{\sigma_2}^2 \end{pmatrix}. \tag{8.10}$$

We diagonalize the mass matrix (8.10):

$$\mathcal{V}^\dagger M^2 \mathcal{V} = \begin{pmatrix} M_{\phi_1^0}^2 & 0 \\ 0 & M_{\phi_2^0}^2 \end{pmatrix}, \tag{8.11}$$

and identify

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \mathcal{V} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}, \tag{8.12}$$

with $\mathcal{V}$ being an orthogonal matrix to make the mass matrix diagonal. Comparing the Higgs couplings in Eq. (8.1) and those in Eq. (8.8), we see $\mathcal{V}$ should be expressed by κ_V,

$$\mathcal{V} = \begin{pmatrix} \kappa_V^{\phi_1^0} & \kappa_V^{\phi_2^0} \\ -\kappa_V^{\phi_2^0} & \kappa_V^{\phi_1^0} \end{pmatrix}. \tag{8.13}$$

We next rewrite

$$\begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix} = \mathcal{V}^\dagger \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}, \qquad \mathcal{V}^\dagger = \begin{pmatrix} \kappa_V^{\phi_1^0} & -\kappa_V^{\phi_2^0} \\ \kappa_V^{\phi_2^0} & \kappa_V^{\phi_1^0} \end{pmatrix}, \tag{8.14}$$

and put Eq. (8.14) into Eq. (8.3). We obtain

$$\begin{aligned}V = &\frac{1}{2}(\sigma_1, \sigma_2)\tilde{M}^2 \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} \\ &+ \frac{\tilde{\lambda}_{111}}{3!}\sigma_1^3 + \frac{\tilde{\lambda}_{112}}{2}\sigma_1^2\sigma_2 + \frac{\tilde{\lambda}_{122}}{2}\sigma_1\sigma_2^2 \\ &+ \frac{\tilde{\lambda}_{222}}{3!}\sigma_2^3 + \frac{\tilde{\lambda}_{1111}}{4!}\sigma_1^4 + \frac{\tilde{\lambda}_{1112}}{3!}\sigma_1^3\sigma_2 \\ &+ \frac{\tilde{\lambda}_{1122}}{2!2!}\sigma_1^2\sigma_2^2 + \frac{\tilde{\lambda}_{1222}}{3!}\sigma_1^1\sigma_2^3 + \frac{\tilde{\lambda}_{2222}}{4!}\sigma_2^4.\end{aligned} \tag{8.15}$$

Here $\tilde{M}^2$ and $\tilde{\lambda}$ are functions of M^2, λ and κ, and are defined in Appendix D. Comparing Eq. (8.15) with Eq. (8.9), we find six constraints:

$$\tilde{\lambda}_{111} = \frac{3}{v}(\tilde{M}^2)_{11}, \tag{8.16}$$

$$\tilde{\lambda}_{1111} = \frac{3}{v^2}(\tilde{M}^2)_{11}, \tag{8.17}$$

$$\tilde{\lambda}_{112} = \frac{1}{v}(\tilde{M}^2)_{12}, \tag{8.18}$$

$$\tilde{\lambda}_{1122} = \frac{1}{v}\tilde{\lambda}_{122}, \tag{8.19}$$

$$0 = \tilde{\lambda}_{1112}, \tag{8.20}$$

$$0 = \tilde{\lambda}_{1222}, \tag{8.21}$$

which should be satisfied to make the model UV-complete one.

IX. EFFECTIVE FIELD THEORY AND CONSTRAINTS ON ITS CUTOFF

Varieties of effective field theory approaches have been proposed to describe the properties of the observed 125 GeV Higgs particle. In the effective field theory approaches, deviations of the 125 GeV Higgs particle are parametrized by the coefficients of higher dimensional operators. These higher dimensional operators violate the perturbative unitarity of the high energy scattering amplitudes. They also conflict with the renormalizability of the model, and we need to introduce a UV cutoff in the loop level analysis of the effective field theory. Perturbative unitarity and the EWPTs are used to constrain the cutoff scale in the effective field theory approaches.

Our approach we adopt in this paper differs from the effective field theory approaches, since we introduce heavier Higgs bosons other than the observed 125 GeV Higgs particle. Moreover, the parameters of our Lagrangian are assumed to satisfy the unitarity sum rules, thus the scattering amplitudes are free from the perturbative unitarity violation even at high energies.

On the other hand, if we integrate out the heavier Higgs bosons from our Lagrangian [e.g., $N_0 = 2$ model, Eq. (8.1)], we obtain an effective field theory of the 125 GeV Higgs particle:

$$\mathcal{L}_{\text{int}} = \frac{v}{2}\kappa_V h \mathrm{tr}[(D_\mu U)^\dagger (D^\mu U)] + \frac{1}{4}\kappa_V^{hh} hh \mathrm{tr}[(D_\mu U)^\dagger (D^\mu U)], \tag{9.1}$$

with h being the 125 GeV Higgs particle $h = \phi_1^0$, and

$$\kappa_V = \kappa_V^{\phi_1^0}, \qquad \kappa_V^{hh} = \kappa_V^{\phi_1^0}\kappa_V^{\phi_1^0} - \frac{\kappa_V^{\phi_2^0}}{M_{\phi_2^0}^2} v\lambda_{112}. \tag{9.2}$$

Here λ_{112} is the Higgs self-interaction coefficient defined in Eq. (8.3).[12] Our approach should be understood as a systematic trial to construct a perturbative UV completion theory (unitary theory) of the light Higgs effective field theory.

In this section, we evaluate the present constraints on the cutoff scale in the effective field theory using the perturbative unitarity and the results of the EWPTs. We then compare the cutoff constraints in the effective field theory method with our findings on the heavy Higgs boson mass bounds in our approach.

A. Unitarity constraints

In the effective field theory [Eq. (9.1)], the deviation of the Higgs coupling κ_V from its SM value affects the longitudinal gauge boson scattering amplitudes to violate the perturbative unitarity constraint at high energy scale. This is one of the reasons why we need to introduce a UV cutoff scale in the effective field theory framework. We estimate the upper bound of the cutoff scale Λ from the S-wave amplitudes,

$$t_0^{W_L^+W_L^- \to W_L^+W_L^-} \simeq \frac{1}{2}\tilde{\mathcal{T}}, \tag{9.3}$$

$$t_0^{Z_LZ_L \to W_L^+W_L^-} \simeq \frac{1}{\sqrt{2}}\tilde{\mathcal{T}}, \tag{9.4}$$

$$t_0^{Z_LZ_L \to Z_LZ_L} \simeq 0, \tag{9.5}$$

for $s \gg M_h^2$. Here $\tilde{\mathcal{T}}$ is given by

$$\tilde{\mathcal{T}} \equiv \frac{G_F}{8\sqrt{2}\pi}(1-\kappa_V^2)s, \tag{9.6}$$

with s being the square of the energy of the scattering.

The S-wave transition matrix among $W_L^+W_L^-$ and Z_LZ_L states is

$$\begin{aligned} \mathrm{T} &= \begin{pmatrix} t_0^{W_L^+W_L^- \to W_L^+W_L^-} & t_0^{W_L^+W_L^- \to Z_LZ_L} \\ t_0^{Z_LZ_L \to W_L^+W_L^-} & t_0^{Z_LZ_L \to Z_LZ_L} \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2}\tilde{\mathcal{T}} & \frac{1}{\sqrt{2}}\tilde{\mathcal{T}} \\ \frac{1}{\sqrt{2}}\tilde{\mathcal{T}} & 0 \end{pmatrix}, \end{aligned} \tag{9.7}$$

and we obtain the maximum eigenvalue of the T matrix

$$t_0^{\max} = \tilde{\mathcal{T}}. \tag{9.8}$$

The perturbative unitarity requires $|t_0^{\max}| < 1/2$, and we thus find

$$(1-\kappa_V^2)\Lambda^2 < 8\pi v^2. \tag{9.9}$$

Here we identified the cutoff scale Λ as the scattering energy scale below which the amplitudes can be safely evaluated by using the effective theory framework.

Comparing Eq. (9.9) with Eq. (7.19), we find the upper bound of the heavy Higgs boson mass M_H as we discussed in Sec. VII A can be related with the upper bound of the effective field theory cutoff scale Λ :

$$M_H^{\text{upper}} = \sqrt{\frac{2}{5}}\Lambda^{\text{upper}}. \tag{9.10}$$

Noting

$$\sqrt{\frac{2}{5}} \simeq 0.63, \tag{9.11}$$

we see that, in our model, the upper bound on the extra Higgs mass M_H is a bit tighter than the estimation of the cutoff scale in the effective field theory framework.

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[12] Using the UV-completeness constraints [Eqs. (8.16)–(8.21)], and the unitarity sum rule $(\kappa_V^{\phi_1^0})^2 + (\kappa_V^{\phi_2^0})^2 = 1$, we are able to show $(\kappa_V - 1) = (\kappa_V^{hh} - 1)/4$ for sufficiently large $M_{\phi_2^0}^2$. The relation is consistent with the findings of Ref. [119] $\kappa_V = 1 - v^2/(2f^2)c_H$, $\kappa_V^{hh} = 1 - 2v^2/f^2 c_H$, derived in the context of the strongly interacting light Higgs effective theory.

B. Electroweak precision tests

We next turn to the electroweak precision constraint on the cutoff Λ scale in the effective field theory approach. Using the results of Sec. VI, it is straightforward to evaluate the oblique correction parameters from the effective field theory Lagrangian (9.1),

$$S = \frac{1}{4\pi}(1-\kappa_V^2)\left[\frac{1}{3}\ln\frac{\Lambda^2}{\mu^2} - G^{Zh'}\right] \tag{9.12}$$

$$T = -\frac{3(1-\kappa_V^2)}{16\pi^2 v^2 \alpha}(M_Z^2 - M_W^2)\left[\ln\frac{\Lambda^2}{\mu^2} - \frac{1}{3} - \frac{1}{3}\frac{G^{Zh} - G^{Wh}}{M_Z^2 - M_W^2} + \frac{1}{3(M_Z^2 - M_W^2)}\left[M_Z^2 \ln\frac{M_Z^2}{\mu^2} - M_W^2 \ln\frac{M_W^2}{\mu^2}\right]\right], \tag{9.13}$$

$$U = \frac{1}{4\pi}(1-\kappa_V^2)[G^{Zh'} - G^{Wh'}], \tag{9.14}$$

with Λ being the UV cutoff scale as we define in Appendix. C. The finite parts in the above formulas can be easily evaluated, and we obtain

$$S \simeq \frac{1}{12\pi}(1-\kappa_V^2)\left[\ln\frac{\Lambda^2}{M_h^2} + 1.69\right], \tag{9.15}$$

$$T \simeq -\frac{3(1-\kappa_V^2)}{16\pi^2\alpha}\frac{M_Z^2 - M_W^2}{v^2}\left[\ln\frac{\Lambda^2}{M_h^2} - 0.22\right], \tag{9.16}$$

$$U \simeq \frac{1-\kappa_V^2}{3\pi} \times (-0.028). \tag{9.17}$$

Again, we used $M_Z = 91.2$ GeV, $M_W = 80.4$ GeV in the estimates of the finite parts.

It should be emphasized, however, that the definition of the cutoff parameter Λ in the loop integrals is not unique. There is a non-negligible ambiguity in the size of finite corrections in Eqs. (9.15) and (9.16). Actually, Refs. [120] and [107] neglect these finite corrections and use a simpler form,

$$S \simeq \frac{1}{12\pi}(1-\kappa_V^2)\ln\frac{\Lambda^2}{M_h^2}, \tag{9.18}$$

$$T \simeq -\frac{3(1-\kappa_V^2)}{16\pi^2\alpha}\frac{M_Z^2 - M_W^2}{v^2}\ln\frac{\Lambda^2}{M_h^2}. \tag{9.19}$$

Note that the T-parameter constraint is more stringent than the S-parameter. Comparing Eq. (9.19) with Eq. (7.26), we find

$$M_H^{\text{upper}} \simeq 1.69 \times \Lambda^{\text{upper}}, \tag{9.20}$$

with $1.69 \simeq e^{1.05/2}$. We see that, in the electroweak precision constraints, the upper bound on M_H is a bit weaker than the corresponding bound on Λ of the effective field theory framework.

X. CONCLUSIONS AND OUTLOOK

In this paper we discussed how the unitarity of the longitudinal gauge boson scattering amplitudes is related with the finiteness of the electroweak oblique parameters S, T, and U. Starting from the general Lagrangian of the electroweak symmetry breaking sector with arbitrary number of neutral Higgs bosons, we (re)derived the unitarity sum rules among Higgs couplings, which should be satisfied to keep the longitudinal gauge boson scattering amplitudes unitary at high energy. The unitarity arguments allow us to show, without invoking the custodial symmetry explicitly, the tree-level ρ parameter to be unity in any unitary EWSB model if it only possesses neutral Higgs bosons. This finding explains the reason of the ρ parameter stability against the radiative corrections in the septet Higgs extension model which does not enjoy explicit custodial symmetry [30–32]. Thanks to the electroweak chiral Lagrangian framework we used, the electroweak gauge symmetry is kept manifest, which allows us to investigate the one loop radiative corrections to the electroweak oblique parameters explicitly at the one loop level. We showed the finiteness of the oblique parameters is automatically guaranteed in our framework, once we impose the unitarity sum rules among various Higgs couplings.

We also derived upper bounds on the second lightest Higgs boson mass M_H as functions of the deviation of the 125 GeV Higgs boson coupling $\Delta\kappa_V$. We found, for $\Delta\kappa_V \lesssim -0.008$ ($\Delta\kappa_V \lesssim -0.03$), the oblique parameter constraint at 95% C.L. (99% C.L.) gives a more stringent bound on M_H than the unitarity bound. The result of the LHC direct search of the second lightest Higgs boson can also be combined, and we found a constraint on $\Delta\kappa_V$ tighter than the present signal strength uncertainty of the 125 GeV Higgs boson measurements. The combined results with the LHC direct search give the strongest bound on κ_V for $M_H \simeq 400$ GeV, while for the wide range of M_H region EWPTs have the best sensitivity.

Finally, we compared our bounds on M_H with the bounds on the UV cutoff Λ of the effective field theory approach. Simple relationships were found between M_H and Λ bounds both in the unitarity and the oblique parameter arguments.

It should be emphasized, however, that our results heavily rely on the assumption we made: the EWSB is perturbatively realized only with additional neutral Higgs bosons. We need to relax our model to include, e.g., charged Higgs bosons so as to make our analysis applicable to a wider class of EWSB models, including the triplet Higgs extensions [100–103] and the septet Higgs extensions [30–32]. It will also be interesting to utilize the Yukawa coupling unitarity sum rules which can be derived from the amplitudes involving heavy fermions. We are now preparing a complete set of the unitarity sum rules and the oblique parameter formulas in the models including arbitrary number of charged Higgs bosons. The results will be published elsewhere.

Possibility of nonperturbative EWSB should also be investigated, since the present experimental results still allow such a possibility. For an example, as we discussed in Sec. IV D, the wrong sign $\kappa_{ZZ}^h\kappa_{WW}^h$ is consistent with the present measurements of the 125 GeV Higgs particle and the EWPTs. The present measurements are sensitive only to $|\kappa_{ZZ}^h|^2$ and $|\kappa_{WW}^h|^2$, not to its relative sign. The sign should be determined by measuring the $WW \to ZZ$ cross section at the future LHC experiments.

We finally emphasize that the 125 GeV Higgs coupling measurements, the precision oblique parameter measurements, and the direct search of the extra Higgs bosons give complementary limits on the model. Future precision measurements of these parameters at the ILC experiments will be able to pin down the direction of the new physics beyond the standard model.

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APPENDIX A: SCATTERING AMPLITUDES AT TREE LEVEL IN THE GAUGELESS LIMIT

In this Appendix, we evaluate the would-be NGB two-body scattering amplitudes in the gaugeless limit ($g = g_Y = 0$) in the model discussed in Sec. II. The equivalence theorem between the longitudinally polarized vector boson amplitudes and the NGB amplitudes then enables us to evaluate the longitudinally polarized vector boson amplitudes in the high energy limit.

We first consider the amplitude

$$w^+(p_1)w^-(p_2) \to w^+(p_3)w^-(p_4). \tag{A1}$$

Note that the NGBs are massless in the gaugeless limit. We find

$$\mathcal{A}_{w^+w^-\to w^+w^-} = -\frac{1}{v^2}\left(4-3\frac{v_Z^2}{v^2}\right)u - \frac{1}{v^2}\sum_{m=1}^{N_0}(\kappa_{WW}^{\phi_m^0})^2\frac{t^2}{t-M_{\phi_m^0}^2} - \frac{1}{v^2}\sum_{m=1}^{N_0}(\kappa_{WW}^{\phi_m^0})^2\frac{s^2}{s-M_{\phi_m^0}^2}, \tag{A2}$$

with s, t and u being the usual Mandelstam variables

$$s \equiv (p_1+p_2)^2 = (p_3+p_4)^2,$$
$$t \equiv (p_1-p_3)^2 = (p_2-p_4)^2,$$
$$u \equiv (p_1-p_4)^2 = (p_2-p_3)^2.$$

The factor $(4-3v_Z^2/v^2)$ in the first term of Eq. (A2) agrees with the low energy theorem of $SU(2)\times U(1)/U(1)$ NGB scattering. It arises from the corresponding factor in the contact four-NGB vertex given in Eq. (2.2). The second and third terms in Eq. (A2) come from the t- and s-channel exchanges of the neutral Higgs bosons, respectively. We next consider the amplitude of

$$w^+(p_1)w^-(p_2) \to z(p_3)z(p_4). \tag{A3}$$

It should be noted the existence of the wwz vertex in the second term of Eq. (2.2) produces t- and u-channel w-exchange (NGB exchange) diagrams when $v_Z^2 \neq v^2$. The NGB pole cancels with the numerator at the on-shell $p_1^2 = p_2^2 = p_3^2 = p_4^2 = 0$ in these NGB exchange amplitudes. Combined with the four-NGB contact interaction (2.2), these NGB exchange amplitudes reproduce the low energy theorem amplitude of $SU(2)\times U(1)/U(1)$ symmetry breaking. We now obtain

$$\mathcal{A}_{w^+w^-\to zz} = \frac{v_Z^2}{v^4}s - \frac{1}{v_Z^2}\sum_{m=1}^{N_0}(\kappa_{WW}^{\phi_m^0})(\kappa_{ZZ}^{\phi_m^0})\frac{s^2}{s-M_{\phi_m^0}^2}, \tag{A4}$$

where the first term is the low energy theorem amplitude, while the second term comes from the s-channel Higgs exchange diagram.

Due to the lack of the low energy theorem amplitude, the amplitude

$$z(p_1)z(p_2) \to z(p_3)z(p_4) \tag{A5}$$

behaves as $\mathcal{O}(E^4)$ at low energy. We find

$$\mathcal{A}_{zz\to zz} = -\frac{v^2}{v_Z^4}\sum_{m=1}^{N_0}(\kappa_{ZZ}^{\phi_m^0})(\kappa_{ZZ}^{\phi_m^0}) \times\left(\frac{s^2}{s-M_{\phi_m^0}^2}+\frac{t^2}{t-M_{\phi_m^0}^2}+\frac{u^2}{u-M_{\phi_m^0}^2}\right). \tag{A6}$$

We next consider the amplitude

$$w^-(p_1)w^+(p_2) \to \phi_{n_1}^0(p_3)\phi_{n_2}^0(p_4), \tag{A7}$$

which can be evaluated from the contact interaction terms [Eqs. (2.34)–(2.35)] and the t- and u-channel w exchange graphs arising from Eq. (2.33).

We also note that there exists an s-channel Higgs exchange contribution arising from triple-Higgs couplings.

The s-channel Higgs exchange graph, however, does not grow up in the high energy limit, and we neglect it in this Appendix. We obtain

$$\begin{aligned}\mathcal{A}_{w^-w^+\to\phi^0_{n_1}\phi^0_{n_2}} &= -\frac{i}{v^2}\kappa_Z^{\phi^0_{n_1}\phi^0_{n_2}}(t-u) - \frac{1}{v^2}\kappa_{WW}^{\phi^0_{n_1}\phi^0_{n_2}}s\\ &\quad - \frac{1}{v^2}\kappa_{WW}^{\phi^0_{n_1}}\kappa_{WW}^{\phi^0_{n_2}}\frac{[t-M^2_{\phi^0_{n_1}}][t-M^2_{\phi^0_{n_2}}]}{t}\\ &\quad - \frac{1}{v^2}\kappa_{WW}^{\phi^0_{n_1}}\kappa_{WW}^{\phi^0_{n_2}}\frac{[u-M^2_{\phi^0_{n_1}}][u-M^2_{\phi^0_{n_2}}]}{u}.\end{aligned} \tag{A8}$$

Note here that the P-wave final state is present when $\kappa_Z^{\phi^0_{n_1}\phi^0_{n_2}} \neq 0$. We also note the imaginary number in the amplitude is the result of CP violation arising from the simultaneous existence of $\kappa_Z^{\phi^0_{n_1}\phi^0_{n_2}} \neq 0$ and $\kappa_{WW}^{\phi^0_{n_1}}\kappa_{WW}^{\phi^0_{n_2}} \neq 0$.

The amplitude

$$z(p_1)z(p_2) \to \phi^0_{n_1}(p_3)\phi^0_{n_2}(p_4) \tag{A9}$$

can also be evaluated in a similar manner. We find

$$\begin{aligned}\mathcal{A}_{zz\to\phi^0_{n_1}\phi^0_{n_2}} &= -\frac{1}{v_Z^2}\kappa_{ZZ}^{\phi^0_{n_1}\phi^0_{n_2}}s\\ &\quad + \frac{1}{v_Z^2}\sum_m \kappa_Z^{\phi^0_{n_1}\phi^0_m}\kappa_Z^{\phi^0_m\phi^0_{n_2}}\frac{[t-M^2_{\phi^0_{n_1}}][t-M^2_{\phi^0_{n_2}}]}{t-M^2_{\phi^0_m}}\\ &\quad + \frac{1}{v_Z^2}\sum_m \kappa_Z^{\phi^0_{n_1}\phi^0_m}\kappa_Z^{\phi^0_m\phi^0_{n_2}}\frac{[u-M^2_{\phi^0_{n_1}}][u-M^2_{\phi^0_{n_2}}]}{u-M^2_{\phi^0_m}}\\ &\quad - \frac{v^2}{v_Z^4}\kappa_{ZZ}^{\phi^0_{n_1}}\kappa_{ZZ}^{\phi^0_{n_2}}\frac{[t-M^2_{\phi^0_{n_1}}][t-M^2_{\phi^0_{n_2}}]}{t}\\ &\quad - \frac{v^2}{v_Z^4}\kappa_{ZZ}^{\phi^0_{n_1}}\kappa_{ZZ}^{\phi^0_{n_2}}\frac{[u-M^2_{\phi^0_{n_1}}][u-M^2_{\phi^0_{n_2}}]}{u}.\end{aligned} \tag{A10}$$

We finally consider the amplitude

$$w^+(p_1)w^-(p_2) \to \phi^0_n(p_3)z(p_4). \tag{A11}$$

Evaluating t- and u-channel w exchange graphs, contact interaction graphs, and the s-channel Higgs exchange graph, we obtain

$$\begin{aligned}\mathcal{A}_{w^+w^-\to\phi^0_n z} &= -\frac{i}{vv_Z}\left(\frac{v_Z^2}{v^2}\kappa_{WW}^{\phi^0_n} - \kappa_{ZZ}^{\phi^0_n}\right)(t-u)\\ &\quad + \frac{1}{vv_Z}\sum_{m=1}^{N_0}\kappa_Z^{\phi^0_n\phi^0_m}\kappa_{WW}^{\phi^0_m}\frac{[s-M^2_{\phi^0_n}]s}{s-M^2_{\phi^0_m}}.\end{aligned} \tag{A12}$$

Again, the imaginary number in the amplitude is a consequence of the CP violating coupling of the Higgs bosons.

In a similar manner,

$$z(p_1)z(p_2) \to \phi^0_n(p_3)z(p_4) \tag{A13}$$

amplitude can be evaluated from the Higgs exchange graphs. We obtain

$$\begin{aligned}\mathcal{A}_{zz\to\phi^0_n z} &= \frac{v}{v_Z^3}\sum_{m=1}^{N_0}\kappa_Z^{\phi^0_n\phi^0_m}\kappa_{ZZ}^{\phi^0_m}\frac{[s-M^2_{\phi^0_n}]s}{s-M^2_{\phi^0_m}}\\ &\quad + \frac{v}{v_Z^3}\sum_{m=1}^{N_0}\kappa_Z^{\phi^0_n\phi^0_m}\kappa_{ZZ}^{\phi^0_m}\frac{[t-M^2_{\phi^0_n}]t}{t-M^2_{\phi^0_m}}\\ &\quad + \frac{v}{v_Z^3}\sum_{m=1}^{N_0}\kappa_Z^{\phi^0_n\phi^0_m}\kappa_{ZZ}^{\phi^0_m}\frac{[u-M^2_{\phi^0_n}]u}{u-M^2_{\phi^0_m}}.\end{aligned} \tag{A14}$$

APPENDIX B: EVALUATING $\tilde{\Pi}_{33}(0)$ AND $\tilde{\Pi}_{11}(0)$

In order to evaluate the vacuum polarization functions $\tilde{\Pi}_{33}(0)$ and $\tilde{\Pi}_{11}(0)$ in the electroweak gauged chiral Lagrangian (2.24) and (2.2), it is convenient to introduce the background field formalism. See, e.g., Appendix A 2 of Ref. [106].

We decompose the chiral field U into background field $\bar{U}$ and dynamical fields u^1, u^2, u^z,

$$U = \bar{U}\exp\left[\frac{i(u_1\tau_1 + u_2\tau_2)}{v}\right]\exp\left[\frac{iu_z\tau_3}{v_Z}\right]. \tag{B1}$$

The gauge fields $\mathbf{W}_\mu$ and $\mathbf{B}_\mu$ are also decomposed as

$$\mathbf{B}_\mu = \bar{\mathbf{B}}_\mu + b_\mu\frac{\tau_3}{2}, \tag{B2}$$

and

$$\mathbf{W}'_\mu = \bar{U}^\dagger\mathbf{W}_\mu\bar{U} - \frac{i}{g}\bar{U}^\dagger\partial_\mu\bar{U} = \bar{\mathbf{W}}_\mu + \sum_{a=1}^{3}w^a_\mu\frac{\tau_a}{2}, \tag{B3}$$

with

$$\bar{\mathbf{B}}_\mu = \bar{B}_\mu\frac{\tau_3}{2}, \qquad \bar{\mathbf{W}}_\mu = \sum_{a=1}^{3}\bar{W}^a_\mu\frac{\tau_a}{2}. \tag{B4}$$

Here the background gauge fields are denoted by $\bar{\mathbf{B}}_\mu$ and $\bar{\mathbf{W}}_\mu$, while the quantum fields are b_μ and w_μ. In order to evaluate radiative corrections, we introduce the gauge fixing Lagrangian,

$$\begin{aligned}\mathcal{L}_{\rm GF} &= -\frac{1}{2\xi}\left[(D_\mu w^\mu)_1 - \xi g\frac{v}{2}u_1\right]^2 - \frac{1}{2\xi}\left[(D_\mu w^\mu)_2 - \xi g\frac{v}{2}u_2\right]^2\\ &\quad - \frac{1}{2\xi}\left[(D_\mu w^\mu)_3 - \xi g\frac{v_Z}{2}u_z\right]^2 - \frac{1}{2\xi}\left[\partial_\mu b^\mu + \xi g_Y\frac{v_Z}{2}u_z\right]^2,\end{aligned} \tag{B5}$$

with

$$(D_\mu w_\nu)_a \equiv \partial_\mu w^a_\nu - g\epsilon^{abc}\bar{W}^b_\mu w^c_\nu. \tag{B6}$$

The Lagrangian $\mathcal{L}_\chi$, Eq. (2.2), is expanded in terms of the fluctuating quantum field u. We find the bilinear terms of u can be summarized in a compact expression,

$$\mathcal{L}_\chi|_{uu} + \mathcal{L}_{\mathrm{GF}}|_{uu} = \frac{1}{2}{}^t(D_\mu u)(D^\mu u) - \frac{1}{2}{}^t u\sigma u, \tag{B7}$$

with

$$u \equiv \begin{pmatrix} u_1 \\ u_2 \\ u_z \end{pmatrix}. \tag{B8}$$

In Eq. (B7), $D_\mu u$ is defined as

$$D_\mu u \equiv \partial_\mu u + \Gamma_\mu u, \tag{B9}$$

with

$$\Gamma_\mu = \begin{pmatrix} \Gamma^{11}_\mu & \Gamma^{12}_\mu & \Gamma^{1z}_\mu \\ \Gamma^{21}_\mu & \Gamma^{22}_\mu & \Gamma^{2z}_\mu \\ \Gamma^{z1}_\mu & \Gamma^{z2}_\mu & \Gamma^{zz}_\mu \end{pmatrix}, \tag{B10}$$

$$\Gamma^{11}_\mu = \Gamma^{22}_\mu = \Gamma^{zz}_\mu = 0, \tag{B11}$$

$$\Gamma^{12}_\mu = -\Gamma^{21}_\mu = \frac{1}{2}\left(2 - \frac{v_Z^2}{v^2}\right) g\bar{W}^3_\mu + \frac{1}{2}\frac{v_Z^2}{v^2} g_Y \bar{B}_\mu, \tag{B12}$$

$$\Gamma^{1z}_\mu = -\Gamma^{z1}_\mu = -\frac{v_Z}{2v} g\bar{W}^2_\mu, \tag{B13}$$

and

$$\Gamma^{2z}_\mu = -\Gamma^{z2}_\mu = \frac{v_Z}{2v} g\bar{W}^1_\mu. \tag{B14}$$

Similarly, the matrix σ is given by

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{1z} \\ \sigma_{21} & \sigma_{22} & \sigma_{2z} \\ \sigma_{z1} & \sigma_{z2} & \sigma_{zz} \end{pmatrix}, \tag{B15}$$

with

$$\begin{aligned}\sigma_{11} = {} & \frac{1}{4}\left(4 - 3\frac{v_Z^2}{v^2}\right) g^2 \bar{W}^2_\mu \bar{W}^{2\mu} \\ & + \frac{1}{4}\frac{v_Z^4}{v^4}(g\bar{W}^3_\mu - g_Y\bar{B}_\mu)(g\bar{W}^{3\mu} - g_Y\bar{B}^\mu) + \xi M_W^2,\end{aligned} \tag{B16}$$

$$\begin{aligned}\sigma_{22} = {} & \frac{1}{4}\left(4 - 3\frac{v_Z^2}{v^2}\right) g^2 \bar{W}^1_\mu \bar{W}^{1\mu} \\ & + \frac{1}{4}\frac{v_Z^4}{v^4}(g\bar{W}^3_\mu - g_Y\bar{B}^3_\mu)(g\bar{W}^{3\mu} - g_Y\bar{B}^\mu) + \xi M_W^2,\end{aligned} \tag{B17}$$

$$\sigma_{zz} = \frac{v_Z^2}{4v^2} g^2 (\bar{W}^1_\mu \bar{W}^{1\mu} + \bar{W}^2_\mu \bar{W}^{2\mu}) + \xi M_Z^2, \tag{B18}$$

$$\sigma_{12} = \sigma_{21} = -\frac{1}{4}\left(4 - 3\frac{v_Z^2}{v^2}\right) g^2 \bar{W}^1_\mu \bar{W}^{2\mu}, \tag{B19}$$

$$\sigma_{1z} = \sigma_{z1} = -\frac{1}{4}\frac{v_Z^3}{v^3} g\bar{W}^1_\mu (g\bar{W}^{3\mu} - g_Y\bar{B}^\mu), \tag{B20}$$

$$\sigma_{2z} = \sigma_{z2} = -\frac{1}{4}\frac{v_Z^3}{v^3} g\bar{W}^2_\mu (g\bar{W}^{3\mu} - g_Y\bar{B}^\mu), \tag{B21}$$

with

$$\begin{aligned} M_W^2 &= \frac{g^2}{4} v^2, \\ M_Z^2 &= \frac{g^2 + g_Y^2}{4} v_Z^2. \end{aligned} \tag{B22}$$

In the derivation of Eq. (B7), we used equations of motion of the background field.

The bilinear terms of w^a_μ and b_μ are

$$\begin{aligned} & \mathcal{L}_\chi|_{vv} + \mathcal{L}_{\mathrm{gauge}}|_{vv} + \mathcal{L}_{\mathrm{GF}}|_{vv} \\ & = -\frac{1}{2}(D_\mu w_\nu)^a (D^\mu w^\nu)^a + \frac{1}{2}\left(1 - \frac{1}{\xi}\right)(D_\mu w^\mu)^a (D_\nu w^\nu)^a \\ & \quad + \epsilon^{abc} g\bar{W}^a_{\mu\nu} w^{b\mu} w^{c\nu} - \frac{1}{2}(\partial_\mu b_\nu)(\partial^\mu b^\nu) \\ & \quad + \frac{1}{2}\left(1 - \frac{1}{\xi}\right)(\partial_\mu b^\mu)(\partial_\nu b^\nu) + \frac{g^2 v^2}{8}\sum_{a=1,2} w^a_\nu w^{a\nu} \\ & \quad + \frac{v_Z^2}{8}(g w^3_\nu - g_Y b_\nu)(g w^{3\nu} - g_Y b^\nu). \end{aligned} \tag{B23}$$

We also find

$$\begin{aligned} & \mathcal{L}_\chi|_{uv} + \mathcal{L}_{\mathrm{GF}}|_{uv} \\ & = -g^2 \frac{v}{2}\left(2 - \frac{v_Z^2}{v^2}\right)(\bar{W}^2_\mu w^{3\mu} u_1 - \bar{W}^1_\mu w^{3\mu} u_2) \\ & \quad - g g_Y \frac{v_Z^2}{2v}(\bar{W}^2_\mu b^\mu u_1 - \bar{W}^1_\mu b^\mu u_2) \\ & \quad - g\frac{v_Z^2}{2v}((g\bar{W}^3_\mu - g_Y\bar{B}_\mu) w^{1\mu} u_2 - (g\bar{W}^3_\mu - g_Y\bar{B}_\mu) w^{2\mu} u_1) \\ & \quad - g^2 \frac{v_Z}{2}(\bar{W}^1_\mu w^{2\mu} u_z - \bar{W}^2_\mu w^{1\mu} u_z). \end{aligned} \tag{B24}$$

We are now ready to evaluate the vacuum polarization functions arising from the bosonic fluctuation field

(u, w_μ and b_μ) loops. We first consider the vacuum polarization functions (at zero momentum) arising from the ${}^t u \sigma u$ term in Eq. (B7) with u boson loop. In the Feynman gauge $\xi = 1$, we obtain

$$\Pi^{u}_{11}(0) = -\frac{1}{4}\left(4 - 3\frac{v_Z^2}{v^2}\right)A(M_W) - \frac{v_Z^2}{4v^2}A(M_Z), \tag{B25}$$

$$\Pi^{u}_{33}(0) = -\frac{v_Z^4}{2v^4}A(M_W), \tag{B26}$$

where the loop integral function A is defined by Eq. (C1). In a similar manner, the u boson loop contributions arising from Γ^μ term in Eq. (B7) can be expressed by using the loop integral functions B_0 [See Eq. (C3) for its definition],

$$\Pi^{uu}_{11}(0) = \frac{v_Z^2}{4v^2}[A(M_W) + A(M_Z) + B_0(M_W, M_Z; 0)], \tag{B27}$$

$$\Pi^{uu}_{33}(0) = \frac{1}{4}\left(2 - \frac{v_Z^2}{v^2}\right)^2 [2A(M_W) + B_0(M_W, M_W; 0)] = 0. \tag{B28}$$

We next consider the gauge boson loop diagrams arising from Eq. (B23). For such a purpose, we first rearrange $w^{3\mu}$ and b^μ to the mass eigenfields (z^μ and a^μ)

$$w^{3\mu} = \frac{1}{g_Z}(g z^\mu + g_Y a^\mu), \tag{B29}$$

$$b^\mu = \frac{1}{g_Z}(-g_Y z^\mu + g a^\mu), \tag{B30}$$

in the Lagrangian (B23). In the Feynman gauge $\xi = 1$, we obtain

$$\Pi^{vv}_{11}(0) = D\left[A(M_W) + \frac{g^2}{g_Z^2}A(M_Z) + \frac{g_Y^2}{g_Z^2}A(0) + \frac{g^2}{g_Z^2}B_0(M_W, M_Z; 0) + \frac{g_Y^2}{g_Z^2}B_0(M_W, 0; 0)\right], \tag{B31}$$

$$\Pi^{vv}_{33}(0) = 2D\left[A(M_W) + \frac{1}{2}B_0(M_W, M_W; 0)\right] = 0. \tag{B32}$$

The effects of the Faddeev-Popov ghost loop can be evaluated in a similar manner, we obtain

$$\Pi^{cc}_{11}(0) = -2\left[A(M_W) + \frac{g^2}{g_Z^2}A(M_Z) + \frac{g_Y^2}{g_Z^2}A(0) + \frac{g^2}{g_Z^2}B_0(M_W, M_Z; 0) + \frac{g_Y^2}{g_Z^2}B_0(M_W, 0; 0)\right], \tag{B33}$$

$$\Pi^{cc}_{33}(0) = -4\left[A(M_W) + \frac{1}{2}B_0(M_W, M_W; 0)\right] = 0. \tag{B34}$$

We next consider the u and gauge boson loop diagrams arising from Eq. (B24). We obtain

$$\begin{aligned}\Pi^{uv}_{11}(0) = &-\frac{1}{g_Z^2}\left[g^2\frac{v}{2}\left(2 - \frac{v_Z^2}{v^2}\right) - g_Y^2\frac{v_Z^2}{2v}\right]^2 B(M_W, M_Z; 0)\\ &-\frac{g^2 g_Y^2}{g_Z^2}\left[\frac{v}{2}\left(2 - \frac{v_Z^2}{v^2}\right) + \frac{v_Z^2}{2v}\right]^2 B(M_W, 0; 0)\\ &-\frac{g^2 v_Z^2}{4}B(M_W, M_Z; 0),\end{aligned} \tag{B35}$$

$$\Pi^{uv}_{33}(0) = -\frac{g^2 v_Z^4}{2v^2}B(M_W, M_W; 0), \tag{B36}$$

with B being defined by Eq. (C2).

It is now easy to evaluate $\tilde{\Pi}_{11}(0)$ and $\tilde{\Pi}_{33}(0)$ as

$$\tilde{\Pi}_{11} = \Pi^{u}_{11} + \Pi^{uu}_{11} + \Pi^{vv}_{11} + \Pi^{uv}_{11}, \tag{B37}$$

$$\tilde{\Pi}_{33} = \Pi^{u}_{33} + \Pi^{uu}_{33} + \Pi^{vv}_{33} + \Pi^{uv}_{33}. \tag{B38}$$

APPENDIX C: LOOP INTEGRALS

We define loop integrals in D dimensions

$$A(m) \equiv \int \frac{d^D k}{(2\pi)^D i}\frac{1}{m^2 - k^2}, \tag{C1}$$

and

$$B(m_1, m_2; p^2) \equiv \int \frac{d^D k}{(2\pi)^D i}\frac{1}{[m_1^2 - (k+p)^2][m_2^2 - k^2]}, \tag{C2}$$

$$g^{\mu\nu}B_0(m_1, m_2; p^2) \equiv \int \frac{d^D k}{(2\pi)^D i}\frac{(2k+p)^\mu (2k+p)^\nu}{[m_1^2 - (k+p)^2][m_2^2 - k^2]}\Bigg|_{g^{\mu\nu}}, \tag{C3}$$

with $I^{\mu\nu}|_{g^{\mu\nu}}$ denoting the $g^{\mu\nu}$ part of integral $I^{\mu\nu}(p)$, i.e.,

$$I^{\mu\nu}(p) = g^{\mu\nu}I|_{g^{\mu\nu}} + p^\mu p^\nu I|_{p^\mu p^\nu}.$$

Note that the above definitions of the loop integrals differ slightly from the definitions of $\mathbf{A}$, $\mathbf{B}_0$, $\mathbf{B}_{22}$ used in Ref. [121]:

$$\mathbf{A}(m^2) = -(4\pi)^2 A(m), \tag{C4}$$

$$\mathbf{B}_0(p^2; m_1, m_2) = (4\pi)^2 B(m_1, m_2; p^2), \tag{C5}$$

$$\mathbf{B}_{22}(p^2; m_1, m_2) = \frac{(4\pi)^2}{4} B_0(m_1, m_2; p^2). \tag{C6}$$

It is easy to see

$$\begin{aligned} &g^{\mu\nu} B_0(m_1, m_2; p^2) \\ &\quad = \int \frac{d^D k}{(2\pi)^D i} \frac{4k^\mu k^\nu}{[m_1^2 - (k+p)^2][m_2^2 - k^2]}\bigg|_{g^{\mu\nu}}. \end{aligned} \tag{C7}$$

In the $D \to 4$ limit, these loop integrals suffer UV divergences. Introducing the UV cutoff momentum Λ, they can be written as

$$A(m) = \frac{\Lambda^2}{(4\pi)^2} - \frac{m^2}{(4\pi)^2} \ln \frac{\Lambda^2}{\mu^2} + A_r(m), \tag{C8}$$

and

$$B(m_1, m_2; p^2) = \frac{1}{(4\pi)^2} \ln \frac{\Lambda^2}{\mu^2} + B_r(m_1, m_2; p^2), \tag{C9}$$

$$\begin{aligned} &B_0(m_1, m_2; p^2) \\ &\quad = -2\frac{\Lambda^2}{(4\pi)^2} + \frac{1}{(4\pi)^2}\left(m_1^2 + m_2^2 - \frac{1}{3}p^2\right) \ln \frac{\Lambda^2}{\mu^2} \\ &\qquad + B_{0r}(m_1, m_2; p^2), \end{aligned} \tag{C10}$$

with μ being a finite scale parameter. Finite functions A_r, B_r, B_{0r} can be expressed as

$$A_r(m) = -\frac{m^2}{(4\pi)^2}\left[\ln \frac{\mu^2}{m^2} + 1\right], \tag{C11}$$

$$\begin{aligned} &B_r(m_1, m_2; p^2) \\ &\quad = \frac{1}{(4\pi)^2} \int_0^1 dx \ln\left(\frac{\mu^2}{m_1^2 x + m_2^2(1-x) - p^2 x(1-x)}\right), \end{aligned} \tag{C12}$$

$$\begin{aligned} &B_{0r}(m_1, m_2; p^2) \\ &\quad = \frac{2}{(4\pi)^2} \int_0^1 dx [m_1^2 x + m_2^2(1-x) - p^2 x(1-x)] \\ &\qquad \times \left[\ln\left(\frac{\mu^2}{m_1^2 x + m_2^2(1-x) - p^2 x(1-x)}\right) + 1\right]. \end{aligned} \tag{C13}$$

Performing the parameter integrals, we find

$$(4\pi)^2 B_r(m_1, m_2; 0) = 1 - \frac{1}{m_1^2 - m_2^2}\left[m_1^2 \ln \frac{m_1^2}{\mu^2} - m_2^2 \ln \frac{m_2^2}{\mu^2}\right], \tag{C14}$$

$$\begin{aligned} &(4\pi)^2 B_r{}'(m_1, m_2; 0) \\ &\quad = \frac{1}{(m_1^2 - m_2^2)^2}\left[\frac{m_1^2 + m_2^2}{2} - \frac{m_1^2 m_2^2}{m_1^2 - m_2^2} \ln \frac{m_1^2}{m_2^2}\right], \end{aligned} \tag{C15}$$

$$\begin{aligned} &(4\pi)^2 B_{0r}(m_1, m_2; 0) \\ &\quad = \frac{3}{2}(m_1^2 + m_2^2) - \frac{1}{m_1^2 - m_2^2}\left[m_1^4 \ln \frac{m_1^2}{\mu^2} - m_2^4 \ln \frac{m_2^2}{\mu^2}\right], \end{aligned} \tag{C16}$$

$$\begin{aligned} &(4\pi)^2 B_{0r}{}'(m_1, m_2; 0) \\ &\quad = -\frac{1}{18}\frac{5m_1^4 - 22m_1^2 m_2^2 + 5m_2^4}{(m_1^2 - m_2^2)^2} + \frac{1}{3}\frac{1}{(m_1^2 - m_2^2)^3} \\ &\qquad \times \left[m_1^4(m_1^2 - 3m_2^2) \ln \frac{m_1^2}{\mu^2} - m_2^4(m_2^2 - 3m_1^2) \ln \frac{m_2^2}{\mu^2}\right], \end{aligned} \tag{C17}$$

with B_r', B_{0r}' being defined by

$$\begin{aligned} B_r'(m_1, m_2; p^2) &\equiv \frac{d}{dp^2} B_r(m_1, m_2; p^2), \\ B_{0r}'(m_1, m_2; p^2) &\equiv \frac{d}{dp^2} B_{0r}(m_1, m_2; p^2). \end{aligned} \tag{C18}$$

The functions used in the expressions of S_f, T_f and U_f are defined as

$$\begin{aligned} F^{\phi_n\phi_m} &\equiv (4\pi)^2[B_{0r}(M_{\phi_n}, M_{\phi_m}; 0) + A_r(M_{\phi_n}) + A_r(M_{\phi_m})] \\ &= \frac{M_{\phi_n}^2 + M_{\phi_m}^2}{2} - \frac{M_{\phi_n}^2 M_{\phi_m}^2}{M_{\phi_n}^2 - M_{\phi_m}^2} \ln \frac{M_{\phi_n}^2}{M_{\phi_m}^2}, \end{aligned} \tag{C19}$$

$$\begin{aligned} F^{\phi_n\phi_m\prime} &\equiv (4\pi)^2 B_{0r}'(M_{\phi_n}, M_{\phi_m}; 0) \\ &= -\frac{1}{3}\left\{\frac{4}{3} - \frac{M_{\phi_n}^2 \ln \frac{M_{\phi_n}^2}{\mu^2} - M_{\phi_m}^2 \ln \frac{M_{\phi_m}^2}{\mu^2}}{M_{\phi_n}^2 - M_{\phi_m}^2} \right. \\ &\qquad \left. - \frac{M_{\phi_n}^2 + M_{\phi_m}^2}{(M_{\phi_n}^2 - M_{\phi_m}^2)^2} F^{\phi_n\phi_m}\right\}, \end{aligned} \tag{C20}$$

$$\begin{aligned} G^{V\phi} &\equiv (4\pi)^2[B_{0r}(M_\phi, M_V; 0) - 4M_V^2 B_r(M_\phi, M_V; 0) \\ &\qquad + A_r(M_\phi) + A_r(M_V)] \\ &= F^{V\phi} + 4M_V^2\left(-1 + \frac{M_\phi^2 \ln \frac{M_\phi^2}{\mu^2} - M_V^2 \ln \frac{M_V^2}{\mu^2}}{M_\phi^2 - M_V^2}\right), \end{aligned} \tag{C21}$$

$$\begin{aligned} G^{V\phi\prime} &\equiv (4\pi)^2[B_{0r}'(M_\phi, M_V; 0) - 4M_V^2 B_r'(M_\phi, M_V; 0)] \\ &= F^{V\phi\prime} - \frac{4M_V^2}{(M_V^2 - M_\phi^2)^2} F^{V\phi}, \end{aligned} \tag{C22}$$

where functions A_r, B_r, B_{0r} B'_r and B'_{0r} are given in Eqs. (C11), (C14), (C15), (C16) and (C17).

For $\Delta M_{\phi_n\phi_m} \equiv |M_{\phi_n} - M_{\phi_m}| \ll M_{\phi_n}, M_{\phi_m}$, we find

$$F^{\phi_n\phi_m} = \frac{2}{3}(\Delta M_{\phi_n\phi_m})^2 - \frac{1}{30}\frac{(\Delta M_{\phi_n\phi_m})^4}{\bar{M}^2_{\phi_n\phi_m}} + \cdots, \tag{C23}$$

$$F^{\phi_n\phi_m}{}' = \frac{1}{3}\ln\frac{\bar{M}^2_{\phi_n\phi_m}}{\mu^2} + \frac{1}{20}\frac{(\Delta M_{\phi_n\phi_m})^2}{\bar{M}^2_{\phi_n\phi_m}} + \cdots, \tag{C24}$$

with

$$\bar{M}_{\phi_n\phi_m} \equiv \frac{M_{\phi_n} + M_{\phi_m}}{2}. \tag{C25}$$

For $M_V \ll M_\phi$, we also note

$$G^{V\phi} = \frac{1}{2}M_\phi^2 + \left(3\ln\frac{M_\phi^2}{\mu^2} + \ln\frac{M_V^2}{\mu^2} - \frac{7}{2}\right)M_V^2 + \cdots, \tag{C26}$$

$$G^{V\phi}{}' = \frac{1}{3}\ln\frac{M_\phi^2}{\mu^2} - \frac{5}{18} - \frac{4}{3}\frac{M_V^2}{M_\phi^2} + \cdots. \tag{C27}$$

APPENDIX D: SELF-INTERACTIONS AMONG HIGGS BOSONS

In this Appendix, we list the formulas of $\tilde{M}$ and $\tilde{\lambda}$ used in Sec. VIII,

$$(\tilde{M}^2)_{11} = (\kappa_V^{\phi_1^0})^2 M^2_{\phi_1^0} + (\kappa_V^{\phi_2^0})^2 M^2_{\phi_2^0}, \tag{D1}$$

$$(\tilde{M}^2)_{12} = (\kappa_V^{\phi_1^0})(\kappa_V^{\phi_2^0})(M^2_{\phi_2^0} - M^2_{\phi_1^0}), \tag{D2}$$

$$(\tilde{M}^2)_{22} = (\kappa_V^{\phi_2^0})^2 M^2_{\phi_1^0} + (\kappa_V^{\phi_1^0})^2 M^2_{\phi_2^0}, \tag{D3}$$

$$\begin{aligned}\tilde{\lambda}_{111} = {}& \lambda_{111}(\kappa_V^{\phi_1^0})^3 + \lambda_{222}(\kappa_V^{\phi_2^0})^3 + 3\lambda_{112}(\kappa_V^{\phi_1^0})^2(\kappa_V^{\phi_2^0}) \\ & + 3\lambda_{122}(\kappa_V^{\phi_1^0})(\kappa_V^{\phi_2^0})^2,\end{aligned} \tag{D4}$$

$$\begin{aligned}\tilde{\lambda}_{112} = {}& -\lambda_{111}(\kappa_V^{\phi_1^0})^2(\kappa_V^{\phi_2^0}) + \lambda_{222}(\kappa_V^{\phi_1^0})(\kappa_V^{\phi_2^0})^2 \\ & + \lambda_{112}[(\kappa_V^{\phi_1^0})^3 - 2(\kappa_V^{\phi_1^0})(\kappa_V^{\phi_2^0})^2] \\ & - \lambda_{122}[(\kappa_V^{\phi_2^0})^3 - 2(\kappa_V^{\phi_1^0})^2(\kappa_V^{\phi_2^0})],\end{aligned} \tag{D5}$$

$$\begin{aligned}\tilde{\lambda}_{122} = {}& \lambda_{111}(\kappa_V^{\phi_1^0})(\kappa_V^{\phi_2^0})^2 + \lambda_{222}(\kappa_V^{\phi_1^0})^2(\kappa_V^{\phi_2^0}) \\ & + \lambda_{112}[(\kappa_V^{\phi_2^0})^3 - 2(\kappa_V^{\phi_1^0})^2(\kappa_V^{\phi_2^0})] \\ & + \lambda_{122}[(\kappa_V^{\phi_1^0})^3 - 2(\kappa_V^{\phi_1^0})(\kappa_V^{\phi_2^0})^2],\end{aligned} \tag{D6}$$

$$\begin{aligned}\tilde{\lambda}_{222} = {}& -\lambda_{111}(\kappa_V^{\phi_2^0})^3 + \lambda_{222}(\kappa_V^{\phi_1^0})^3 + 3\lambda_{112}(\kappa_V^{\phi_1^0})(\kappa_V^{\phi_2^0})^2 \\ & - 3\lambda_{122}(\kappa_V^{\phi_1^0})^2(\kappa_V^{\phi_2^0}),\end{aligned} \tag{D7}$$

$$\begin{aligned}\tilde{\lambda}_{1111} = {}& \lambda_{1111}(\kappa_V^{\phi_1^0})^4 + \lambda_{2222}(\kappa_V^{\phi_2^0})^4 \\ & + 6\lambda_{1122}(\kappa_V^{\phi_1^0})^2(\kappa_V^{\phi_2^0})^2 \\ & + 4\lambda_{1112}(\kappa_V^{\phi_1^0})^3(\kappa_V^{\phi_2^0}) + 4\lambda_{1222}(\kappa_V^{\phi_1^0})(\kappa_V^{\phi_2^0})^3,\end{aligned} \tag{D8}$$

$$\begin{aligned}\tilde{\lambda}_{1112} = {}& -\lambda_{1111}(\kappa_V^{\phi_1^0})^3(\kappa_V^{\phi_2^0}) + \lambda_{2222}(\kappa_V^{\phi_1^0})(\kappa_V^{\phi_2^0})^3 \\ & + 3\lambda_{1122}(\kappa_V^{\phi_1^0})(\kappa_V^{\phi_2^0})[(\kappa_V^{\phi_1^0})^2 - (\kappa_V^{\phi_2^0})^2] \\ & + \lambda_{1112}(\kappa_V^{\phi_1^0})^4 - 3\lambda_{1112}(\kappa_V^{\phi_1^0})^2(\kappa_V^{\phi_2^0})^2 \\ & - \lambda_{1222}(\kappa_V^{\phi_2^0})^4 + 3\lambda_{1222}(\kappa_V^{\phi_1^0})^2(\kappa_V^{\phi_2^0})^2,\end{aligned} \tag{D9}$$

$$\begin{aligned}\tilde{\lambda}_{1122} = {}& (\lambda_{1111} + \lambda_{2222})(\kappa_V^{\phi_1^0})^2(\kappa_V^{\phi_2^0})^2 \\ & + \lambda_{1122}[(\kappa_V^{\phi_1^0})^4 - 4(\kappa_V^{\phi_1^0})^2(\kappa_V^{\phi_2^0})^2 + (\kappa_V^{\phi_2^0})^4] \\ & - 2(\lambda_{1112} - \lambda_{1222})(\kappa_V^{\phi_1^0})(\kappa_V^{\phi_2^0}) \\ & \times [(\kappa_V^{\phi_1^0})^2 - (\kappa_V^{\phi_2^0})^2],\end{aligned} \tag{D10}$$

$$\begin{aligned}\tilde{\lambda}_{1222} = {}& -\lambda_{1111}(\kappa_V^{\phi_1^0})(\kappa_V^{\phi_2^0})^3 f + \lambda_{2222}(\kappa_V^{\phi_1^0})^3(\kappa_V^{\phi_2^0}), \\ & - 3\lambda_{1122}[(\kappa_V^{\phi_1^0})^2 - (\kappa_V^{\phi_2^0})^2](\kappa_V^{\phi_1^0})(\kappa_V^{\phi_2^0}) \\ & + \lambda_{1112}[3(\kappa_V^{\phi_1^0})^2 - (\kappa_V^{\phi_2^0})^2](\kappa_V^{\phi_2^0})^2 \\ & + \lambda_{1222}[(\kappa_V^{\phi_1^0})^2 - 3(\kappa_V^{\phi_2^0})^2](\kappa_V^{\phi_1^0})^2,\end{aligned} \tag{D11}$$

$$\begin{aligned}\tilde{\lambda}_{2222} = {}& \lambda_{1111}(\kappa_V^{\phi_2^0})^4 + \lambda_{2222}(\kappa_V^{\phi_1^0})^4 \\ & + 6\lambda_{1122}(\kappa_V^{\phi_1^0})^2(\kappa_V^{\phi_2^0})^2 \\ & - 4\lambda_{1112}(\kappa_V^{\phi_1^0})(\kappa_V^{\phi_2^0})^3 - 4\lambda_{1222}(\kappa_V^{\phi_1^0})^3(\kappa_V^{\phi_2^0}).\end{aligned} \tag{D12}$$

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