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Hard scale dependent gluon density, saturation, and forward-forward dijet production at the LHC

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We propose a method to introduce Sudakov effects to the unintegrated gluon density, promoting it to be hard scale dependent. The advantage of the approach is that it guarantees that the gluon density is positive definite and that the Sudakov effects cancel on the integrated level. As a case study, we apply the method to calculate angular correlations and the R_{pA} ratio for $\mathrm{p+p}$ vs $\mathrm{p+Pb}$ collision in the production of forward-forward dijets.

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I. INTRODUCTION

In perturbative QCD, the theoretical construction that is used to evaluate cross sections for hadron-hadron collisions is a factorization [1] that allows one to split the cross section into parton densities characterizing the incoming hadrons and hard subprocess. In particular, high-energy factorization [2,3] is a prescription for such a decomposition that allows for taking into account the off-shellness of incoming partons carrying low longitudinal momentum fraction x of hadron already at the lowest-order accuracy both in matrix elements [4–9] and parton densities. Its applications to situations in which saturation effects are relevant is a phenomenologically useful. There are already results that generalize it in some limit of phase space to include saturation [10]. The basic ingredient of the formula for factorization is the unintegrated gluon density. In the high-energy limit, it comes from resummation of emission of gluons emitted in the s channel that are ordered in the longitudinal momentum fractions and unordered in the transversal momenta. When the longitudinal momentum fraction is $x \ll 1$, one argues that the nonlinear effects start to show up to tame the rapid power-like growth of the gluon density [2], and there are indications that indeed saturation occurs in nature [11–14]. Resummation of relevant contributions for introducing unitarity corrections can be achieved conveniently in the coordinate space in the so-called dipole picture (the virtual probe interacting with target is represented as a color dipole) and leads to the Balitsky–Kovchegov (BK) equation [15–17] or its generalizations [18]. The unintegrated gluon density (also called the dipole gluon density) used in the high-energy factorization can be obtained as the Fourier transform of a dipole amplitude N,

$$\mathcal{F}(x,k^2) = \frac{C_F}{\alpha_s(2\pi)^3}\int d^2\mathbf{b}\, d^2\mathbf{r}\, e^{-i\mathbf{k}\cdot\mathbf{r}}\nabla_r^2 N(\mathbf{r},\mathbf{b},x), \quad (1)$$

where N is the solution of the coordinate space BK equation and $\mathbf{b}$ is the impact parameter at which a color dipole collides with the target, the size of a dipole is $r = |\mathbf{r}|$, and $k = |\mathbf{k}|$ is the momentum. The two-dimensional vectors $|\mathbf{r}|$ and $|\mathbf{k}|$ lie in the plane transversal to the collision axis.

It turns out that in order for the BK equation to be applicable for processes at LHC one needs to include resummed corrections of higher orders among which the kinematical constraint [19,20] is the most dominant. It softens the singularities of the evolution kernel and therefore slows down the evolution. Its inclusion in the Balitsky-Fadin-Kuraev-Lipatov (BFKL) kernel allows for a reasonably good description of p_T spectra of the forward-central dijet at the LHC [21,22]. Another type of effect that is beyond the BK is the angular ordering leading to the dependence of the gluon density on the scale of the hard process. At the linear level, inclusion of such effects leads to the Catani-Ciafaloni-Fiarani-Marchesini (CCFM) evolution equation[1] [24–26], while at the nonlinear level, it leads to equations introduced in Refs. [27–29]. Importance of the hard scale dependence has been also recognized by Refs. [30,31], in which the effects of coherence were introduced in the last step of the evolution. The framework developed in [30,31] is particularly interesting as it is relatively straightforward to apply since it uses parton densities that might come from a collinear framework on top of which the Sudakov effects are applied [32] in a factorized form. Furthermore, it has been noticed in Ref. [33] that in order to obtain a description of the data in a wider domain of the $\Delta\phi$ one needs to include Sudakov effects in the low-x framework to ensure no emissions between the scale k of the gluon transverse momentum and the scale μ of the hard process. In the method described in Ref. [33] the Sudakov effects were imposed on the cross section level, i.e., generated events were weighted with a Sudakov form factor preserving unitarity, assuring that the total cross section will not be affected.[2] Another approach

[1] Recently fitted to F_2 data in Ref. [23].

[2] The method used in Ref. [33] will be soon available within the LxJet program [34].

to introduce the Sudakov effect is to include it directly as a part of the evolution equation, i.e., at all steps in the evolution. Such an approach leads to the already-mentioned CCFM evolution, and the Sudakov form factor gets the interpretation of an object that resumms virtual and unresolved real corrections relevant when the scale of the harder process is larger than the local k of the gluon.

In the present paper, we start directly from the gluon density summing low-x logarithms, accounting for nonlinearities, and promote it to depend on the hard scale. This method is attractive since it provides gluon density that, once constructed, can be used in various phenomenological applications. We perform our construction for proton and lead and apply the resulting gluon density to provide estimates of relevance of coherence for the nuclear modification ratio R_{pA} in the production of forward-forward dijet.

II. SUDAKOV EFFECTS AND UNINTEGRATED GLUON DENSITY

The solutions of the evolution equation combining the physics of saturation and coherence show that the saturation scale gets nontrivial dependence on the scale of the hard process [35,36] and leads, for instance, to the effect called saturation of the saturation scale [37]. Because of its numerical complexity, the equation has still not been applied to phenomenology. Below, we propose a model prescription of how to introduce hard scale dependence on top of the unintegrated gluon density $\mathcal{F}(x,k^2)$ obtained from solutions of the BK or BFKL evolution equation.

The prescription is motivated by the method developed in Ref. [33] but formulated in terms of the unintegrated gluon density. Therefore, the methods are formally not equivalent. The comparison of the methods is postponed for future studies. The basic assumptions are the following:

(i) On the integrated level, the gluon densities obtained from the hard scale dependent gluon density $\mathcal{F}(x,k^2,\mu^2)$ and $\mathcal{F}(x,k^2)$ are the same. This guarantees that the Sudakov form factor just modifies the shape of the gluon density, but on the inclusive level, the distribution is the same.

(ii) The contribution with $k > \mu$ is given by the unintegrated gluon density $\mathcal{F}(x,k^2)$, which could be obtained by solving the BK equation.

The assumptions above lead to the formula

$$\mathcal{F}(x,k^2,\mu^2) \coloneqq \theta(\mu^2-k^2)T_s(\mu^2,k^2)\frac{xg(x,\mu^2)}{xg_{hs}(x,\mu^2)}\mathcal{F}(x,k^2) + \theta(k^2-\mu^2)\mathcal{F}(x,k^2), \tag{2}$$

where

$$xg_{hs}(x,\mu^2) = \int^{\mu^2} dk^2 T_s(\mu^2,k^2)\mathcal{F}(x,k^2),\ xg(x,\mu^2) = \int^{\mu^2} dk^2\mathcal{F}(x,k^2), \tag{3}$$

and the Sudakov form factor assumes the form

$$T_s(\mu^2,k^2) = \exp\left(-\int_{k^2}^{\mu^2}\frac{dk'^2}{k'^2}\frac{\alpha_s(k'^2)}{2\pi} \times \sum_{a'}\int_0^{1-\Delta} dz' P_{a'a}(z')\right), \tag{4}$$

where $\Delta = \frac{\mu}{\mu+k}$ and $P_{a'a}$ is a splitting function with subscripts $a'a$ specifying the type of transition. In the gg channel, one multiplies $P_{gg}(z)$ by z due to symmetry arguments [30].

The construction guarantees that at the integrated level the number of gluons does not change since, after integration up to the hard scale in Eq. (2) and application of Eq. (3), the terms xg_{hs} cancel and the part with $\theta(k^2-\mu^2)$ drops. The Sudakov form factor just makes the shape of the gluon density scale dependent but does not modify its integral.

To study properties of the introduced hard scale dependent unintegrated gluon density, we use the gluon density obtained from the momentum space version of the BK equation in the large target approximation. At leading order in $\alpha_s \ln(1/x)$, it reads

$$\mathcal{F}(x,k^2) = \mathcal{F}^{(0)}(x,k^2) + \frac{\alpha_s(k^2)N_c}{\pi}\int_x^1\frac{dz}{z}\int_{k_0^2}^{\infty}\frac{dl^2}{l^2}\left\{\frac{l^2\mathcal{F}(\frac{x}{z},l^2)-k^2\mathcal{F}(\frac{x}{z},k^2)}{|l^2-k^2|} + \frac{k^2\mathcal{F}(\frac{x}{z},k^2)}{|4l^4+k^4|^{\frac{1}{2}}}\right\} - \frac{2\alpha_s^2(k^2)}{R^2}\left[\left(\int_{k^2}^{\infty}\frac{dl^2}{l^2}\mathcal{F}(x,l^2)\right)^2 + \mathcal{F}(x,k^2)\int_{k^2}^{\infty}\frac{dl^2}{l^2}\ln\left(\frac{l^2}{k^2}\right)\mathcal{F}(x,l^2)\right], \tag{5}$$

where R is the radius of the hadronic target and $\mathcal{F}^{(0)}(x,k^2)$ is the starting distribution. The linear part of Eq. (5) is given by the BFKL kernel, while the nonlinear part is proportional to the triple pomeron vertex [38,39], which allows for the recombination of gluons. In Ref. [40], it has been shown that in order to apply the BK equation to dijet physics one has to go beyond the equation with just running coupling corrections included, i.e., the running coupling Balitsky-Kovchegov equation [41]. Therefore, to be realistic with applications

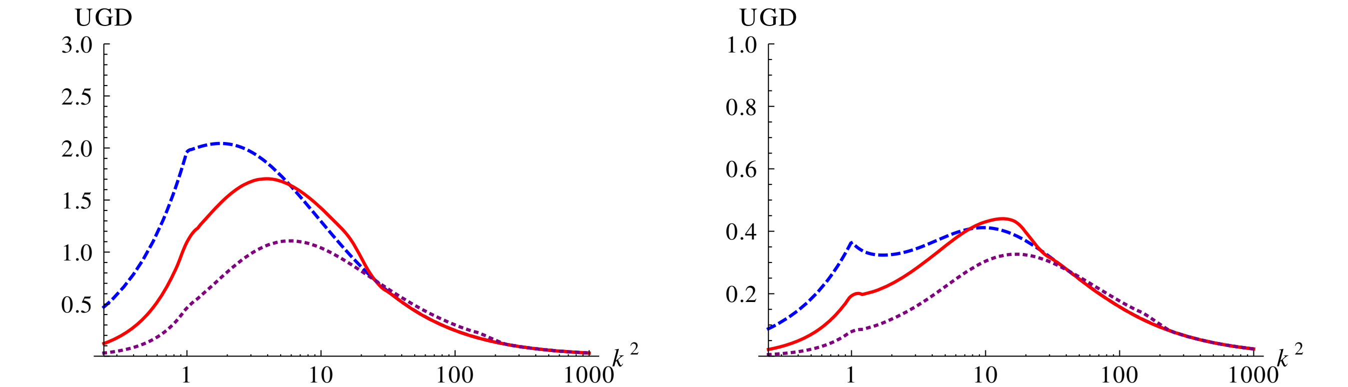

FIG. 1 (color online). We abbreviate the unintegrated gluon density by UGD. Left: unintegrated gluon density of the proton with Sudakov effects evaluated at $x = 10^{-5}$ at hard scale $\mu^2 = 20\ \mathrm{GeV}^2$ (continuous red line), hard scale $\mu^2 = 200\ \mathrm{GeV}^2$ (purple dotted line), and unintegrated gluon density without Sudakov effects evaluated at $x = 10^{-5}$ (blue dashed line). Right: unintegrated gluon density of Pb with Sudakov effects evaluated at $x = 10^{-5}$ at hard scale $\mu^2 = 20\ \mathrm{GeV}^2$ (continuous red line), hard scale $\mu^2 = 200\ \mathrm{GeV}^2$ (purple dotted line), and unintegrated gluon density without Sudakov effects evaluated at $x = 10^{-5}$ (blue dotted line).

for the LHC, we use the momentum space BK equation with corrections formulated in Refs. [42–44]. Those corrections include:

(i) kinematic effects limiting the l integration enforcing the virtuality of the exchanged t-channel gluon to be dominated by its transversal component.

(ii) the running coupling.

(iii) pieces of the splitting function subleading at low-x important at larger values of splitting ratio z and the contribution of sea quarks [indicated below by $\Sigma(x,k^2)$].

The final equation assumes the form

$$\mathcal{F}(x,k^2) = \mathcal{F}^{(0)}(x,k^2) + \frac{\alpha_s(k^2)N_c}{\pi}\int_x^1 \frac{dz}{z}\int_{k_0^2}^{\infty}\frac{dl^2}{l^2}\left\{\frac{l^2\mathcal{F}(\frac{x}{z},l^2)\theta(\frac{k^2}{z}-l^2) - k^2\mathcal{F}(\frac{x}{z},k^2)}{|l^2-k^2|} + \frac{k^2\mathcal{F}(\frac{x}{z},k^2)}{|4l^4+k^4|^{\frac{1}{2}}}\right\}$$
$$+ \frac{\alpha_s(k^2)}{2\pi k^2}\int_x^1 dz\left[\left(P_{gg}(z) - \frac{2N_c}{z}\right)\int_{k_0^2}^{k^2} dl^2\mathcal{F}\left(\frac{x}{z},l^2\right) + zP_{gq}(z)\Sigma\left(\frac{x}{z},k^2\right)\right]$$
$$- \frac{2\alpha_s^2(k^2)}{R^2}\left[\left(\int_{k^2}^{\infty}\frac{dl^2}{l^2}\mathcal{F}(x,l^2)\right)^2 + \mathcal{F}(x,k^2)\int_{k^2}^{\infty}\frac{dl^2}{l^2}\ln\left(\frac{l^2}{k^2}\right)\mathcal{F}(x,l^2)\right], \quad (6)$$

where the input gluon density is given by

$$\mathcal{F}^{(0)}(x,k^2) = \frac{\alpha_s(k^2)}{k^2}\int_x^1 P_{gg}(z)\frac{x}{z}g\left(\frac{x}{z},k_0^2\right),$$
$$xg(x,k_0^2 = 1) = 0.994(1+82.1x)^{18.6}. \quad (7)$$

The plots of the gluon density obtained from solving Eq. (6) and its extension for the Pb target is shown on Fig. 1. The dashed-blue lines correspond to the situation in which the hard scale effects are not taken into account. The kink at $k_0 = 1\ \mathrm{GeV}^2$ is an artifact of the matching condition between the model extension (needed for numerical purposes) below $k < k_0 = 1$ GeV and the evolution for $k > k_0 = 1$ GeV. In the region below $k < k_0 = 1$ GeV, which cannot be accessed with the used numerical framework, the gluon density is assumed to behave like $\mathcal{F} \sim k^2$. The maximum of the distribution signals the emergence of the saturation scale. The gluon density from Eq. (6) has been successfully applied to the description of F_2 structure function data [21] and, after accounting for Sudakov effects (at the cross section level), for the description of azimuthal angle correlations of forward-central dijets in the inclusive and inside jet tag scenario [33].

III. APPLICATIONS

In this section, we apply the hard scale dependent gluon density to study angular correlations of the forward-forward dijet and to calculate the R_{pA} ratio for p + Pb collision.[3] As argued in Ref. [40], this observable is particularly interesting for testing low-x effects since the kinematical configuration of two jets probes the gluon density at $x \approx 10^{-5}$. Furthermore, the distance in rapidity of produced jets is small, and therefore the phase space for

[3]The calculation has been done within Mathematica package MATH4JET, available form the author on request.

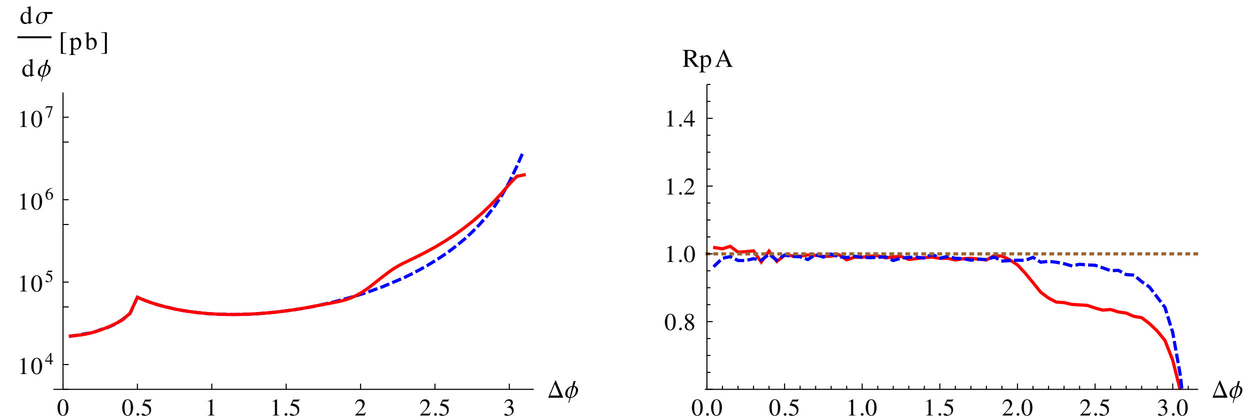

FIG. 2 (color online). Left: cross section for decorrelations in the production of the forward-forward dijet in p + p collision at 7 TeV. The rapidities of produced jets satisfy $p_{t1} > p_{t2} > 20$ GeV. The continuous red line corresponds to the situation with Sudakov effects included, while the blue dashed line omits Sudakov effects. Right: the R_{pA} ratio for p + p v. p + Pb. The continuous red line corresponds to the situation with Sudakov effects included, while the blue dashed line omits Sudakov effects, and the brown line just helps to see the deviation from unity.

emission of further jets is suppressed. To calculate the cross section we are after, we use the hybrid high-energy factorization [45]:

$$\frac{d\sigma}{dy_1 dy_2 dp_{1t} dp_{2t} d\Delta\phi} = \sum_{a,c,d} \frac{p_{t1} p_{t2}}{8\pi^2 (x_1 x_2 S)^2} \times |\overline{\mathcal{M}}_{ag\bar{c}d}|^2 x_1 f_{a/A}(x_1, \mu^2) \times \mathcal{F}_{g/B}(x_2, k^2, \mu^2) \frac{1}{1+\delta_{cd}}, \qquad (8)$$

with $k^2 = p_{t1}^2 + p_{t2}^2 + 2p_{t1}p_{t2}\cos\Delta\phi$ and

$$x_1 = \frac{1}{\sqrt{S}}(p_{t1}e^{y_1} + p_{t2}e^{y_2}),$$

$$x_2 = \frac{1}{\sqrt{S}}(p_{t1}e^{-y_1} + p_{t2}e^{-y_2}).$$

In the formulas above, S is the squared energy in the center-of-mass system of the incoming hadrons (for p + p, energy is 7 TeV, while for p + Pb, it is 5.02 TeV), the matrix elements correspond to processes $qg^* \to qg$, $gg^* \to gg$, $gg^* \to \bar{q}q$, and $f(x_1, \mu^2)$ is a collinear parton density, while the hard scale is given by $\mu = (p_{t1} + p_{t2})/2$. To visualize the role of the Sudakov effect, we calculate the cross section for angular correlations of produced jets Fig. 2. The kinematical cuts are $p_{t1}, p_{t2} > 20$ GeV, $4.9 > y_1, y_2 > 3.2$, and we use the jet algorithm in a form $R = \sqrt{(\Delta y)^2 + (\Delta\phi)^2} > 0.5$; i.e., if the distance between two partons is larger than R, the partons form two jets, while if the distance between R is smaller than R, the events are rejected. The jet algorithm serves as a regulator of the collinear singularity of the off-shell matrix element that arises when at small rapidity distance the azimuthal angle between produced partons is small. We see that the Sudakov effect suppresses the cross section when the jets are close to back-to-back configurations, while it enhances the cross section in the region dominated by configurations

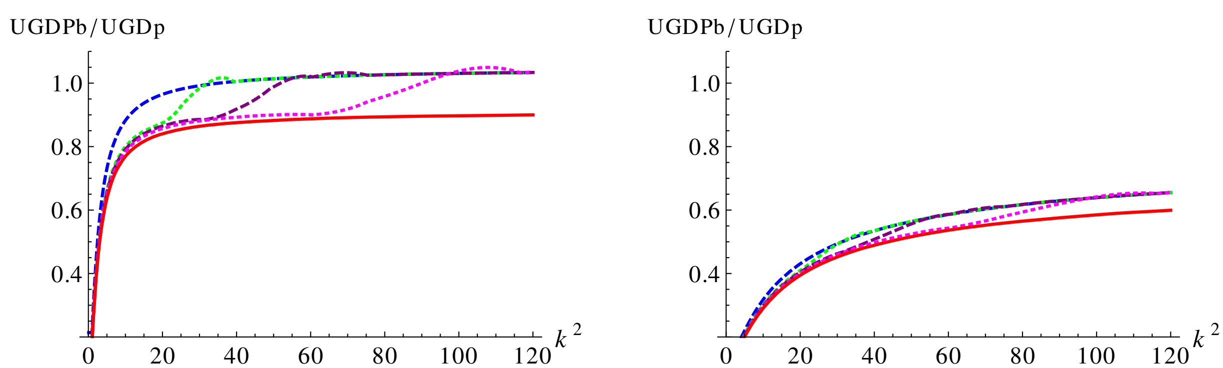

FIG. 3 (color online). Left: ratio of the unintegrated gluon density of lead to the unintegrated gluon density of the proton evaluated at $x = 10^{-3}$ at hard scale $\mu^2 = 25$ GeV^2 (green dotted line), $\mu^2 = 45$ GeV^2 (purple dashed line), $\mu^2 = 80$ GeV^2 (magenta dotted line), $\mu^2 = 400$ GeV^2 (red continuous line), and no hard scale dependence (blue dashed line). Right: ratio of the gluon density of lead to the gluon density of the proton evaluated at $x = 10^{-5}$ at hard scale $\mu^2 = 25$ GeV^2 (green dotted line), $\mu^2 = 45$ GeV^2 (purple dashed line), $\mu^2 = 80$ GeV^2 (magenta dotted line), $\mu^2 = 400$ GeV^2 (red continuous line), and no hard scale dependence (blue dashed line).

in which the hard scale of the process is a bit larger than the k of the incoming off-shell gluon (a similar effect was observed earlier in studies of forward-central jets in Ref. [33]). What is novel is that now one can attribute the enhancement phenomenon to the hard scale dependent gluon density dominating over the regular gluon density at regions in which the hard scale is approaching k as seen in Fig. 1. The kink visible at small values of $\Delta\phi$ is due to the jet definition, which introduces a sharp cutoff of the events not classified as jets.

To finalize our study, we investigate the R_{pA}, i.e., the ratio of the cross section for decorrelations of the dijet produced in $\mathrm{p}+\mathrm{p}$ and $\mathrm{p}+\mathrm{Pb}$. We see that the hard scale dependence leads to the ratio of considered cross sections being smaller where the saturation effects play a role, i.e., at values of large $\Delta\phi$. By inspecting the plots of unintegrated gluon densities and their ratios in Fig. 3, we see that the gluon density of the proton is more affected by Sudakov effects than the lead gluon density. Therefore, the ratio is smaller than one in a wider range of k and therefore in a larger range of $\Delta\phi$. This is because, in the case of lead, the saturation effects are larger, and the suppression of the low-k region is more significant already for the hard scale independent gluon density.

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