PHYSICAL REVIEW D **91,** 025027 (2015)

Gauge dependence of the effective potential for Horava-Lifshitz-like theories

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(Received 17 December 2014; published 28 January 2015)

We calculate the one-loop effective potential for Horava-Lifshitz-like QED with an arbitrary critical exponent within different approaches and discuss its gauge dependence.

DOI: 10.1103/PhysRevD.91.025027 PACS numbers: 11.10.Ef

The effective potential is known to be a key object of the quantum field theory allowing us to make conclusions about many issues related to the low-energy effective behavior of the corresponding theory such as spontaneous symmetry breaking, phase transitions, Green functions, and many other aspects. It has been studied within numerous contexts and for different field theory models. Certainly, it is interesting to study the effective potential also in the theories where Lorentz symmetry is broken—in particular, in the theories with time-space asymmetry—that is, those ones characterized by different orders in space and time derivatives. First attempts of studies of such theories [1], motivated further by the Horava gravity concept [2], called attention to investigation of different properties of other Horava-Lifshitz-like (HL-like) field theory models, with two main lines of study: first, their renormalization aspects [3]; and second, their effective potential, which has been studied for different models including HL-like QED, HL-like Yukawa, and different scalar theories in Refs. [4–7]. At the same time, one could note that, within studies of the effective potential in the HL-like QED [5–7], a special gauge has been employed—that is, the HL-like generalization of the Feynman gauge. Certainly, it simplifies the calculations essentially. However, the problem of the gauge dependence of the effective potential is still open. In this paper we try to answer this problem.

Our starting point is the Lagrangian of the scalar QED with an arbitrary z [5]:

$$L = \frac{1}{2}F_{0i}F_{0i} + (-1)^z\frac{1}{4}F_{ij}\Delta^{z-1}F_{ij} + D_0\phi(D_0\phi)^* - D_{i_1}D_{i_2}\ldots D_{i_z}\phi(D_{i_1}D_{i_2}\ldots D_{i_z}\phi)^*, \tag{1}$$

where $D_0 = \partial_0 - ieA_0$, $D_i = \partial_i - ieA_i$ is a gauge covariant derivative. For the sake of simplicity, we suggest that there is no self-coupling of the matter field, the theory is massless, and the critical exponents for scalar and gauge fields are the same (the generalization for the case of their difference is straightforward, as well as for the case of the massive theory). Here we have used slightly different definitions in comparison with Refs. [5–7] for convenience. Our signature is $(-+++)$.

We introduce the canonical momenta conjugated to A_i, ϕ, ϕ^*, respectively:

$$\Pi_i = F_{0i}, \qquad \pi = (\partial_0 + ieA_0)\phi^* = (D_0\phi)^*,$$
$$\pi^* = (\partial_0 - ieA_0)\phi = D_0\phi. \tag{2}$$

At this time we note the presence of the primary constraint

$$\Phi^{(1)} = \Pi_0 \simeq 0. \tag{3}$$

For didactic reasons, we introduce an intermediate object $L[\Pi,\pi]$—that is, the Lagrangian where the velocities are expressed in terms of momenta:

$$L[\Pi,\pi] = \frac{1}{2}\Pi_i\Pi_i + (-1)^z\frac{1}{4}F_{ij}\Delta^{z-1}F_{ij} + \pi\pi^* - D_{i_1}D_{i_2}\ldots D_{i_z}\phi(D_{i_1}D_{i_2}\ldots D_{i_z}\phi)^*. \tag{4}$$

The Hamiltonian density is defined as

$$H = \Pi_i\dot{A}_i + \pi\dot{\phi} + \pi^*\dot{\phi}^* - L[\Pi,\pi]. \tag{5}$$

Its explicit form, after one integration by parts, is

$$H = \frac{1}{2}\Pi_i\Pi_i + \pi\pi^* - A_0(\partial_i\Pi_i - ie(\pi\phi - \pi^*\phi^*)) - (-1)^z\frac{1}{4}F_{ij}\Delta^{z-1}F_{ij} + D_{i_1}D_{i_2}\ldots D_{i_z}\phi(D_{i_1}D_{i_2}\ldots D_{i_z}\phi)^*. \tag{6}$$

The secondary constraint has the role of the Gauss law:

$$\Phi^{(2)} = \{\Phi^{(1)}, H\} = \partial_i\Pi_i + \rho, \tag{7}$$

where $\rho = -ie(\pi\phi - \pi^*\phi^*)$ is a charge density. One can conclude that there are no other constraints (indeed, the time dependence in our theory is just the same as in the usual QED). So, our Hamiltonian density is rewritten as

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$$H = \frac{1}{2}\Pi_i\Pi_i + \pi\pi^* - A_0\Phi^{(2)} - (-1)^z\frac{1}{4}F_{ij}\Delta^{z-1}F_{ij} + \\ + D_{i_1}D_{i_2}\ldots D_{i_z}\phi(D_{i_1}D_{i_2}\ldots D_{i_z}\phi)^*. \quad (8)$$

Now, let us follow the algorithm of Ref. [8]. We introduce the transverse projector $P^{\perp}_{ij}$ and the longitudinal one $P^{\parallel}_{ij}$ (we note that for a pure HL-like QED without scalar matter, the Hamiltonian analysis has been performed in Ref. [9]):

$$P^{\perp}_{ij} = \delta_{ij} - \frac{\partial_i\partial_j}{\Delta}, \qquad P^{\parallel}_{ij} = \frac{\partial_i\partial_j}{\Delta}. \quad (9)$$

It is clear that $P^{\perp}_{ij}P^{\perp}_{jk} = P^{\perp}_{ik}$, $P^{\perp}_{ij}P^{\parallel}_{jk} = 0$, and $\partial_i P^{\perp}_{ij} = 0$. Other important properties of the projectors are also valid. Then, we introduce the transverse and longitudinal momenta:

$$\Pi^t_i = P^{\perp}_{ij}\Pi_j, \qquad \Pi^l_i = P^{\parallel}_{ij}\Pi_j. \quad (10)$$

In the same manner, we can introduce transverse and longitudinal fields A^t_i and A^l_i. We note that the transverse field A^t_i is invariant under the gauge transformations. It is easy to show that $F_{ij}\Delta^{z-1}F_{ij} = -2A^t_i\Delta^z A^t_i$ (up to the additive total derivative). So, we arrive at the following form for the Hamiltonian density:

$$H = \frac{1}{2}\Pi^t_i\Pi^t_i + \frac{1}{2}\Pi^l_i\Pi^l_i + \pi\pi^* - A_0\Phi^{(2)} + \frac{1}{2}A^t_i(-\Delta)^z A^t_i \\ + D_{i_1}D_{i_2}\ldots D_{i_z}\phi(D_{i_1}D_{i_2}\ldots D_{i_z}\phi)^*. \quad (11)$$

We note that the secondary constraint now can be rewritten as $\Phi^{(2)} = -(\partial_i\Pi^l_i + \rho)$, so it involves only the longitudinal part of the vector field as it must be. Now, let us carry out the same trick as in Ref. [10]—that is, we make the change of variables

$$\phi \to \tilde{\phi} = e^{ieR}\phi, \qquad \phi^* \to \tilde{\phi}^* = \phi^* e^{-ieR}. \quad (12)$$

Here R does not depend on ϕ, ϕ^*. It is clear that the conjugated momenta are defined as

$$\tilde{\pi} = e^{-ieR}\pi, \qquad \tilde{\pi}^* = e^{ieR}\pi^*, \quad (13)$$

and thus $\pi\pi^* = \tilde{\pi}\tilde{\pi}^*$. Then, it is clear that

$$(\partial_i + ieA_i)\phi = (\partial_i + ieA_i)(e^{-ieR}\tilde{\phi}) \\ = (\partial_i\tilde{\phi} + ie[A_i - \partial_i R]\tilde{\phi})e^{-ieR}.$$

If one changes $R(x) = -\int d^d y(\partial_j A^l_j(y))G(x-y)$, with $\nabla^2 G(x-y) = -\delta^d(x-y)$, and A^l_j is a longitudinal part of A_j (so $\frac{\partial_i\partial_j}{\nabla^2}A^l_j = A^l_i$), one has $A^l_i - \partial_i R = 0$; therefore,

$$(\partial_i + ieA_i)\phi = [(\partial_i + ieA^t_i)\tilde{\phi}]e^{-ieR}. \quad (14)$$

Using the mathematical induction method together with the relation $A^l_i - \partial_i R = 0$, one can show that for any integer n,

$$(\partial_{i_n} + ieA_{i_n})\ldots(\partial_{i_1} + ieA_{i_1})\phi \\ = [(\partial_{i_n} + ieA^t_{i_n})\ldots(\partial_{i_1} + ieA^t_{i_1})\tilde{\phi}]e^{-ieR}. \quad (15)$$

Therefore, our Hamiltonian density is

$$H = \frac{1}{2}\Pi^t_i\Pi^t_i + \frac{1}{2}\Pi^l_i\Pi^l_i + \tilde{\pi}\tilde{\pi}^* + A_0\Phi^{(2)} + \frac{1}{2}A^t_i(-\Delta)^z A^t_i \\ + [(\partial_{i_n} + ieA^t_{i_n})\ldots(\partial_{i_1} + ieA^t_{i_1})\tilde{\phi}] \\ \times [(\partial_{i_n} + ieA^t_{i_n})\ldots(\partial_{i_1} + ieA^t_{i_1})\tilde{\phi}]^*. \quad (16)$$

Now, it is the time to remember that our aim consists in the calculation of the one-loop effective potential. So, we make the shift $\tilde{\phi} \to \tilde{\Phi} + \tilde{\phi}$, $\tilde{\phi}^* \to \tilde{\Phi}^* + \tilde{\phi}^*$ and suggest first that the field A_i is a purely quantum one, and second that the background fields $\tilde{\Phi}, \tilde{\Phi}^*$ are constants (which in terms of the original fields is equivalent to the condition that $\Phi\Phi^* = \tilde{\Phi}\tilde{\Phi}^*$ is a constant), so $\partial_i\tilde{\Phi} = \partial_i\tilde{\Phi}^* = 0$. Restricting ourselves by the terms of the second order in quantum fields and integrating by parts where it is necessary, we find the following Hamiltonian density:

$$H = \frac{1}{2}\Pi^t_i\Pi^t_i + \frac{1}{2}\Pi^l_i\Pi^l_i + \tilde{\pi}\tilde{\pi}^* + \frac{1}{2}A^t_i(-\Delta)^z A^t_i + \tilde{\phi}(-\Delta)^z\tilde{\phi}^* \\ + e^2\Phi\Phi^* A^t_i(-\Delta)^{z-1}A^t_i - A_0\Phi^{(2)}. \quad (17)$$

Now, the transverse and longitudinal parts are completely separated. The condition $\Phi^{(2)} \simeq 0$ emerges as a consequence of the corresponding constraint. Then, we proceed as in Ref. [8]: we can solve the secondary constraint (7) as

$$\Pi^l_i(x) = \partial_i \int d^3y G(x-y)\rho(y), \quad (18)$$

where $G(x-y)$ is a Green function for the Laplace operator, such as $\nabla^2 G(x-y) = -\delta(x-y)$, and $\rho(y)$ is a (gauge invariant) charge density. In this case, we can eliminate the longitudinal momenta Π^l_i so that

$$\int d^3x \frac{1}{2}\Pi^l_i\Pi^l_i = \frac{1}{2}\int d^3x d^3y \rho(x)G(x-y)\rho(y). \quad (19)$$

So, our Hamiltonian, on the surface of the constraint, takes the form

$$H = \int d^3x \left(\frac{1}{2}\Pi_i^t\Pi_i^t + \frac{1}{2}A_i^t(-\Delta)^z A_i^t + e^2\Phi\Phi^* A_i^t(-\Delta)^{z-1}A_i^t + \tilde{\pi}\tilde{\pi}^* + \tilde{\phi}(-\Delta)^z\tilde{\phi}^* \right) + \frac{1}{2}\int d^3x d^3y \rho(x)G(x-y)\rho(y). \quad (20)$$

It is clear that the dynamics of scalar fields is completely factorized out, and this Hamiltonian yields the well-known contribution to the one-loop effective potential [5–7]:

$$U_t^{(1)} = d\int \frac{d^dk}{(2\pi)^d}[\vec{k}^{2z} + 2e^2\Phi\Phi^*\vec{k}^{2z-2}]^{1/2}, \quad (21)$$

whose result has been found in Ref. [5] to be

$$U_t^{(1)} = -\frac{d\pi^{\frac{d-1}{2}}}{4(2\pi)^d}(2e^2\Phi\Phi^*)^{\frac{d+z}{2}}\frac{\Gamma(-\frac{d+z}{2})\Gamma(\frac{d+z-1}{2})}{\Gamma(\frac{d}{2})}. \quad (22)$$

So, we reproduced the result found in Ref. [5] for a HL-like analogue of the Feynman gauge. In other words, it is clear within this formalism that the coupling of the gauge field to quantum scalar fields contributes only to the gauge dependent part.

Finally, we conclude that the only contribution to the effective potential is just (21). Actually, we have shown that this result does not depend on the gauge choice.

With the other approach, we start again with the expression (1) and note that it can be in principle rewritten in terms of the real fields ϕ_1 and ϕ_2, such as

$$\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}, \qquad \phi^* = \frac{\phi_1 - i\phi_2}{\sqrt{2}}. \quad (23)$$

However, we postpone introduction of ϕ_1, ϕ_2 up to a certain step, since the formulation with ϕ, ϕ^* is much more convenient for the quantum calculations. Moreover, the only place where we will actually use the fields ϕ_1 and ϕ_2 rather than ϕ, ϕ^* now will be the gauge condition.

Then, we introduce the following analogue for the R_ξ gauge (cf. Ref. [8]) by modifying the gauge-fixing Lagrangian from the form used in Ref. [8] to

$$L_{gf} = \frac{1}{2\xi}(-1)^z[(-1)^z\Delta^{-\frac{z-1}{2}}\partial_0 A_0 + \Delta^{\frac{z-1}{2}}\partial_i A_i + e\epsilon_{ab}v_a\phi_b]^2. \quad (24)$$

The v_i actually is an isovector in a two-dimensional space. We choose $v_i = (v, 0)$, so $\epsilon_{ab}v_a\phi_b = v\phi_2 = v\frac{\phi-\phi^*}{i\sqrt{2}}$. Also, we choose the background $\Phi_i = (\Phi, 0)$ to provide $\epsilon_{ij}v_i\Phi_j = 0$ (this relation is required by the gauge invariance; cf. Ref. [8]), which in terms of the fields ϕ, ϕ^* will mean that the background scalar field is real, $\Phi^* = \Phi$. Since the gauge transformations are as usual

$$\delta A_{0,i} = \partial_{i,0}\omega, \qquad \delta\phi_a = -e\epsilon_{ab}\omega\phi_b, \quad (25)$$

we should also introduce a Lagrangian for the corresponding ghosts c, c':

$$L_{gh} = \frac{1}{\sqrt{\xi}}c[(-1)^z\Delta^{-\frac{z-1}{2}}\partial_0\partial_0 + \Delta^{\frac{z+1}{2}} + e^2v\Phi]c'. \quad (26)$$

The total Lagrangian of the gauge field will take the form (cf. Refs. [5–7])

$$L_{\text{gauge}} = \frac{1}{2}A_0\left(-\Delta + \frac{1}{\xi}\partial_0^2(-\Delta)^{-(z-1)}\right)A_0 - \partial_0 A_0\partial_i A_i\left(1 - \frac{1}{\xi}\right) - \frac{1}{2}A_j[\partial_0^2 + (-\Delta)^z]A_j + \frac{1}{2}\left(1 - \frac{1}{\xi}\right)\partial_i A_i(-\Delta)^{z-1}\partial_j A_j + \frac{1}{2\xi}e^2v^2\phi_2^2(-1)^z + \frac{1}{\xi}ev[\Delta^{-\frac{z-1}{2}}\partial_0 A_0 + (-1)^z\Delta^{\frac{z-1}{2}}\partial_i A_i]\phi_2, \quad (27)$$

where $\phi_2 = \frac{\phi-\phi^*}{2i}$. Since our aim consists in calculating the effective potential, we as usual carry out the background quantum splitting by the rule $\phi \to \Phi + \phi$, $\phi^* \to \Phi^* + \phi^*$ (with Φ, Φ^* as the background fields, and ϕ, ϕ^* the quantum ones). We get the following quadratic Lagrangian of quantum fields from the scalar sector:

$$L_{sc} = \partial_0\phi\partial_0\phi^* + ie[(\partial_0 A_0) - (-\Delta)^{z-1}\partial_i A_i](\Phi\phi^* - \Phi^*\phi) + e^2A_0A_0\Phi\Phi^* - \phi(-\Delta)^z\phi^* - e^2A_i(-\Delta)^{z-1}A_i\Phi\Phi^*. \quad (28)$$

After we impose the condition of reality for the background, $\Phi^* = \Phi$, and introduce the ϕ_1, ϕ_2 fields as above, we get

$$L_{sc} = \frac{1}{2}[\partial_0\phi_1\partial_0\phi_1 - \phi_1(-\Delta)^z\phi_1 + \partial_0\phi_2\partial_0\phi_2 - \phi_2(-\Delta)^z\phi_2 + 2e^2\Phi^2A_0A_0 - 2e^2\Phi^2A_i(-\Delta)^{z-1}A_i] + (\sqrt{2})e\Phi[(\partial_0 A_0) - (-\Delta)^{z-1}(\partial_i A_i)]\phi_2. \quad (29)$$

We sum L_{sc}, L_{gauge}, and L_{gh}. As a result, the total quadratic action is

$$L_{\text{total}} = -\frac{1}{2}\phi_1[\partial_0^2 + (-\Delta)^z]\phi_1 - \frac{1}{2}\phi_2\left[\partial_0^2 + (-\Delta)^z + \frac{e^2v^2}{\xi}(-1)^{z-1}\right]\phi_2 + \frac{1}{2}A_0\left(\frac{\xi^{-1}\partial_0^2 + (-\Delta)^z + 2e^2\Phi^2(-\Delta)^{z-1}}{(-\Delta)^{z-1}}\right)A_0$$
$$-\frac{1}{2}A_j[\partial_0^2 + (-\Delta)^z + 2e^2\Phi^2(-\Delta)^{z-1}]A_j + \left((\sqrt{2})e\Phi + \frac{1}{\xi}ev\Delta^{-\frac{z-1}{2}}\right)[(\partial_0 A_0) - (-\Delta)^{z-1}(\partial_i A_i)]\cdot\phi_2$$
$$+\frac{1}{2}\left(1-\frac{1}{\xi}\right)\partial_i A_i(-\Delta)^{z-1}\partial_j A_j - \partial_0 A_0\partial_i A_i\left(1-\frac{1}{\xi}\right) + c[(-1)^z\Delta^{-\frac{z-1}{2}}\partial_0\partial_0 + \Delta^{\frac{z+1}{2}} + e^2v\Phi]c'. \qquad (30)$$

Here we have reabsorbed the factor $\frac{1}{\sqrt{\xi}}$ into the redefinition of the ghosts. We note that the ghost contribution is completely factorized, as it must be in the one-loop order, and we will consider it in the final step.

In principle, we can write down the nonghost contribution to the corresponding one-loop effective potential as a trace of the logarithm of some operator:

$$\Gamma^{(1)}_{\phi,A} = \frac{i}{2}\operatorname{tr}\ln\begin{pmatrix} -\Box_z & 0 & 0 & 0 \\ 0 & -\Box_z + \frac{1}{\xi}e^2v^2(-1)^z & T_0 & T_i \\ 0 & -T_0 & Q & \partial_0\partial_i(1-\frac{1}{\xi}) \\ 0 & -T_i & \partial_0\partial_i(1-\frac{1}{\xi}) & -H_{ij}\end{pmatrix}. \qquad (31)$$

Here $\partial_0^2 + (-\Delta)^z \equiv \Box_z$, $P_z = \Box_z + 2e^2\Phi^2(-\Delta)^{z-1}$, $Q = -\Delta + \frac{1}{\xi}\partial_0^2(-\Delta)^{-(z-1)} + 2e^2\Phi^2$, $H_{ij} = \delta_{ij}P_z + (1-\frac{1}{\xi})(-\Delta)^{z-1}\partial_i\partial_j$, $T_0 = ((\sqrt{2})e\Phi + \frac{1}{\xi}ev\Delta^{-\frac{z-1}{2}})\partial_0$, $T_i = -((\sqrt{2})e\Phi + \frac{1}{\xi}ev\Delta^{-\frac{z-1}{2}})(-\Delta)^{z-1}\partial_i$. Already at this step, the field ϕ_1 (corresponding to the first line/column) completely decouples, so one rests with

$$\Gamma^{(1)}_{\phi,A} = \frac{i}{2}\operatorname{tr}\ln\begin{pmatrix} -\Box_z + \frac{1}{\xi}e^2v^2(-1)^z & T_0 & T_i \\ -T_0 & Q & \partial_0\partial_i(1-\frac{1}{\xi}) \\ -T_i & \partial_0\partial_i(1-\frac{1}{\xi}) & -H_{ij}\end{pmatrix}. \qquad (32)$$

This is a result for the one-loop effective potential in an arbitrary gauge. In principle, one can reduce even this determinant through the following formula for the determinant of the block matrix:

$$\ln\det\begin{pmatrix} A & B \\ C & D\end{pmatrix} = \ln\det A + \ln\det(D - CA^{-1}B), \qquad (33)$$

and if we choose $A = -\Box_z + \frac{1}{\xi}e^2v^2(-1)^z$, the first term of this logarithm yields a mere constant and thus can be thrown away. However, the second term, in the case of the arbitrary gauge, is very complicated (the same situation takes place in Ref. [8]).

So, let us choose some gauge in which our one-loop effective potential is radically simplified. It is easy to see that, first, the cancellation of the "mixed" scalar-vector term requires an essentially nonlocal condition

$$\left((\sqrt{2})e\Phi + \frac{1}{\xi}ev\Delta^{-\frac{z-1}{2}}\right)[(\partial_0 A_0) - (-\Delta)^{z-1}(\partial_i A_i)] = 0. \qquad (34)$$

Actually, this is a generalization of the Feynman gauge, which can be treated as an equation on the A_0, A_i [really, it is weaker than the usual Feynman-like gauge condition $(\partial_0 A_0) - (-\Delta)^{z-1}(\partial_i A_i) = 0$]. In what rests, we can put also $\xi = 1$. We rest with the quadratic action of quantum fields (except that of ghosts):

$$L_{\text{total}} = \frac{1}{2}[\partial_0\phi_1\partial_0\phi_1 - \phi_1(-\Delta)^z\phi_1 + \partial_0\phi_2\partial_0\phi_2$$
$$-\phi_2(-\Delta)^z\phi_2 + e^2v^2\phi_2^2(-1)^z]$$
$$+\frac{1}{2}A_0(-\Delta + \partial_0^2(-\Delta)^{-(z-1)} + 2e^2\Phi^2)A_0$$
$$-\frac{1}{2}A_j[\partial_0^2 + (-\Delta)^z + 2e^2\Phi^2(-\Delta)^{z-1}]A_j. \qquad (35)$$

It is clear that the contribution to the one-loop effective action from the scalar fields is trivial, since it does not involve any background fields (note that v is a constant, not a field), and from the gauge fields one has

$$\Gamma^{(1)}_{\phi,A} = \frac{i}{2}(d+1)\mathrm{tr}\ln[\partial_0^2 + (-\Delta)^z + 2e^2\Phi^2(-\Delta)^{z-1}]. \quad (36)$$

It is clear that at $z=1$, the expression is Lorentz invariant, and the usual result for the QED is restored. After the Fourier transform and Wick rotation, we have

$$\Gamma^{(1)}_{\phi,A} = \frac{1}{2}(d+1)\int\frac{dk_{0E}d^dk}{(2\pi)^{d+1}}\ln(k_{0E}^2 + \vec{k}^{2z} + 2e^2\Phi^2\vec{k}^{2z-2}), \quad (37)$$

which, with use of Ref. [5], is

$$\Gamma^{(1)}_{\phi,A} = -\frac{(d+1)\pi^{\frac{d-1}{2}}}{4(2\pi)^d}(2e^2\Phi^2)^{\frac{d+z}{2}}\frac{\Gamma(-\frac{d+z}{2})\Gamma(\frac{d+z-1}{2})}{\Gamma(\frac{d}{2})}. \quad (38)$$

The only difference is the overall factor $d+1$ instead of d in Refs. [5,6]. However, this is a natural impact of the difference of the gauge choice.

Now, recalling Ref. [6], we can briefly describe the dependence of this result on d and z. It is easy to see that when $d+z=2n+1$ is odd, the one-loop effective potential is essentially finite. Moreover, if in this case the n is even, the factor $-\Gamma(-\frac{d+z}{2}) = -\Gamma(-n-\frac{1}{2})$ in (38) is positive; therefore, the effective potential is non-negative, having the minimum at $\Phi=0$, and if n is odd, the effective potential is negative and the theory is unstable at one loop. At the same time, if $d+z=2l$ is even, the one-loop effective potential diverges and requires an introduction of a corresponding counterterm—that is, the self-coupling of the scalar field, with additional one-loop contributions [7].

It remains for us to treat the ghost contribution to the one-loop effective action. In this case, it is nontrivial, being equal to

$$\Gamma^{(1)}_{gh} = -\frac{i}{2}\mathrm{tr}\ln[(-1)^z\Delta^{-\frac{z-1}{2}}\partial_0\partial_0 + \Delta^{\frac{z+1}{2}} + e^2v\Phi], \quad (39)$$

or, as is the same,

$$\Gamma^{(1)}_{gh} = -\frac{i}{2}\mathrm{tr}\ln[\partial_0\partial_0 + (-1)^z\Delta^z + e^2v\Phi(-1)^z\Delta^{\frac{z-1}{2}}], \quad (40)$$

which after Fourier transform and Wick rotation yields

$$\Gamma^{(1)}_{gh} = -\frac{1}{2}\int\frac{d^d\vec{k}dk_0}{(2\pi)^{d+1}}\ln[k_0^2 + \vec{k}^{2z} + e^2v\Phi(-1)^{\frac{3z-1}{2}}|\vec{k}|^{z-1}]. \quad (41)$$

To avoid the problems with reality of the expression, we can suggest that $(-1)^{\frac{3z-1}{2}} = \pm 1$ (that is, z must be odd), with the sign of v chosen in an appropriate manner. It remains for us to integrate, which we can do following the lines of Ref. [5]. Afterwards, we arrive at

$$\Gamma^{(1)}_{gh} = \frac{1}{(4\pi)^{\frac{2d+z+1}{4}}}\frac{1}{z+1}\frac{\Gamma(\frac{2d+z-1}{2z+2})}{\Gamma(\frac{2d+z-1}{4})} \times\Gamma\left(-\frac{1}{2}-\frac{2d+z-1}{2z+2}\right)(e^2|v\Phi|)^{\frac{d+z}{z+1}}. \quad (42)$$

This contribution diverges if $\frac{2d+z-1}{z+1} = 2n-1$, with n a non-negative integer (in particular, if $z=1$, it corresponds to an odd d, as it must be). The whole result is a sum of (38) and (42). We note that first, it diverges at certain values of d and z; and second, while (38) does not depend on v, (42) essentially depends on it, which means that these two contributions have essentially distinct structure.

Let us compare the results obtained within the two approaches—that is, (22) and the sum of (38) with (42), respectively. It is clear that the latter result is obtained in some special gauge—that is, the analogue of R_ξ gauge allowing for removal of nondiagonal terms of the action by paying the price of introducing the extra parameters v and ξ, and further, the nontrivial coupling of the ghosts to the scalar field. It is clear that this gauge is much more generic than the usual Feynman-like gauge used in the first part of the paper. However, if we suggest that we impose several special restrictions on these parameters—that is, choose $\xi=1$ to remove the nondiagonal terms in the purely gauge sector, and $v=0$ to remove the ghost-matter coupling together with its consequence—that is, the contribution of (42). [Recall that within usual gauges, which do not involve scalar fields, this coupling does not arise—the results, (22) and (38), will have exactly the same functional form.] The only difference is in the overall factor (d or $d+1$), which is caused by the fact that while within the first manner of calculation we absorbed A_0 (so-called "scalar photon") into the charge density ρ, which has no contribution at the one-loop order, within the second manner we treated A_0 on the same base as the physical A_i components, which yields a contribution similar to that of A_i. Therefore, we conclude that, if we restrict ourselves to purely physical variables—that is, throw away the contribution of a nonphysical A_0—the results will coincide. Actually, the difference of results within two methods is caused by the fact that, in the R_ξ gauge, the result for the one-loop effective potential is strongly gauge dependent. Nevertheless, the physical variables should be gauge independent.

We considered two different approaches to the study of the one-loop effective potential in the HL-like QED. Within the first of them, we implemented the gauge invariant physical variables and obtained the one-loop potential expressed in these variables. The result coincided with the previous result of Ref. [5]. Within the second one, we used a special gauge—that is, the R_ξ gauge, known as an efficient tool in simplification of the classical action. We showed that using this gauge with an appropriate fixation of the free parameters allows us to obtain a result which differs from that of Ref. [5] only by a numerical factor, plus some extra contribution generated by ghosts which do not decouple in this case because of the unusual structure of the gauge-fixing function.

This work was partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq). A. Yu. P. has been supported by CNPq Project No. 303438-2012/6.

[1] D. Anselmi, Ann. Phys. (Amsterdam) **324**, 874 (2009); **324**, 1058 (2009).
[2] P. Horava, Phys. Rev. D **79**, 084008 (2009).
[3] R. Iengo, J. Russo, and M. Serone, J. High Energy Phys. 11 (2009) 020; R. Iengo and M. Serone, Phys. Rev. D **81**, 125005 (2010); J. M. Romero, J. A. Santiago, O. Gonzalez-Gaxiola, and A. Zamora, Mod. Phys. Lett. A **25**, 3381 (2010); Pedro R. S. Gomes and M. Gomes, Phys. Rev. D **85**, 085018 (2012); **85**, 065010 (2012); T. Kanazawa and A. Yamamoto, arXiv:1411.4667.
[4] M. Eune, W. Kim, and E. J. Son, Phys. Lett. B **703**, 100 (2011); J. Alexandre, K. Farakos, and A. Tsapalis, Phys. Rev. D **81**, 105029 (2010); K. Farakos and D. Metaxas, Phys. Lett. B **707**, 562 (2012); **711**, 76 (2012); K. Farakos, Int. J. Mod. Phys. A **27**, 1250168 (2012).
[5] C. F. Farias, M. Gomes, J. R. Nascimento, A. Yu. Petrov, and A. J. da Silva, Phys. Rev. D **85**, 127701 (2012).
[6] C. F. Farias, J. R. Nascimento, and A. Yu. Petrov, Phys. Lett. B **719**, 196 (2013).
[7] C. F. Farias, M. Gomes, J. R. Nascimento, A. Yu. Petrov, and A. J. da Silva, Phys. Rev. D **89**, 025014 (2014).
[8] D. Bazeia and J. R. S. Nascimento, Phys. Rev. D **35**, 2490 (1987); **55**, 1105 (1997).
[9] A. Gaona and J. M. Romero, arXiv:1411.5927.
[10] D. Boyanovsky, D. Brahm, R. Holman, and D. S. Lee, Phys. Rev. D **54**, 1763 (1996).